

# Aggregation, Capital Heterogeneity, and the Investment CAPM

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## Abstract

This paper provides a careful treatment of aggregation, and to a lesser extent, capital heterogeneity in the investment CAPM. Firm-level investment returns are constructed from firm-level variables, and then aggregated to the portfolio level to match with portfolio-level stock returns. Current assets form a separate production input besides physical capital. The model fits well the value, momentum, investment, and profitability premiums simultaneously, and partially explains the positive stock-investment return correlations, the procyclical and short-term dynamics of the momentum and profitability premiums, as well as the countercyclical and long-term dynamics of the value and investment premiums. However, the model fails to explain momentum crashes.

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# 1 Introduction

Aggregation and heterogeneity have long been recognized as thorny problems for empirical studies of the investment behavior. Nickell (1978) identifies three major problems on aggregation and heterogeneity. First, “the question arises as to whether one can construct aggregates for the outputs, the capital good inputs and the labour inputs so that it is possible to define a production function which gives aggregate output as a function of the aggregate capital and aggregate labour inputs. The answer, in any realistic case, is that it is not (p. 229–230).” Second, even if the empirical relations at the firm level are good approximations of reality, “it is difficult to develop structural restrictions on the aggregate relationships corresponding to those which theory imposes on the micro-level equations (p. 230).” This difficulty is especially acute, if the micro-level equations are nonlinear. Third, there are serious problems associated with measuring investment and capital stock. Investment data can be “based on orders, deliveries or payments or some mixture of all three (p. 231)” that are not additions to a firm’s capital stock. The key problem measuring the capital stock is “the evaluation of old capital goods for which there exist no active markets (p. 231).”<sup>1</sup>

This paper provides a careful treatment of aggregation, which is the second major problem identified by Nickell (1978). We also address, albeit to a lesser extent, the problem of capital heterogeneity and the measurement of investment and capital, which are the first and third problems in Nickell. We do so in the context of the Investment Capital Asset Pricing Model (the investment CAPM).

Prior studies implement the investment CAPM via structural estimation at the portfolio level (Liu, Whited, and Zhang 2009). Firm-level accounting variables are aggregated to portfolio-level variables, from which portfolio-level investment returns are constructed to match with portfolio-level stock returns. While a useful first stab, this procedure has a couple of drawbacks. First, on economic grounds, it assumes that firms within a given portfolio all make the identical investment choice. This assumption is clearly counterfactual. Second, on econometric grounds, this procedure

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<sup>1</sup>Aggregation and heterogeneity pose even more challenging problems for empirical studies of the consumption behavior. Blundell and Stoker (2007) provide a recent review of this important literature.

misses a substantial amount of heterogeneity in firm-level variables that can help identify structural parameters. We instead use firm-level variables to construct firm-level investment returns, which are then aggregated to the portfolio level to match with portfolio-level stock returns.

In addition, prior studies ignore capital heterogeneity, with physical capital (net property, plant, and equipment) as the single production input. However, net property, plant, and equipment is only a small fraction of total assets on firms' balance sheet. While many choices exist to introduce an additional production input, we settle on current assets, and impose zero adjustment costs on current assets (an assumption that we verify empirically). Consequently, the resulting two-capital model is as parsimonious as the baseline, physical capital model, with only two parameters.

Our benchmark model with two capital goods estimated at the firm level goes a long way in resolving the empirical difficulties in prior studies. Liu, Whited, and Zhang (2009) estimate the physical capital model at the portfolio level, but find that the marginal product and adjustment costs parameters vary greatly across the value and momentum deciles. If the model is well specified, or "structural," the parameter estimates should be mostly invariant across different testing portfolios. Consequently, the baseline model fails to explain value and momentum simultaneously. This weakness applies to the investment CAPM literature more broadly. In a prominent asset pricing textbook, Campbell (2018, p. 213) writes: "This problem, that different parameters are needed to fit each anomaly, is a pervasive one in the  $q$ -theoretic asset pricing literature."

The parameter estimates in our benchmark model are relatively stable across the testing deciles on value, momentum, asset growth, and return on equity, separately or jointly. When fitting value and momentum deciles together, with and without also adding the asset growth and return on equity deciles, the scatter plots of average predicted stock returns versus average realized stock returns are mostly aligned with the 45-degree line. When fitting the 40 value-weighted deciles together, the model predicts a value premium of 4.96% per annum, with a pricing error of 1.6% ( $t = 0.46$ ), a momentum premium of 16.22%, with an error of  $-0.75\%$  ( $t = -0.25$ ), an investment premium of

−4.63%, with an error of −0.48% ( $t = -0.23$ ), and a return on equity premium of 9.22%, with an error of −0.95% ( $t = -0.39$ ). However, the model is still rejected by the test of overidentification.

Aggregation is important for the benchmark model’s performance. When implemented at the portfolio level, the parameter estimates are less stable, and the model yields larger pricing errors, especially for the value premium. In particular, with the 40 value-weighted deciles together, the alternative aggregation yields a value premium of only 1.56% per annum, with an error of 5% ( $t = 1.85$ ).

Introducing current assets is also important for the benchmark model’s performance. Although the physical capital model implemented at the firm level yields largely stable parameter estimates, it is severely misspecified. In the data, the fraction of physical capital in the sum of physical capital and current assets averages only 38%. Accordingly, the average product in the physical capital model, mismeasured as sales-to-physical capital, averages 9.59, with a large standard deviation of 14.46 in the data. In contrast, the average product in the benchmark model, measured as sales scaled by the sum of physical capital and current assets, averages only 1.67, with a modest standard deviation of 1.05. The measurement errors in the average product translate to large pricing errors for the physical capital model, errors that are avoided in the benchmark model.

Finally, we use the “fundamental” returns (the predicted stock returns from the benchmark model) to study the dynamics of factor premiums. Because the benchmark model’s parameters are estimated from the average returns only, the dynamics serve as separate diagnostics on the model’s performance. The model predicts significantly positive stock-fundamental return correlations, overcoming another weakness in prior studies that report weakly negative correlations. The stock-fundamental correlations of factor premiums are all positive, ranging from 0.18 to 0.54. The model is consistent with the short-lived nature of the momentum and return on equity premiums, as well as the long-lasting nature of the value and investment premiums. The model also partially explains the procyclical variation of the momentum and return on equity premiums as well as the countercyclical variation of the value and investment premiums. However, the model underestimates

the volatility, skewness, and kurtosis of factor premiums, and fails to explain momentum crashes.

Cochrane (1991) applies the  $q$ -theory of investment to study aggregate asset prices, and Cochrane (1996) specifies the stochastic discount factor as a linear function of aggregate investment returns in cross-sectional tests. Belo (2010) uses the marginal rate of transformation as the stochastic discount factor. Jermann (2010) examines the equity premium implied from the first principle of investment. Tuzel (2010) constructs a two-capital model with real estate and physical capital, and shows that firms with higher real estate holdings are riskier, and earn higher expected returns. Cooper and Priestley (2016) use the investment CAPM to study the cost of capital for private firms. Li (2017) constructs a quantitative, theoretical model to explain value and momentum jointly. In contrast, we conduct structural estimation of a different model on the real data. Aggregation has been largely ignored so far, and except for Tuzel, capital heterogeneity as well. We fill this gap.<sup>2</sup>

The rest of the paper is organized as follows. Section 2 sets up the model of the firms. Section 3 presents our econometric methods. Section 4 describes our data. Section 5 discusses our structural estimation results, and Section 6 the separate diagnostics. Finally, Section 7 concludes.

## 2 The Model of the Firms

Firms use both short-term capital (current assets) and physical capital (long-term assets) to produce a homogeneous output. Let  $\Pi_{it} \equiv \Pi(K_{it}, C_{it}, X_{it})$  denote the operating profits of firm  $i$  at time  $t$ , in which  $K_{it}$  is physical capital,  $C_{it}$  current assets, and  $X_{it}$  a vector of exogenous aggregate and firm-specific shocks. We assume that  $\Pi_{it}$  exhibits constant returns to scale, i.e.,  $\Pi_{it} = K_{it} \partial \Pi_{it} / \partial K_{it} + C_{it} \partial \Pi_{it} / \partial C_{it}$ . We also assume that firms have a Cobb-Douglas production function. The marginal product of physical capital can then be parameterized as  $\partial \Pi_{it} / \partial K_{it} = \gamma_K Y_{it} / K_{it}$ , in which  $\gamma_K > 0$  is a technological parameter, and  $Y_{it}$  sales (Gilchrist and Himmelberg

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<sup>2</sup>Outside asset pricing, Wildasin (1984) derives optimal investment conditions with many capital goods. Schaller (1990) shows that aggregation is partially responsible for large adjustment costs from aggregate time series. Hayashi and Inoue (1991) derive a one-to-one relation between the growth rate of the capital aggregate and Tobin's  $q$  in an investment model with multiple capital goods, and estimate this relation on Japanese firms. Chirinko (1993) estimates the investment model with multiple capital inputs that differ in adjustment technologies. Doyle and Whited (1998) show that smooth industry-level investment results from aggregating asynchronous and lumpy micro-level investment.

1998). Similarly, the marginal product of current assets is  $\partial\Pi_{it}/\partial C_{it} = \gamma_C Y_{it}/C_{it}$ , in which  $\gamma_C > 0$ .

Taking operating profits as given, firms choose investments in both short- and long-term capital stocks to maximize the market value of equity. Physical capital evolves as  $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$ , in which  $I_{it}$  is investment in physical capital, and  $\delta_{it}$  the rate of depreciation that firm  $i$  takes as given. We allow  $\delta_{it}$  to be firm-specific and time-varying. Current assets evolve as  $C_{it+1} = J_{it} + C_{it}$ , in which  $J_{it}$  is investment in current assets. We assume that current assets do not depreciate. Firms incur adjustment costs when investing in physical capital, but not in current assets.<sup>3</sup> The adjustment costs function, denoted  $\Phi(I_{it}, K_{it})$ , is increasing and convex in  $I_{it}$ , decreasing in  $K_{it}$ , and of constant returns to scale in  $I_{it}$  and  $K_{it}$ , i.e.,  $\Phi(I_{it}, K_{it}) = I_{it} \partial\Phi(I_{it}, K_{it})/\partial I_{it} + K_{it} \partial\Phi(I_{it}, K_{it})/\partial K_{it}$ . We adopt the standard quadratic functional form:

$$\Phi_{it} \equiv \Phi(I_{it}, K_{it}) = \frac{a}{2} \left( \frac{I_{it}}{K_{it}} \right)^2 K_{it}, \quad (1)$$

in which  $a > 0$  is the adjustment costs parameter of physical capital.

At the beginning of time  $t$ , firm  $i$  issues debt,  $B_{it+1}$ , which must be repaid at the beginning of  $t+1$ . When borrowing, firms take as given the gross cost of debt on  $B_{it}$ , denoted  $r_{it}^B$ , which varies across firms and over time. Taxable corporate profits equal operating profits less physical capital depreciation, adjustment costs, and interest expenses,  $\Pi_{it} - \delta_{it}K_{it} - \Phi_{it} - (r_{it}^B - 1)B_{it}$ . Let  $\tau_t$  be the corporate tax rate,  $\tau_t \delta_{it}K_{it}$  be depreciation tax shield, and  $\tau_t (r_{it}^B - 1)B_{it}$  be interest tax shield. Firm  $i$ 's net payout is given by  $D_{it} \equiv (1 - \tau_t)(\Pi_{it} - \Phi_{it}) - I_{it} - J_{it} + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it}K_{it} + \tau_t (r_{it}^B - 1)B_{it}$ .

Let  $M_{t+1}$  be the stochastic discount factor from  $t$  to  $t+1$ . Taking  $M_{t+1}$  as given, firm  $i$  chooses the streams of  $I_{it}$ ,  $K_{it+1}$ ,  $J_{it}$ ,  $C_{it+1}$ , and  $B_{it+1}$  to maximize its cum-dividend market value of equity,  $V_{it} \equiv E_t [\sum_{s=0}^{\infty} M_{t+s} D_{it+s}]$ , subject to a transversality condition,  $\lim_{T \rightarrow \infty} E_t [M_{t+T} B_{it+T+1}] = 0$ , which prevents the firm from borrowing an infinite amount of debt. The firm's first-order condition

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<sup>3</sup>In the appendix, we document in detail that the adjustment costs estimates on current assets are insignificantly different from zero for most testing deciles, especially in the joint estimation.

for physical investment implies  $E_t[M_{t+1}r_{it+1}^I] = 1$ , in which  $r_{it+1}^I$  is the physical investment return:

$$r_{it+1}^I \equiv \frac{(1 - \tau_{t+1}) \left[ \gamma_K \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2} \left( \frac{I_{it+1}}{K_{it+1}} \right)^2 \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1}) a \left( \frac{I_{it+1}}{K_{it+1}} \right) \right]}{1 + (1 - \tau_t) a \left( \frac{I_{it}}{K_{it}} \right)}. \quad (2)$$

Intuitively, the physical investment return is the marginal benefit of physical investment at  $t+1$  divided by its marginal cost at  $t$ . In the numerator of equation (2),  $(1 - \tau_{t+1})\gamma_K(Y_{it+1}/K_{it+1})$  is the after-tax marginal product of physical capital,  $(1 - \tau_{t+1})(a/2)(I_{it+1}/K_{it+1})^2$  is the after-tax marginal reduction in physical adjustment costs, and  $\tau_{t+1}\delta_{it+1}$  is the marginal depreciation tax shield. The last term in the numerator is the marginal continuation value of an extra unit of physical capital net of depreciation, in which the marginal continuation value equals the marginal cost of physical investment in the next period,  $1 + (1 - \tau_{t+1})a(I_{it+1}/K_{it+1})$ . Finally,  $E_t[M_{t+1}r_{it+1}^I] = 1$  says that the marginal cost equals the next period marginal benefit discounted to  $t$ .

Similarly, the firm's first-order condition for investment in current assets is  $E_t[M_{t+1}r_{it+1}^J] = 1$ , in which  $r_{it+1}^J$  is the current (assets) investment return:

$$r_{it+1}^J \equiv 1 + (1 - \tau_{t+1})\gamma_C \frac{Y_{it+1}}{C_{it+1}}. \quad (3)$$

The current investment return is again the marginal benefit of current (assets) investment at  $t+1$  divided by its marginal cost at  $t$ . The marginal cost equals one because of no adjustment costs on current assets. For the marginal benefit,  $(1 - \tau_{t+1})\gamma_C(Y_{it+1}/C_{it+1})$  is the after-tax marginal product of current assets, and without adjustment costs or depreciation, the marginal continuation value of an extra unit of current assets net of (zero) depreciation equals one.

Define the after-tax cost of debt as  $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$ . The firm's first-order condition for new debt implies  $E_t[M_{t+1}r_{it+1}^{Ba}] = 1$ . Define  $P_{it} \equiv V_{it} - D_{it}$  as the ex-dividend market value of equity,  $r_{it+1}^S \equiv (P_{it+1} + D_{it+1})/P_{it}$  as the stock return, and  $w_{it}^B \equiv B_{it+1}/(P_{it} + B_{it+1})$  as the market leverage. Also, the shadow price of physical capital is marginal  $q$ ,  $q_{it} = 1 + (1 - \tau_t)a(I_{it}/K_{it})$ , which

in the optimum equals the marginal cost of physical investment. The shadow price of current assets equals one. Finally, define  $w_{it}^K \equiv q_{it}K_{it+1}/(q_{it}K_{it+1} + C_{it+1})$  as the weight of the firm's market value attributed to physical capital. Then the weighted average of the two investment returns equals the weighted average of the cost of equity and the after-tax cost of debt (Appendix A):

$$w_{it}^K r_{it+1}^I + (1 - w_{it}^K) r_{it+1}^J = w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S. \quad (4)$$

Solving for the stock return from equation (4) yields the investment CAPM:

$$r_{it+1}^S = r_{it+1}^F \equiv \frac{w_{it}^K r_{it+1}^I + (1 - w_{it}^K) r_{it+1}^J - w_{it}^B r_{it+1}^{Ba}}{1 - w_{it}^B}, \quad (5)$$

in which  $r_{it+1}^F$  is the “fundamental” return as a nonlinear function of firm characteristics. If  $w_{it}^K = 1$ , equation (4) collapses to the equivalence between the physical investment return and the weighted average cost of capital, as in Liu, Whited, and Zhang (2009). If  $w_{it}^K = 1$  and  $w_{it}^B = 0$ , equation (5) reduces to the equivalence between the stock and physical investment returns as in Cochrane (1991).

Equation (5) clearly shows that even without adjustment costs, current assets help describe the cost of capital distribution across firms more accurately. As such, current assets are different from labor, which does not appear on firms' balance sheet as assets. Firms hire, but do not own, workers. As a result, without adjustment costs on labor hiring, the labor input can be absorbed into the operating profits function, and does not affect the cost of capital distribution.

### 3 Econometric Methods

We describe our structural estimation in Section 3.1 and aggregation in Section 3.2.

#### 3.1 Generalized Method of Moments (GMM)

We use GMM to test the ex ante restriction implied by equation (5):

$$E[r_{pt+1}^S - r_{pt+1}^F] = 0, \quad (6)$$



in which  $r_{pt+1}^S$  is the stock return of testing portfolio  $p$ , and  $r_{pt+1}^F$  is portfolio  $p$ 's fundamental return given by the right hand side of equation (5). In particular, the pricing error from the investment CAPM is defined as  $e_p \equiv E_T[r_{pt+1}^S - r_{pt+1}^F]$ , in which  $E_T[\cdot]$  is the sample mean.

Although the model has three parameters ( $\gamma_K, \gamma_C$ , and  $a$ ),  $\gamma_K$  and  $\gamma_C$  enter the moment condition (6) only in the form of  $\gamma \equiv \gamma_K + \gamma_C$ . To see this point, we use equations (2) and (3) to rewrite:

$$w_{it}^K r_{it+1}^I + (1 - w_{it}^K) r_{it+1}^J = \frac{(1 - \tau_{t+1})(\gamma_K + \gamma_C) Y_{it+1} / (K_{it+1} + C_{it+1})}{q_{it} K_{it+1} / (K_{it+1} + C_{it+1}) + C_{it+1} / (K_{it+1} + C_{it+1})} + w_{it}^K \frac{(1 - \tau_{t+1})(a/2) (I_{it+1} / K_{it+1})^2 + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) q_{it+1}}{q_{it}} + (1 - w_{it}^K). \quad (7)$$

As such,  $\gamma_K$  and  $\gamma_C$  are not separately identifiable, and only their sum,  $\gamma$ , can be estimated. With only two parameters,  $\gamma$  and  $a$ , the two-capital model with physical capital and current assets is as parsimonious as the baseline model with only physical capital.

Also, the numerator of the first term in the right hand side of equation (7) shows that the marginal product in the two-capital model should be measured as proportional to the ratio of sales to the sum of physical capital and current assets,  $Y_{it+1} / (K_{it+1} + C_{it+1})$ , as opposed to sales-to-physical capital,  $Y_{it+1} / K_{it+1}$ , in the physical capital model. Finally, the denominator of the first term can be interpreted as the weighted average of the marginal  $q$  of physical capital and that of current assets (one), with the weight given by  $K_{it+1} / (K_{it+1} + C_{it+1})$  and  $C_{it+1} / (K_{it+1} + C_{it+1})$ , respectively.

Formally, let  $\mathbf{c} \equiv (\gamma, a)$  denote the model's parameter, and  $\mathbf{g}_T$  the sample moments. The GMM objective function is a weighted sum of squares of the errors across a set of testing portfolios,  $\mathbf{g}'_T \mathbf{W} \mathbf{g}_T$ , in which we set  $\mathbf{W} = \mathbf{I}$ , the identity matrix (Cochrane 1996). Let  $\mathbf{D} = \partial \mathbf{g}_T / \partial \mathbf{c}$  and  $\mathbf{S}$  be a consistent estimate of the variance-covariance matrix of the sample errors,  $\mathbf{g}_T$ . The  $\mathbf{S}$  estimate accounts for autocorrelations of up to 12 lags. The estimate of  $\mathbf{c}$ , denoted  $\hat{\mathbf{c}}$ , is asymptotically normal with the variance-covariance matrix given by  $\text{var}(\hat{\mathbf{c}}) = (\mathbf{D}' \mathbf{W} \mathbf{D})^{-1} \mathbf{D}' \mathbf{W} \mathbf{S} \mathbf{W} \mathbf{D} (\mathbf{D}' \mathbf{W} \mathbf{D})^{-1} / T$ . To construct the standard errors for the pricing errors of individual portfolios, we use the variance-covariance matrix for  $\mathbf{g}_T$ ,  $\text{var}(\mathbf{g}_T) = [\mathbf{I} - \mathbf{D} (\mathbf{D}' \mathbf{W} \mathbf{D})^{-1} \mathbf{D}' \mathbf{W}] \mathbf{S} [\mathbf{I} - \mathbf{D} (\mathbf{D}' \mathbf{W} \mathbf{D})^{-1} \mathbf{D}' \mathbf{W}]' / T$ . Fi-

nally, we form a  $\chi^2$  test on the null hypothesis that all the pricing errors are jointly zero,  $\mathbf{g}'_T [\text{var}(\mathbf{g}_T)]^+ \mathbf{g}_T \sim \chi^2(\# \text{moments} - \# \text{parameters})$ , in which  $\chi^2$  is the chi-square distribution with the degrees of freedom given by the number of moments minus the number of parameters, and the superscript  $+$  denotes pseudo-inversion (Hansen 1982).

### 3.2 Aggregation

Prior studies estimate the physical capital model with accounting data aggregated to the portfolio level. Portfolio-level fundamental returns are constructed from portfolio-level characteristics to match with portfolio-level stock returns. Formally, the prior studies estimate:

$$E \left[ \sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^S - r_{pt+1}^F (\gamma_K, a; Y_{pt+1}, K_{pt+1}, I_{pt+1}, \delta_{pt+1}, I_{pt}, K_{pt}, r_{pt+1}^{Ba}, w_{pt}^B) \right] = 0, \quad (8)$$

in which  $N_{pt}$  is the number of firms in portfolio  $p$  at the beginning of period  $t$ ,  $w_{ipt}$  is the weight of stock  $i$  in portfolio  $p$  at the beginning of period  $t$ ,  $r_{ipt+1}^S$  is the return of stock  $i$  in portfolio  $p$  over period  $t$ , and  $r_{pt+1}^F$  is the fundamental return for portfolio  $p$ . For equal-weighted portfolios,  $w_{ipt} = 1/N_{pt}$ , and for value-weighted portfolios,  $w_{ipt}$  is the market value-weights at the beginning of period  $t$ .  $r_{pt+1}^F$  is constructed from portfolio-level characteristics aggregated from firm-level characteristics, and its functional form does not change with  $w_{ipt}$ . To aggregate accounting variables from the firm level to the portfolio level,  $I_{pt+1} = \sum_{i=1}^{N_{pt}} I_{ipt+1}$ , in which  $I_{ipt+1}$  is investment of firm  $i$  in portfolio  $p$  over period  $t + 1$ ,  $w_{pt}^B = \sum_{i=1}^{N_{pt}} B_{ipt+1} / \sum_{i=1}^{N_{pt}} (P_{ipt} + B_{ipt+1})$ , and  $r_{pt+1}^{Ba} = (1/N_{pt}) \sum_{i=1}^{N_{pt}} r_{ipt+1}^{Ba}$ . Other portfolio-level variables are constructed analogously.

Working with this aggregation procedure, Liu, Whited, and Zhang (2009) show that the physical capital model explains value and momentum separately, but the parameter estimates vary greatly across the two sets of deciles. In addition, Liu and Zhang (2014) show that when forced to use the same parameter values in the joint estimation, the physical capital model manages to capture the momentum premium, but fails to explain the value premium altogether.

We explore a new, exact aggregation procedure. We first construct firm-level fundamental re-

turns from firm-level accounting variables, and then aggregate to portfolio-level fundamental returns to match with portfolio-level stock returns. Formally, we estimate:

$$E \left[ \sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^S - \sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^F (\gamma, \alpha; Y_{ipt+1}, K_{ipt+1}, I_{ipt+1}, \delta_{ipt+1}, I_{ipt}, K_{ipt}, r_{ipt+1}^{Ba}, w_{ipt}^B) \right] = 0, \quad (9)$$

in which  $r_{ipt+1}^F$  is the fundamental return for firm  $i$ . As such, aggregating  $r_{ipt+1}^S$  and  $r_{ipt+1}^F$  is symmetric, and the portfolio-level fundamental return,  $r_{pt+1}^F \equiv \sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^F$ , varies with  $w_{ipt}$ .

## 4 Data

We obtain firm-level data from the Center for Research in Security Prices (CRSP) monthly stock file and the annual Standard and Poor's Compustat industrial files. We exclude firms with primary standard industrial classifications between 6000 and 6999 (financial firms) when forming testing portfolios. When calculating the fundamental returns, we further exclude firms for which total assets, net property, plant, and equipment, or sales are either zero or negative at each portfolio formation. The sample for stock and fundamental returns is from January 1967 to December 2015.

### 4.1 Testing Portfolios

We use 40 testing deciles formed on book-to-market equity, momentum, asset growth, and return on equity, either separately or jointly. Book-to-market and momentum are the anomalies that underpin the popular Carhart (1997) four-factor model. We also include asset growth and return on equity, both of which feature prominently in the new generation of factor pricing models (Hou, Xue, and Zhang 2015, see also Fama and French 2015).

