The CAPM Strikes Back?
An Equilibrium Model with Disasters

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Abstract

Embedding disasters into a general equilibrium production economy with heterogeneous firms induces strong nonlinearity in the pricing kernel, helping explain the empirical failure of the (consumption) CAPM. Our *single*-factor model reproduces the failure of the CAPM in explaining the value premium in finite samples without disasters, and its relative success in samples with disasters. The standard consumption CAPM fails in simulations, even though a nonlinear model with the true pricing kernel holds exactly by construction. Due to beta measurement errors, the relation between the pre-ranking beta and the average return is flat in simulations, consistent with the beta "anomaly," even though the true beta-expected return relation is strongly positive. In all, the empirical failures of standard asset pricing models should be interpreted with caution.

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1 Introduction

Despite similar market betas, firms with high book-to-market (value firms) earn higher average stock returns than firms with low book-to-market (growth firms). This stylized fact is commonly referred to as the value premium puzzle. In the U.S. sample from July 1963 to June 2017, the high-minus-low book-to-market decile return is 0.47% per month ($t = 2.53$). However, its market beta is only 0.07 ($t = 0.86$), giving rise to an economically large alpha of 0.43% ($t = 1.89$) in the Capital Asset Pricing Model (CAPM) (Fama and French 1992). However, the CAPM performs better in explaining the value premium in the long sample from July 1926 onward that contains the Great Depression (Ang and Chen 2007). The high-minus-low return is on average 0.48% ($t = 2.5$), but its CAPM alpha is only 0.19% ($t = 0.99$), with a large market beta of 0.45 ($t = 3.87$).

This paper studies whether incorporating rare disasters helps explain the value premium puzzle. To this end, we embed disasters into a general equilibrium model with heterogeneous firms. The resulting model features three key ingredients, including rare but severe declines in aggregate productivity growth, asymmetric adjustment costs, and recursive utility. We calibrate the model to disaster moments estimated from a historical, cross-country panel dataset (Nakamura, Steinsson, Barro, and Ursua 2013). We quantify the model's properties on simulated samples in which disasters are not realized, as well as on samples in which disasters are realized.

We report three key quantitative results. First, our equilibrium model succeeds in explaining the failure of the CAPM in explaining the value premium in finite samples in which disasters are not materialized, as well as its better performance in samples in which disasters are materialized. Intuitively, with asymmetric adjustment costs, when a disaster hits, value firms are burdened with more unproductive capital, and find it more difficult to reduce capital than growth firms. As such, value firms are more exposed to the disaster risk than growth firms. Combined with the household's high marginal utility in disasters, the model implies a sizeable value premium.

More important, the disaster risk induces strong nonlinearity in the pricing kernel, making the linear CAPM a poor empirical proxy for the pricing kernel. When disasters are not realized in a finite sample, the estimated market beta only measures the weak covariation of the value-minus-growth return with the market excess return in normal times. However, the value premium is primarily driven by the higher exposures of value stocks to disasters than growth stocks. Consequently, the CAPM fails to explain the value premium in normal times. In contrast, when disasters are realized, the estimated market beta provides an adequate account for the large covariation between the value-minus-growth return and the pricing kernel. As such, the CAPM does better in capturing the value premium in samples with disasters. In all, disasters help explain the value premium puzzle.

Second, our equilibrium model is also consistent with the beta "anomaly" that the empirical relation between the market beta and the average return is too flat to be consistent with the CAPM (Frazzini and Pedersen 2014). In simulated samples, with and without disasters, sorting on the pre-ranking market beta yields an average return spread that is economically small and statistically insignificant, a post-ranking beta spread that is economically large and significantly positive, and a CAPM alpha spread that is economically large and often significantly negative.

The crux is that the estimated market beta is a poor proxy for the true beta. Intuitively, based on prior 60-month rolling windows, the pre-ranking beta is the average beta over the prior five years. In contrast, the true beta accurately reflects changes in aggregate and firm-specific state variables. In simulations, the true beta often mean-reverts within a given rolling window, giving rise to a negative correlation with the rolling beta, especially in samples without disasters. However, while the realization of disasters makes the rolling beta more aligned with the true beta, the measurement errors remain large, and the beta "anomaly" persists even in the disaster samples.

Third, our equilibrium model, in which a nonlinear consumption CAPM holds by construction, also largely replicates the poor empirical performance of the standard, linearized consumption CAPM in the data. In simulations, with and without disasters, the consumption betas from regressing excess returns on the aggregate consumption growth in the first-stage regressions are mostly insignificant and often even negative. In the second-stage cross-sectional regressions, the slopes for the price of consumption risk are significantly negative, but the intercepts are significantly positive. Intuitively, the aggregate consumption growth is a poor proxy for the pricing kernel based on recursive utility. The true pricing kernel performs substantially better in the linearized consumption CAPM tests, especially in the disaster samples. However, without the extreme observations from disasters, even the true price kernel encounters difficulty in the linear tests. Finally, as a byproduct from using the 25 size and book-to-market portfolios as testing assets for the consumption CAPM, our equilibrium model also reproduces the stylized fact that the average value premium is stronger in small firms than in big firms. Decreasing returns to scale and the disaster risk drive this result in our model, without any limit to arbitrage per Shleifer and Vishny (1997).

Our work contributes to investment-based asset pricing theories. Building on Cochrane (1991) and Berk, Green, and Naik (1999), early models explain the value premium with only one aggregate shock. Carlson, Fisher, and Giammarino (2004) highlight operating leverage. Zhang (2005) emphasizes asymmetric adjustment costs, which make assets in place harder to reduce, and cause the assets to be riskier than growth options, especially in bad times. We extend the asymmetry mechanism to disasters. Cooper (2006) examines nonconvex adjustment costs and investment irreversibility. Tuzel (2010) studies real estate capital, and shows that firms with high real estate are riskier than firms with low real estate, as it depreciates more slowly. A limitation of these one-shock models is that the CAPM roughly holds in simulations, as the CAPM alpha of the value premium is economically too small relative to that in the post-1963 sample (Lin and Zhang 2013).

Several recent studies try to explain the failure of the CAPM by breaking the tight link between the pricing kernel and the market excess return via multiple aggregate shocks, including short-run and long-run shocks (Ai and Kiku 2013), investment-specific technological shocks (Kogan and Papanikolaou 2013), stochastic adjustment costs (Belo, Lin, and Bazdresch 2014), and uncertainty shocks in (Koh 2015). Although successful in explaining the failure of the CAPM in the post-1963 sample, these two-shock models contradict the long sample evidence by construction. We retain the single-factor structure, and fail the CAPM via disaster-induced nonlinearity in the pricing kernel.

Methodologically, most prior models are partial equilibrium in nature, with exogenous pricing kernels. We instead construct a general equilibrium model with heterogenous firms, in which consumption and the pricing kernel are endogenously determined. A major challenge in solving the general equilibrium model is that the infinite-dimensional cross-sectional distribution of firms is an endogenous, aggregate state variable. We adapt the approximate aggregation algorithm of Krusell and Smith (1997, 1998) to overcome the computational difficulty. Substantively, the general equilibrium allows us to explain the poor performance of the consumption CAPM in the data.