To control for microcaps (stocks smaller than the 20th percentile of market equity of NYSE stocks), we form two sets of testing deciles. In the first set, we sort stocks with NYSE breakpoints, and calculate value-weighted decile returns. In the second set, we first exclude microcaps from our sample. We then sort the remaining stocks into deciles, and calculate equal-weighted decile returns.

To form the book-to-market (Bm) deciles, at the end of June of each year  $t$ , we sort stocks on Bm, which is the book equity for the fiscal year ending in calendar year  $t - 1$  divided by the market equity (from CRSP) at the end of December of  $t - 1$ . For firms with more than one share class, we merge the market equity for all share classes before computing Bm. Monthly decile returns are calculated from July of year  $t$  to June of  $t + 1$ , and the deciles are rebalanced in June of  $t + 1$ .<sup>4</sup>

To form the momentum ( $R^{11}$ ) deciles, we split all stocks at the beginning of each month  $t$  based on their prior 11-month returns from month  $t - 12$  to  $t - 2$ . Skipping month  $t - 1$ , we calculate monthly decile returns for month  $t$ , and rebalance the deciles at the beginning of month  $t + 1$ , following Fama and French (1996). Liu and Zhang (2014) instead follow Jegadeesh and Titman (1993), sort on the prior six-month return, skipping one month, and hold the deciles for the subsequent six-month period. We avoid the resulting six overlapping sets of momentum deciles with only the one-month holding period. In any event, the momentum profits from the  $R^{11}$  deciles are higher than those in Liu and Zhang, raising the hurdle for the investment model to explain.

To form the asset growth (I/A) deciles, at the end of June of each year  $t$ , we sort stocks into deciles based on I/A, which is measured as total assets (Compustat annual item AT) for the fiscal year ending in calendar year  $t - 1$  divided by total assets for the fiscal year ending in  $t - 2$  (Cooper, Gulen, and Schill 2008). Monthly decile returns are calculated from July of year  $t$  to June of  $t + 1$ , and the deciles are rebalanced in June of  $t + 1$ .

We measure return on equity (Roe) as income before extraordinary items (Compustat quarterly item IBQ) divided by one-quarter-lagged book equity (Hou, Xue, and Zhang 2015).<sup>5</sup> At the

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<sup>4</sup>Book equity is stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (Compustat annual item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if it is available. If not, we measure stockholders' equity as the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock.

<sup>5</sup>From 1972 onward, quarterly book equity is shareholders' equity, plus balance sheet deferred taxes and investment tax credit (item TXDITCQ) if available, minus the book value of preferred stock (item PSTKQ). Depending on availability, we use stockholders' equity (item SEQQ), or common equity (item CEQQ) plus the book value of preferred stock, or total assets (item ATQ) minus total liabilities (item LTQ) in that order as shareholders' equity. Prior to 1972, we expand the sample coverage by using book equity from Compustat annual files and imputing quarterly book equity with clean surplus accounting.

beginning of each month  $t$ , we sort all stocks into deciles based on their most recent past Roe. Before 1972, we use the most recent Roe computed with quarterly earnings from fiscal quarters ending at least four months ago. Starting from 1972, we use Roe computed with quarterly earnings from the most recent quarterly earnings announcement dates (Compustat quarterly item RDQ). For a firm to enter the portfolio formation, we require the end of the fiscal quarter that corresponds to its most recent Roe to be within six months prior to the portfolio formation, and its earnings announcement date to be after the corresponding fiscal quarter end. Monthly decile returns are calculated for the current month  $t$ , and the deciles are rebalanced at the beginning of month  $t + 1$ .

Table 1 reports the descriptive statistics of the 40 testing deciles as well as the high-minus-low deciles. From Panel A, the value premium (the average return of the high-minus-low Bm decile) is 0.47% per month ( $t = 2.07$ ) with NYSE breakpoints and value-weights and 0.66% ( $t = 2.8$ ) with all-but-micro breakpoints and equal-weights. Panel B shows that the momentum premium (the average return of the high-minus-low  $R^{11}$  decile) is much larger, 1.2% ( $t = 4.1$ ) with value-weights and 1.26% ( $t = 4.21$ ) with equal-weights. The investment premium (the average return of the high-minus-low I/A decile) is  $-0.37\%$  ( $t = -2.22$ ) with value-weights and  $-0.52\%$  ( $t = -3.39$ ) with equal-weights (Panel C). The Roe premium (the average return of the high-minus-low Roe decile) is 0.69% ( $t = 2.98$ ) with value-weights and 0.95% ( $t = 4.13$ ) with equal-weights (Panel D).<sup>6</sup>

## 4.2 Components of the Fundamental Return

This subsection describes firm-level accounting variables used to construct the fundamental return.

### Variable Measurement

We largely follow Liu, Whited, and Zhang (2009) and Liu and Zhang (2014), but offer several refinements. In the model, time- $t$  stock variables are at the beginning of period  $t$ , and time- $t$  flow variables are over the course of period  $t$ . In Compustat both stock and flow variables are recorded

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<sup>6</sup>For completeness, Table 1 also reports that the Carhart (1997) four-factor model, the Fama-French (2015) five-factor model, and the  $q$ -factor model largely succeed in capturing the value and investment premiums. However, while the Carhart and  $q$ -factor models capture the momentum premium, the five-factor model cannot. Finally, while the  $q$ -factor model captures the Roe premium, the Carhart and five-factor models cannot.

at the end of period  $t$ . As such, for the year 2010, for example, we take time- $t$  stock variables from the 2009 balance sheet, and time- $t$  flow variables from the 2010 income or cash flow statement.

We measure output,  $Y_{it}$ , as sales (Compustat annual item SALE) and short-term capital as current assets (item ACT). Total debt,  $B_{it+1}$ , is long-term debt (item DLTT, zero if missing) plus short-term debt (item DLC, zero if missing). The market leverage,  $w_{it}^B$ , is the ratio of total debt to the sum of total debt and market equity (from CRSP). The tax rate,  $\tau_t$ , is the statutory corporate income tax rate from the Commerce Clearing House's annual publications. The physical capital,  $K_{it}$ , is net property, plant, and equipment (item PPENT).

Departing from the prior studies, we offer several refinements in measurement. First, the prior studies measure the depreciate rate of physical capital,  $\delta_{it}$ , as the amount of depreciation and amortization (Compustat annual item DP) divided by physical capital (item PPENT). We subtract the amortization of intangibles (item AM, zero if missing) from item DP, before scaling the difference by item PPENT. This measure is more accurate. In the data, the AM/DP ratio is on average 6.3%, with a standard deviation of 13.9%. The AM/DP distribution has a long right tail. Its median is 0%, but the 75th, 90th, and 95th percentiles are 4.2%, 24.2%, and 39.7%, respectively.

Second, the prior studies measure investment,  $I_{it}$ , as capital expenditures (item CAPX) minus sales of property, plant, and equipment (item SPPE, zero if missing). Despite its simplicity, this  $I_{it}$  measure can violate the capital accumulation equation,  $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$ , in the data. At the portfolio level, across the 40 testing deciles, the difference is more than 5%, 15%, and 25% of  $K_{it}$  for 14.2%, 1.2%, and 0.3% of the observations, respectively. The violation is more severe at the firm level. The difference is more than 5%, 15%, and 25% of  $K_{it}$  for 35.5%, 18.2%, and 12.1% of the observations, respectively. As such, we measure  $I_{it}$  directly as  $K_{it+1} - (1 - \delta_{it})K_{it}$ .

Finally, to measure the firm-level pre-tax cost of debt in a broad sample, the prior studies impute credit ratings for firms with no credit ratings data in Compustat, and then assign the corporate bond returns for a given credit rating to all the firms with the same credit rating. We instead

measure the pre-tax cost of debt as the ratio of total interest and related expenses (item XINT) scaled by total debt,  $B_{it+1}$ . Doing so increases the sample coverage by 12.7%.

### **Timing Alignment**

We follow Liu and Zhang (2014) in aligning the timing of stock returns and accounting variables. In particular, the momentum and Roe deciles are rebalanced monthly, but accounting variables in Compustat are annual.<sup>7</sup> We construct monthly fundamental returns from annual accounting variables to match with monthly stock returns. For each month, we take firm-level accounting variables from the fiscal year end that is closest to the month in question to measure (flow) variables dated  $t$  in the model, and to take accounting variables from the subsequent fiscal year end to measure (flow) variables dated  $t + 1$  in the model. Because the portfolio composition can change monthly, the portfolio fundamental returns also change monthly.

While portfolio stock returns are in monthly terms and in monthly frequency, portfolio fundamental returns are in monthly frequency but in annual terms, constructed from annual accounting variables. To align the units, Liu and Zhang (2014) annualize monthly portfolio stock returns to match with portfolio fundamental returns. This procedure creates potential timing mismatch, as portfolio stock returns are for a given month, but fundamental returns are constructed from annual accounting variables both prior to and after the month. To better align the timing, we instead compound the portfolio stock returns within a 12-month rolling window with the month in question in the middle of the window. In particular, we multiply simple gross portfolio stock returns from month  $t - 5, t - 4, \dots, t, t + 1, \dots$ , and  $t + 6$  to match with the fundamental return for month  $t$ .

### **Descriptive Properties of the Accounting Variables**

Table 2 reports descriptive statistics for firm-level accounting variables in the fundamental return.

The mean physical investment-to-capital,  $I_{it}/K_{it}$ , is 0.38 in the full sample, with a large standard

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<sup>7</sup>Due to the large number of data items required to construct the fundamental return, we do not work with the Compustat quarterly files because of their limited coverage for many of these data items.

deviation of 0.56. In the all-but-micro sample, the dispersion in  $I_{it}/K_{it}$  is dampened, with a standard deviation of 0.48. For comparison, the mean investment rate in current assets,  $J_{it}/C_{it}$ , is 0.14, and its standard deviation is 0.39 in the full sample. Disinvestment in current assets is much more frequent than that in physical capital. The 5th percentile of  $J_{it}/C_{it}$  is  $-0.3$  and  $-0.2$ , with and without microcaps, in contrast to  $-0.03$  and  $0.02$  for  $I_{it}/K_{it}$ , respectively.

On average, physical capital accounts for only 38% of the sum of physical capital and current assets in the full sample, and the 25th and 75th percentiles of this fraction are 18% and 55%, respectively. The average fraction is slightly higher, 44%, in the all-but-micro sample. This evidence indicates the importance of accounting for capital heterogeneity in our estimation. The ratio of sales to the sum of the two assets,  $Y_{it+1}/(K_{it+1} + C_{it+1})$ , is on average 1.67, which is close to the median of 1.51 in the full sample. Its standard deviation is only 1.05. The moments without microcaps are close, with an average of 1.58, a median of 1.44, and a standard deviation of 0.99.

In contrast, microcaps have a large impact on sales-to-physical capital,  $Y_{it+1}/K_{it+1}$ . Its mean is 9.59, median 5.21, and standard deviation 14.46 in the full sample, and the moments are 6.74, 4.28, and 8.99, respectively, without microcaps. As such,  $Y_{it+1}/K_{it+1}$  is much more skewed than  $Y_{it+1}/(K_{it+1} + C_{it+1})$ . The rate of physical capital depreciation is on average 20%, with a standard deviation of 13% in the full sample. The market leverage,  $w_{it}^B$ , is on average 0.26, with a standard deviation of 0.22. For the pre-tax cost of debt, the mean is 10%, and the standard deviation 10%. The moments without microcaps are largely similar.

Table 2 also reports pairwise correlations of the accounting variables. In the full sample, the investment rate in physical capital,  $I_{it}/K_{it}$ , and the investment rate in current assets,  $J_{it}/C_{it}$ , have a positive correlation of 0.29.  $I_{it}/K_{it}$  has an autocorrelation of 0.26. However,  $J_{it}/C_{it}$  has an autocorrelation of only 0.03, which is consistent with our assumption of zero adjustment costs on current assets.  $I_{it+1}/K_{it+1}$  has positive correlations of 0.23 and 0.19 with two marginal product measures, sales-to-physical capital,  $Y_{it+1}/K_{it+1}$ , and sales over the sum of physical capital



and current assets,  $Y_{it+1}/(K_{it+1} + C_{it+1})$ , respectively. However,  $I_{it+1}/K_{it+1}$  is uncorrelated with  $Y_{it+1}/C_{it+1}$ . Similarly,  $J_{it+1}/C_{it+1}$  have positive correlations of 0.22 and 0.18 with  $Y_{it+1}/C_{it+1}$  and  $Y_{it+1}/(K_{it+1} + C_{it+1})$ , respectively, but a small correlation of 0.05 with  $Y_{it+1}/K_{it+1}$ . The fraction of physical capital in its sum with current assets,  $K_{it+1}/(K_{it+1} + C_{it+1})$ , has negative correlations of  $-0.26$ ,  $-0.53$ , and  $-0.31$  with  $I_{it+1}/K_{it+1}$ ,  $Y_{it+1}/K_{it+1}$ , and  $Y_{it+1}/(K_{it+1} + C_{it+1})$ , respectively, but a positive correlation of 0.44 with  $Y_{it+1}/C_{it+1}$ . The results without microcaps are largely similar.

Figure 1 reports the histograms of the accounting variables both at the firm level and at the portfolio level. Aggregating firm-level variables to the portfolio level eliminates a great deal of heterogeneity. Firm-level  $I_{it}/K_{it}$  varies from  $-0.5$  to  $2.5$ , but the portfolio-level  $I_{it}/K_{it}$  lies between  $-0.5$  and one, while centering about 0.25. Firm-level  $Y_{it+1}/(K_{it+1} + C_{it+1})$  varies from zero to 6.5, whereas its portfolio-level variable from 0.4 to 2.4. The firm-level  $Y_{it+1}/K_{it+1}$  distribution is even more dispersed, ranging from zero to 50, whereas the portfolio-level  $Y_{it+1}/K_{it+1}$  ranges from zero to only seven. The firm-level pre-tax cost of debt,  $r_{it+1}^B$ , varies from zero to slightly above 0.4, whereas the portfolio-level  $r_{it+1}^B$  mostly from zero to 0.15. The firm-level distribution of  $r_{it+1}^B$  has a spike at zero, since we treat all firms without debt as having zero cost of debt.

## 5 GMM Estimation and Tests

We first replicate the key findings from the prior studies that estimate the physical capital model at the portfolio level in Section 5.1. In Section 5.2, we report the results from the benchmark two-capital model estimated at the firm level. In Section 5.3, we quantify the impact of aggregation by estimating the two-capital model at the portfolio level. Finally, in Section 5.4, we quantify the impact of capital heterogeneity by estimating the physical capital model at the firm level.

### 5.1 Replicating the Prior Studies

Panel A of Table 3 reports the GMM estimation and tests for the physical capital model estimated directly at the portfolio level, without constructing firm-level fundamental returns. Consistent with

the prior literature, the physical capital model does a good job in accounting for value and momentum separately, but fails to do so jointly. The failure in the joint estimation is reflected in the parameter instability across the testing deciles when estimated separately. With value-weighted returns, the marginal product parameter,  $\gamma_K$ , is 0.168 with the book-to-market deciles, but 0.12 with the momentum deciles. For the adjustment costs parameter,  $a$ , the contrast is between 6.33 and 1.27. The average absolute high-minus-low error in the joint value and momentum estimation is 6.97% per annum, which is substantially larger than 1.24% and 1.56% in the separate estimation.

The joint estimation failure is more severe with the equal-weighted testing deciles. The marginal product parameter,  $\gamma_K$ , is estimated to be 0.72, and the adjustment costs parameter,  $a$ , 63.4 with the book-to-market deciles, in contrast to 0.129 and 1.34, respectively, with the momentum deciles. In the joint estimation,  $\gamma_K$  is 0.14, and  $a$  is 2.85. Consequently, the average absolute high-minus-low error in the joint value and momentum estimation is 12.24% per annum, which is substantially higher than 3.79% and 0.14% in the separate estimation.

Figure 2 reports individual errors by plotting average predicted stock returns against average realized stock returns across the value and momentum deciles as well as across all the 40 testing deciles in the joint estimation. The physical capital model manages to fit the momentum premium, but fails entirely to fit the value premium. With the value-weighted value and momentum deciles jointly, the model predicts a *negative* value premium of  $-2.89\%$  per annum, in contrast to  $6.56\%$  in the data (Panel A). The pricing error is large,  $9.45\%$  ( $t = 2.83$ ). The model also predicts a momentum premium of  $19.96\%$ , overshooting the data moment of  $15.48\%$ , with an error of  $-4.48\%$  ( $t = -2.36$ ). The failure in fitting the equal-weighted deciles is more severe. From Panel B, the model predicts a large, negative value premium of  $-7.48\%$ , in contrast to an observed premium of  $8.95\%$ , giving rise to a massive error of  $16.79\%$  ( $t = 4.96$ ). The model implied momentum premium is  $24.55\%$ , relative to the data moment of  $16.86\%$ , with an error of  $-7.68\%$  ( $t = -3.61$ ).

From Panels C and D, adding the asset growth and Roe deciles exacerbates the model's failure

in explaining the value premium. With value-weighted returns, the model predicts a value premium of  $-4.41\%$  per annum, with a large error of  $10.96\%$  ( $t = 3.63$ ). The model does well in predicting a momentum premium of  $16.78\%$ , with a small error of  $-1.31\%$  ( $t = -0.53$ ). With equal-weighted returns, the value premium is even more negative,  $-8.68\%$ , giving rise to a massive error of  $17.62\%$  ( $t = 5.61$ ). The fit across the momentum deciles also deteriorates. The model predicts a momentum premium of  $23.04\%$ , with an error of  $-6.18\%$  ( $t = -2.15$ ). The model does well in fitting the investment premium,  $-6.58\%$  with value-weights and  $-6.23\%$  with equal-weights, with small errors of  $1.47\%$  ( $t = 0.68$ ) and  $-1.05\%$  ( $t = -0.38$ ), respectively. However, the errors are larger for the Roe deciles. The model predicts an Roe premium of  $11.71\%$  with value-weights and  $16.92\%$  with equal-weights, and the errors are  $-3.44\%$  ( $t = -1.4$ ) and  $-4.87\%$  ( $t = -1.96$ ), respectively.

## 5.2 The Benchmark Estimation

Panel B of Table 3 reports that our benchmark two-capital model estimated at the firm level succeeds in explaining value and momentum simultaneously. A first indication is that the parameter estimates are relatively stable across the testing deciles. In particular, with value-weighted returns, the marginal product parameter,  $\gamma$ , is  $0.152$  with the book-to-market deciles, and  $0.163$  with the momentum deciles. For the adjustment costs parameter,  $a$ , the contrast is between  $5.37$  and  $3.74$ .

When estimating value and momentum jointly, the average absolute high-minus-low error is only  $1.06\%$  per annum in value-weighted returns, and is a small fraction of  $6.97\%$  from the physical capital model estimated at the portfolio level. With equal-weighted returns,  $\gamma$  is  $0.156$  and  $0.157$ , and  $a$   $3.6$  and  $2.65$ , with the book-to-market and momentum deciles, respectively. The average absolute high-minus-low error is  $1.84\%$ , which is small relative to  $12.24\%$  from the physical capital model. The mean absolute error is also smaller in the benchmark estimation than in the physical capital model,  $1\%$  versus  $2.86\%$  with value-weighted returns, and  $0.93\%$  versus  $4.05\%$  with equal-weighted returns. However, the benchmark model is still rejected by the overidentification test. Finally, adding the asset growth and Roe deciles does not materially change the results.

Figure 3 plots average predicted stock returns from the benchmark estimation against average realized stock returns across the testing deciles. The model performs well in all specifications, and the scatter points are mostly aligned with the 45-degree line. In particular, when fitting value-weighted value and momentum deciles (Panel A), the model predicts a value premium of 5.56% per annum (6.56% in the data), giving rise to a pricing error of 1% ( $t = 0.66$ ). The model also predicts a momentum premium of 16.6% (15.48% in the data), with an error of  $-1.13\%$  ( $t = -0.37$ ). The errors are slightly larger in magnitude in equal-weighted returns (Panel B). The value premium is 6.86% in the model (8.95% in the data), giving rise to an error of 2.09% ( $t = 1.17$ ). The momentum premium is 18.47% (16.86% in the data), with an error of  $-1.6\%$  ( $t = -0.5$ ).

Panels C and D show that the errors from the benchmark model increase only slightly in magnitude after adding the asset growth and Roe deciles. The scatter plots continue to align largely along the 45-degree line with all the 40 testing deciles. With value-weighted returns (Panel C), the model predicts a value premium of 4.96% per annum, with a pricing error of 1.6% ( $t = 0.46$ ) and a momentum premium of 16.22%, with an error of  $-0.75\%$  ( $t = -0.25$ ). The investment premium is  $-4.63\%$  in the model ( $-5.11\%$  in the data), giving rise to an error of  $-0.48\%$  ( $t = -0.23$ ). Finally, the Roe premium is 9.22% in the model (8.27% in the data), with an error of  $-0.95\%$  ( $t = -0.39$ ).

With equal-weighted returns (Panel D), the value premium is 4.85% per annum in the model, with an error of 4.1% ( $t = 1.86$ ). The momentum premium is 16.98% in the model, with an error of only  $-0.11\%$  ( $t = -0.04$ ). The investment premium is  $-8.41\%$  in the model ( $-7.28\%$  in the data), with an error of 1.13% ( $t = 0.71$ ). Finally, the Roe premium is 10.13% in the model (12.05% in the data), with an error of 1.93% ( $t = 0.84$ ). As such, the benchmark model does a good job in jointly explaining momentum, investment, and Roe premiums. Although the value premium continues to be the most challenging for the benchmark model to explain, its improvement over the physical capital model from the prior studies (Panel D of Figure 2) is substantial.

## Intuition

What are the economic mechanisms behind the value, momentum, investment, and Roe premiums in the investment CAPM? To shed light on the intuition, we conduct comparative statics on key variables in the benchmark specification, including the current investment-to-physical capital,  $I_{it}/K_{it}$ , the next period investment-to-physical capital,  $I_{it+1}/K_{it+1}$ , and the next period average product,  $Y_{it+1}/(K_{it+1} + C_{it+1})$ . Other variables also matter, but their impact is not nearly as important.

In the experiment on  $I_{it}/K_{it}$ , we set  $I_{it}/K_{it}$  to be its cross-sectional median at period  $t$  across all the firms. We use the parameter estimates from the benchmark estimation with all the 40 deciles together to reconstruct the fundamental returns, and recalculate the model's errors as the average portfolio stock minus fundamental returns. If the resulting errors are large relative to those from the benchmark estimation, we can infer that the  $I_{it}/K_{it}$  spread is quantitatively important to explain the average return spreads. The other comparative statics are designed analogously.

Table 4 shows that  $I_{it}/K_{it}$  and  $I_{it+1}/K_{it+1}$  are the two most important drivers of the expected return.  $I_{it}/K_{it}$  is more important than  $I_{it+1}/K_{it+1}$  for the value and investment premiums, but  $I_{it+1}/K_{it+1}$  is more important than  $I_{it}/K_{it}$  for the momentum and Roe premiums. Although less important than both,  $Y_{it+1}/(K_{it+1} + C_{it+1})$  also plays a role, especially for the Roe deciles.

From Panel A,  $I_{it}/K_{it}$  is essential to the value premium. Removing the cross-sectional variation of  $I_{it}/K_{it}$  gives rise to an error of 40.9% per annum for the value-weighted value premium, and an error of 44.7% for the equal-weighted. Intuitively, firms that invest more are growth firms with high marginal  $q$ , which in the optimum equals the marginal cost of physical investment,  $q_{it} = 1 + (1 - \tau)a(I_{it}/K_{it})$ . In the data, the average cross-sectional correlation between  $I_{it}/K_{it}$  and book-to-market is  $-0.2$  with microcaps, and  $-0.26$  without microcaps. Since the marginal cost is in the denominator of the physical investment return equation (2), growth firms with high  $I_{it}/K_{it}$  have lower fundamental returns than value firms with low  $I_{it}/K_{it}$ . Once we fix  $I_{it}/K_{it}$ , we hold the denominator of equation (2) constant to shut down this mechanism, yielding large errors.

The next period investment-to-physical capital,  $I_{it+1}/K_{it+1}$ , is the countervailing force of the current investment-to-physical capital,  $I_{it}/K_{it}$ . Fixing  $I_{it+1}/K_{it+1}$  across firms yields an error of  $-36.1\%$  per annum for the value-weighted value premium, and  $-29.6\%$  for the equal-weighted. Intuitively, growth firms also invest more and have higher marginal  $q$  next period than value firms. In the data, the average cross-sectional correlation between  $I_{it+1}/K_{it+1}$  and the current book-to-market is  $-0.17$  with microcaps, and  $-0.24$  without microcaps. Since  $I_{it+1}/K_{it+1}$  appears in the numerator of equation (2), the  $I_{it+1}/K_{it+1}$  spread implies that growth firms should have higher fundamental returns than value firms, countervailing the impact of  $I_{it}/K_{it}$  in the denominator. On net, the denominator effect dominates the numerator effect, and allows the model to yield a positive value premium.