We also contribute to the disaster literature, which uses disasters to explain the equity premium puzzle, so far mostly in endowment economies. Barro (2006, 2009) revives the idea of Rietz (1988), by calibrating the disaster model to a long, cross-country panel dataset. Wachter (2013) uses time-varying disaster probability to explain the market volatility. In an endowment economy with multiple assets, Martin (2013) shows that return correlations arise endogenously to spike in disasters. Gourio (2012) embeds disasters into an aggregate production economy to jointly explain asset prices and business cycles. We differ by studying the cross section. Integrating the disaster literature with investment-based asset pricing, we show how disasters help resolve a long-standing difficulty in the latter literature in explaining the failure of the (consumption) CAPM.

The rest of the paper is organized as follows. Section 2 presents the stylized facts, Section 3 constructs the equilibrium model, Section 4 reports the quantitative results, and Section 5 concludes.

2 Stylized Facts

This section documents the stylized facts to be explained. Section 2.1 presents the CAPM performance, Section 2.2 on the beta "anomaly," and Section 2.3 on the consumption CAPM.

Table 1 : The CAPM Regressions for the Book-to-market Deciles

This table reports the average excess return, denoted m, the CAPM alpha, the market beta, their t-values adjusted for heteroscedasticity and autocorrelations, and the goodness-of-fit from the CAPM regression. L, H, and H−L are the growth, value, and value-minus-growth deciles, respectively.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: July 1963 to June 2017										
m	0.44	0.54	0.59	0.54	0.55	0.66	0.62	0.70	0.86	0.91	0.47
t_m	2.22	3.00	3.26	2.98	3.14	3.88	3.49	3.88	4.41	3.80	2.53
α	−0.11	0.02	0.07	0.03	0.07	0.20	0.15	0.23	0.35	0.32	0.43
t_α	−1.23	0.44	1.17	0.39	0.80	2.21	1.23	2.00	3.03	2.04	1.89
β	1.06	1.00	0.99	0.98	0.91	0.88	0.92	0.91	0.98	1.13	0.07
t_β	41.66	42.06	40.88	32.43	28.19	23.30	19.35	18.26	22.65	17.47	0.86
R^2	0.86	0.91	0.91	0.87	0.83	0.80	0.78	0.76	0.77	0.68	0.00
	Panel B: July 1926 to June 2017										
m	0.59	0.69	0.69	0.66	0.72	0.79	0.72	0.91	1.06	1.07	0.48
t_m	3.40	4.28	4.23	3.71	4.19	4.35	3.73	4.49	4.55	3.84	2.50
α	−0.08	0.07	0.05	−0.02	0.07	0.11	0.00	0.16	0.22	0.11	0.19
t_α	−1.21	1.46	1.02	−0.38	0.92	1.32	0.02	1.82	1.94	0.74	0.99
β	1.01	0.95	0.97	1.05	1.00	1.03	1.10	1.14	1.28	1.46	0.45
t_β	52.73	27.62	59.98	22.11	27.29	14.85	17.73	16.11	14.32	14.49	3.87
R^2	0.90	0.91	0.93	0.90	0.89	0.85	0.84	0.83	0.80	0.72	0.14

2.1 The Performance of the CAPM

Table 1 reports the monthly CAPM regressions for the book-to-market deciles. The monthly returns data for the deciles, the value-weighted market portfolio, and the one-month Treasury bill rate are from Kenneth French's data library. The data are from July 1926 to June 2017.

Panel A shows that consistent with Fama and French (1992), the CAPM has difficulty in explaining the value premium (the value-minus-growth decile return) in the sample after July 1963. Moving from the growth decile to the value decile, the average excess return rises from 0.44% per month to 0.91%, and the average return spread is 0.47% ($t = 2.53$). Despite the increasing relation between book-to-market and the average excess return, the market beta is largely flat across the deciles. The value-minus-growth decile has only a small market beta of 0.07 ($t = 0.86$). Accordingly, its CAPM alpha is economically large, 0.43%, albeit marginally significant ($t = 1.89$). The CAPM alpha is nearly identical in magnitude to the average value premium. The regression R^2 is essentially zero.

In the original July 1963–December 1990 sample in Fama and French (1992), the average excess return goes from 0.22% per month for the growth decile to 0.81% for the value decile, and the value premium is on average 0.59% ($t = 2.41$) (untabulated). However, the market beta decreases slightly from 1.08 for the growth decile to 1.05 for the value decile. As a result, the CAPM alpha for the value-minus-growth decile is 0.6% ($t = 2.17$).

Panel B shows that the CAPM explains the value premium in the long sample from July 1926 to June 2017, consistent with Ang and Chen (2007). Their sample ends in December 2001, and we replicate their result in our extended sample. The average excess return varies from 0.59% per month for the growth decile to 1.07% for the value decile. The value premium is on average 0.48% ($t = 2.5$), which is close to 0.47% in the post-1963 sample. More important, the CAPM explains the value premium, with a small alpha of 0.19% ($t = 0.99$) and a large market beta of 0.45 ($t = 3.87$). Relative to the post-1963 sample, the regression R^2 rises considerably from zero to 14%.

To shed light on the differences across the pre- and post-1963 samples, Table 2 reports large market swings with market excess returns below 1.5 and above 98.5 percentiles of the empirical distribution, as well as the corresponding months and value-minus-growth decile returns. There are in total 32 such observations, 23 of which are from the Great Depression. When the market excess return is very low, the value-minus-growth return tends to be very low, and when the market excess return is very high, the value-minus-growth return tends to be very high. Their correlation is 0.72 across these observations. In particular, the lowest value premium is −20.35% in March 1938, which comes with an abysmally low market excess return of −23.82%. The highest value premium is 67.95% in August 1932, which comes with an exuberantly high market excess return of 37.06%. More recently, following the bankruptcy of Lehman Brothers, the market excess return is −17.23% in October 2008, in which the value-minus-growth return is −9.64%.

Figure 1 presents the scatter plots and fitted market regression lines for the value-minus-growth decile return for the long sample (Panel A) and the post-1963 sample (Panel B). Panel A highlights in red the observations with monthly market excess returns below 1.5 and above 98.5 percentiles

Table 2 : Large Swings in the Stock Market Returns and the Corresponding Value-minus-growth Decile Returns, July 1926–June 2017

This table reports market excess returns, MKT, below 1.5 and above 98.5 percentiles in the long U.S. sample. H−L is the value-minus-growth decile return. Returns are in monthly percent.