Panel B shows that the next period investment-to-physical capital,  $I_{it+1}/K_{it+1}$ , is the most important driver of momentum, and the current  $I_{it}/K_{it}$  is the offsetting force. Fixing  $I_{it+1}/K_{it+1}$  across firms yields an error of  $22.5\%$  per annum for the value-weighted momentum premium, and  $23.5\%$  for the equal-weighted. Intuitively, winners invest more, and have higher marginal  $q$  next period than losers. In the data, the average cross-sectional correlation between  $I_{it+1}/K_{it+1}$  and prior 11-month returns,  $R^{11}$ , is  $0.18$  in the full sample, and  $0.23$  without microcaps. This expected investment mechanism implies that winners should have higher fundamental returns than losers. The current  $I_{it}/K_{it}$  is the countervailing force, but weaker. In the data, the average cross-sectional correlation between  $I_{it}/K_{it}$  and  $R^{11}$  is lower,  $0.09$  with microcaps and  $0.13$  without microcaps. On net, the numerator effect dominates the denominator effect, yielding positive momentum profits.

Not surprisingly, Panel C shows that the current investment-to-physical capital,  $I_{it}/K_{it}$ , is the most important driver for the investment premium. Fixing  $I_{it}/K_{it}$  across firms yields an error about  $-26.7\%$  per annum for both the value- and equal-weighted investment premiums. In the data, the average cross-sectional correlation between  $I_{it}/K_{it}$  and asset growth is  $0.17$  with microcaps, and  $0.27$  without microcaps. The next period  $I_{it+1}/K_{it+1}$  is the countervailing force, and its comparative statics yield an error of  $16.4\%$  for the value- and  $17.7\%$  for the equal-weighted investment premiums. The average cross-sectional correlations of asset growth with  $I_{it+1}/K_{it+1}$  are  $0.08$  and  $0.16$  with and

without microcaps, which are lower than the correlations with  $I_{it}/K_{it}$ , 0.17 and 0.27, respectively. The economic mechanism for the investment premium is similar to that for the value premium.

Finally, Panel D shows that the Roe premium is similar to momentum in its mechanism. The next period investment-to-physical capital,  $I_{it+1}/K_{it+1}$ , is the most important driver. Fixing  $I_{it+1}/K_{it+1}$  across firms yields an error of 14.2% per annum for the value-weighted Roe premium, and 18.8% for the equal-weighted. The average product,  $Y_{it+1}/(K_{it+1} + C_{it+1})$ , reinforces  $I_{it+1}/K_{it+1}$  in the numerator of equation (2). Removing its dispersion yields a value-weighted error of 4.5% per annum and an equal-weighted error of 10.2%. The current  $I_{it}/K_{it}$  is the countervailing force. Removing its dispersion yields an error of  $-8\%$  for the value-weighted premium, and  $-11.3\%$  for the equal-weighted. On net, the numerator effect from  $I_{it+1}/K_{it+1}$  and  $Y_{it+1}/(K_{it+1} + C_{it+1})$  dominates the denominator effect from  $I_{it}/K_{it}$ , yielding a positive Roe premium.

### 5.3 The Impact of Aggregation

We seek to understand the sources of the improvement of the benchmark model estimated at the firm level over the physical capital model estimated at the portfolio level. This subsection quantifies the impact of aggregation, whereas the next subsection quantifies the impact of capital heterogeneity.

Panel A of Table 5 reports the GMM estimation and tests for the two-capital model but implemented at the portfolio level. In particular, instead of constructing firm-level fundamental returns per the benchmark estimation, we aggregate firm-level characteristics to the portfolio level first, and then construct fundamental returns directly at the portfolio level. Comparing Panel A of Table 5 with Panel B of Table 3 shows that the parameter estimates are less stable from the portfolio-level implementation. With the firm-level estimation, across the value-weighted value and momentum deciles, the marginal product parameter,  $\gamma$ , varies from 0.152 to 0.163, and the adjustment costs parameter,  $a$ , from 3.74 to 5.37 (Panel B of Table 3). In contrast,  $\gamma$  varies from 0.192 to 0.227, and  $a$  from 2.52 to 5.62 with the portfolio-level estimation (Panel A of Table 5). Across the equal-weighted value and momentum deciles,  $\gamma$  varies from 0.156 to 0.157, and  $a$  from 2.65 to 3.6 with

the firm-level estimation (Panel B of Table 3), whereas  $\gamma$  varies from 0.215 to 0.271, and  $a$  from 3.26 to 8.9 with the portfolio-level estimation (Panel A of Table 5).

Consequently, the portfolio-level estimation yields larger pricing errors when fitting the value and momentum deciles simultaneously. With value-weighted returns, the mean absolute error is 1.56% per annum, and the average absolute high-minus-low return 2.75% with the portfolio-level estimation (Panel A of Table 5). These errors are larger than 1% and 1.06%, respectively, with the firm-level estimation (Panel B of Table 3). With equal-weighted returns, the mean absolute error is 2.04%, and the average absolute high-minus-low return 4.91% with the portfolio-level estimation, both of which are higher than 0.93% and 1.84%, respectively, from the firm-level estimation. Finally, the results are not materially changed once the asset growth and Roe deciles are added.

Figure 4 shows the scatter plots of average predicted stock returns from the portfolio-level estimation of the two-capital model versus average realized stock returns. The model struggles to fit the value premium in the joint estimations with momentum. With value-weighted value and momentum deciles (Panel A), the value premium is only 2.45% in the model, with an error of 4.11%, albeit insignificant ( $t = 1.4$ ). In contrast, the momentum premium is 14.08%, with a small error of 1.39% ( $t = 0.89$ ). With equal-weighted value and momentum deciles (Panel B), the model implied value premium is even negative,  $-0.79\%$ , giving rise to a large error of 9.73% ( $t = 2.98$ ). The momentum premium is 16.94%, with a small error of  $-0.08\%$  ( $t = -0.05$ ).

Adding the asset growth and Roe deciles further increases the pricing errors for the value premium. With the 40 value-weighted deciles (Panel C), the value premium is only 1.56% per annum in the model, and the error is 5% ( $t = 1.85$ ), whereas the momentum premium is 11.93%, with an error of 3.54% ( $t = 1.57$ ). With the 40 equal-weighted deciles (Panel D), the value premium is again negative,  $-1.13\%$ , which gives rise to a large error of 10.08% ( $t = 3.32$ ). The momentum premium is 15.63%, with a small error of 1.24% ( $t = 0.51$ ). Finally, the model does a reasonable job in fitting the investment and Roe premiums. In particular, with the 40 equal-weighted deciles



(Panel D), the investment premium is  $-5.73\%$  in the model, with an error of  $-1.55\%$  ( $t = -0.7$ ), and the Roe premium is  $9.74\%$ , with an error of  $2.31\%$  ( $t = 1.03$ ).

The contrast between Panel B of Table 3 and Panel A of Table 5 as well as that between Figures 3 and 4 quantify the impact of aggregation in the two-capital model. Figure 1 shows that the amount of heterogeneity in accounting variables in the fundamental return is substantial at the firm level. This heterogeneity is dampened greatly once the variables are aggregated to the portfolio level. As such, estimating the two-capital model at the firm level is more “structural” than at the portfolio level, making the parameter estimates more stable, and the pricing errors smaller when explaining value and momentum simultaneously in the firm-level estimation.

#### 5.4 The Impact of Capital Heterogeneity

In this subsection, we quantify the impact of introducing current assets as a production input in addition to physical capital in the benchmark two-capital model. To this end, we report detailed the GMM estimation results from the physical capital model (without current assets) implemented at the firm level, and compare them with those from the two-capital model estimated at the firm level.

Panel B of Table 5 reports the point estimates and overall performance of the physical capital model estimated at the firm level, departing from the prior studies that estimate this model at the portfolio level. The firm-level estimation again yields relatively stable parameter estimates across the value and momentum deciles. With value-weighted returns, the marginal product parameter,  $\gamma_K$ , varies from 0.049 to 0.071, and the adjustment costs parameter,  $a$ , from 0.58 to 3.11. With equal-weighted returns,  $\gamma_K$  varies from 0.038 to 0.057, and  $a$  from 0.29 to 3.26.

These  $\gamma_K$  estimates are lower than those from the portfolio-level estimation (Panel A of Table 3). The crux is that the firm-level distribution of sales-to-capital,  $Y_{it+1}/K_{it+1}$ , is highly skewed, with a mean of 9.59, a median of 5.21, a 5th percentile of 0.46, and a 95th percentile of 35 (Table 2). In contrast, the portfolio-level  $Y_{it+1}/K_{it+1}$  distribution is less dispersed, and shifted to the left, with a mean of only 2.53, a median of 2.39, a 5th percentile of 1.23, and a 95th percentile of 4.21. The lower

$\gamma_K$  estimates in Panel B of Table 5 reflect the different  $Y_{it+1}/K_{it+1}$  distribution at the firm level.

The  $\gamma_K$  estimates are also lower than the estimates of the marginal product parameter,  $\gamma$ , in the two-capital model estimated at the firm level. The reason is that the average product in the two-capital model, measured as the ratio of sales to the sum of physical capital and current assets,  $Y_{it+1}/(K_{it+1} + C_{it+1})$ , is much less dispersed than the average product in the physical capital model, measured as sales-to-physical capital,  $Y_{it+1}/K_{it+1}$ . Relative to the  $Y_{it+1}/K_{it+1}$  distribution, the  $Y_{it+1}/(K_{it+1} + C_{it+1})$  is also shifted to the left, with a mean of 1.67, a median of 1.51, a 5th percentile of 0.3, and a 95th percentile of 3.81 (Table 2). As noted, the fraction of physical capital in its sum with current assets,  $K_{it+1}/(K_{it+1} + C_{it+1})$ , is on average only 0.38, ranging from 0.07 in the 5th percentile to 0.88 in the 95th. As such, incorporating current assets better characterizes the firm-level distribution of the average product and the fundamental return.

Incorporating current assets clearly helps the model's performance. Without current assets, when fitting the value-weighted value and momentum deciles jointly, Panel B of Table 5 shows that the physical capital model yields a mean absolute error of 2.05% per annum and an average absolute high-minus-low error of 3.25%. These errors are much larger than 1% and 1.06%, respectively, from the two-capital model (Table 3). With equal-weighted deciles, the physical capital model yields a mean absolute error of 2.69% and an average absolute high-minus-low error of 11.82%, in contrast to 0.93% and 1.84%, respectively, from the two-capital model. Adding the asset growth and Roe deciles does not change the results materially. With value-weighted returns, the mean absolute error is 2.78%, and the average absolute high-minus-low error 4.17% in the physical capital model, relative to 1.29% and 0.94%, respectively, in the benchmark model. With equal-weighted returns, the mean absolute error is 2.79%, and the average absolute high-minus-low error 9.95% in the physical capital model, in contrast to 0.91% and 1.82%, respectively, in the two-capital model.

Figure 5 shows the scatter plots of average predicted stock returns from the firm-level estimation of the physical capital model versus average realized stock returns. The model struggles to explain

the average returns across the testing deciles, especially the value premium. With value-weighted value and momentum deciles (Panel A), the value premium is 4.4% per annum in the model, with an error of 2.16% ( $t = 0.35$ ). The model also exaggerates the momentum premium to 19.81%, with an error of -4.34% ( $t = -0.84$ ). With equal-weighted value and momentum deciles (Panel B), the value premium is negative and large, -10.29%, in the model, with a massive error of 19.24% ( $t = 8.16$ ). The momentum premium is 21.26%, with an error of -4.4% ( $t = -6$ ).

Adding the asset growth and Roe deciles further increases the pricing errors for the value premium. With the 40 value-weighted deciles, the value premium is -2.85% per annum in the model, with a large error of 9.14% ( $t = 3.36$ ). The momentum premium is 20.57%, with an error of -5.09% ( $t = -1.43$ ). With equal-weighted deciles, the value premium is again negative, -9.45%, which gives rise to a massive error of 18.39% ( $t = 8.29$ ). The momentum premium is 26.37%, with a large error of -9.5% ( $t = -3.9$ ). Finally, the physical capital model estimated at the firm level does a good job in fitting the value-weighted investment and Roe premiums, but not equal-weighted. With the 40 value-weighted deciles, the investment premium is -6.91% in the model, with an error of 1.8% ( $t = 0.7$ ), and the Roe premium is 8.65%, with an error of -0.39% ( $t = -0.14$ ). However, with equal-weighted deciles, the investment premium is -1.89%, with an error of -5.39% ( $t = -2.67$ ), and the Roe premium is 18.55%, with an error of -6.5% ( $t = -2.77$ ).

## 6 The Dynamics of Factor Premiums

In this section, we use the fundamental return implied from the benchmark two-capital model estimated at the firm level to examine the dynamics of the value, momentum, investment, and Roe premiums. Because the model parameters are estimated from matching only the average returns across the testing portfolios, the dynamics are important in serving as separate diagnostics on the model's performance. We study calendar-time as well as event-time dynamics. Since the focus is on comparing the dynamics of fundamental returns with those of stock returns, we winsorize the

firm-level fundamental returns at the 1–99% level each month to alleviate the impact of outliers.<sup>8</sup> Finally, to construct the fundamental returns, we always use the parameter estimates from the joint estimation of all the 40 testing deciles. We report both value- and equal-weighted results.

## 6.1 Correlations between Stock and Fundamental Returns

Taken literally, equation (5) implies that the stock and fundamental returns are equal ex post. However, Liu, Whited, and Zhang (2009) document a correlation puzzle, i.e., the contemporaneous correlations between the stock and fundamental returns are weakly negative, but the correlations between the one-year-lagged stock returns and the fundamental returns are significantly positive.

Liu, Whited, and Zhang (2009) align the timing of annual stock returns from July of year  $t$  to June of  $t + 1$  with the fundamental returns constructed from the accounting variables at fiscal year end of  $t$  and  $t + 1$ . We instead follow Liu and Zhang (2014) in constructing monthly fundamental returns from annual accounting data. As noted, for each month, we take accounting variables from the fiscal year end that is closest to the month to measure current-period variables in the model, and to take accounting variables from the subsequent fiscal year end to measure next-period variables in the model. However, differing from Liu and Zhang, we match the fundamental returns for the month in equation (aggregated to the portfolio level) with portfolio stock returns compounded across the 12-month rolling window with the month in question in the middle of the window. This rolling window measurement of portfolio stock returns helps resolve the correlation puzzle.

Table 6 shows that the contemporaneous correlations between the stock returns and the fundamental returns from the benchmark model are significantly positive. From Panel A, the time series average of cross-sectional correlations of the two types of returns across all firms is 0.14, with a  $p$ -value less than 1%. Excluding microcaps yields the same correlation. The stock-fundamental return correlation is 0.22 across the 40 value-weighted deciles, and 0.37 across the equal-weighted

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<sup>8</sup>We do not winsorize the firm-level fundamental returns in the GMM estimation and tests reported in Section 5. Doing so complicates the computation of standard errors of point estimates because fundamental returns depend on the estimates. However, implementing the winsorization will most likely reduce the pricing errors, and improve the benchmark model’s performance in matching the average returns across the testing portfolios.

deciles, both of which are highly significant. At the firm level, the lead-lag correlations are all positive within the 12-month horizon, but turn negative at longer horizons. At the portfolio level, the lead-lag correlations are all positive across the horizons within 60 months.

Panel B shows the time series correlation between the stock and fundamental returns for each testing decile. The correlations are significantly positive for the extreme deciles and high-minus-low deciles, but those for the middle deciles are weaker. In particular, the correlation is 0.32 for the value premium with value-weights, and 0.54 with equal-weights. For the momentum premium, the correlation is 0.18 with value-weights, and 0.28 with equal-weights. The correlation is 0.39 for the investment premium with value-weights, and 0.44 with equal-weights. For the Roe premium, the correlation is 0.21 with value-weights, and 0.34 with equal-weights. Finally, we emphasize that equation (5) predicts perfect stock-fundamental return correlations across firms and across portfolios. The correlations in Table 6, while mostly positive, are far from perfect.

## 6.2 Market States and Factor Premiums

Cooper, Gutierrez, and Hameed (2004) show that momentum is large and positive following nonnegative prior 36-month market returns (UP markets), but negative following negative prior 36-month market returns (DOWN markets). Liu and Zhang (2014) show that the physical capital model estimated at the portfolio level fails to explain this evidence. Our benchmark model makes some progress in predicting procyclical momentum profits, but the procyclical variation is still weaker than that in the data. We also extend the evidence to the value, investment, and Roe premiums.

From Panel A of Table 7, the value-weighted value premium is stronger following DOWN than UP markets (identified with prior 36-month market returns): 16.68% versus 4.7% per annum. The model succeeds in capturing the countercyclical pattern: 20.89% versus  $-0.02\%$ . The variation is more dampened with the equal-weighted value premium both in the data and in the model.

From Panel B, the momentum premium is stronger following UP than DOWN markets. With the market states identified with prior 36-month market returns, the momentum premium is 20.12%

per annum following UP markets, but  $-9.39\%$  following DOWN markets. In the model, the contrast is  $16.9\%$  versus  $11.31\%$ . The results for the equal-weighted momentum premium in both the data and the model are quantitatively similar. As such, although the model explains the procyclical variation of momentum, its magnitude is substantially weaker than that in the data.

Panel C shows that the investment premium is stronger following DOWN than UP markets. With prior 12-month market returns categorizing the market states, the value-weighted investment premium is  $-11.77\%$  per annum following DOWN markets, but  $-3.14\%$  following UP markets. In the model, the contrast is only  $-6.69\%$  versus  $-5.37\%$ . The results for the equal-weighted investment premium are largely similar. As such, although going in the right direction, the model falls short to explain the countercyclical variation of the investment premium.

Finally, Panel D shows that the Roe premium is stronger following UP than DOWN markets. With prior 36-month market returns identifying the market states, the value-weighted Roe premium is  $11.07\%$  per annum following UP markets, but  $-6.02\%$  following DOWN markets. In the model, the comparison is between  $7.07\%$  and  $1.94\%$ . The equal-weighted Roe premium is  $14.16\%$  following UP markets, but  $-0.11\%$  following DOWN markets. The model predicts  $10.44\%$  and  $8.8\%$  following the two market states, respectively. As such, although the model explains the procyclical variation of the Roe premium, its magnitude is substantially weaker than that in the data.

### **6.3 Persistence of Factor Premiums**

Prior studies show that value is more persistent than momentum. Fama and French (1995) show that the value premium subsists for three to five years after the portfolio formation, whereas Chan, Jegadeesh, and Lokonishok (1996) show that momentum profits are short-lived, positive within the 12-month horizon, but negative afterward. Liu and Zhang (2014) show that the physical capital model estimated at the portfolio level explains the short-lived dynamics of momentum. We extend the persistence evidence to the investment and Roe premiums, and show that the benchmark model succeeds in explaining the short-lived nature of the momentum and Roe premiums as well as the

long-lived nature of the value and investment premiums.

Figure 6 reports the event-time dynamics of stock and fundamental returns of the high and low deciles during 36 months after the portfolio formation. The value premium persists even after three years, whereas the momentum premium converges to zero after only about ten months (Panels A–D). From Panels E and F, the value premium in the model is also long-lasting. Panels G and H show that the model also explains the short-lived nature of momentum. Also, Panels I and J show that the investment premium in the data is persistent, but less so than the value premium. The value-weighted investment premium converges in about two years after the portfolio formation, and the equal-weighted investment premium in three years. From Panels M and N, the investment premiums in the model exhibit largely similar dynamics.

Finally, the Roe premium is as short-lived in the data as momentum (Panels K and L). In particular, the value-weighted stock returns of the high and low deciles start at 13.44% and 5.47% per annum at month one, and converge to 10.45% and 10.32%, respectively, at month seven. Similarly, the value-weighted fundamental returns of the two deciles start at 14.82% and 8.5% at month one, and converge to 12.73% and 12.41%, respectively, at month eight. However, while the stock returns of the two extreme deciles are indistinguishable afterward, the fundamental return of the low decile outperforms that of the high decile. The results are largely similar with equal-weighted returns.

## 6.4 Higher Moments

Table 8 compares the higher moments, including volatility, skewness, and kurtosis, of the stock returns with those of the fundamental returns for each testing decile.

Several important patterns emerge from this table. First, fundamental returns are less volatile than stock returns, echoing Cochrane’s (1991) results at the aggregate level. In particular, except for the value-weighted value premium, which shows volatilities about 19.5% per annum for both stock and fundamental returns (Panel A of Table 8), the fundamental volatilities of factor premiums are often less than one half of their stock volatilities. For example, the contrast is 9.49% versus 20.98%

for the equal-weighted value premium, 13.19% versus 27.48% for the value-weighted momentum premium, and 10.51% versus 30.34% for the equal-weighted momentum premium. The fundamental return volatilities of individual deciles are also low relative to their stock return volatilities.

Second, the benchmark model largely fails to explain the negative skewness of momentum. Daniel and Moskowitz (2016) document that momentum can experience infrequent and persistent negative returns. Such momentum crashes yield a negative skewness for the momentum premium. Panel B largely replicates their results. The value-weighted momentum premium has a skewness of  $-1.83$ , but is only significant at the 10% level, and the equal-weighted momentum premium has a skewness of  $-0.7$ , which is insignificant. However, the fundamental momentum premium in the model shows a positive, albeit small, skewness of 0.57 with value-weights, and 0.62 with equal-weights, and both are highly significant. In addition, Panel D extends the Daniel-Moskowitz evidence to the Roe premium. The skewness is  $-0.84$  with value-weights, and  $-1.95$  with equal-weights. Both are significant at the 10% level. In contrast, the model predicts insignificant skewness of  $-0.24$  and 0.3 for the value- and equal-weighted Roe premium, respectively.

Finally, the model does better in explaining the kurtosis of factor premiums. For the value-weighted value premium, the kurtosis is 3.4 for stock returns, relative to 4.37 for fundamental returns. The contrast is 4.92 versus 4.81 for the equal-weighted value premium. The fundamental returns also match the kurtosis of the stock returns for the investment premium, 3.72 versus 3.44 with value-weights, and 2.99 versus 3.28 with equal-weights. However, the fundamental returns fall short for momentum, 4.71 versus 12.08 with value-weights, and 3.45 versus 11.39 with equal-weights, as well as for the equal-weighted Roe premium, 3.82 versus 16.9. The model does better for the value-weighted Roe premium, 4.35 versus 5.78.

Figure 7 plots the time series of stock and fundamental returns of the factor premiums. The fundamental returns track the stock returns well, reflecting the economically large and statistically significant correlations in Table 6. However, the fundamental returns clearly fall short in explaining



the extreme movements in the momentum and Roe premiums. In particular, the value-weighted momentum premium experiences a crash of more than  $-150\%$  in 2009, but its fundamental return falls no more than  $50\%$  (Panel C). In addition, the equal-weighted Roe premium experiences a crash of more than  $-150\%$  in 1999, but its fundamental return falls less than  $5\%$ .

## 7 Conclusion

Aggregation and heterogeneity are thorny challenges for empirical investment studies. This paper provides a careful treatment of aggregation, and to a lesser extent, capital heterogeneity in the context of the investment CAPM. Instead of forming fundamental returns from portfolio-level accounting variables aggregated from the firm level, we construct firm-level fundamental returns from firm-level variables, and then aggregate firm-level fundamental returns to the portfolio level to match with portfolio-level stock returns. In addition, we introduce current assets as a separate production input from physical capital. Both innovations make the empirical specification of the investment CAPM more “structural,” help stabilize parameter estimates, and better describe cross-sectional expected returns. Most important, our benchmark specification with two capital goods estimated at the firm level goes a long way in explaining value and momentum simultaneously.

The empirical success of the investment CAPM suggests that it can be adopted more broadly. While factor models are effective in describing the common variation in the cross section, facilitating practical risk management for portfolio managers, the cost of capital estimates from factor models are noisy, yielding little out-of-sample predictive power (Fama and French 1997). In response, a voluminous literature has emerged to estimate the implied cost of capital from accounting-based valuation models (Gebhardt, Lee, and Swaminathan 2001), with many applications. However, the implied cost of capital estimated as the internal rate of return does not seem to forecast one-period-ahead realized returns (Easton and Monahan 2005). In contrast, the fundamental return from our economic model provides a detailed, theoretical description of the one-period-ahead expected return. The fact that the fundamental return is less volatile than the realized stock return might be

a blessing in disguise. The evidence suggests that the expected fundamental return might be a less noisy proxy for the expected return than the average realized return. Future work can develop the expected fundamental return as a new class of the implied cost of capital.

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**Table 1 : Descriptive Properties of Testing Deciles, January 1967–December 2015**

For each testing decile as well as the high-minus-low decile (H–L), we report its average return in excess of the one-month Treasury bill rate,  $m$ , the alpha from the Carhart (1997) four-factor model,  $\alpha_C$ , the alpha from the Fama-French (2015) five-factor model,  $\alpha_{FF}$ , and the Hou-Xue-Zhang (2015)  $q$ -factor model,  $\alpha_q$ , as well as their  $t$ -statistics adjusted for heteroscedasticity and autocorrelations,  $t_m, t_C, t_{FF}$ , and  $t_q$ , respectively.