Month	MKT	H−L	Month	MKT	H−L
November 1928	11.81	−0.29	August 1933	12.05	3.76
October 1929	−20.12	7.60	January 1934	12.60	35.20
June 1930	−16.27	−3.60	September 1937	−13.61	−10.56
May 1931	−13.24	−3.37	March 1938	−23.82	−20.35
June 1931	13.90	14.57	April 1938	14.51	9.16
September 1931	−29.13	−4.03	June 1938	23.87	11.15
December 1931	−13.53	−16.22	September 1939	16.88	57.22
April 1932	−17.96	−2.65	May 1940	−21.95	−15.59
May 1932	−20.51	4.09	October 1974	16.10	−13.57
July 1932	33.84	44.54	January 1975	13.66	19.72
August 1932	37.06	67.95	January 1976	12.16	15.03
October 1932	−13.17	−12.80	March 1980	−12.90	−8.78
February 1933	−15.24	−5.70	January 1987	12.47	−2.83
April 1933	38.85	20.04	October 1987	−23.24	−1.20
May 1933	21.43	44.85	August 1998	−16.08	−3.27
June 1933	13.11	10.40	October 2008	−17.23	−9.64

of the empirical distribution. These observations clearly contribute to the market beta of 0.45 ($t = 3.87$) for the value-minus-growth decile in the long sample. In contrast, Panel B shows that large swings in the stock market are scarce in the post-1963 sample, giving rise to a largely flat regression line. In all, the CAPM does a good job in explaining the value premium in the long sample that includes the Great Depression, but largely fails in the short post-1963 sample.

2.2 The Beta "Anomaly"

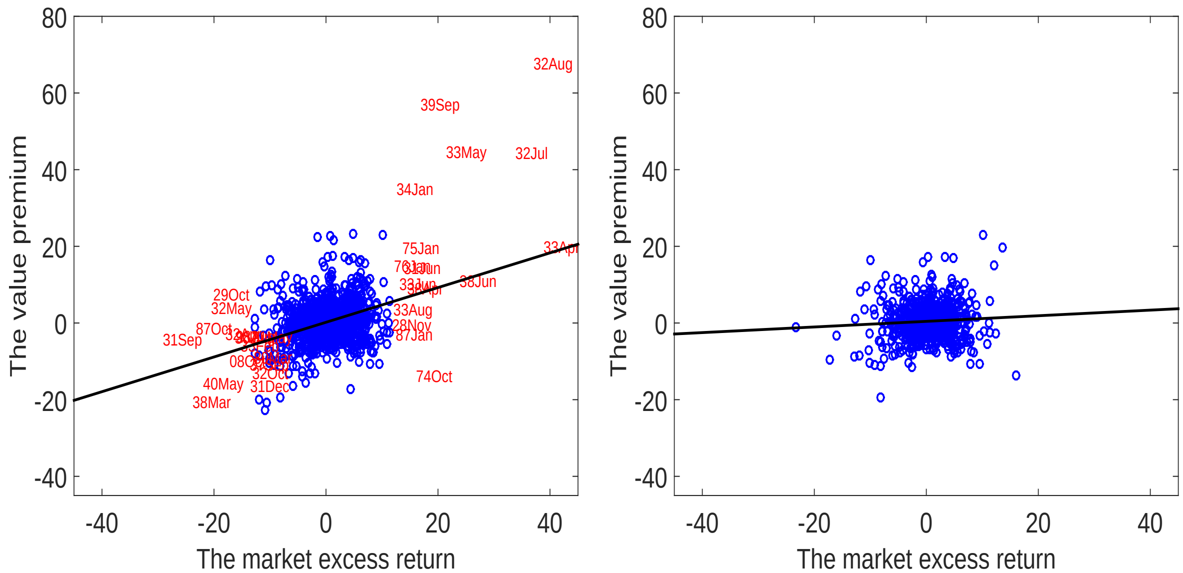
Refuting Ang and Chen (2007) who argue that the CAPM explains the value premium in the long sample, Fama and French (2006) emphasize the CAPM's problem that the cross-sectional variation in the market beta goes unrewarded. This flat relation between the market beta and the average return, known as the beta "anomaly," has a long tradition in empirical asset pricing (Fama and MacBeth 1973; Fama and French 1992; and Frazzini and Pedersen 2014).

Table 3 presents the average excess returns and CAPM regressions across the market beta deciles. At the end of June of each year t, NYSE, Amex, and NASDAQ stocks are sorted into deciles based on the NYSE breakpoints of the pre-ranking betas from rolling-window CAPM regressions

Figure 1 : The CAPM Regressions for the Value-minus-growth Decile, July 1926–June 2017

The figure presents the scatter plot and fitted line for the CAPM regression of the value premium, measured as the value-minus-growth decile return. In Panel A, the monthly market excess returns below the 1.5 and above 98.5 percentiles are dated in red. Returns are in monthly percent.

Panel A: July 1926–June 2017 | Panel B: July 1963–June 2017



in the prior 60 months (24 months minimum). Monthly value-weighted returns are calculated from July of year t to June of $t+1$, and the deciles are rebalanced in June. The sample starts in July 1928 because we use the data from the first 24 months to estimate the pre-ranking betas in June 1928.

Panel A shows that, contradicting the CAPM, the relation between the market beta and the average return in the data is largely flat. Moving from the low to high beta decile, the average excess return rises from 0.52% per month to 0.55%, and the tiny spread of 0.03% is within 0.2 standard errors from zero. Sorting on the pre-ranking beta yields an economically large post-ranking beta spread of 1.06 ($t = 11.81$) across the extreme deciles. As such, the CAPM alpha for the high-minus-low market beta decile is economically large, -0.52%, albeit marginally significant ($t = -1.94$).

From Panel B, the sample from July 1928 onward yields largely similar results. The average excess return varies from 0.58% per month for the low beta decile to 0.75% for the high beta decile, and the small spread of 0.16% is within one standard error from zero. The pre-ranking beta sort

Table 3 : The CAPM Regressions for the Pre-ranking Market Beta Deciles

This table reports the average excess return (m), the CAPM alpha, the post-ranking market beta, their t-statistics adjusted for heteroscedasticity and autocorrelations, and the goodness-of-fit from the CAPM regressions. L, H, and H−L are the low, high, and high-minus-low market beta decile.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: July 1963–June 2017										
m	0.52	0.52	0.56	0.58	0.69	0.55	0.67	0.55	0.57	0.55	0.03
t_m	3.85	3.64	3.45	3.38	3.75	2.86	3.14	2.42	2.23	1.72	0.11
α	0.22	0.17	0.13	0.12	0.18	0.01	0.07	−0.08	−0.13	−0.29	−0.52
t_α	2.11	1.76	1.69	1.42	2.17	0.18	0.85	−0.82	−1.10	−1.49	−1.94
β	0.57	0.68	0.82	0.87	0.98	1.03	1.15	1.22	1.34	1.62	1.06
t_β	12.39	17.21	20.57	20.68	28.13	31.21	50.25	41.76	35.41	30.92	11.81
R^2	0.53	0.68	0.77	0.79	0.86	0.86	0.88	0.86	0.84	0.77	0.43
	Panel B: July 1928–June 2017										
m	0.58	0.63	0.65	0.74	0.83	0.72	0.79	0.73	0.77	0.75	0.16
t_m	5.03	4.66	4.41	4.46	4.54	3.71	3.74	3.11	2.94	2.44	0.66
α	0.22	0.16	0.13	0.14	0.17	0.01	0.02	−0.13	−0.17	−0.33	−0.55
t_α	2.87	2.22	2.21	2.31	2.49	0.20	0.27	−1.51	−1.68	−2.29	−2.81
β	0.57	0.73	0.83	0.94	1.05	1.11	1.22	1.36	1.48	1.70	1.13
t_β	22.86	30.50	36.61	40.31	41.41	39.61	48.26	36.17	26.65	40.93	18.82
R^2	0.66	0.81	0.85	0.88	0.90	0.90	0.91	0.90	0.88	0.84	0.57

again yields an economically large spread of 1.13 ($t = 18.82$) in the post-ranking beta across the extreme deciles. As such, the CAPM alpha for the high-minus-low beta decile is negative, both economically large, −0.55%, and statistically significant, $t = -2.81$.