	L	2	3	4	5	6	7	8	9	H	H–L	L	2	3	4	5	6	7	8	9	H	H–L
	NYSE breakpoints and value-weighted returns											All-but-micro breakpoints and equal-weighted returns										
Panel A: The book-to-market (Bm) deciles																						
$m$	0.42	0.50	0.57	0.46	0.51	0.54	0.66	0.61	0.72	0.89	0.47	0.24	0.36	0.51	0.56	0.62	0.70	0.73	0.71	0.73	0.89	0.66
$t_m$	1.74	2.54	2.93	2.18	2.73	3.01	3.46	3.20	3.85	3.76	2.07	0.77	1.36	2.06	2.40	2.71	3.27	3.52	3.32	3.46	3.87	2.80
$\alpha_C$	0.13	0.08	0.06	-0.13	-0.06	-0.03	0.02	-0.08	-0.01	0.03	-0.10	-0.07	0.00	0.11	0.06	0.05	0.10	0.10	0.05	0.03	0.09	0.16
$t_C$	2.05	1.40	0.93	-1.60	-0.70	-0.47	0.31	-1.08	-0.17	0.29	-0.74	-0.64	-0.02	1.45	0.85	0.77	1.46	1.57	0.59	0.43	0.97	1.27
$\alpha_{FF}$	0.11	-0.04	-0.04	-0.22	-0.17	-0.05	-0.01	-0.15	-0.05	-0.04	-0.14	-0.04	-0.05	-0.01	-0.06	-0.13	-0.09	-0.06	-0.12	-0.09	-0.08	-0.05
$t_{FF}$	1.54	-0.62	-0.58	-2.39	-2.14	-0.63	-0.11	-1.96	-0.57	-0.32	-1.15	-0.27	-0.50	-0.12	-0.61	-1.62	-1.00	-0.72	-1.48	-1.35	-0.92	-0.37
$\alpha_q$	0.10	-0.06	-0.02	-0.20	-0.16	-0.01	0.08	-0.10	0.09	0.18	0.08	0.03	0.04	0.11	0.06	-0.01	0.01	0.05	0.00	0.02	0.12	0.09
$t_q$	1.06	-0.76	-0.32	-1.90	-1.87	-0.17	0.84	-0.93	0.82	1.55	0.48	0.17	0.28	0.88	0.49	-0.14	0.09	0.52	0.00	0.18	1.16	0.39
Panel B: The momentum ( $R^{11}$ ) deciles																						
$m$	-0.12	0.36	0.43	0.45	0.42	0.46	0.45	0.62	0.67	1.08	1.20	-0.07	0.37	0.49	0.50	0.55	0.63	0.71	0.77	0.99	1.19	1.26
$t_m$	-0.35	1.33	1.93	2.24	2.23	2.35	2.52	3.11	3.13	3.88	4.10	-0.21	1.40	2.13	2.32	2.78	3.19	3.57	3.58	3.93	3.65	4.21
$\alpha_C$	-0.04	0.36	0.28	0.18	0.07	-0.03	-0.07	-0.05	-0.11	0.09	0.13	-0.15	0.13	0.11	0.00	-0.01	-0.04	-0.04	-0.07	0.02	0.05	0.20
$t_C$	-0.42	3.68	3.19	1.76	0.82	-0.33	-0.91	-0.69	-1.64	0.90	1.20	-1.37	1.61	1.48	0.02	-0.11	-0.54	-0.62	-1.26	0.23	0.46	1.47
$\alpha_{FF}$	-0.68	-0.26	-0.18	-0.20	-0.16	-0.15	-0.21	-0.01	0.05	0.63	1.31	-0.68	-0.32	-0.24	-0.24	-0.17	-0.14	-0.02	0.11	0.39	0.70	1.38
$t_{FF}$	-2.57	-1.28	-1.35	-1.76	-1.74	-2.03	-2.00	-0.11	0.45	3.73	3.32	-2.75	-1.69	-1.98	-2.75	-2.14	-1.89	-0.25	1.18	3.14	3.44	3.34
$\alpha_q$	0.01	0.20	0.13	-0.09	-0.04	-0.08	-0.25	-0.14	-0.16	0.30	0.28	-0.04	0.06	-0.02	-0.12	-0.09	-0.12	-0.08	-0.02	0.18	0.37	0.41
$t_q$	0.05	0.91	0.79	-0.58	-0.32	-0.94	-2.29	-1.80	-1.64	1.50	0.65	-0.17	0.27	-0.17	-1.08	-0.95	-1.46	-1.31	-0.20	1.20	1.44	0.88
Panel C: The asset growth (I/A) deciles																						
$m$	0.68	0.67	0.61	0.51	0.52	0.54	0.58	0.46	0.57	0.31	-0.37	0.69	0.77	0.78	0.70	0.74	0.72	0.64	0.59	0.45	0.17	-0.52
$t_m$	2.85	3.28	3.64	3.01	2.95	2.92	3.07	2.31	2.28	1.15	-2.22	2.65	3.76	4.01	3.74	3.63	3.66	2.92	2.56	1.75	0.57	-3.39
$\alpha_C$	-0.04	0.13	0.03	0.04	0.01	0.01	0.10	0.01	0.16	-0.14	-0.11	0.01	0.17	0.16	0.14	0.15	0.16	0.06	0.03	-0.04	-0.29	-0.29
$t_C$	-0.35	1.55	0.49	0.50	0.18	0.10	1.58	0.19	2.07	-1.65	-0.75	0.06	2.07	2.39	2.43	2.43	2.40	0.93	0.45	-0.43	-2.77	-2.21
$\alpha_{FF}$	-0.18	-0.13	-0.14	0.00	-0.07	-0.10	0.04	0.03	0.30	-0.09	0.09	-0.17	-0.05	-0.04	-0.04	-0.01	0.03	-0.04	-0.03	-0.13	-0.38	-0.21
$t_{FF}$	-1.79	-1.69	-2.10	-0.05	-1.04	-1.56	0.50	0.44	3.46	-1.03	0.69	-1.88	-0.52	-0.62	-0.60	-0.12	0.45	-0.71	-0.40	-1.23	-2.62	-1.89
$\alpha_q$	-0.14	-0.06	-0.17	0.00	-0.08	-0.10	0.03	0.03	0.36	0.00	0.14	-0.02	0.05	0.02	0.03	0.07	0.10	0.03	0.07	0.07	-0.20	-0.18
$t_q$	-1.35	-0.75	-2.16	0.04	-1.01	-1.27	0.38	0.40	3.93	-0.02	1.22	-0.18	0.43	0.20	0.43	0.90	1.23	0.40	0.83	0.62	-1.30	-1.64
Panel D: The return on equity (Roe) deciles																						
$m$	0.04	0.22	0.38	0.40	0.53	0.42	0.54	0.50	0.56	0.73	0.69	0.12	0.30	0.50	0.50	0.60	0.62	0.71	0.74	0.82	1.07	0.95
$t_m$	0.12	0.88	1.79	2.15	2.84	2.07	2.85	2.70	2.83	3.29	2.98	0.34	1.08	2.26	2.47	2.92	2.87	3.46	3.40	3.72	4.23	4.13
$\alpha_C$	-0.51	-0.25	-0.08	-0.09	0.08	-0.09	0.06	0.01	0.09	0.28	0.78	-0.48	-0.25	-0.05	-0.09	-0.03	0.00	0.10	0.12	0.21	0.42	0.90
$t_C$	-3.32	-2.51	-0.94	-1.15	1.06	-1.11	0.94	0.21	1.50	3.75	4.09	-3.26	-2.45	-0.67	-1.32	-0.41	0.04	1.37	1.75	3.24	5.23	4.68
$\alpha_{FF}$	-0.40	-0.34	-0.10	-0.14	-0.03	-0.09	-0.02	-0.04	0.15	0.20	0.60	-0.26	-0.13	-0.12	-0.22	-0.17	-0.14	-0.02	0.02	0.12	0.35	0.60
$t_{FF}$	-3.25	-3.21	-1.06	-1.36	-0.31	-1.17	-0.25	-0.63	2.07	2.59	4.08	-2.28	-1.08	-1.41	-2.68	-2.37	-1.84	-0.24	0.25	1.81	4.73	4.02
$\alpha_q$	0.03	0.01	0.25	0.00	0.06	-0.06	0.00	-0.13	0.08	0.05	0.02	0.14	0.20	0.10	-0.11	-0.11	-0.13	-0.06	-0.03	0.04	0.22	0.08
$t_q$	0.25	0.08	3.14	-0.02	0.65	-0.77	0.01	-1.87	1.03	0.65	0.15	1.07	2.03	1.30	-1.25	-1.29	-1.53	-0.71	-0.46	0.49	2.51	0.47

**Table 2 : Descriptive Statistics of Firm-level Accounting Variables in the Fundamental Return, January 1967–December 2015**

For all the components in the fundamental return, we report the time series averages of cross-sectional statistics, including mean ( $m$ ), standard deviation ( $\sigma$ ), percentiles (5%, 25%, 50%, 75%, and 95%), as well as their pairwise correlations. The statistics are computed after the 1%–99% winsorization for all the listed variables except for the market leverage,  $w_{it}^B$ , at each portfolio formation.  $I_{it}/K_{it}$  is time- $t$  physical investment-to-physical capital,  $J_{it}/C_{it}$  is the time- $t$  ratio of current assets investment divided by current assets.  $Y_{it+1}/K_{it+1}$  is the sales-to-physical capital in time  $t + 1$ .  $Y_{it+1}/C_{it+1}$  is the sales-to-current assets in time  $t + 1$ .  $K_{it+1}/(K_{it+1} + C_{it+1})$  is the fraction of physical capital in the sum of physical capital and current assets.  $\delta_{it+1}$  is the rate of physical capital depreciation.  $r_{it+1}^B$  is the pre-tax cost of debt.

	Panel A: The full sample								Panel B: The all-but-micro sample																					
	$m$	$\sigma$	5%	25%	50%	75%	95%		$m$	$\sigma$	5%	25%	50%	75%	95%		$m$	$\sigma$	5%	25%	50%	75%	95%							
$\frac{I_{it}}{K_{it}}$	0.38	0.56	-0.03	0.11	0.23	0.44	1.32		0.37	0.48	0.02	0.13	0.23	0.42	1.14		0.37	0.48	0.02	0.13	0.23	0.42	1.14							
$\frac{J_{it}}{C_{it}}$	0.14	0.39	-0.30	-0.05	0.07	0.23	0.82		0.17	0.36	-0.20	-0.01	0.10	0.24	0.82		0.17	0.36	-0.20	-0.01	0.10	0.24	0.82							
$\frac{Y_{it+1}}{K_{it+1}}$	9.59	14.46	0.46	2.38	5.21	10.10	35.00		6.74	8.99	0.44	1.81	4.28	7.66	21.92		6.74	8.99	0.44	1.81	4.28	7.66	21.92							
$\frac{Y_{it+1}}{C_{it+1}}$	3.17	2.26	0.78	1.79	2.62	3.84	7.47		3.33	2.08	1.06	1.99	2.85	4.06	7.35		3.33	2.08	1.06	1.99	2.85	4.06	7.35							
$\frac{Y_{it+1}}{K_{it+1}+C_{it+1}}$	1.67	1.05	0.30	0.97	1.51	2.11	3.81		1.58	0.99	0.34	0.91	1.44	1.99	3.55		1.58	0.99	0.34	0.91	1.44	1.99	3.55							
$\frac{K_{it+1}}{K_{it+1}+C_{it+1}}$	0.38	0.25	0.07	0.18	0.32	0.55	0.88		0.44	0.25	0.11	0.24	0.39	0.65	0.90		0.44	0.25	0.11	0.24	0.39	0.65	0.90							
$w_{it}^B$	0.26	0.22	0.00	0.07	0.22	0.42	0.68		0.25	0.20	0.00	0.07	0.21	0.39	0.61		0.25	0.20	0.00	0.07	0.21	0.39	0.61							
$\delta_{it+1}$	0.20	0.13	0.05	0.11	0.16	0.25	0.49		0.17	0.11	0.04	0.10	0.14	0.20	0.40		0.17	0.11	0.04	0.10	0.14	0.20	0.40							
$r_{it+1}^B$	0.10	0.10	0.00	0.06	0.08	0.11	0.25		0.10	0.09	0.01	0.06	0.08	0.10	0.21		0.10	0.09	0.01	0.06	0.08	0.10	0.21							
	$\frac{I_{it+1}}{K_{it+1}}$	$\frac{J_{it}}{C_{it}}$	$\frac{J_{it+1}}{C_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}}$	$\frac{Y_{it+1}}{C_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}+C_{it+1}}$	$\frac{K_{it+1}}{K_{it+1}+C_{it+1}}$	$w_{it}^B$	$\delta_{it+1}$	$r_{it+1}^B$	$\frac{I_{it+1}}{K_{it+1}}$	$\frac{J_{it}}{C_{it}}$	$\frac{J_{it+1}}{C_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}}$	$\frac{Y_{it+1}}{C_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}+C_{it+1}}$	$\frac{K_{it+1}}{K_{it+1}+C_{it+1}}$	$w_{it}^B$	$\delta_{it+1}$	$r_{it+1}^B$	$\frac{I_{it+1}}{K_{it+1}}$	$\frac{J_{it}}{C_{it}}$	$\frac{J_{it+1}}{C_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}}$	$\frac{Y_{it+1}}{C_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}+C_{it+1}}$	$\frac{K_{it+1}}{K_{it+1}+C_{it+1}}$	$w_{it}^B$	$\delta_{it+1}$	$r_{it+1}^B$
$\frac{I_{it}}{K_{it}}$	0.26	0.29	0.09	0.07	-0.05	0.05	-0.15	-0.15	0.23	0.07	0.35	0.36	0.12	0.16	-0.07	0.07	-0.21	-0.20	0.33	0.08	0.35	0.36	0.12	0.16	-0.07	0.07	-0.21	-0.20	0.33	0.08
$\frac{I_{it+1}}{K_{it+1}}$		0.21	0.29	0.23	0.00	0.19	-0.26	-0.26	0.53	0.20	0.26	0.32	0.32	-0.02	0.20	-0.30	-0.30	0.57	0.21	0.26	0.32	0.32	-0.02	0.20	-0.30	-0.30	0.57	0.21		
$\frac{J_{it}}{C_{it}}$			0.03	0.04	-0.05	0.00	-0.05	-0.07	0.05	0.03		0.03	0.08	-0.08	-0.01	-0.10	-0.08	0.13	0.04		0.03	0.08	-0.08	-0.01	-0.10	-0.08	0.13	0.04		
$\frac{J_{it+1}}{C_{it+1}}$				0.05	0.22	0.18	0.08	-0.12	0.09	0.15			0.09	0.20	0.15	0.04	-0.12	0.12	0.15			0.09	0.20	0.15	0.04	-0.12	0.12	0.15		
$\frac{Y_{it+1}}{K_{it+1}}$					0.08	0.53	-0.53	-0.16	0.49	0.07				0.08	0.64	-0.61	-0.27	0.49	0.07					0.08	0.64	-0.61	-0.27	0.49	0.07	
$\frac{Y_{it+1}}{C_{it+1}}$						0.57	0.44	0.18	-0.16	0.05					0.50	0.46	0.17	-0.20	0.01						0.50	0.46	0.17	-0.20	0.01	
$\frac{Y_{it+1}}{K_{it+1}+C_{it+1}}$							-0.31	-0.07	0.22	0.12						-0.40	-0.23	0.26	0.10							-0.40	-0.23	0.26	0.10	
$\frac{K_{it+1}}{K_{it+1}+C_{it+1}}$								0.36	-0.56	-0.05							0.49	-0.58	-0.09								0.49	-0.58	-0.09	
$w_{it}^B$									-0.32	-0.07									-0.39	-0.12								-0.39	-0.12	
$\delta_{it+1}$										0.12																			0.12	

**Table 3 : GMM Estimation and Tests, the Physical Capital Model Estimated at the Portfolio Level versus the Benchmark Two-capital Model Estimated at the Firm Level, January 1967–December 2015**

This table reports GMM estimation and tests for the 40 testing deciles formed on book-to-market (Bm), prior 11-month returns ( $R^{11}$ ), asset growth (I/A), and return on equity (Roe), separately and jointly (Bm and  $R^{11}$ , I/A and Roe, and all 40 deciles together). d.f. is the degrees of freedom in the GMM test of overidentification. In Panel A,  $\gamma_K$  is the technological parameter on the marginal product of physical capital as a fraction of sales-to-physical capital,  $Y_{it+1}/K_{it+1}$ . In Panel B,  $\gamma$  is the technological parameter on the joint marginal product of current assets and physical capital as a fraction of the ratio of sales divided by the sum of the two assets,  $Y_{it+1}/(K_{it+1} + C_{it+1})$ .  $a$  is the adjustment costs parameter of physical capital.  $[\gamma]$ ,  $[\gamma_K]$ , and  $[a]$  are the standard errors of the point estimates of these parameters. m.a.e. is the mean absolute error across a given set of testing portfolios,  $\overline{|e_{H-L}|}$  is the average absolute high-minus-low error, and  $p$  is the  $p$ -value of the overidentification test across a given set of testing portfolios.  $\gamma$ ,  $\gamma_K$ ,  $[\gamma]$ ,  $[\gamma_K]$ , m.a.e.,  $\overline{|e_{H-L}|}$ , and  $p$  are in percent.

Panel A: The physical capital model estimated at the portfolio level																
	NYSE breakpoints and value-weighted returns								All-but-micro breakpoints and equal-weighted returns							
	d.f.	$\gamma_K$	$[\gamma_K]$	$a$	$[a]$	m.a.e.	$\overline{ e_{H-L} }$	$p$	$\gamma_K$	$[\gamma_K]$	$a$	$[a]$	m.a.e.	$\overline{ e_{H-L} }$	$p$	
Bm	8	16.78	[2.41]	6.33	[1.93]	2.34	1.24	0.00	72.08	[12.75]	63.40	[0.51]	3.65	3.79	7.31	
$R^{11}$	8	11.99	[1.14]	1.27	[0.53]	1.38	1.56	14.90	12.93	[1.29]	1.34	[0.58]	1.31	0.14	34.03	
I/A	8	12.28	[1.08]	1.13	[0.40]	2.07	0.21	0.00	14.72	[1.46]	2.24	[0.52]	2.50	1.33	0.00	
Roe	8	10.34	[0.98]	0.00	[0.05]	3.18	0.25	0.00	11.54	[1.11]	0.00	[0.04]	2.90	0.29	0.00	
Bm- $R^{11}$	18	13.26	[1.18]	2.30	[0.48]	2.86	6.97	0.00	14.04	[1.39]	2.85	[0.52]	4.05	12.24	0.00	
I/A-Roe	18	11.59	[1.02]	0.85	[0.35]	2.78	1.60	0.00	13.75	[1.33]	1.75	[0.40]	2.97	3.24	0.00	
Bm- $R^{11}$ -I/A-Roe	38	12.55	[1.09]	1.73	[0.35]	2.88	4.30	0.00	14.09	[1.34]	2.50	[0.37]	3.50	7.43	0.00	

Panel B: The benchmark two-capital model estimated at the firm level																
	NYSE breakpoints and value-weighted returns								All-but-micro breakpoints and equal-weighted returns							
	d.f.	$\gamma$	$[\gamma]$	$a$	$[a]$	m.a.e.	$\overline{ e_{H-L} }$	$p$	$\gamma$	$[\gamma]$	$a$	$[a]$	m.a.e.	$\overline{ e_{H-L} }$	$p$	
Bm	8	15.17	[2.55]	5.37	[0.00]	0.74	2.37	97.81	15.60	[1.99]	3.60	[0.01]	0.78	1.82	2.47	
$R^{11}$	8	16.32	[2.06]	3.74	[0.00]	0.86	0.20	77.48	15.69	[1.97]	2.65	[0.97]	0.58	0.29	41.86	
I/A	8	17.17	[1.80]	1.56	[0.69]	0.96	2.63	0.78	16.48	[1.79]	1.99	[0.47]	0.64	0.88	0.70	
Roe	8	15.10	[2.76]	6.07	[0.01]	0.94	1.93	49.13	14.82	[1.98]	3.74	[0.01]	0.34	0.24	40.99	
Bm- $R^{11}$	18	16.68	[2.09]	3.60	[0.01]	1.00	1.06	2.35	15.52	[2.09]	3.28	[0.26]	0.93	1.84	0.00	
I/A-Roe	18	17.01	[1.84]	1.65	[0.70]	1.15	2.28	0.00	16.17	[1.84]	2.05	[0.43]	0.70	1.30	0.00	
Bm- $R^{11}$ -I/A-Roe	38	16.69	[2.05]	3.55	[0.00]	1.29	0.94	0.00	15.91	[1.96]	2.78	[0.27]	0.91	1.82	0.00	

**Table 4 : Comparative Statics, the Benchmark Two-capital Model Estimated at the Firm Level, January 1967–December 2015**

This table reports the pricing errors from three comparative statics:  $\overline{I_{it}/K_{it}}$ ,  $\overline{I_{it+1}/K_{it+1}}$ , and  $\overline{Y_{it+1}/(K_{it+1} + C_{it+1})}$ . In the experiment denoted  $\overline{I_{it}/K_{it}}$ ,  $I_{it}/K_{it}$  is set to be its cross-sectional average at period  $t$  across all the firms. The parameters from the benchmark GMM estimation (with all 40 Bm,  $R^{11}$ , I/A, and Roe deciles together) are used to reconstruct the fundamental returns, with all the other characteristics unchanged. The other experiments are designed analogously. The pricing error is the average difference between portfolio stock returns and reconstructed fundamental returns. The “Benchmark” rows report the benchmark model’s errors.

		NYSE breakpoints and value-weighted returns										All-but-micro breakpoints and equal-weighted returns											
		L	2	3	4	5	6	7	8	9	H	H-L	L	2	3	4	5	6	7	8	9	H	H-L
		Panel A: The book-to-market (Bm) deciles																					
Benchmark		-0.82	-0.91	0.40	-0.58	-0.97	2.27	0.93	1.27	-1.94	0.78	1.60	-2.11	-1.64	-0.04	-0.43	-0.83	1.78	1.60	1.55	1.52	1.99	4.10
$\overline{I_{it}/K_{it}}$		-11.16	-6.51	-2.56	0.51	3.10	8.87	14.13	17.47	20.83	29.79	40.94	-19.06	-13.19	-9.39	-7.02	-3.99	0.31	3.55	7.91	13.03	25.66	44.72
$\overline{I_{it+1}/K_{it+1}}$		8.20	5.22	4.21	0.20	-1.34	-2.60	-6.95	-11.47	-17.26	-27.85	-36.06	9.70	7.43	7.45	5.06	3.27	2.82	0.96	-3.33	-7.23	-19.86	-29.56
$\overline{Y_{it+1}/(K_{it+1} + C_{it+1})}$		0.19	0.01	1.12	-0.93	-2.78	-0.77	-4.00	-5.42	-8.76	-6.94	-7.13	-0.03	0.81	2.38	2.43	1.44	3.22	1.92	0.24	-1.52	-3.08	-3.05
		Panel B: The momentum ( $R^{11}$ ) deciles																					
Benchmark		0.90	2.53	1.27	0.73	2.57	-1.08	-1.12	-1.05	-2.36	0.15	-0.75	0.35	0.42	0.02	-1.13	-0.94	-0.77	-1.02	-0.89	0.05	0.24	-0.11
$\overline{I_{it}/K_{it}}$		3.16	4.36	4.34	4.02	3.01	3.57	1.59	1.54	-2.95	-9.11	-12.28	0.53	0.83	1.52	0.99	1.53	1.06	-0.61	-2.97	-5.51	-12.25	-12.78
$\overline{I_{it+1}/K_{it+1}}$		-9.72	-2.10	-1.68	-1.55	0.87	-3.67	-1.98	-0.43	2.18	12.77	22.49	-7.48	-2.50	-2.03	-2.68	-2.34	-1.13	0.84	3.41	8.86	16.00	23.48
$\overline{Y_{it+1}/(K_{it+1} + C_{it+1})}$		-2.28	0.33	-0.68	-1.29	1.02	-2.86	-2.44	-2.02	-2.29	1.48	3.77	-1.16	0.18	-0.03	-1.15	-0.87	-0.41	-0.03	0.78	2.56	3.51	4.68
		Panel C: The asset growth (I/A) deciles																					
Benchmark		-1.58	-2.74	-1.54	-1.49	0.58	1.05	2.01	0.99	3.81	-2.07	-0.48	-1.60	-0.39	-0.60	-0.51	1.63	2.12	1.47	0.80	1.03	-0.48	1.13
$\overline{I_{it}/K_{it}}$		7.34	8.75	9.60	9.05	7.40	4.82	2.85	-2.31	-4.93	-19.30	-26.64	8.34	10.22	9.18	6.19	5.08	3.14	-1.13	-5.03	-9.27	-18.60	-26.94
$\overline{I_{it+1}/K_{it+1}}$		-6.28	-8.09	-7.29	-6.90	-3.99	-1.54	1.66	3.82	9.11	10.13	16.41	-7.94	-7.54	-6.11	-4.07	-0.51	1.19	3.01	5.02	7.62	9.78	17.72
$\overline{Y_{it+1}/(K_{it+1} + C_{it+1})}$		-3.23	-4.25	-4.04	-4.70	-1.76	-0.32	0.92	0.61	4.02	-1.59	1.64	-2.37	-0.71	-1.51	-0.75	1.70	2.86	2.74	2.46	2.47	0.34	2.71
		Panel D: The return on equity (Roe) deciles																					
Benchmark		-0.05	-2.20	0.25	0.58	2.65	0.24	0.39	-1.29	-0.41	-1.00	-0.95	-1.46	-0.37	0.53	-0.26	0.19	-0.72	-1.26	-1.09	-0.18	0.47	1.93
$\overline{I_{it}/K_{it}}$		0.19	3.99	9.95	9.21	8.82	3.15	1.73	-2.75	-4.45	-7.77	-7.96	2.36	3.88	5.17	3.23	0.49	-2.06	-4.63	-6.36	-7.78	-8.96	-11.31
$\overline{I_{it+1}/K_{it+1}}$		-6.27	-9.43	-8.20	-6.55	-2.79	-1.68	0.23	1.42	4.87	7.94	14.21	-7.60	-5.80	-3.83	-3.17	0.11	1.52	3.57	4.98	7.72	11.23	18.83
$\overline{Y_{it+1}/(K_{it+1} + C_{it+1})}$		-3.60	-6.68	-4.96	-4.05	-0.94	-1.86	-0.51	-0.64	0.43	0.90	4.50	-4.84	-3.05	-1.48	-1.30	0.70	0.78	1.45	2.35	3.87	5.35	10.19

**Table 5 : GMM Estimation and Tests, the Two-capital Model Estimated at the Portfolio Level and the Physical Capital Model Estimated at the Firm Level, January 1967–December 2015**

This table reports GMM estimation and tests for the 40 testing deciles formed on book-to-market (Bm), prior 11-month returns ( $R^{11}$ ), asset growth (I/A), and return on equity (Roe), separately and jointly (Bm and  $R^{11}$ , I/A and Roe, and all 40 deciles together). d.f. is the degrees of freedom in the GMM test of overidentification. In Panel A,  $\gamma$  is the technological parameter on the joint marginal product of current and physical assets as a fraction of the ratio of sales divided by the sum of the two assets,  $Y_{it+1}/(K_{it+1} + C_{it+1})$ . In Panel B,  $\gamma_K$  is the technological parameter on the marginal product of physical capital as a fraction of sales-to-physical capital,  $Y_{it+1}/K_{it+1}$ .  $a$  is the adjustment costs parameter of physical capital.  $[\gamma]$ ,  $[\gamma_K]$ , and  $[a]$  are the standard errors of the point estimates of these parameters. m.a.e. is the mean absolute error across a given set of testing portfolios,  $\overline{|e_{H-L}|}$  is the average absolute high-minus-low error, and  $p$  is the  $p$ -value of the overidentification test across a given set of testing portfolios.  $\gamma$ ,  $\gamma_K$ ,  $[\gamma]$ ,  $[\gamma_K]$ , m.a.e.,  $\overline{|e_{H-L}|}$ , and  $p$  are in percent.