2.3 The Performance of the Consumption CAPM

To test the consumption CAPM, we use two-stage Fama-MacBeth (1973) cross-sectional regressions because the aggregate consumption growth is not tradable (Breeden, Gibbons, and Litzenberger 1989; and Jagannathan and Wang 2007). To ensure a sufficient number of observations in the second-stage regressions, we use the 25 size and book-to-market portfolios as testing assets (Fama-French 1996). In the first stage, we regress excess returns on the aggregate consumption growth, g_{Ct}:

$$R^e_{it} = \alpha_i + \beta^C_i g_{Ct} + u^1_{it}, \tag{1}$$

in which R^e_{it} is portfolio i's excess return, β^C_i the consumption beta, and u^1_{it} the residuals. In the second stage, we regress portfolio excess returns on the consumption betas cross-sectionally:

$$R^e_{it} = \phi_0 + \phi_1 \beta^C_i + u^2_{it}, \tag{2}$$

in which ϕ_0 is the intercept, ϕ_1 the price of consumption risk, and u^2_{it} the residuals. The consumption CAPM predicts that $\phi_0 = 0$, ϕ_1 is significantly positive, and the expected risk premium equals $\phi_1 \beta^C_i$.

We obtain consumption data from National Income and Product Accounts (NIPA) Table 7.1 from Bureau of Economic Analysis. Consumption is the sum of per capita nondurables plus services in chained dollars. The annual series is from 1929 to 2016, and the quarterly series from the first quarter (Q1) of 1947 to the second quarter (Q2) of 2017. The annual series contains the Great Depression, but the quarterly series does not. We test the consumption CAPM with both annual and quarterly data. We also implement the Jagannathan-Wang (2007) fourth-quarter consumption growth model, in which annual consumption growth is calculated with only the fourth-quarter consumption data. The rationale is that investors are more likely to make their consumption and portfolio choice decisions simultaneously in the fourth-quarter because the tax year ends in December.

Table 4 reports average excess returns and consumption betas for the 25 size and book-to-market portfolios. The portfolio returns data are from Kenneth French's Web site. Panel A shows that in the 1930–2016 annual sample, the average value premium is stronger in small firms than in big firms. In the smallest quintile, the value-minus-growth quintile return is on average 12.52% per annum ($t = 4.31$), whereas in the biggest quintile, only 4.12% ($t = 1.74$). The pattern is similar in the 1947:Q2–2017:Q2 quarterly sample. The value premium is on average 2.4% per quarter ($t = 5.01$) in the smallest quintile, but only 0.57% ($t = 1.33$) in the biggest quintile. The results from the shorter 1948–2016 annual sample are largely similar.

Panel A also shows that the consumption betas estimated from annual consumption growth do not align with the average returns across the 25 portfolios. For example, despite the high average

Table 4 : Average Excess Returns and Consumption Betas for the 25 Size and Book-to-market Portfolios

For each portfolio, this table reports average excess return, m, and consumption beta, β^C, and their t-values adjusted for heteroscedasticity and autocorrelations, t_m and t_{β^C}, respectively. Returns in Panels A and C are in annual percent, and those in Panel B in quarterly percent.

	L	2	3	4	H	L	2	3	4	H
	Panel A: Annual consumption growth, 1930–2016									
	m					t_m				
Small	6.04	10.65	13.73	16.82	18.56	1.48	2.44	3.85	4.44	4.57
2	9.02	12.32	13.33	14.90	16.03	2.51	4.00	4.25	4.51	4.67
3	9.27	11.83	11.88	13.73	14.72	3.09	4.35	4.38	4.69	4.34
4	8.82	9.68	11.49	12.83	13.16	3.48	3.76	4.16	4.45	3.69
Big	7.46	7.38	8.90	8.36	11.58	3.44	3.62	3.92	3.12	3.72
	β^C					t_{β^C}				
Small	2.80	0.66	1.63	1.86	1.58	1.52	0.19	0.70	0.69	0.57
2	1.25	1.72	0.88	1.25	1.68	0.54	0.83	0.41	0.53	0.78
3	0.29	1.11	1.77	2.12	2.15	0.14	0.64	0.99	1.15	0.94
4	0.38	0.37	1.32	1.36	0.47	0.25	0.20	0.70	0.66	0.18
Big	1.05	0.59	1.79	2.26	−0.88	0.93	0.47	1.18	1.19	−0.28
	Panel B: Quarterly consumption growth, 1947:Q2–2017:Q2									
	m					t_m				
Small	1.25	2.58	2.57	3.23	3.65	1.39	3.36	3.78	4.93	5.06
2	1.74	2.58	2.86	3.01	3.38	2.21	3.90	4.78	5.02	5.00
3	1.96	2.61	2.54	2.99	3.26	2.79	4.40	4.63	5.26	5.08
4	2.18	2.18	2.60	2.74	2.93	3.41	3.97	4.83	5.06	4.45
Big	1.90	1.90	2.18	1.98	2.47	3.74	4.10	4.99	3.91	4.26
	β^C					t_{β^C}				
Small	4.22	4.73	3.43	3.63	3.94	2.46	3.23	2.54	2.84	2.63
2	3.01	2.89	2.91	3.07	3.60	2.08	2.34	2.65	2.62	2.66
3	2.85	2.59	2.57	2.63	2.99	2.02	2.18	2.43	2.22	2.55
4	2.47	2.16	2.54	2.39	3.77	1.86	1.92	1.94	2.04	2.59
Big	2.62	1.94	1.97	2.60	2.80	2.54	1.93	2.09	1.99	2.44
	Panel C: Fourth-quarter consumption growth, 1948–2016									
	m					t_m				
Small	5.38	11.47	11.21	14.25	16.17	1.30	3.14	3.61	4.69	4.77
2	6.95	10.71	12.30	13.18	14.48	2.08	3.93	4.53	4.86	4.82
3	7.72	11.03	10.74	13.14	14.25	2.74	4.42	4.57	4.78	4.85
4	8.77	9.00	11.21	12.00	12.73	3.35	3.97	4.45	4.74	4.25
Big	7.95	7.74	9.41	8.54	10.81	3.47	3.92	4.59	3.58	3.94
	β^C					t_{β^C}				
Small	3.83	5.50	4.35	5.05	6.09	1.43	2.32	2.01	2.73	2.69
2	3.07	3.17	4.48	5.08	6.34	1.36	1.60	2.58	3.07	3.50
3	2.64	3.89	4.03	4.50	5.68	1.20	2.13	2.45	2.26	3.06
4	2.22	3.02	4.23	5.03	5.95	1.06	1.60	2.02	2.78	2.77
Big	3.04	2.86	3.34	5.19	5.12	1.67	1.84	2.11	2.89	2.66

excess return, 18.56% per annum, of the small-value portfolio, relative to only 6.04% of the small-growth portfolio, the consumption beta of the former is lower than that of the latter, 1.58 versus 2.8. Similarly, Panel B shows that the consumption betas estimated from quarterly consumption growth do not align either with the average returns. The contrast in the average return between the small-growth and small-value portfolios is 1.25% versus 3.65% per quarter, but the consumption beta goes in the wrong direction, 4.22 versus 3.94. Finally, consistent with Jagannathan and Wang (2007), the consumption betas estimated from fourth-quarter consumption growth align better with the average returns. The small-value portfolio has a consumption beta of 6.09, which is higher than 3.83 of the small-growth portfolio, going in the right direction in explaining the average returns.