Panel A: The two-capital model estimated at the portfolio level															
	NYSE breakpoints and value-weighted returns								All-but-micro breakpoints and equal-weighted returns						
	d.f.	$\gamma$	$[\gamma]$	$a$	$[a]$	m.a.e.	$\overline{ e_{H-L} }$	$p$	$\gamma$	$[\gamma]$	$a$	$[a]$	m.a.e.	$\overline{ e_{H-L} }$	$p$
Bm	8	22.73	[2.79]	5.62	[2.16]	1.49	1.38	0.04	27.09	[4.20]	8.90	[3.58]	2.90	5.75	0.09
$R^{11}$	8	19.16	[2.14]	2.52	[0.92]	1.06	3.25	12.79	21.49	[2.59]	3.26	[1.05]	0.61	1.72	21.93
I/A	8	18.69	[1.81]	1.45	[0.64]	1.05	2.12	0.06	22.01	[2.28]	2.85	[0.70]	1.69	2.20	0.16
Roe	8	17.09	[2.09]	1.06	[1.16]	1.59	3.62	0.07	21.17	[2.83]	2.65	[1.57]	1.40	3.22	0.00
Bm- $R^{11}$	18	20.24	[1.96]	3.10	[0.79]	1.56	2.75	0.00	22.08	[2.45]	3.90	[0.89]	2.04	4.91	0.00
I/A-Roe	18	18.09	[1.80]	1.46	[0.56]	1.42	2.71	0.00	21.61	[2.28]	2.78	[0.60]	1.56	2.72	0.00
Bm- $R^{11}$ -I/A-Roe	38	19.32	[1.86]	2.44	[0.57]	1.60	2.80	0.00	21.96	[2.32]	3.43	[0.61]	1.86	3.79	0.00

Panel B: The physical capital model estimated at the firm level															
	NYSE breakpoints and value-weighted returns								All-but-micro breakpoints and equal-weighted returns						
	d.f.	$\gamma_K$	$[\gamma_K]$	$a$	$[a]$	m.a.e.	$\overline{ e_{H-L} }$	$p$	$\gamma_K$	$[\gamma_K]$	$a$	$[a]$	m.a.e.	$\overline{ e_{H-L} }$	$p$
Bm	8	5.40	[0.94]	2.99	[0.00]	1.37	1.62	22.46	3.79	[0.89]	3.26	[0.01]	2.29	7.08	0.33
$R^{11}$	8	6.79	[0.59]	0.58	[0.43]	1.81	0.75	0.47	5.56	[0.47]	0.29	[0.25]	0.81	0.35	10.79
I/A	8	6.24	[0.84]	2.73	[0.00]	2.75	7.21	0.04	4.61	[0.70]	2.73	[0.00]	2.08	6.73	2.00
Roe	8	6.44	[0.82]	2.65	[0.00]	1.59	4.39	1.15	5.00	[0.58]	1.82	[0.35]	1.91	4.38	0.00
Bm- $R^{11}$	18	4.92	[1.01]	3.11	[0.00]	2.05	3.25	0.00	5.67	[0.48]	0.54	[0.24]	2.69	11.82	0.00
I/A-Roe	18	6.20	[0.88]	2.74	[0.00]	2.47	2.37	0.00	5.43	[0.56]	1.51	[0.23]	2.31	3.54	0.00
Bm- $R^{11}$ -I/A-Roe	38	7.06	[0.66]	1.34	[0.19]	2.78	4.17	0.00	5.66	[0.51]	0.92	[0.14]	2.79	9.95	0.00



**Table 6 : Correlations between Stock Returns and Fundamental Returns**

Panel A reports the firm-level and portfolio-level correlations between the stock returns of various leads and lags and fundamental returns,  $r_{it}^F$ . For instance, the column denoted  $r_{it}^S$  reports contemporaneous correlations, and the column  $r_{it-3}^S$  the correlations between three-month-lagged stock returns and fundamental returns. Other columns are defined analogously. “vw-portfolios” means the 40 value-weighted deciles formed on book-to-market, prior 11-month returns, asset growth, and return on equity, and “ew-portfolios” the 40 equal-weighted deciles. The correlations are time series averages of cross-sectional correlations, and their  $p$ -values are calculated as the Fama-MacBeth (1973)  $p$ -values adjusted for autocorrelations of up to 12 lags. Panel B reports for each of the 40 value- and equal-weighted decile as well as the high-minus-low decile, the time series contemporaneous correlations between the stock and fundamental returns. Their  $p$ -values are calculated as the  $p$ -values of the slopes from regressing the stock returns on the contemporaneous fundamental returns, and are adjusted for autocorrelations of up to 12 lags. The correlations that are significant at the 1%, 5%, and 10% levels are denoted with three stars, two stars, and one star, respectively. All the correlations are calculated after winsorizing the firm-level fundamental returns at the 1–99% level each month. The first two rows in Panel A and the value-weighted results in Panel B are based on the parameter values from estimating the benchmark model on all the 40 value-weighted testing deciles jointly, and the other results use the parameter values from jointly estimating all the 40 equal-weighted testing deciles.

Panel A: Correlations with the fundamental returns, $r_{it}^F$											
	$r_{it-60}^S$	$r_{it-36}^S$	$r_{it-24}^S$	$r_{it-12}^S$	$r_{it-3}^S$	$r_{it}^S$	$r_{it+3}^S$	$r_{it+12}^S$	$r_{it+24}^S$	$r_{it+36}^S$	$r_{it+60}^S$
All firms	-0.02***	-0.04***	-0.03***	0.02***	0.12***	0.14***	0.14***	0.05***	-0.01	0.00	-0.01
vw-portfolios	0.06**	0.09***	0.05*	0.10***	0.20***	0.22***	0.21***	0.12***	0.08***	0.13***	0.12***
No microcaps	-0.01	-0.02**	-0.01*	0.06***	0.14***	0.14***	0.13***	0.04***	-0.01*	-0.01	0.00
ew-portfolios	0.26***	0.25***	0.23***	0.27***	0.36***	0.37***	0.36***	0.27***	0.22***	0.26***	0.23***
Panel B: Contemporaneous correlations between the stock and fundamental returns across the testing deciles											
	L	2	3	4	5	6	7	8	9	H	H-L
NYSE breakpoints and value-weighted deciles											
Bm	0.13	0.20	0.12	0.02	0.12**	0.20	0.01	-0.02	0.03	0.25**	0.32***
$R^{11}$	0.25**	0.12	0.07	-0.04	-0.03	0.02	0.02	0.09	0.09	0.22	0.18**
I/A	0.19**	0.10	0.12	-0.03	0.10	-0.01	0.07	0.01	0.10	0.29***	0.39***
Roe	0.25**	0.19*	0.12	0.13*	-0.02	0.00	0.07	0.02	-0.01	0.10	0.21**
All-but-micro breakpoints and equal-weighted deciles											
Bm	0.38***	0.28**	0.23	0.14	0.17	0.20**	0.16*	0.14*	0.17*	0.13	0.54***
$R^{11}$	0.23**	0.13	0.09	0.04	0.06	0.05	0.16	0.22**	0.27***	0.42***	0.28***
I/A	0.18	0.13	-0.04	0.05	0.08	0.11	0.12	0.13	0.16	0.34***	0.44***
Roe	0.34***	0.22*	0.12	0.02	0.01	0.09	0.13	0.09	0.12	0.24**	0.34***

**Table 7 : Market States and Factor Premiums**

For each month  $t$ , we categorize the market state as UP (DOWN) if the value-weighted CRSP index returns from month  $t - N$  to  $t - 1$ , with  $N = 12, 24$ , or  $36$  are nonnegative (negative). The table reports the high-minus-low returns averaged across UP (DOWN) market states.  $r^S$  denotes the stock return, and  $r^F$  the fundamental returns, both of which are in percent per annum. The  $t$ -values are adjusted for heteroscedasticity and autocorrelations of up to 12 lags. We winsorize the firm-level fundamental returns at the 1–99% level each month. The value-weighted results use the parameter values from estimating the benchmark model on all the 40 value-weighted testing deciles jointly, and the equal-weighted results use those from all the 40 equal-weighted testing deciles.

$N$	MKT	$r^S$	$t_{r^S}$	$r^F$	$t_{r^F}$	$r^S$	$t_{r^S}$	$r^F$	$t_{r^F}$
		Value-weighted returns				Equal-weighted returns			
Panel A: The high-minus-low Bm decile									
12	DOWN	11.84	4.19	3.76	0.64	12.35	4.39	5.15	3.45
12	UP	5.04	1.72	3.14	1.34	7.94	2.67	1.73	1.27
24	DOWN	13.17	2.57	16.42	3.00	12.11	2.67	5.10	3.23
24	UP	5.38	1.94	0.85	0.37	8.36	2.95	2.03	1.46
36	DOWN	16.68	3.22	20.89	3.79	11.60	2.47	4.50	2.51
36	UP	4.70	1.80	-0.02	-0.01	8.45	2.94	2.14	1.51
Panel B: The high-minus-low $R^{11}$ decile									
12	DOWN	0.89	0.09	18.23	6.85	3.22	0.28	18.36	6.50
12	UP	19.76	7.65	15.37	12.12	20.74	7.07	14.94	13.08
24	DOWN	-7.23	-0.62	14.68	4.15	-7.58	-0.61	15.25	5.22
24	UP	19.65	7.79	16.27	12.68	21.24	7.18	15.81	12.69
36	DOWN	-9.39	-0.99	11.31	5.63	-9.95	-1.06	11.91	9.59
36	UP	20.12	7.96	16.90	11.93	21.75	7.15	16.43	12.11
Panel C: The high-minus-low I/A decile									
12	DOWN	-11.77	-4.31	-6.69	-2.60	-14.24	-5.32	-9.83	-4.17
12	UP	-3.14	-1.69	-5.37	-3.43	-5.34	-3.00	-7.34	-6.41
24	DOWN	-11.66	-5.04	-6.93	-2.45	-15.95	-5.16	-7.63	-2.69
24	UP	-3.89	-1.92	-5.43	-3.31	-5.78	-3.21	-7.96	-6.57
36	DOWN	-7.64	-3.06	-5.27	-1.91	-10.97	-4.25	-4.22	-2.10
36	UP	-4.63	-2.20	-5.74	-3.44	-6.69	-3.51	-8.59	-6.77
Panel D: The high-minus-low Roe decile									
12	DOWN	2.34	0.51	6.19	2.66	5.76	0.89	10.12	5.87
12	UP	10.16	3.68	6.29	4.54	13.73	5.44	10.20	10.63
24	DOWN	-3.26	-0.54	4.21	1.54	2.52	0.31	9.58	5.36
24	UP	10.53	4.02	6.64	4.80	13.65	5.65	10.29	10.59
36	DOWN	-6.02	-1.27	1.94	0.85	-0.11	-0.02	8.80	6.00
36	UP	11.07	4.23	7.07	5.04	14.16	5.59	10.44	10.63

**Table 8 : Higher Moments of Stock Returns and Fundamental Returns**

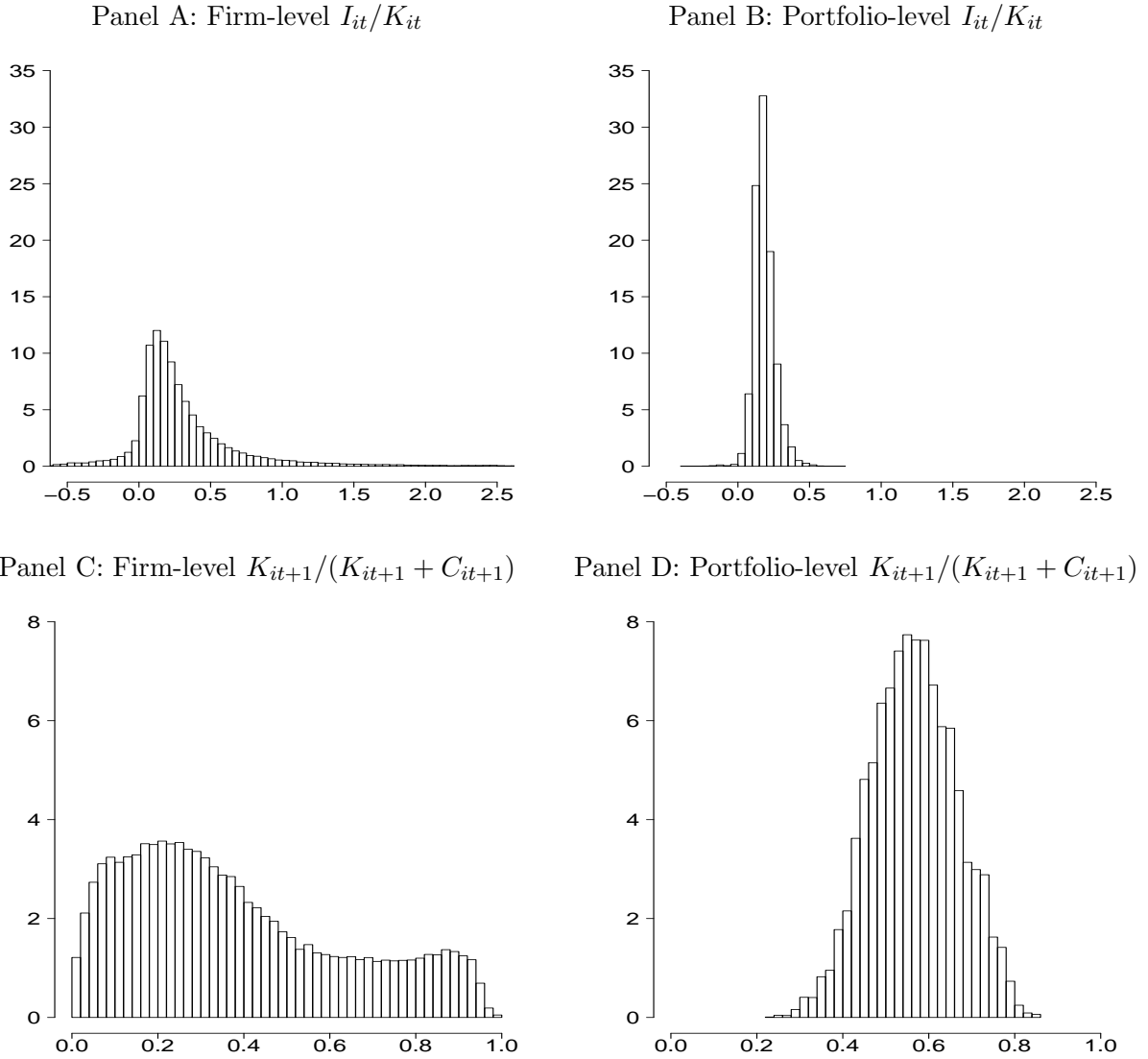
For each decile, we report the volatility in percent,  $\sigma$ , skewness,  $S_k$ , and kurtosis,  $K_u$ , of its stock returns,  $r^S$ , and fundamental returns,  $r^F$ . For each high-minus-low decile, the volatility and skewness significantly different from zero and the kurtosis significantly different from three at the 1%, 5%, and 10% levels are denoted with three stars, two stars, and one star, respectively. The  $p$ -values are based on 5,000 block bootstrapped samples, with each block containing 60 months. We winsorize the firm-level fundamental returns at the 1–99% level each month. The value-weighted results use the parameter values from estimating the benchmark model on all the 40 value-weighted deciles jointly, and the equal-weighted results use those from all the 40 equal-weighted deciles.

		L	2	3	4	5	6	7	8	9	H	H–L
Panel A: The book-to-market deciles												
Value-weighted returns												
$\sigma$	$r^S$	20.50	18.04	17.64	18.90	16.82	16.37	17.38	16.85	17.09	21.37	19.99***
	$r^F$	5.49	7.40	7.80	8.67	9.69	10.82	8.76	10.03	14.36	19.71	19.47***
$S_k$	$r^S$	−0.25	0.01	−0.07	−0.06	−0.18	−0.07	−0.22	−0.47	−0.12	0.09	0.45
	$r^F$	−0.90	−0.69	2.21	0.97	1.14	−1.85	0.70	0.28	0.44	0.41	0.02
$K_u$	$r^S$	2.98	3.08	2.75	3.38	3.19	3.62	3.48	4.35	3.96	4.42	3.40
	$r^F$	3.86	5.35	13.81	6.69	5.39	8.17	3.01	3.44	4.59	4.82	4.37*
Equal-weighted returns												
$\sigma$	$r^S$	26.66	23.31	22.56	21.18	21.20	19.88	19.39	19.37	19.45	21.38	20.98***
	$r^F$	7.10	5.51	5.74	4.54	4.76	4.60	5.08	4.90	6.29	8.67	9.49***
$S_k$	$r^S$	0.06	0.03	0.09	0.03	0.09	0.19	0.19	0.16	0.13	0.56	0.38
	$r^F$	−1.55	−1.76	−1.16	−0.61	−0.36	−0.55	0.23	0.37	−0.02	−0.38	0.45
$K_u$	$r^S$	3.14	3.00	3.39	3.18	3.63	3.82	3.94	4.50	4.02	6.65	4.92***
	$r^F$	6.70	7.11	5.14	2.91	2.41	3.29	3.19	2.60	2.32	3.79	4.81**
Panel B: The momentum deciles												
Value-weighted returns												
$\sigma$	$r^S$	29.59	24.18	19.87	18.20	16.55	17.22	15.87	17.85	19.52	26.05	27.48***
	$r^F$	11.65	9.13	9.21	7.90	8.36	7.49	7.50	7.64	7.90	7.77	13.19***
$S_k$	$r^S$	1.54	0.98	0.15	0.46	−0.10	−0.14	−0.23	−0.19	−0.13	−0.06	−1.83*
	$r^F$	−0.90	−0.17	0.05	0.40	0.95	0.99	1.18	0.72	0.60	−0.18	0.57**
$K_u$	$r^S$	10.43	8.21	3.84	4.11	3.65	3.49	2.96	3.03	3.48	3.16	12.08***
	$r^F$	6.20	5.61	7.42	4.95	6.29	6.21	7.00	5.53	5.03	3.94	4.71**
Equal-weighted returns												
$\sigma$	$r^S$	30.22	23.13	20.78	19.22	18.14	17.94	18.66	20.16	24.46	33.10	30.34***
	$r^F$	9.26	6.22	5.40	4.99	4.71	4.55	4.37	4.73	5.00	6.76	10.51***
$S_k$	$r^S$	1.51	0.32	0.31	0.05	0.07	0.04	0.17	0.03	0.44	1.02	−0.70
	$r^F$	−0.85	−0.63	−0.26	−0.33	−0.17	0.02	−0.12	−0.01	−0.50	−0.61	0.62***
$K_u$	$r^S$	9.41	4.72	4.14	3.69	3.75	3.66	3.93	3.39	4.42	6.72	11.39***
	$r^F$	4.08	3.44	3.26	2.76	2.84	3.02	2.82	2.67	3.64	4.76	3.45*

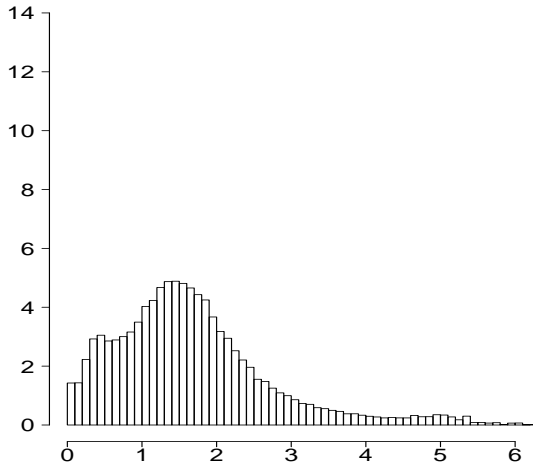
		L	2	3	4	5	6	7	8	9	H	H-L
Panel C: The asset growth deciles												
Value-weighted returns												
$\sigma$	$r^S$	22.11	18.51	15.78	15.38	15.68	16.60	16.68	17.38	21.58	22.88	14.90***
	$r^F$	8.82	9.90	9.33	7.37	6.14	8.67	6.53	5.83	8.18	7.94	11.46***
$S_k$	$r^S$	0.46	-0.05	-0.01	-0.17	-0.27	-0.18	-0.19	-0.17	-0.30	-0.22	0.06
	$r^F$	0.14	1.68	0.68	0.65	0.47	-0.52	-0.07	0.43	0.21	-0.26	0.33
$K_u$	$r^S$	4.42	3.57	3.14	3.45	3.51	3.16	3.15	3.11	3.27	3.09	3.44
	$r^F$	3.03	9.44	3.25	4.34	2.95	5.35	4.52	4.15	4.26	4.09	3.72
Equal-weighted returns												
$\sigma$	$r^S$	24.36	19.07	18.32	17.28	18.70	18.44	19.71	20.94	23.30	26.02	13.16***
	$r^F$	6.35	4.73	4.66	4.15	4.60	4.51	4.74	5.28	6.24	8.61	9.24***
$S_k$	$r^S$	0.61	0.33	0.26	0.11	-0.02	0.08	-0.03	0.06	0.12	0.08	0.04
	$r^F$	0.60	0.08	-0.32	-0.17	0.34	-0.10	-0.40	-0.51	-0.74	-0.71	-0.28
$K_u$	$r^S$	4.80	4.15	3.99	4.08	3.89	3.71	3.90	3.47	3.46	3.24	3.28
	$r^F$	2.89	3.18	2.69	2.54	3.04	2.30	2.58	3.01	3.18	3.01	2.99
Panel D: The return on equity deciles												
Value-weighted returns												
$\sigma$	$r^S$	27.67	22.03	18.82	16.40	16.75	17.64	16.66	16.81	17.54	20.15	20.41***
	$r^F$	14.08	14.14	12.87	10.15	8.66	8.08	7.34	5.96	5.70	5.99	14.01***
$S_k$	$r^S$	0.19	0.24	-0.03	-0.05	-0.22	-0.35	-0.41	-0.11	-0.23	-0.09	-0.84*
	$r^F$	0.38	0.57	1.37	0.68	0.48	1.39	-0.03	-0.03	0.07	0.02	-0.24
$K_u$	$r^S$	3.69	3.96	4.11	3.31	3.08	3.53	3.11	2.87	3.33	2.66	5.78***
	$r^F$	4.65	6.22	11.05	6.37	4.99	7.60	3.65	4.21	3.38	2.89	4.35**
Equal-weighted returns												
$\sigma$	$r^S$	31.58	25.49	20.04	18.23	18.58	19.49	18.81	20.15	20.65	23.90	21.21***
	$r^F$	8.56	6.72	5.68	5.24	4.60	4.90	4.72	4.36	4.76	5.54	8.14***
$S_k$	$r^S$	1.17	0.61	0.32	-0.02	-0.05	-0.22	-0.18	0.01	-0.01	0.00	-1.95*
	$r^F$	-0.41	-0.60	-0.48	-0.08	-0.01	-0.33	-0.40	-0.29	-0.39	-0.61	0.30
$K_u$	$r^S$	8.16	5.24	4.06	3.54	3.51	3.45	3.44	3.70	3.38	3.46	16.90**
	$r^F$	4.10	3.98	3.62	2.43	2.41	2.83	3.06	2.93	2.94	3.30	3.82*

**Figure 1 : Firm-level versus Portfolio-level Accounting Variables, Histograms, 1967–2015**

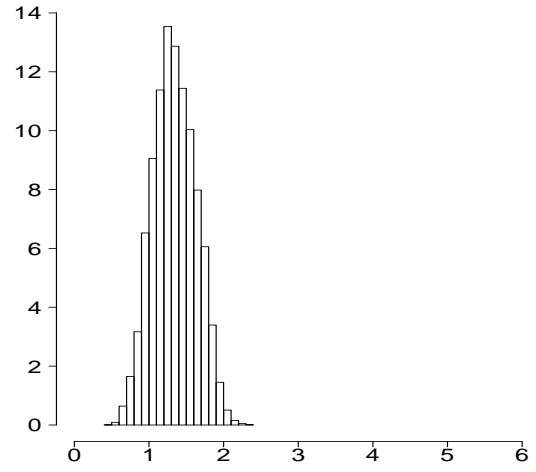
$I_{it}/K_{it}$  is physical investment-to-capital,  $K_{it+1}/(K_{it+1} + C_{it+1})$  the fraction of physical capital in the sum of physical capital and current assets,  $Y_{it+1}/(K_{it+1} + C_{it+1})$  the ratio of sales over the sum of physical capital and current assets,  $Y_{it+1}/K_{it+1}$  sales-to-physical capital,  $Y_{it+1}/C_{it+1}$  sales-to-current assets,  $\delta_{it+1}$  the rate of physical capital depreciation,  $w_{it}^B$  the market leverage, and  $r_{it+1}^B$  the pre-tax cost of debt. The left column of panels reports the firm-level histograms, and the right column the portfolio-level histograms across the 40 testing deciles.



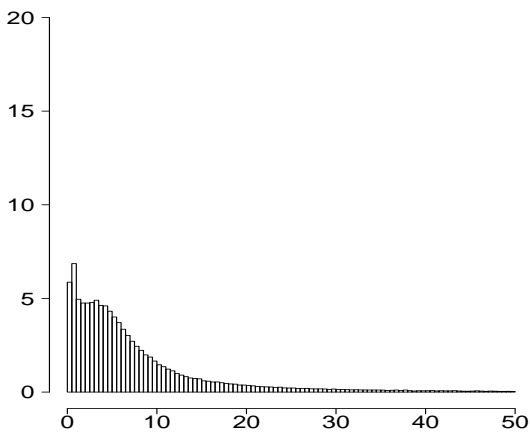
Panel E: Firm-level  $Y_{it+1}/(K_{it+1} + C_{it+1})$



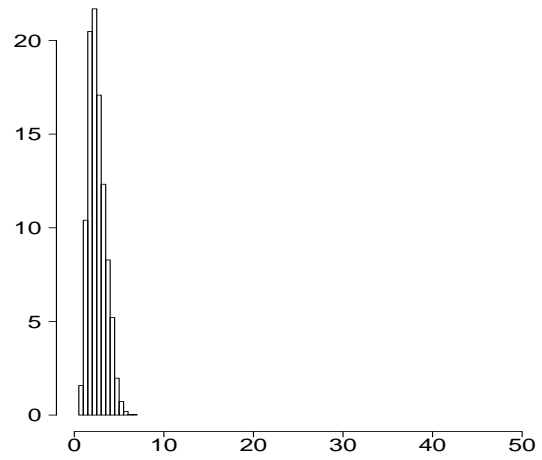
Panel F: Portfolio-level  $Y_{it+1}/(K_{it+1} + C_{it+1})$



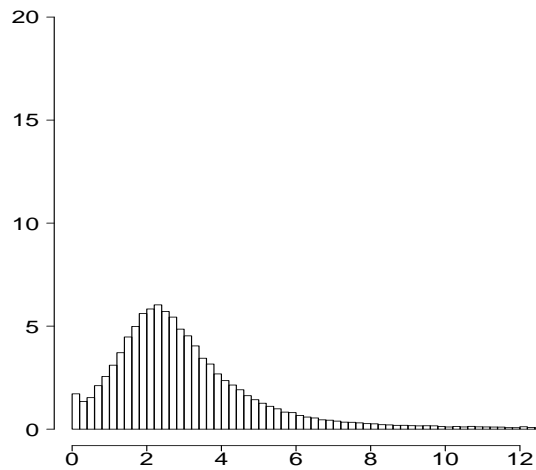
Panel G: Firm-level  $Y_{it+1}/K_{it+1}$



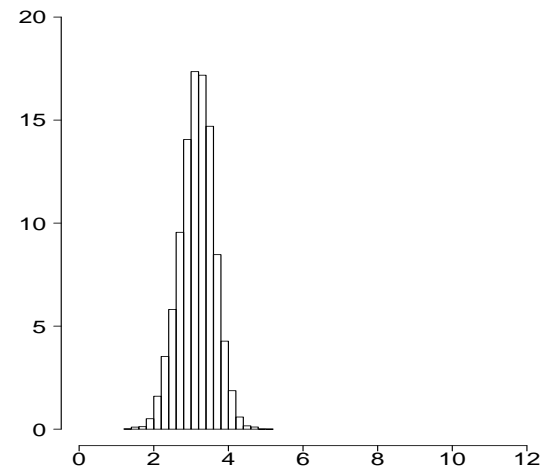
Panel H: Portfolio-level  $Y_{it+1}/K_{it+1}$



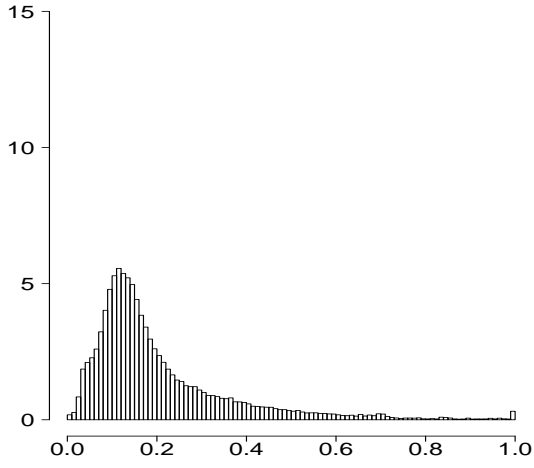
Panel I: Firm-level  $Y_{it+1}/C_{it+1}$



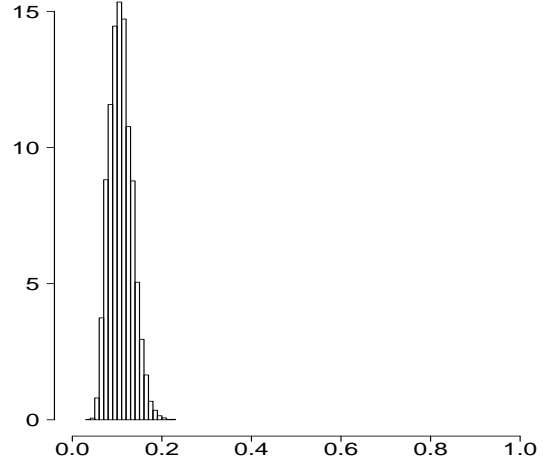
Panel J: Portfolio-level  $Y_{it+1}/C_{it+1}$



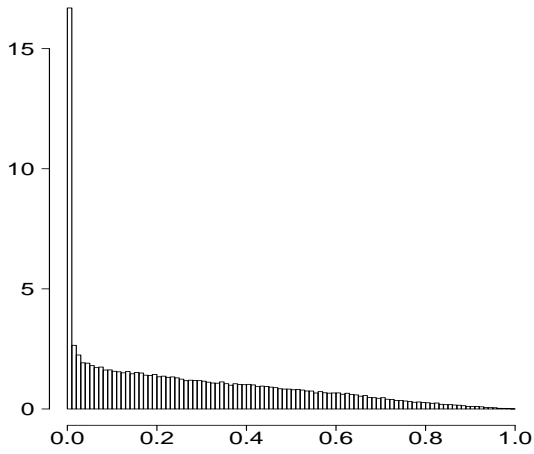
Panel K: Firm-level  $\delta_{it+1}$



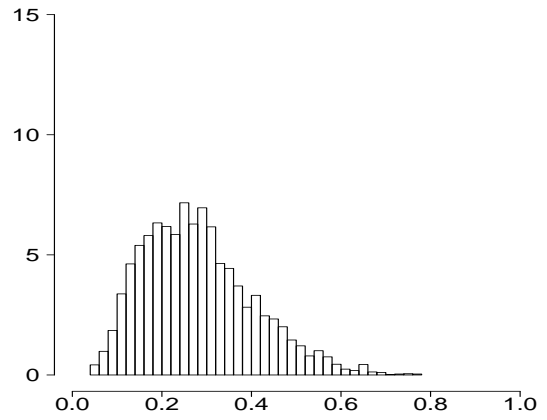
Panel L: Portfolio-level  $\delta_{it+1}$



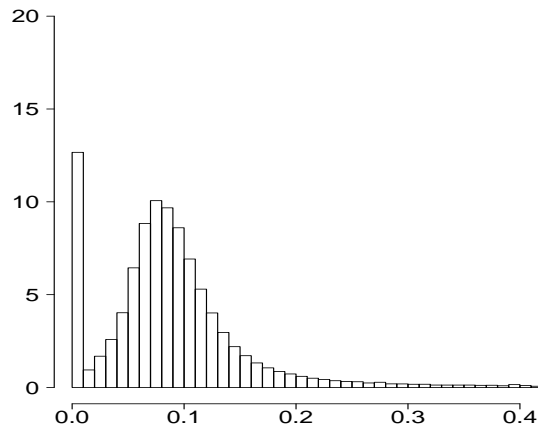
Panel M: Firm-level  $w_{it}^B$



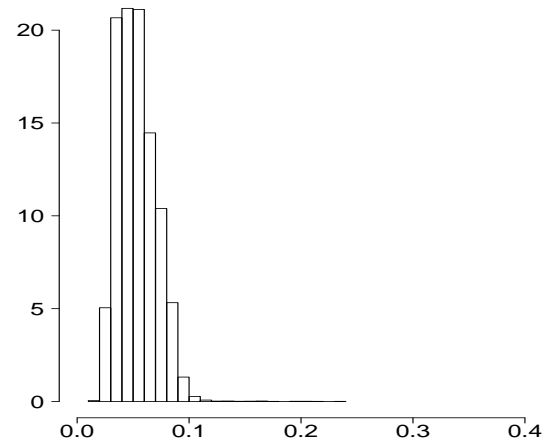
Panel N: Portfolio-level  $w_{it}^B$



Panel O: Firm-level  $r_{it+1}^B$



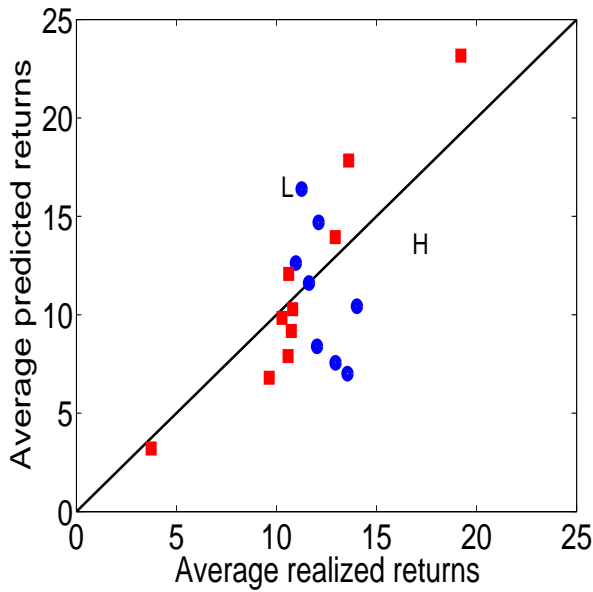
Panel P: Portfolio-level  $r_{it+1}^B$



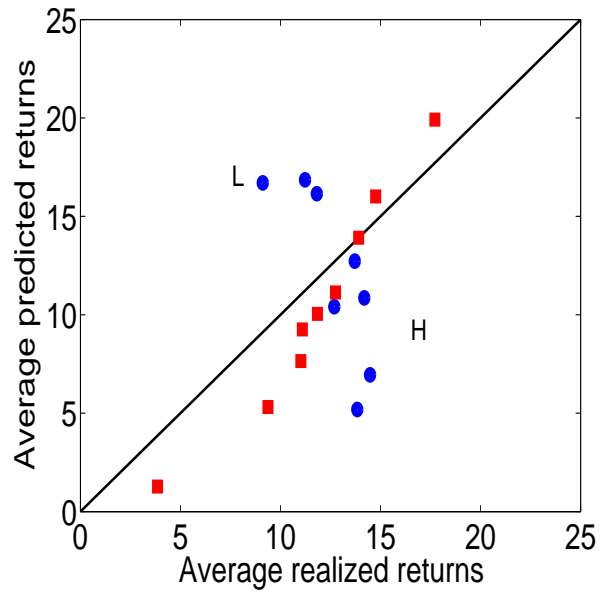
**Figure 2 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Physical Capital Model Estimated at the Portfolio Level**

Both average predicted and realized stock returns are in percent. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum ( $R^{11}$ ) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The lowest Bm decile is denoted “L,” and the highest Bm decile “H.” Panels A and B fit the Bm and  $R^{11}$  deciles jointly, and Panels C and D fit all the 40 deciles together.

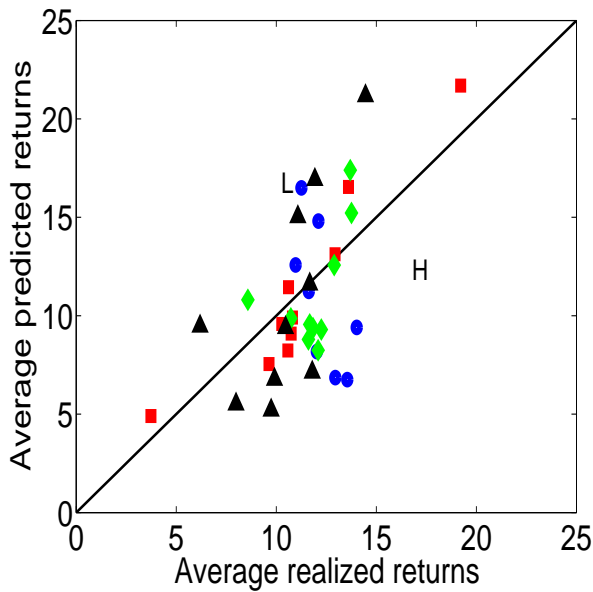
Panel A: Bm- $R^{11}$ , NYSE breakpoints and value-weighted returns



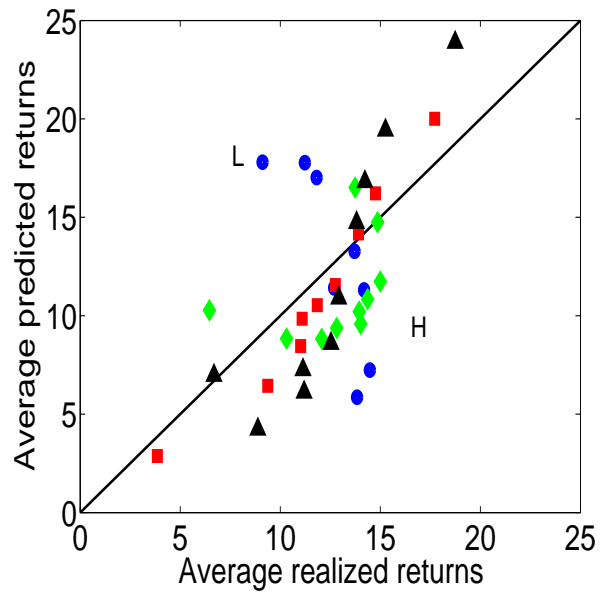
Panel B: Bm- $R^{11}$ , all-but-micro breakpoints and equal-weighted returns



Panel C: Bm- $R^{11}$ -I/A-Roe, NYSE breakpoints and value-weighted returns



Panel D: Bm- $R^{11}$ -I/A-Roe, all-but-micro breakpoints and equal-weighted returns

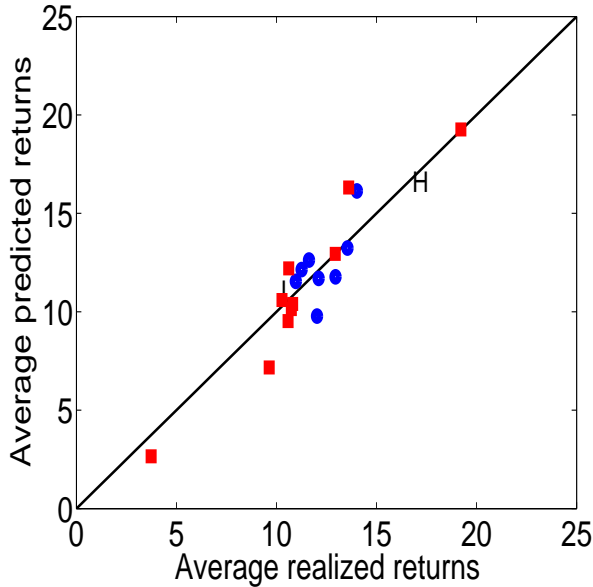




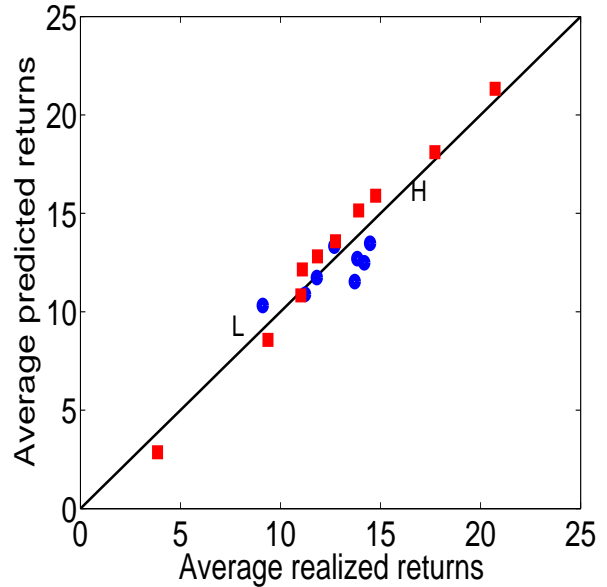
**Figure 3 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Benchmark Two-capital Model Estimated at the Firm Level**

Both average predicted and realized stock returns are in percent. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum ( $R^{11}$ ) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The lowest Bm decile is denoted “L,” and the highest Bm decile “H.” Panels A and B fit the Bm and  $R^{11}$  deciles jointly, and Panels C and D fit all the 40 deciles together.

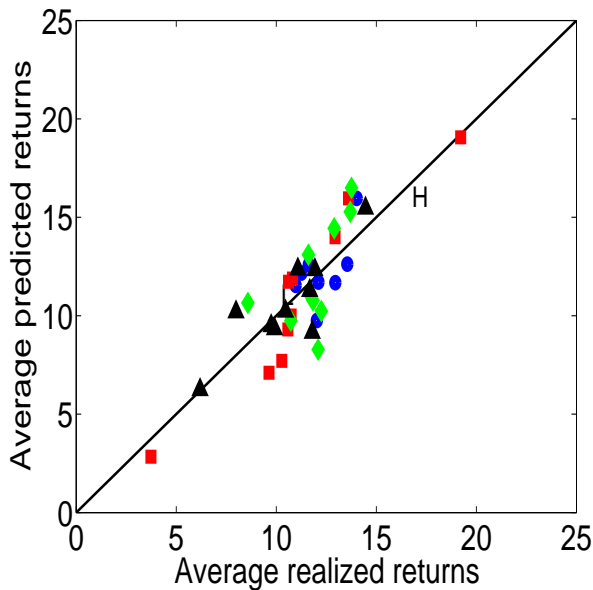
Panel A: Bm- $R^{11}$ , NYSE breakpoints and value-weighted returns



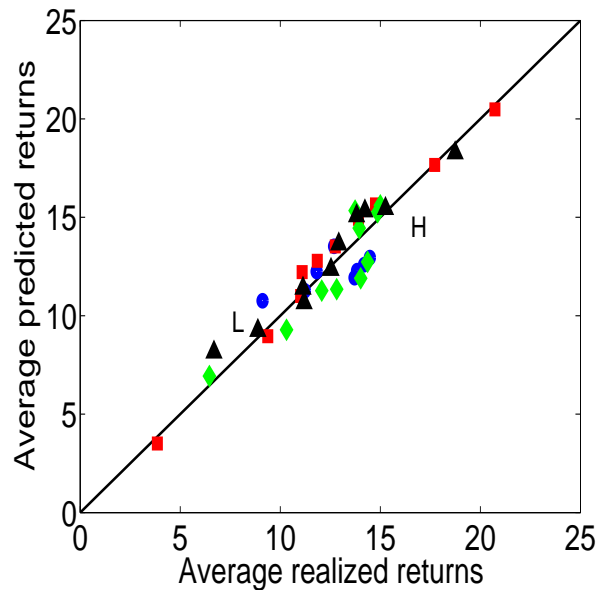
Panel B: Bm- $R^{11}$ , all-but-micro breakpoints and equal-weighted returns



Panel C: Bm- $R^{11}$ -I/A-Roe, NYSE breakpoints and value-weighted returns



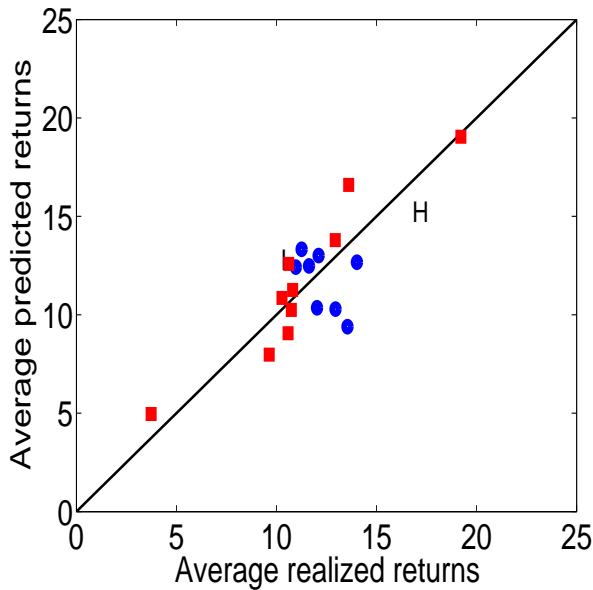
Panel D: Bm- $R^{11}$ -I/A-Roe, all-but-micro breakpoints and equal-weighted returns



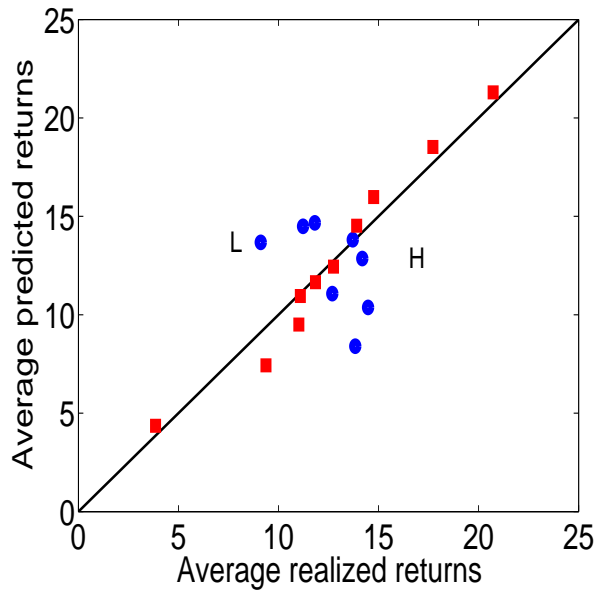
**Figure 4 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Two-capital Model Estimated at the Portfolio Level**

Both average predicted and realized stock returns are in percent. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum ( $R^{11}$ ) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The lowest Bm decile is denoted “L,” and the highest Bm decile “H.” Panels A and B fit the Bm and  $R^{11}$  deciles jointly, and Panels C and D fit all the 40 deciles together.