Table 5 reports the second-stage cross-sectional tests of the consumption CAPM. From Panel A, the consumption CAPM does not perform well in the annual sample from 1930 to 2016. The estimate of the price of consumption risk, ϕ_1, is economically small, 0.58% per annum, and statistically insignificant, with both the Fama-MacBeth t-value and Shanken (1992) adjusted t-value below 1.2. In contrast, the intercept, ϕ_0, is economically large, 10.97%, and highly significant, with both t-values around four. Finally, the Fama-MacBeth R^2 is only 2.13%, indicating that average excess returns and consumption betas are poorly aligned across the testing assets.

The poor alignment is shown in Panel A of Figure 2, which plots average excess returns predicted by the consumption CAPM estimated from the annual data against average realized excess returns. The scatter plot is largely horizontal, indicating little explanatory power. In particular, the small-growth portfolio (denoted "11") earns on average only 6.04% per annum, and the small-value portfolio ("15") 18.56%. In contrast, the small-growth portfolio has a higher consumption beta than the small-value portfolio, 2.8 versus 1.58. Combined with the ϕ_1 estimate of 0.58%, the consumption CAPM predicts a negative small-stock value premium of -0.71%, in contrast to 12.52% in the data.

Using the quarterly sample from 1947 onward yields largely similar results. Panel B of Table 5 shows that the price of consumption risk, ϕ_1, is estimated to be 0.22% per quarter, which is

Table 5 : Cross-sectional Regression Tests of the Consumption CAPM

This table reports the Fama-MacBeth cross-sectional regression tests of the consumption CAPM in equation (2). Testing assets are the 25 Fama-French size and book-to-market portfolios. Consumption betas are estimated from time-series regressions of portfolio excess returns on the aggregate consumption growth. Panel A uses annual consumption growth from 1930 to 2016, Panel B quarterly consumption growth from the second quarter (Q2) of 1947 to the second quarter of 2017, and Panel C the fourth-quarter consumption growth from 1948 to 2016. ϕ_0 is the intercept, and ϕ_1 the slope, which provides the price of the consumption risk, in the second-stage cross-sectional regressions. t_{FM} is the Fama-MacBeth t-statistics, and t_{S} the Shanken-adjusted t-statistics. R^2 is the average goodness-of-fit coefficient of the cross-sectional regressions. The estimates of ϕ_0 and ϕ_1 are annual percent in Panels A and C, and in quarterly percent in Panel B.

	Panel A: Annual, 1930–2016		Panel B: Quarterly, 1947:Q2–2017:Q2		Panel C: Fourth-quarter, 1948–2016	
	ϕ_0	ϕ_1	ϕ_0	ϕ_1	ϕ_0	ϕ_1
Estimates	10.97	0.58	1.88	0.22	3.30	1.75
t_{FM}	4.14	1.16	3.73	1.12	1.23	3.44
t_{S}	3.99	1.13	3.42	1.03	0.77	2.23
R^2		0.02		0.07		0.60

economically small and statistically insignificant, with the Fama-MacBeth and Shanken-adjusted t-values both below 1.2. In contrast, the intercept, ϕ_0, is 1.88%, which is economically large and highly significant, with t-values above 3.4. The cross-sectional regression R^2 remains low, 7.11%. Panel B of Figure 2 again shows the poor alignment between average predicted and average realized excess returns. The small-growth portfolio earns on average 1.25%, and the small-value portfolio 3.65%. However, the small-growth portfolio has a higher consumption beta than the small-value portfolio, 4.22 versus 3.94. Combined with the ϕ_1 estimate of 0.22%, the consumption CAPM predicts a negative small-stock value premium of -0.06%, in contrast to 2.4% in the data.

With an extended sample from 1948 to 2016, we replicate the superior performance of the fourth-quarter consumption growth model that Jagannathan and Wang (2007) document in their 1954–2003 sample. Panel C of Table 5 reports that the price of consumption risk is 1.75% per annum, with a Fama-MacBeth t-value of 3.44 and a Shanken-adjusted t-value of 2.23. The intercept of cross-sectional regressions is only 3.3%, which is insignificant with t-values below 1.3. More impressively, the regression R^2 is 60%. Panel C of Figure 2 shows further that the scatter plot of average predicted versus average realized excess returns is better aligned with the 45-degree

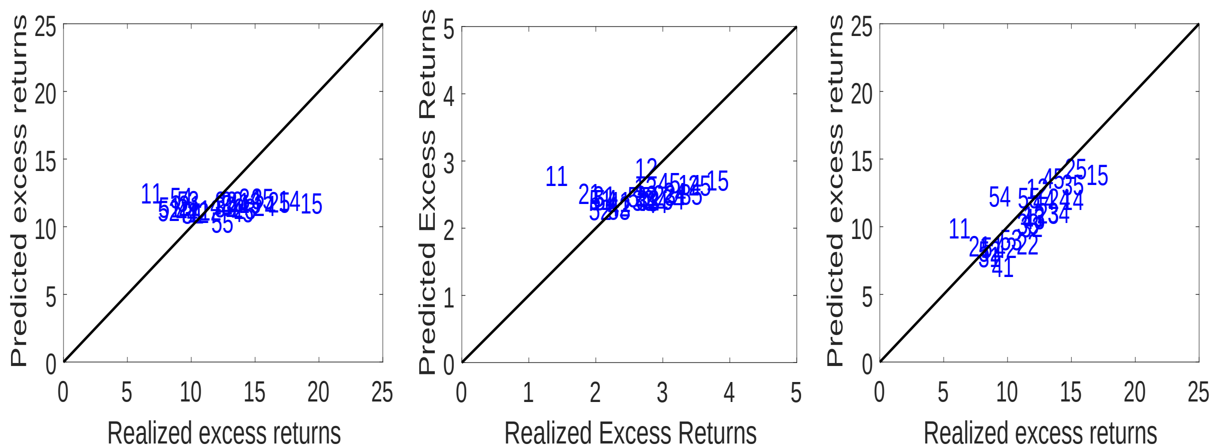
Figure 2 : Average Predicted Excess Returns versus Average Realized Excess Returns, the Consumption CAPM

This figure plots average predicted against average realized excess returns of the 25 size and book-to-market portfolios. Each two-digit number represents one portfolio, with the first digit referring to the size quintile ("1" the smallest, "5" the biggest), and the second digit the book-to-market quintile ("1" the lowest, "5" the highest). Panel A uses annual consumption growth from 1930 to 2016, Panel B quarterly consumption growth from the second quarter (Q2) of 1947 to the second quarter of 2017, and Panel C the fourth-quarter consumption growth from 1948 to 2016. The predicted excess return of portfolio i is $\phi_1 \beta_i^C$, in which β_i^C is its consumption beta from the first-stage regression, and ϕ_1 the price of consumption risk from the second-stage regression.