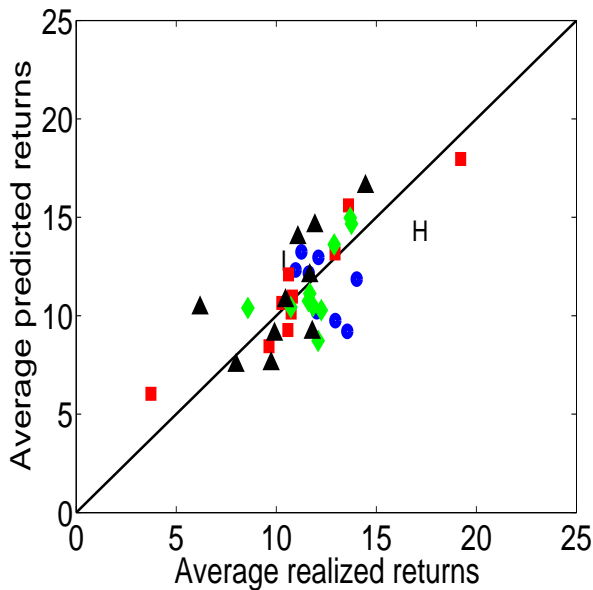
Panel A: Bm- $R^{11}$ , NYSE breakpoints and value-weighted returns



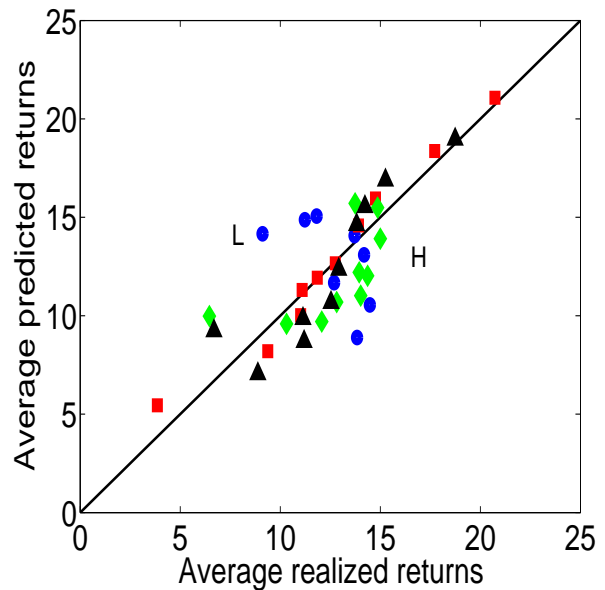
Panel B: Bm- $R^{11}$ , all-but-micro breakpoints and equal-weighted returns



Panel C: Bm- $R^{11}$ -I/A-Roe, NYSE breakpoints and value-weighted returns



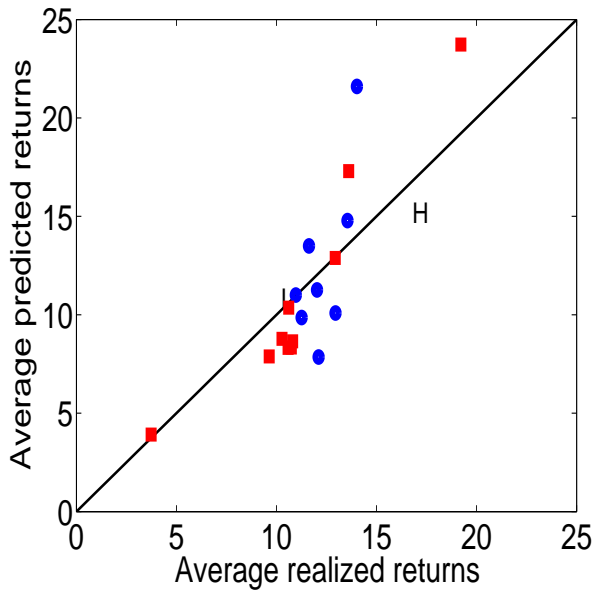
Panel D: Bm- $R^{11}$ -I/A-Roe, all-but-micro breakpoints and equal-weighted returns



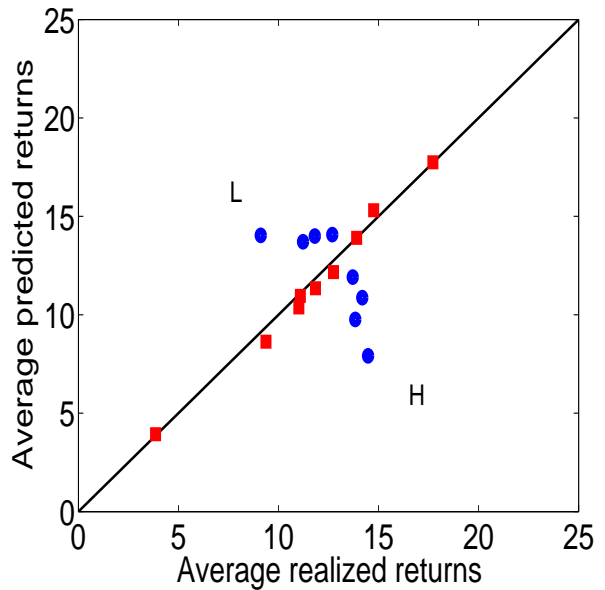
**Figure 5 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Physical Capital Model Estimated at the Firm Level**

Both average predicted and realized stock returns are in percent. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum ( $R^{11}$ ) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The lowest Bm decile is denoted “L,” and the highest Bm decile “H.” Panels A and B fit the Bm and  $R^{11}$  deciles jointly, and Panels C and D fit all the 40 deciles together.

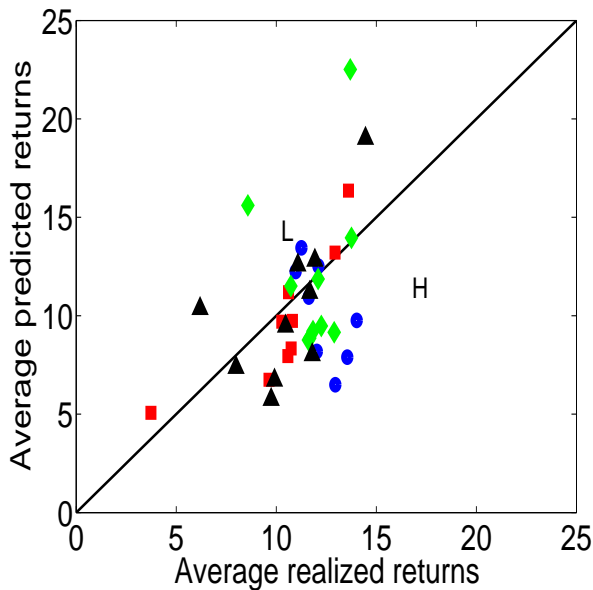
Panel A: Bm- $R^{11}$ , NYSE breakpoints and value-weighted returns



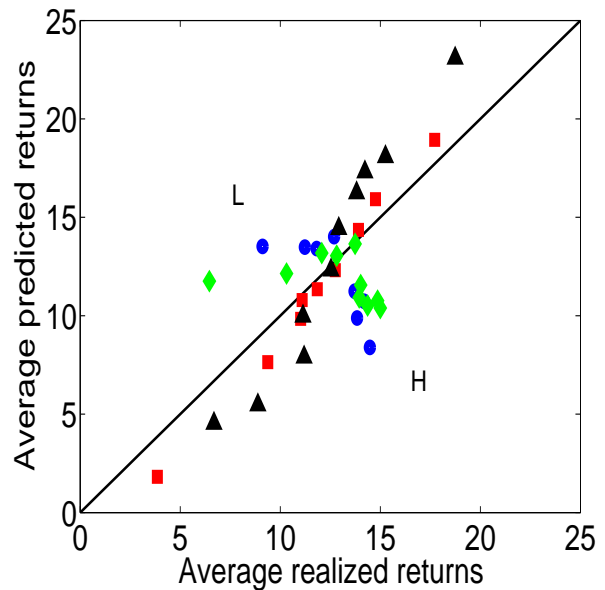
Panel B: Bm- $R^{11}$ , all-but-micro breakpoints and equal-weighted returns



Panel C: Bm- $R^{11}$ -I/A-Roe, NYSE breakpoints and value-weighted returns



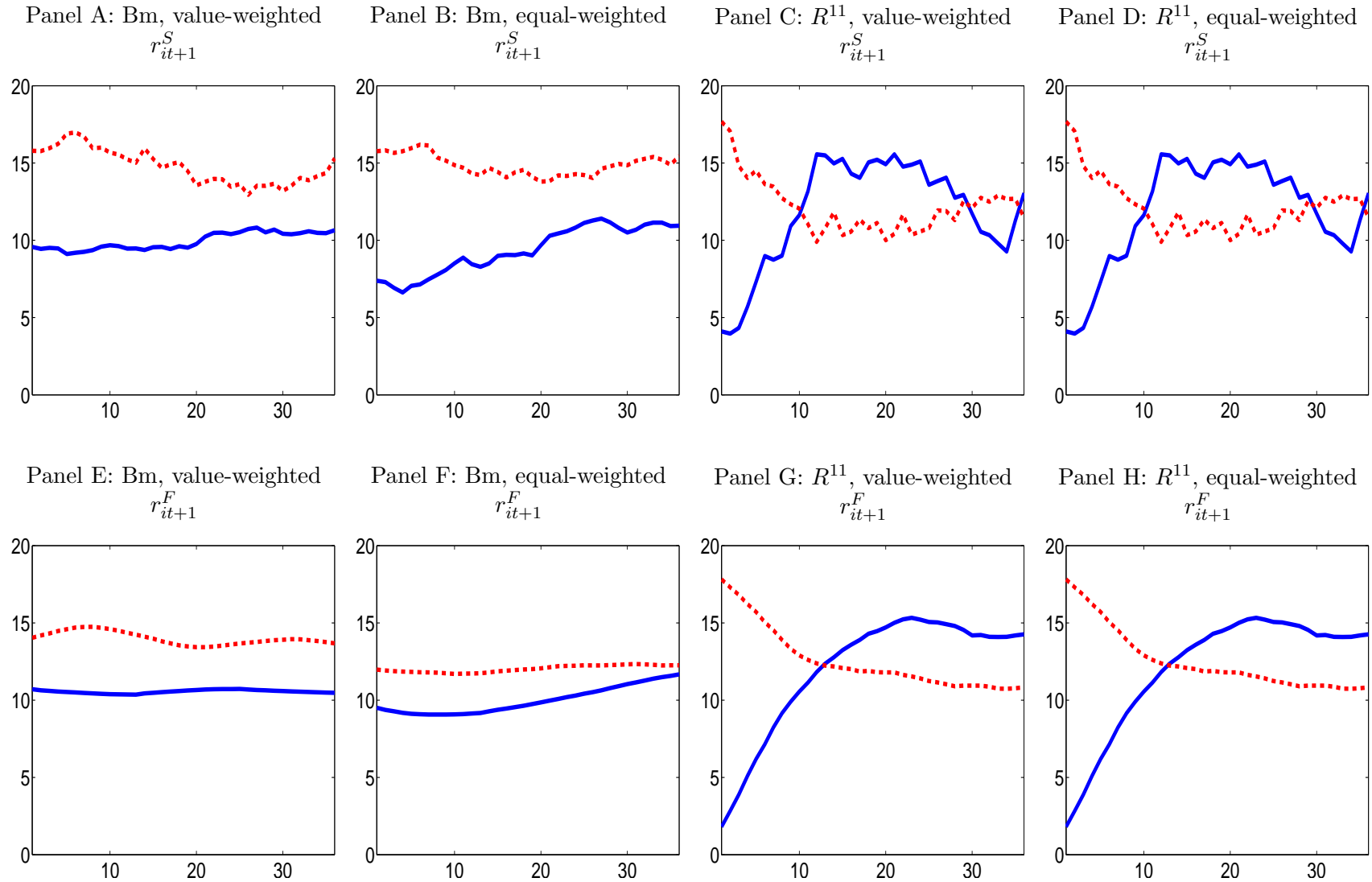
Panel D: Bm- $R^{11}$ -I/A-Roe, all-but-micro breakpoints and equal-weighted returns



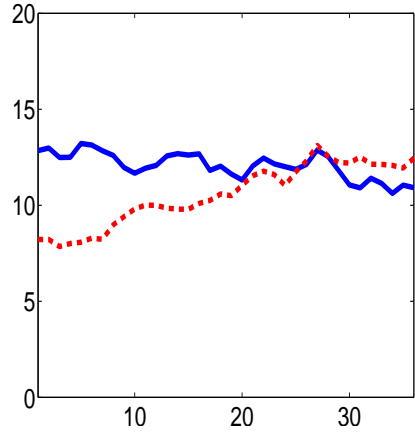
**Figure 6 : Event-time Dynamics of Stock and Fundamental Returns of the High and Low Deciles**

For 36 months after the portfolio formation, this figure plots event-time evolution of the stock return,  $r_{it+1}^S$ , and the fundamental return,  $r_{it+1}^F$ , for the high and low deciles. Bm denotes the book-to-market deciles,  $R^{11}$  the momentum deciles, I/A the asset growth deciles, and Roe the Roe deciles. The blue solid lines represent the low deciles, and the red broken lines the high deciles. We winsorize the firm-level fundamental returns at the 1–99% level each month before aggregating them to the portfolio level. The value-weighted results on the fundamental returns use the parameter values from estimating the benchmark model on all the 40 value-weighted testing deciles jointly, and the equal-weighted results use those from all the 40 equal-weighted testing deciles.

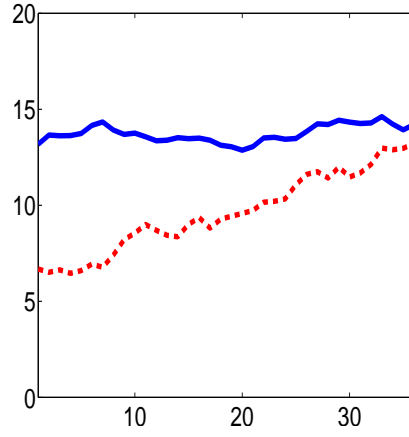
51



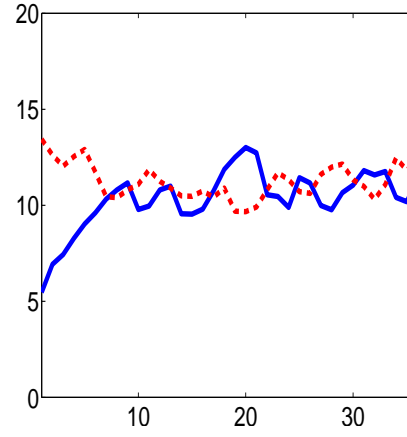
Panel I: I/A, value-weighted  
 $r_{it+1}^S$



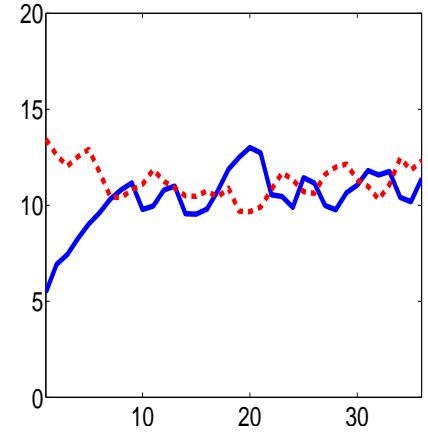
Panel J: I/A, equal-weighted  
 $r_{it+1}^S$



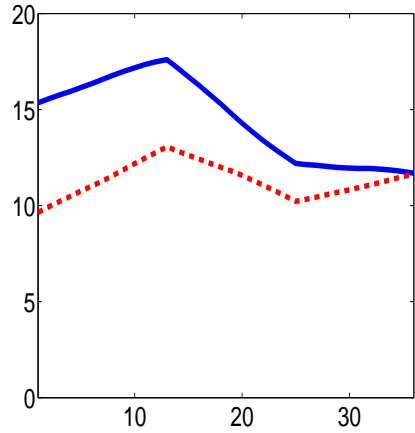
Panel K: Roe, value-weighted  
 $r_{it+1}^S$



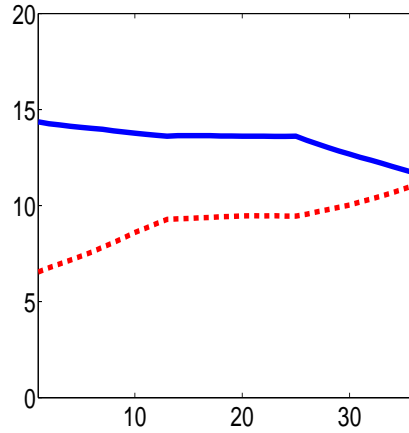
Panel L: Roe, equal-weighted  
 $r_{it+1}^S$



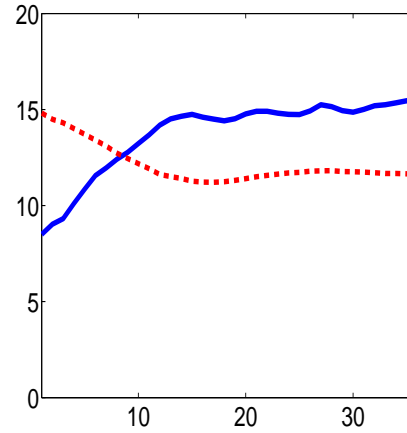
Panel M: I/A, value-weighted  
 $r_{it+1}^F$



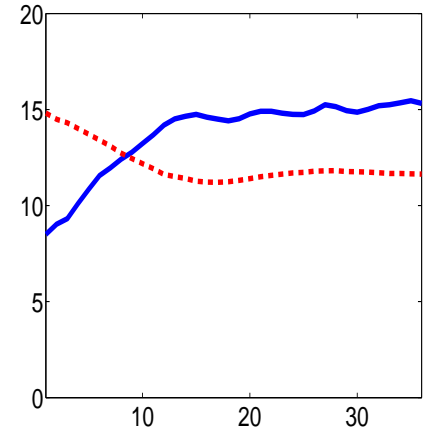
Panel N: I/A, equal-weighted  
 $r_{it+1}^F$



Panel O: Roe, value-weighted  
 $r_{it+1}^F$

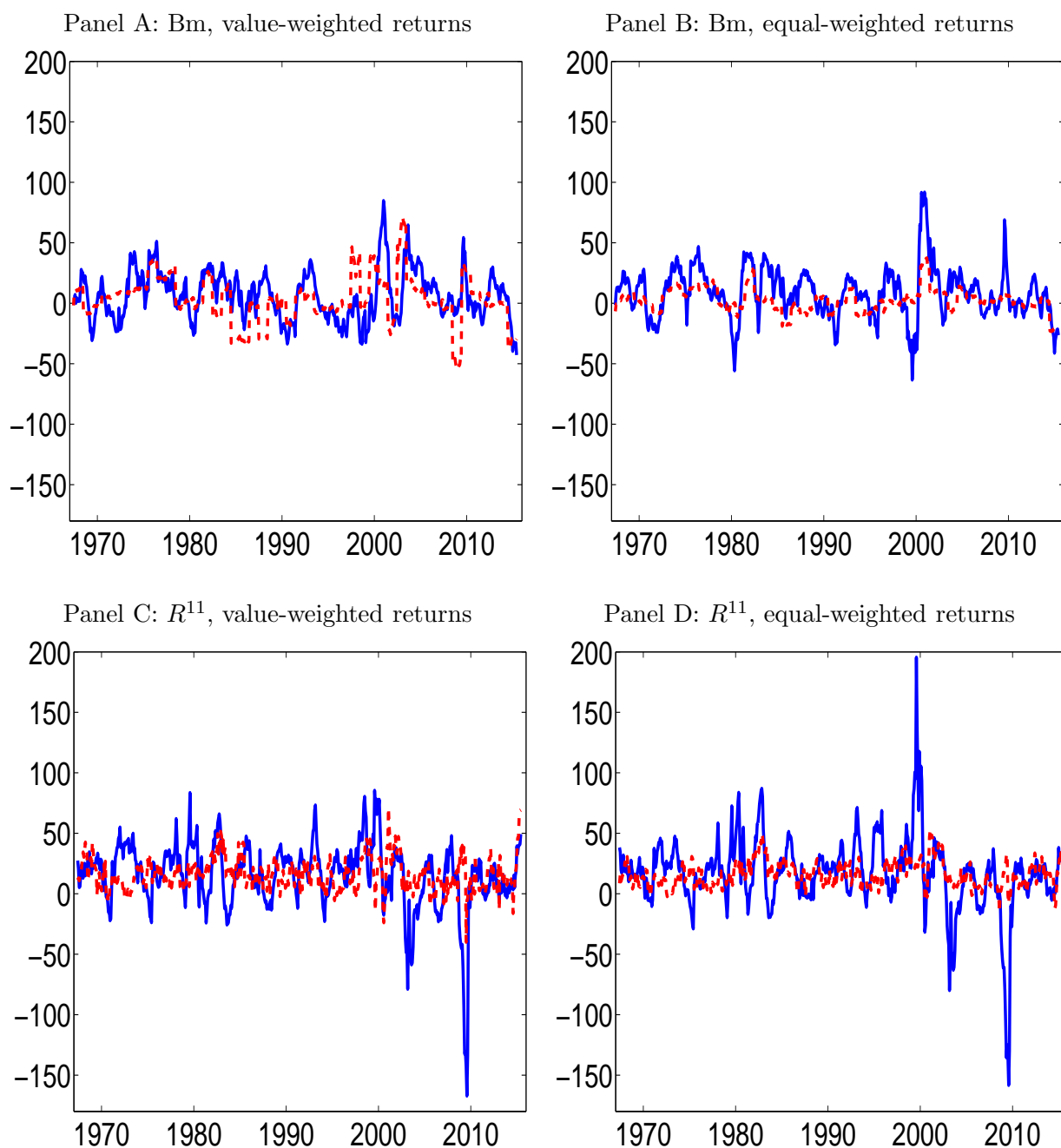


Panel P: Roe, equal-weighted  
 $r_{it+1}^F$

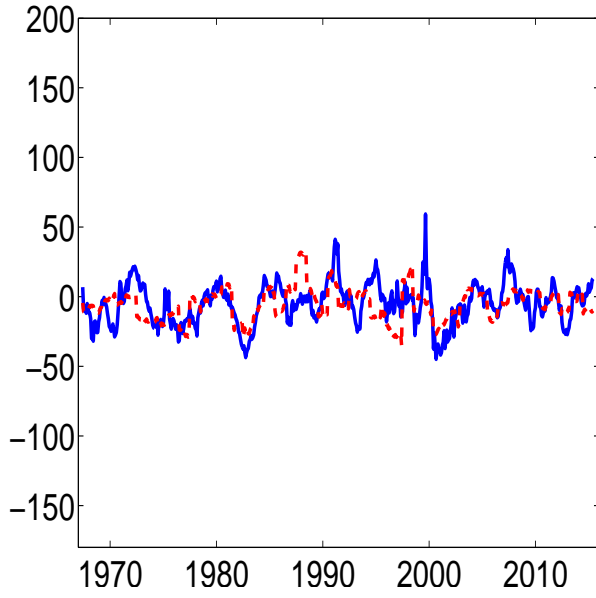


**Figure 7 : Time Series of Stock and Fundamental Returns of the Factor Premiums**

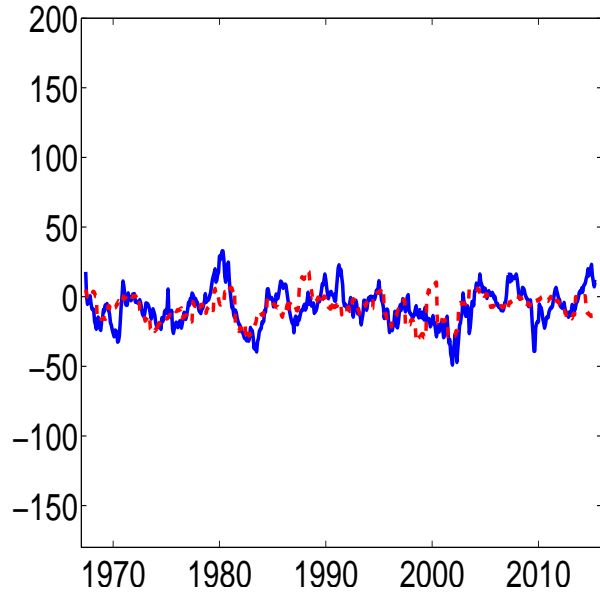
The blue solid lines represent the stock returns of the high-minus-low deciles, and the red broken lines the corresponding fundamental returns. Bm denotes book-to-market,  $R^{11}$  prior 11-month returns (momentum), I/A asset growth, and Roe return on equity. We winsorize the firm-level fundamental returns at the 1–99% level each month before aggregating them to the portfolio level.