Panel A: Annual, 1930–2016

Panel B: Quarterly, 1947:Q2–2017Q2

Panel C: Fourth-quarter, 1948–2016



line. In particular, the small-growth portfolio earns on average 5.38%, in contrast to 16.17% for the small-value portfolio. Going in the right direction as the average returns, the small-growth portfolio has a lower consumption beta than the small-value portfolio, 3.83 versus 6.09. Combined with the ϕ_1 estimate of 1.75%, the Jagannathan-Wang consumption CAPM predicts a positive small-stock value premium of 3.96%. Although its magnitude is lower than 10.79% in the data, the model is a substantial improvement over the standard consumption CAPM.

3 An Equilibrium Model with Disasters and Heterogeneous Firms

The general equilibrium production economy draws elements from the disaster model of Rietz (1988) and Barro (2006, 2009) as well as the investment model of Zhang (2005). The economy is populated by a representative agent with recursive utility and heterogenous firms. The firms take the representative agent's intertemporal rate of substitution as given when determining optimal policies. The production technology is subject to both aggregate and firm-specific shocks. The aggregate shock contains normally distributed states as well as a disaster state and a recovery state.

3.1 Preferences

The representative agent has recursive utility, U_t, defined over aggregate consumption, C_t:

$$U_t = \left[(1-\iota)C_t^{1-\frac{1}{\psi}} + \iota\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{3}$$

in which ι is the time discount factor, ψ is the intertemporal elasticity of substitution, and γ is the relative risk aversion (Epstein and Zin 1989). The pricing kernel is given by:

$$M_{t+1} = \iota\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}^{1-\gamma}}{E_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{4}$$

We adopt the recursive utility to delink the relative risk aversion, γ, from the intertemporal elasticity of substitution, ψ. Their values are both higher than unity in our calibration (Section 4.1). Nakamura, Steinsson, Barro, and Ursua (2013) show that a low value of ψ less than unity implies counterfactually a surge in stock prices at the onset of disasters. The reason is that entering a (persistent) disaster state generates a strong desire to save, as consumption is expected to fall substantially in the future. With a small ψ, this effect dominates the negative effect of the disaster state on firms' cash flows, raising their stock prices. Gourio (2012) makes a similar point in a production economy that when $\psi < 1$, the onset of disasters counterfactually increases investment.

3.2 Technology

Firms produce output with capital, and are subject to both aggregate and firm-specific shocks. Output for firm i at time t, denoted $Y_{it} \equiv Y(K_{it}, Z_{it}, X_t)$, is given by:

$$Y_{it} = (X_t Z_{it})^{1-\xi} K_{it}^{\xi}, \tag{5}$$

in which $\xi > 0$ is the curvature parameter, X_t is the aggregate productivity, Z_{it} is the firm-specific productivity, and K_{it} is capital. Operating profits are defined as:

$$\Pi_{it} = Y_{it} - fK_{it}, \tag{6}$$

in which fK_{it}, with $f > 0$, is the fixed costs of production. The fixed costs are scaled by capital to ensure that the costs do not become trivially small along the balanced growth path.

The aggregate productivity growth, $g_{xt} \equiv \log(X_t/X_{t-1})$, is specified as:

$$g_{xt} = \overline{g} + g_t, \tag{7}$$

in which $\overline{g}$ is the constant mean. We assume that g_t follows a first-order autoregressive process:

$$g_{t+1} = \rho_g g_t + \sigma_g \epsilon_{t+1}, \tag{8}$$

in which ϵ_{t+1} is a standard normal shock, and the unconditional mean of g_t is zero.

The firm-specific productivity for firm i, Z_{it}, has a transition function given by:

$$z_{it+1} = (1 - \rho_z)\overline{z} + \rho_z z_{it} + \sigma_z e_{it+1}, \tag{9}$$

in which $z_{it} \equiv \log Z_{it}$, $\overline{z}$ is the unconditional mean of z_{it} common to all firms, and e_{it+1} is an independently and identically distributed standard normal shock. We assume that e_{it+1} and e_{jt+1} are uncorrelated for any $i \neq j$, and ϵ_{t+1} and e_{it+1} are uncorrelated for all i.

3.3 Disasters

We follow Rouwenhorst (1995) to discretize the demeaned aggregate productivity growth, g_t, into a five-point grid, $\{g_1, g_2, g_3, g_4, g_5\}$.[1] The grid is symmetric around the long-run mean of zero, and even-spaced. The distance between any two adjacent grid point is given by $2\sigma_g/\sqrt{(1-\rho_g^2)(n_g-1)}$,

[1] Kopecky and Suen (2010) show that the Rouwenhorst (1995) method dominates other popular methods in the Markov-chain approximation to autoregressive processes in the context of the stochastic growth model. Petrosky-Nadeau and Zhang (2017) show similar results in the search model of equilibrium unemployment.

in which $n_g = 5$. The Rouwenhorst procedure also produces a transition matrix, $\widetilde{P}$, given by:

$$\widetilde{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{15} \\ p_{21} & p_{22} & \cdots & p_{25} \\ \vdots & \vdots & \ddots & \vdots \\ p_{51} & p_{52} & \cdots & p_{55} \end{bmatrix}, \tag{10}$$

in which p_{ij}, for $i, j = 1, \ldots, 5$, is the probability of $g_{t+1} = g_j$ conditional on $g_t = g_i$.[2]

To incorporate disasters into the model, we modify directly the discretized g_t grid and its transition matrix, following Danthine and Donaldson (1999). In particular, we insert into the g_t grid a disaster state, $g_0 = \lambda_D$, in which $\lambda_D < 0$ is the disaster size, as well as a recovery state, $g_6 = \lambda_R$, in which $\lambda_R > 0$ is the recovery size. Accordingly, we form the transition matrix, P, by modifying $\widetilde{P}$ to incorporate the disaster and recovery states as follows:

$$P = \begin{bmatrix} \theta & 0 & 0 & \ldots & 0 & 1-\theta \\ \eta & p_{11}-\eta & p_{12} & \ldots & p_{15} & 0 \\ \eta & p_{21} & p_{22}-\eta & \ldots & p_{25} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta & p_{51} & p_{52} & \ldots & p_{55}-\eta & 0 \\ 0 & (1-\nu)/5 & (1-\nu)/5 & \ldots & (1-\nu)/5 & \nu \end{bmatrix}. \tag{13}$$