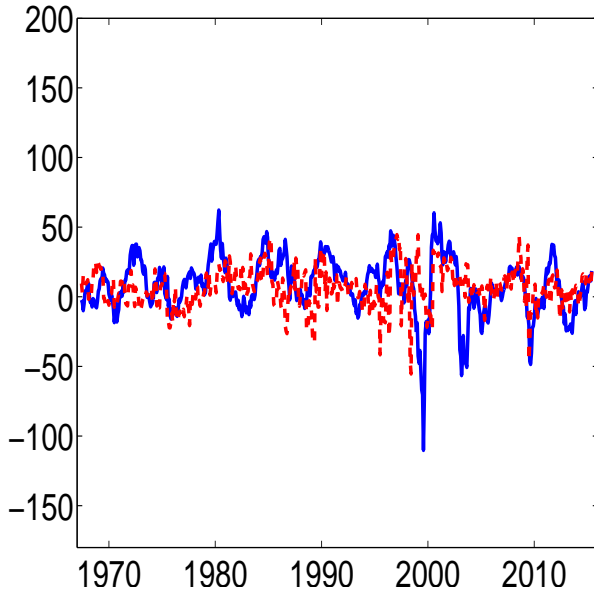
Panel E: I/A, value-weighted returns



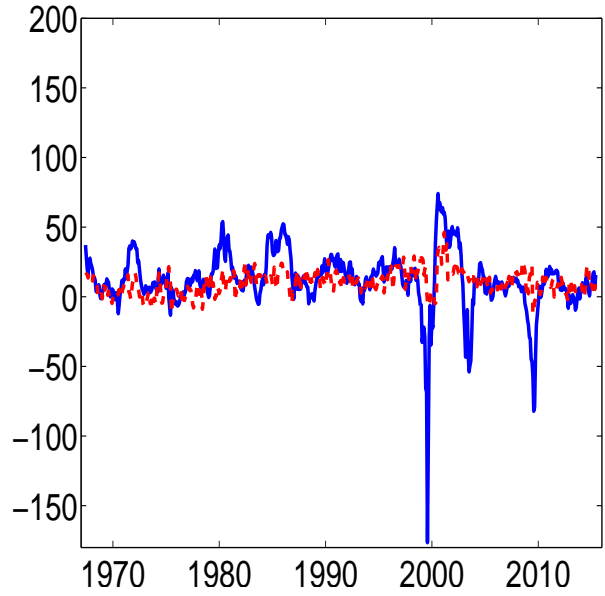
Panel F: I/A, equal-weighted returns



Panel G: Roe, value-weighted returns



Panel H: Roe, equal-weighted returns



## A Derivations

Let  $q_{it}$  and  $q_{it}^C$  be the Lagrangian multipliers associated with  $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$  and  $C_{it+1} = C_{it} + J_{it}$ , respectively. Form the Lagrangian function:

$$\begin{aligned} \mathcal{L} = & \dots + (1 - \tau_t)(\Pi_{it} - \Phi_{it}) - I_{it} - J_{it} + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it} K_{it} + \tau_t (r_{it}^B - 1) B_{it} \\ & - q_{it}(K_{it+1} - (1 - \delta_{it})K_{it} - I_{it}) - q_{it}^C(C_{it+1} - C_{it} - J_{it}) \\ & + E_t[M_{t+1}[(1 - \tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) - I_{it+1} - J_{it+1} + B_{it+2} - r_{it+1}^B B_{it+1} + \tau_{t+1} \delta_{it+1} K_{it+1} \\ & + \tau_{t+1}(r_{it+1}^B - 1)B_{it+1} - q_{it+1}(K_{it+2} - (1 - \delta_{it+1})K_{it+1} - I_{it+1}) - q_{it+1}^C(C_{it+2} - C_{it+1} - J_{it+1})]] \\ & + \dots \end{aligned} \quad (\text{A1})$$

Setting the first-order derivatives of  $\mathcal{L}$  with respect to  $I_{it}$ ,  $J_{it}$ ,  $K_{it+1}$ ,  $C_{it+1}$ , and  $B_{it+1}$  to zero yields, respectively:

$$q_{it} = 1 + (1 - \tau_t) \frac{\partial \Phi_{it}}{\partial I_{it}} \quad (\text{A2})$$

$$q_{it}^C = 1 \quad (\text{A3})$$

$$q_{it} = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left[ \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial K_{it+1}} \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) q_{it+1}^K \right] \right] \quad (\text{A4})$$

$$q_{it}^C = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} + q_{it+1}^C \right] \right] \quad (\text{A5})$$

$$1 = E_t[M_{t+1}(r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1})] = E_t[M_{t+1}r_{it+1}^{Ba}] \quad (\text{A6})$$

Equations (A2) and (A4) yield  $E_t[M_{t+1}r_{it+1}^I] = 1$ , in which  $r_{it+1}^I$  is given by equation (2), and equations (A19) and (A20) yield  $E_t[M_{t+1}r_{it+1}^J] = 1$ , in which  $r_{it+1}^J$  is given by equation (3).

To prove equation (4), we first show  $P_{it} + B_{it+1} = q_{it}K_{it+1} + C_{it+1}$ . We proceed with a guess-and-verify approach. We first assume that this equation holds for period  $t + 1$ , and then show it also holds for period  $t$ . It then follows that the equation must hold for all periods. We start with:

$$P_{it} + B_{it+1} = E_t[M_{t+1}(P_{it+1} + D_{it+1})] + B_{it+1} \quad (\text{A7})$$

Using  $P_{it+1} + B_{it+2} = q_{it+1}K_{it+2} + C_{it+2}$  to rewrite the right hand side yields:

$$P_{it} + B_{it+1} = E_t[M_{t+1}(q_{it+1}K_{it+2} + C_{it+2} - B_{it+2} + D_{it+1})] + B_{it+1} \quad (\text{A8})$$

Using the definition of  $D_{it+1} \equiv (1 - \tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) - I_{it+1} - J_{it+1} + B_{it+2} - r_{it+1}^B B_{it+1} + \tau_{t+1} \delta_{it+1} K_{it+1} + \tau_{t+1}(r_{it+1}^B - 1)B_{it+1}$  to write the right hand side yields:

$$\begin{aligned} P_{it} + B_{it+1} = & E_t[M_{t+1}[(1 - \tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}K_{it+2} - I_{it+1}]] \\ & + E_t[M_{t+1}(C_{it+2} - J_{it+1})] - B_{it+1} E_t[M_{t+1}[r_{it+1}^B - \tau_{t+1}(r_{it+1}^B - 1)]] + B_{it+1} \end{aligned} \quad (\text{A9})$$

The constant returns to scale for  $\Pi_{it}$  and equation (A6) then imply:

$$\begin{aligned} P_{it} + B_{it+1} = & E_t \left[ M_{t+1} \left[ K_{it+1}(1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}}{K_{it+1}} \right) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}[(1 - \delta_{it+1})K_{it+1} + I_{it+1}] - I_{it+1} \right] \right] \\ & + E_t \left[ M_{t+1} \left[ C_{it+1}(1 - \tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} + (C_{it+1} + J_{it+1}) - J_{it+1} \right] \right] \end{aligned} \quad (\text{A10})$$



Using the first-order conditions in equations (A2) and (A19) to rewrite the right hand side yields:

$$P_{it} + B_{it+1} = E_t \left[ M_{t+1} \left[ K_{it+1}(1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}}{K_{it+1}} + \frac{I_{it+1}}{K_{it+1}} \frac{\partial \Phi_{it+1}}{\partial I_{it+1}} \right) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}(1 - \delta_{it+1}) K_{it+1} \right] \right. \\ \left. + E_t \left[ M_{t+1} \left[ C_{it+1}(1 - \tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} + C_{it+1} \right] \right] \right] \quad (\text{A11})$$

Constant returns to scale mean that  $\Phi_{it} = I_{it} \partial \Phi_{it}^K / \partial I_{it} + K_{it} \partial \Phi_{it}^K / \partial K_{it}$ . Equation (A25) becomes:

$$P_{it} + B_{it+1} = K_{it+1} E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial K_{it+1}} \right) + \tau_{t+1} \delta_{it+1} + q_{it+1}(1 - \delta_{it+1}) \right] \right] \\ + C_{it+1} E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} + 1 \right] \right] = q_{it} K_{it+1} + C_{it+1}, \quad (\text{A12})$$

in which the last equality follows from equations (A4) and (A20).

Finally, we are ready to prove equation (4),

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{B_{it+1}}{P_{it} + B_{it+1}} [r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1}] + \frac{P_{it}}{P_{it} + B_{it+1}} \frac{(P_{it+1} + D_{it+1})}{P_{it}} \\ = \frac{B_{it+1} [r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1}] + q_{it+1} K_{it+2} + C_{it+2} - B_{it+2} + D_{it+1}}{P_{it} + B_{it+1}} \quad (\text{A13})$$

Using the definition of  $D_{it+1}$  yields:

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{(1 - \tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1} K_{it+2} + C_{it+2} - I_{it+1} - J_{it+1}}{P_{it} + B_{it+1}} \quad (\text{A14})$$

Using the constant returns to scale for  $\Pi_{it+1}$  yields:

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{K_{it+1}(1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}}{K_{it+1}} \right) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}(I_{it+1} + (1 - \delta_{it+1}) K_{it+1}) - I_{it+1}}{P_{it} + B_{it+1}} \\ + \frac{C_{it+1}(1 - \tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} + (C_{it+1} + J_{it+1}) - J_{it+1}}{P_{it} + B_{it+1}} \quad (\text{A15})$$

Using the constant returns to scale for  $\Phi_{it+1}$  and equations (A2) and (A19), we obtain:

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{K_{it+1}}{q_{it} K_{it+1} + C_{it+1}} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial K_{it+1}} \right) + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) q_{it+1} \right] \\ + \frac{C_{it+1}}{q_{it} K_{it+1} + C_{it+1}} \left[ (1 - \tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} + 1 \right] \quad (\text{A16})$$

Using equations (A4) and (A20) yields the desired result:

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{q_{it} K_{it+1}}{q_{it} K_{it+1} + C_{it+1}} r_{it+1}^I + \frac{C_{it+1}}{q_{it} K_{it+1} + C_{it+1}} r_{it+1}^J. \quad (\text{A17})$$

## B Estimates of the Adjustment Costs on Current Assets

In this appendix, we examine an extended two-capital model with adjustment costs on current assets. We lay out the model in Appendix B.1, and present its estimation results in Appendix B.2.

### B.1 An Extended Model

We continue to build on the setup of the benchmark two-capital model described in Section 2. However, we assume that the adjustment costs function depends on current assets and their investment. We adopt the quadratic functional form, which is also separate in the two capital inputs:

$$\Phi_{it} \equiv \Phi(I_{it}, K_{it}, J_{it}, C_{it}) = \Phi^K(I_{it}, K_{it}) + \Phi^C(J_{it}, C_{it}) = \frac{a}{2} \left( \frac{I_{it}}{K_{it}} \right)^2 K_{it} + \frac{b}{2} \left( \frac{J_{it}}{C_{it}} \right)^2 C_{it}, \quad (\text{A18})$$

The first-order conditions with respect to  $I_{it}$  and  $K_{it+1}$  continue to be equations (A2) and (A4), respectively. However, the first-order conditions for  $J_{it}$  and  $C_{it+1}$  become:

$$q_{it}^C = 1 + (1 - \tau_t) \frac{\partial \Phi_{it}}{\partial J_{it}} \quad (\text{A19})$$

$$q_{it}^C = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left[ \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial C_{it+1}} \right] + q_{it+1}^C \right] \right] \quad (\text{A20})$$

Combining the two equations yields  $E_t[M_{t+1}r_{it+1}^J] = 1$ , in which  $r_{it+1}^J$  is given by:

$$r_{it+1}^J \equiv \frac{1 + (1 - \tau_{t+1}) \left[ \gamma_C \frac{Y_{it+1}}{C_{it+1}} + b \left( \frac{J_{it+1}}{C_{it+1}} \right) + \frac{b}{2} \left( \frac{J_{it+1}}{C_{it+1}} \right)^2 \right]}{1 + (1 - \tau_t) b \left( \frac{J_{it}}{C_{it}} \right)}. \quad (\text{A21})$$

To show  $P_{it} + B_{it+1} = q_{it}^K K_{it+1} + q_{it}^C C_{it+1}$ , we use  $P_{it+1} + B_{it+2} = q_{it+1}^K K_{it+2} + q_{it+1}^W W_{it+2}$  to rewrite the right hand side of equation (A7) as:

$$P_{it} + B_{it+1} = E_t[M_{t+1}(q_{it+1}^K K_{it+2} + q_{it+1}^C C_{it+2} - B_{it+2} + D_{it+1})] + B_{it+1} \quad (\text{A22})$$

Using the definition of  $D_{it+1} \equiv (1 - \tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) - I_{it+1} - J_{it+1} + B_{it+2} - r_{it+1}^B B_{it+1} + \tau_{t+1} \delta_{it+1} K_{it+1} + \tau_{t+1} (r_{it+1}^B - 1) B_{it+1}$  to rewrite the right hand side yields:

$$\begin{aligned} P_{it} + B_{it+1} &= E_t[M_{t+1}[(1 - \tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}^K K_{it+2} - I_{it+1}]] \\ &+ E_t[M_{t+1}(q_{it+1}^C C_{it+2} - J_{it+1})] - B_{it+1} E_t[M_{t+1}[r_{it+1}^B - \tau_{t+1} (r_{it+1}^B - 1)]] + B_{it+1} \end{aligned} \quad (\text{A23})$$

The constant returns to scale for  $\Pi_{it}$  and equation (A6) then imply:

$$\begin{aligned} P_{it} + B_{it+1} &= E_t \left[ M_{t+1} \left[ K_{it+1} (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}^K}{K_{it+1}} \right) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}^K [(1 - \delta_{it+1}) K_{it+1} + I_{it+1}] - I_{it+1} \right] \right] \\ &+ E_t \left[ M_{t+1} \left[ C_{it+1} (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} - \frac{\Phi_{it+1}^C}{C_{it+1}} \right) + q_{it+1}^C (C_{it+1} + J_{it+1}) - J_{it+1} \right] \right] \end{aligned} \quad (\text{A24})$$

Using the first-order conditions in equations (A2) and (A19) to rewrite the right hand side yields:

$$P_{it} + B_{it+1} = E_t \left[ M_{t+1} \left[ K_{it+1}(1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}^K}{K_{it+1}} + \frac{I_{it+1}}{K_{it+1}} \frac{\partial \Phi_{it+1}}{\partial I_{it+1}} \right) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}^K (1 - \delta_{it+1}) K_{it+1} \right] \right. \\ \left. + E_t \left[ M_{t+1} \left[ C_{it+1}(1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} - \frac{\Phi_{it+1}^C}{C_{it+1}} + \frac{J_{it+1}}{C_{it+1}} \frac{\partial \Phi_{it+1}}{\partial J_{it+1}} \right) + q_{it+1}^C C_{it+1} \right] \right] \right] \quad (\text{A25})$$

The constant returns to scale for  $\Phi_{it}$  mean that  $\Phi_{it}^K = I_{it} \partial \Phi_{it}^K / \partial I_{it} + K_{it} \partial \Phi_{it}^K / \partial K_{it}$  and  $\Phi_{it}^C = J_{it} \partial \Phi_{it}^C / \partial J_{it} + C_{it} \partial \Phi_{it}^C / \partial C_{it}$ . As such, equation (A25) becomes:

$$P_{it} + B_{it+1} = K_{it+1} E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial K_{it+1}} \right) + \tau_{t+1} \delta_{it+1} + q_{it+1}^K (1 - \delta_{it+1}) \right] \right. \\ \left. + C_{it+1} E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial C_{it+1}} \right) + q_{it+1}^C \right] \right] \right] \quad (\text{A26})$$

$$= q_{it}^K K_{it+1} + q_{it}^C C_{it+1}, \quad (\text{A27})$$

in which the last equality follows from equations (A4) and (A20). To show equation (4),

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{B_{it+1}}{P_{it} + B_{it+1}} [r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1}] + \frac{P_{it}}{P_{it} + B_{it+1}} \frac{(P_{it+1} + D_{it+1})}{P_{it}} \\ = \frac{B_{it+1} [r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1}] + q_{it+1}^K K_{it+2} + q_{it+1}^C C_{it+2} - B_{it+2} + D_{it+1}}{P_{it} + B_{it+1}} \quad (\text{A28})$$

Using the definition of  $D_{it+1}$  yields:

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{(1 - \tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}^K K_{it+2} + q_{it+1}^C C_{it+2} - I_{it+1} - J_{it+1}}{P_{it} + B_{it+1}} \quad (\text{A29})$$

Using the constant returns to scale for  $\Pi_{it+1}$  yields:

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{K_{it+1}(1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}^K}{K_{it+1}} \right) + \tau_{t+1} \delta_{it+1} K_{it+1} + q_{it+1}^K (I_{it+1} + (1 - \delta_{it+1}) K_{it+1}) - I_{it+1}}{P_{it} + B_{it+1}} \\ + \frac{C_{it+1}(1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} - \frac{\Phi_{it+1}^C}{C_{it+1}} \right) + q_{it+1}^C (C_{it+1} + J_{it+1}) - J_{it+1}}{P_{it} + B_{it+1}} \quad (\text{A30})$$

Using the constant returns to scale for  $\Phi_{it+1}$  and equations (A2) and (A19), we obtain:

$$w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S = \frac{K_{it+1}}{q_{it}^K K_{it+1} + q_{it}^C C_{it+1}} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial K_{it+1}} \right) + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) q_{it+1}^K \right] \\ + \frac{C_{it+1}}{q_{it}^K K_{it+1} + q_{it}^C C_{it+1}} \left[ (1 - \tau_{t+1}) \left( \frac{\partial \Pi_{it+1}}{\partial C_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial C_{it+1}} \right) + q_{it+1}^C \right] \quad (\text{A31})$$

$$= \frac{q_{it}^K K_{it+1}}{q_{it}^K K_{it+1} + q_{it}^C C_{it+1}} r_{it+1}^I + \frac{q_{it}^C C_{it+1}}{q_{it}^K K_{it+1} + q_{it}^C C_{it+1}} r_{it+1}^J. \quad (\text{A32})$$

## B.2 Estimation Results

We continue to test the moment condition given by equation (6), in which the fundamental return is given by equation (5), except that the current investment return is now given by equation (A21).

Table A1 reports GMM estimation and tests of the extended two-capital model. The table shows that many estimates of the adjustment costs parameter,  $b$ , for current assets are insignificant, including all seven estimates with equal-weighted deciles and three out of seven with value-weighted deciles. In particular, in the joint estimation of value and momentum, the  $b$  estimate is 1.4, with a standard error of 0.06, with value-weights, but is 0.71, with a standard error of 0.97, with equal-weights. With all 40 testing deciles,  $b$  is never significant: 0.3 with a standard error of 0.76 with value-weights, and 0.16 with a standard error of 0.51 with equal-weights. The marginal product parameter,  $\gamma$ , and the adjustment costs parameter,  $a$ , for physical capital are largely similar to those in the benchmark estimation without  $b$  as a separate parameter.

The mean absolute errors, m.a.e., and average absolute high-minus-low errors,  $\overline{|e_{H-L}|}$ , are also largely comparable. In particular, the m.a.e. is 0.77% per annum, and  $\overline{|e_{H-L}|}$  0.89% with value-weights, and the errors are 0.78% and 1.03%, respectively, with equal-weights. These errors are all smaller than those in the benchmark estimation. However, when all 40 testing deciles are included in the joint estimation, the m.a.e. is 1.27%, and  $\overline{|e_{H-L}|}$  2.21% with value-weights, and with equal-weights the errors are 0.91% and 2%, respectively. While the mean absolute errors are comparable with those in the benchmark estimation, the  $\overline{|e_{H-L}|}$  errors are larger.

Finally, Figure A1 reports detailed individual pricing errors by plotting average predicted stock returns against average realized stock returns. Similar to the benchmark estimation reported in Figure 3, the scatter points are all largely aligned with the 45-degree line. The errors in the joint value and momentum estimation are smaller than those in the benchmark estimation, but the errors from all 40 testing deciles are somewhat larger. The bottomline is that adding the extra parameter,  $b$ , does not yield a significant improvement in the model's performance. The evidence lends support to our modeling choice of setting  $b = 0$  in the benchmark estimation for parsimony.

**Table A1 : GMM Estimation and Tests, the Extended Two-capital Model with Adjustment Costs on Current Assets  
Estimated at the Firm Level, January 1967–December 2015**

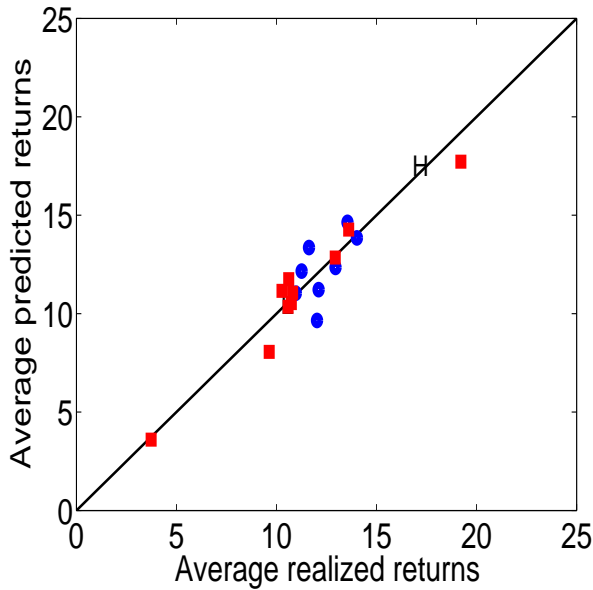
This table reports GMM estimation and tests for the 40 testing deciles formed on book-to-market (Bm), prior 11-month returns ( $R^{11}$ ), asset growth (I/A), and return on equity (Roe), separately and jointly (Bm and  $R^{11}$ , I/A and Roe, and all 40 deciles together). d.f. is the degrees of freedom in the GMM test of overidentification.  $\gamma$  is the technological parameter on the joint marginal product of current assets and physical capital as a fraction of the ratio of sales divided by the sum of the two assets,  $Y_{it+1}/(K_{it+1} + C_{it+1})$ .  $a$  is the adjustment costs parameter of physical capital, and  $b$  is that of current assets.  $[\gamma]$ ,  $[a]$ , and  $[b]$  are the standard errors of the point estimates of these parameters. m.a.e. is the mean absolute error across a given set of testing portfolios,  $\overline{|e_{H-L}|}$  is the average absolute high-minus-low error, and  $p$  is the  $p$ -value of the overidentification test across a given set of testing portfolios.  $\gamma$ ,  $[\gamma]$ , m.a.e.,  $\overline{|e_{H-L}|}$ , and  $p$  are in percent.

	NYSE breakpoints and value-weighted returns										All-but-micro breakpoints and equal-weighted returns								
	d.f.	$\gamma$	$[\gamma]$	$a$	$[a]$	$b$	$[b]$	m.a.e.	$\overline{ e_{H-L} }$	$p$	$\gamma$	$[\gamma]$	$a$	$[a]$	$b$	$[b]$	m.a.e.	$\overline{ e_{H-L} }$	$p$
Bm	8	16.66	2.17	3.40	0.00	0.99	0.01	0.70	0.45	39.82	15.95	2.14	3.28	0.44	0.70	1.50	0.72	0.75	2.79
$R^{11}$	8	13.13	2.23	3.53	0.00	2.62	0.00	0.46	0.29	93.66	13.94	2.37	2.31	0.95	1.76	0.66	0.32	0.11	38.88
I/A	8	17.34	1.82	1.52	0.69	0.47	0.84	0.84	2.73	0.54	16.68	1.79	1.79	0.76	0.33	0.93	0.57	0.64	4.18
Roe	8	15.23	2.57	4.69	0.00	1.91	0.00	0.76	1.25	30.82	15.59	2.00	2.70	1.77	0.00	0.02	0.57	1.89	39.33
Bm- $R^{11}$	18	16.09	2.14	3.36	0.03	1.40	0.06	0.77	0.89	47.03	15.60	1.96	3.23	0.42	0.71	0.97	0.78	1.03	0.00
I/A-Roe	18	17.11	1.86	1.65	0.69	0.20	0.61	1.11	2.57	0.00	16.24	1.92	2.00	0.48	0.07	0.52	0.70	1.42	0.00
Bm- $R^{11}$ -I/A-Roe	38	17.30	1.96	2.59	0.41	0.30	0.76	1.27	2.21	0.00	16.07	1.95	2.72	0.36	0.16	0.51	0.91	2.00	0.00

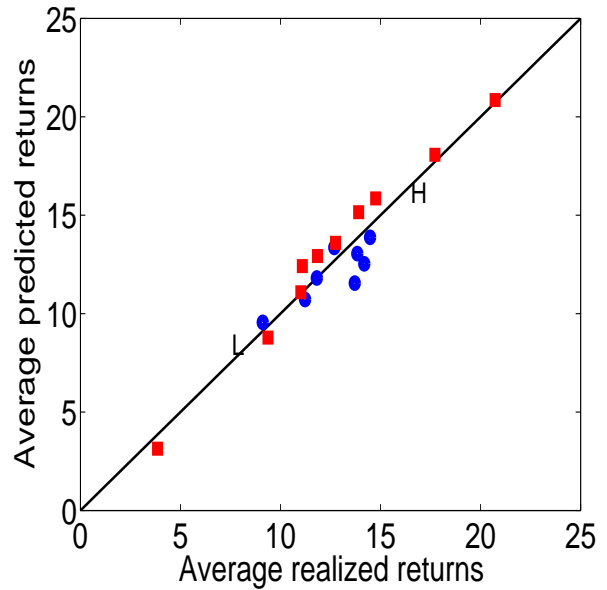
**Figure A1 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Extended Two-capital Model with Adjustment Costs on Current Assets Estimated at the Firm Level**

Both average predicted and realized stock returns are in percent. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum ( $R^{11}$ ) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The lowest Bm decile is denoted “L,” and the highest Bm decile “H.” Panels A and B fit the Bm and  $R^{11}$  deciles jointly, and Panels C and D fit all the 40 deciles together.

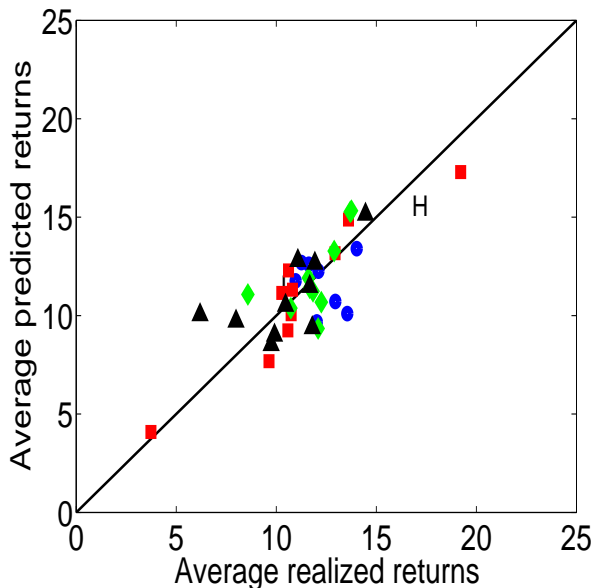
Panel A: Bm- $R^{11}$ , NYSE breakpoints and value-weighted returns



Panel B: Bm- $R^{11}$ , all-but-micro breakpoints and equal-weighted returns



Panel C: Bm- $R^{11}$ -I/A-Roe, NYSE breakpoints and value-weighted returns



Panel D: Bm- $R^{11}$ -I/A-Roe, all-but-micro breakpoints and equal-weighted returns