In the modified transition matrix, P, η is the probability of entering the disaster state from any of the normal states, and θ is the probability of remaining in the disaster state next period conditional on the economy in the disaster state in the current period. As such, θ is the persistence of the disaster state. Similarly, ν is the persistence of the recovery state. In addition, in constructing the transition matrix, we have implicitly assumed that the economy can only enter the recovery

[2] To construct the $\widetilde{P}$ matrix, we set $p = (\rho_g + 1)/2$, and define the transition matrix for $n_g = 3$ as:

$$\widetilde{P}^{(3)} \equiv \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-p) & p^2+(1-p)^2 & p(1-p) \\ (1-p)^2 & 2p(1-p) & p^2 \end{bmatrix}. \tag{11}$$

To obtain $\widetilde{P} = \widetilde{P}^{(5)}$, we use the following recursion:

$$p \begin{bmatrix} \widetilde{P}^{(n_g)} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} + (1-p) \begin{bmatrix} \mathbf{0} & \widetilde{P}^{(n_g)} \\ 0 & \mathbf{0}' \end{bmatrix} + (1-p) \begin{bmatrix} \mathbf{0}' & 0 \\ \widetilde{P}^{(n_g)} & \mathbf{0} \end{bmatrix} + p \begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & \widetilde{P}^{(n_g)} \end{bmatrix}, \tag{12}$$

in which $\mathbf{0}$ is a $n_g \times 1$ column vector of zeros. We then divide all but the top and bottom rows by two to ensure that the conditional probabilities sum up to one in $\widetilde{P}^{(n_g+1)}$ (see Rouwenhorst, 1995, p. 306–307 and p. 325–329).

state following a disaster. Once in the recovery state, the economy can enter any of the normal states with an equal probability, $(1-\nu)/5$, but cannot fall immediately back into the disaster state.

The modeling of disasters as large drops in total factor productivity, and consequently, in output and consumption is motivated by Barro (2006, 2009), Barro and Ursua (2008), and Nakamura, Steinsson, Barro and Ursua (2013). These studies document evidence on consumption and output disasters in a historical, cross-country panel. In addition, Cole and Ohanian (1999, 2007) show that negative shocks to total factor productivity can account for over half of the 1929–1933 downturn in the Great Depression in the United States. Kehoe and Prescott (2007) also show that productivity shocks play an important role during economic disasters around the world.

3.4 Adjustment Costs

Let I_{it} denote firm i's investment at time t. Capital accumulates as follows:

$$K_{it+1} = I_{it} + (1-\delta)K_{it}, \tag{14}$$

in which δ is the capital depreciation rate. Real investment entails asymmetric adjustment costs:

$$\Phi_{it} \equiv \Phi(I_{it}, K_{it}) = \begin{cases} a^+ K_{it} + \frac{c^+}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & \text{for} \quad I_{it} > 0 \\ 0 & \text{for} \quad I_{it} = 0 \\ a^- K_{it} + \frac{c^-}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & \text{for} \quad I_{it} < 0 \end{cases}, \tag{15}$$

in which $a^- > a^+ > 0$ and $c^- > c^+ > 0$ capture the asymmetry (Abel and Eberly 1994).

3.5 Firms' Problem

Let μ_t denote the bivariate cross-sectional distribution of capital, K_{it}, and firm-specific productivity, Z_{it}. Because of the aggregate shocks, μ_t is time-varying. We denote its equilibrium law of motion as:

$$\mu_{t+1} = \Upsilon(\mu_t, X_t, X_{t+1}). \tag{16}$$

The infinite-dimensional μ_t is an endogenous, aggregate state variable because it is relevant for firms to forecast the future aggregation consumption, C_{t+1}, and consequently, the pricing kernel, M_{t+1}.

Upon observing the exogenous aggregate state, X_t, the endogenous aggregate state, μ_t, the exogenous firm-specific state, Z_{it}, and the endogenous firm-specific state, K_{it}, firm i makes optimal investment decision, I_{it}, and optimal exit decision, χ_{it}, to maximize its market value of equity. Let $D_{it} \equiv \Pi_{it} - I_{it} - \Phi(I_{it}, K_{it})$ be dividends. The cum-dividend market equity, V_{it}, is given by:

$$V_{it} \equiv V(K_{it}, Z_{it}; X_t, \mu_t) = \max_{\{\chi_{it}\}} \left(\max_{\{I_{it}\}} \; D_{it} + E_t \left[M_{t+1} V(K_{it+1}, Z_{it+1}; X_{t+1}, \mu_{t+1}) \right], \; sK_{it} \right), \quad (17)$$

in which $s > 0$ is the liquidation value parameter, subject to the capital accumulation equation (14) and the equilibrium law of motion for μ_t in equation (16).

When $V_{it} \geq sK_{it}$, which is the exit threshold, firm i stays in the economy, i.e., $\chi_{it} = 0$. For all the incumbent firms, evaluating the value function at the optimum yields $V_{it} = D_{it} + E_t[M_{t+1}V_{it+1}]$. Equivalently, $E_t[M_{t+1}R_{it+1}] = 1$, in which $R_{it+1} \equiv V_{it+1}/(V_{it} - D_{it})$ is the stock return. Using the definition of covariance, we can rewrite $E_t[M_{t+1}R_{it+1}] = 1$ as:

$$E_t[R_{it+1}] = r_{ft} + \left(-\frac{\mathrm{Cov}_t[R_{it+1}, M_{t+1}]}{\mathrm{Var}_t[M_{t+1}]} \right) \frac{\mathrm{Var}_t[M_{t+1}]}{E_t[M_{t+1}]} = r_{ft} + \beta_{it}^M \phi_{Mt} \quad (18)$$

in which $r_{ft} \equiv 1/E_t[M_{t+1}]$ is the real interest rate, $\beta_i^M \equiv -\mathrm{Cov}_t[R_{it+1}, M_{t+1}]/\mathrm{Var}_t[M_{t+1}]$ the true beta, and $\phi_{Mt} \equiv \mathrm{Var}_t[M_{t+1}]/E_t[M_{t+1}]$ the price of consumption risk. When $V_{it} < sK_{it}$, firm i exits from the economy at the beginning of time t, i.e., $\chi_{it} = 1$. We set its stock return over period $t-1$, R_{it}, to be a predetermined, constant delisting return, denoted $\widetilde{R}$.

When firm i exits from the economy at the beginning of time t, we assume that the firm enters an immediate reorganization process. The current shareholders of the firm receive sK_{it} as the liquidation value, and the old firm ceases to exit. New shareholders take over the remainder of the firm's capital, $(1 - s - \kappa)K_{it}$, in which $0 < \kappa < 1 - s$ is the reorganization cost parameter. For computational tractability, we assume that the reorganization process occurs instantaneously. At the beginning of t, the old firm is replaced by a new firm with an initial capital of $(1-s-\kappa)K_{it}$ and a new firm-specific log productivity, z_{it}, that equals its unconditional mean of zero. This modeling of entry and exit keeps the number of firms constant in the economy. Prior theoretical models, all of which have no disasters, have largely ignored the exit decision (Zhang 2005). With disasters, firms are more likely to exit in the disaster state, especially when the liquidation value parameter, s, is high. As such, we incorporate the exit decision, and the related entry decision, into the model to better quantify the impact of disaster dynamics on the cross section.

3.6 Competitive Equilibrium

A recursive competitive equilibrium consists of an optimal investment rule, $I(K_{it}, Z_{it}; X_t, \mu_t)$; an optimal exit rule, $\chi(K_{it}, Z_{it}; X_t, \mu_t)$; a value function, $V(K_{it}, Z_{it}; X_t, \mu_t)$; and an equilibrium law of motion for the firm distribution, $\Upsilon(\mu_t, X_t, X_{t+1})$, such that the following conditions hold.

- Optimality: $I(K_{it}, Z_{it}; X_t, \mu_t)$, $\chi(K_{it}, Z_{it}; X_t, \mu_t)$, and $V(K_{it}, Z_{it}; X_t, \mu_t)$ solve the value maximization problem in equation (17) for each firm.

- Consistency: The aggregate behavior of the economy is consistent with the optimal behavior of all firms in the economy. Let Y_t, I_t, K_t, Φ_t denote the aggregate output, investment, capital, and adjustment costs, respectively, then:

$$Y_t = \int Y_{it}\, \mu_t(dK_{it}, dZ_{it}); \tag{19}$$

$$I_t = \int I_{it}\, \mu_t(dK_{it}, dZ_{it}); \tag{20}$$

$$K_t = \int K_{it}\, \mu_t(dK_{it}, dZ_{it}); \tag{21}$$

$$\Phi_t = \int \Phi_{it}\, \mu_t(dK_{it}, dZ_{it}). \tag{22}$$

  Also, the law of motion for the firm distribution, Υ, is consistent with the optimal decisions of firms. Let Θ be any measurable set in the product space of K_{it+1} and Z_{it+1}, then Υ is given by:

$$\mu_{t+1}(\Theta, X_{t+1}) = T(\Theta, (K_{it}, Z_{it}), X_t)\, \mu_t(K_{it}, Z_{it}, X_t), \tag{23}$$

in which

$$T(\Theta, (K_{it}, Z_{it}), X_t) \equiv \iint \mathbf{1}_{\{(I_{it}+(1-\delta)K_{it}, Z_{it+1}) \in \Theta\}} \, Q_Z(dZ_{it+1}|Z_{it}) Q_X(dX_{t+1}|X_t), \tag{24}$$

and $\mathbf{1}_{\{\cdot\}}$ is an indicator function that takes the value of one if the event described in $\{\cdot\}$ is true, and zero otherwise, and Q_Z and Q_X are the transition functions for Z_{it} and X_t, respectively.

- Market clearing: Aggregate consumption equals aggregate output minus investment:

$$C_t = Y_t - I_t \qquad \Rightarrow \qquad C_t = D_t + fK_t + \Phi_t. \tag{25}$$

We treat the fixed costs of production, fK_t, and capital adjustment costs, Φ_t, as compensation to labor, and include their sum as part of consumption. Doing so drives a wedge between consumption and aggregate dividends to help raise risk premiums (Abel 1999).

3.7 Solving for the Competitive Equiibrium

Because the model features a balanced growth path, we first detrend it before solving for its competitive equilibrium. We define the following stationary variables: $\widehat{U}_t \equiv U_t/C_t$, $\widehat{\Pi}_{it} \equiv \Pi_{it}/X_{t-1}$, $\widehat{V}_{it} \equiv V_{it}/X_{t-1}$, $\widehat{K}_{it} \equiv K_{it}/X_{t-1}$, $\widehat{I}_{it} \equiv I_{it}/X_{t-1}$, $\widehat{\Phi}_{it} \equiv \Phi_{it}/X_{t-1}$, $\widehat{C}_t \equiv C_t/X_{t-1}$, and $\widehat{D}_{it} \equiv D_{it}/X_{t-1}$. We then rewrite the model's key equations in terms of the detrended, stationary variables:

- The log utility-to-consumption ratio, $\widehat{u}_t \equiv \log(\widehat{U}_t)$:

$$\exp(\widehat{u}_t) = \left[(1-\iota) + \iota \left(E_t \left[\exp\left[(1-\gamma)(\widehat{u}_{t+1} + \widehat{g}_{ct+1} + g_{xt})\right]\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{26}$$

in which $\widehat{g}_{ct+1} \equiv \log(\widehat{C}_{t+1}/\widehat{C}_t)$ is the log growth rate of detrended consumption.

- The pricing kernel:

$$M_{t+1} = \iota \exp\left[-\frac{1}{\psi}\left(\widehat{g}_{ct+1} + g_{xt}\right)\right] \left[\frac{\exp\left[(1-\gamma)(\widehat{u}_{t+1} + \widehat{g}_{ct+1})\right]}{E_t\left[\exp\left[(1-\gamma)(\widehat{u}_{t+1} + \widehat{g}_{ct+1})\right]\right]}\right]^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{27}$$

- Profits: $\widehat{\Pi}_{it} \equiv \exp[(1-\xi)g_{xt}] Z_{it}^{1-\xi} \widehat{K}_{it}^{\xi} - f\widehat{K}_{it}$.

- Capital accumulation: $\widehat{K}_{it+1} \exp(g_{xt}) = (1-\delta)\widehat{K}_{it} + \widehat{I}_{it}$.

- The adjustment costs function:

$$\widehat{\Phi}_{it} = \begin{cases} a^{+}\widehat{K}_{it} + \frac{c^{+}}{2}\left(\frac{\widehat{I}_{it}}{\widehat{K}_{it}}\right)^{2}\widehat{K}_{it} & \text{for} \quad \widehat{I}_{it} > 0 \\ 0 & \text{for} \quad \widehat{I}_{it} = 0 \\ a^{-}\widehat{K}_{it} + \frac{c^{-}}{2}\left(\frac{\widehat{I}_{it}}{\widehat{K}_{it}}\right)^{2}\widehat{K}_{it} & \text{for} \quad \widehat{I}_{it} < 0 \end{cases}. \tag{28}$$

- The cross-sectional distribution of $\widehat{K}_{it}$ and Z_{it}, $\widehat{\mu}_t$, and its equilibrium law of motion, $\widehat{\Upsilon}_t$.

- The value function, $\widehat{V}_{it} \equiv \widehat{V}\left(\widehat{K}_{it}, Z_{it}, g_t, \widehat{\mu}_t\right)$:

$$\widehat{V}_{it} = \max_{\{\chi_{it}\}} \left[\max_{\{\widehat{I}_{it}\}} \ \widehat{D}_{it} + E_t\left[M_{t+1}\widehat{V}\left(\widehat{K}_{it+1}, Z_{it+1}, g_{t+1}, \widehat{\mu}_{t+1}\right)\right]\exp(g_{xt}), \ s\widehat{K_{it}}\right]. \tag{29}$$

- The stock return for an incumbent firm: $R_{it+1} \equiv \widehat{V}_{it+1}\exp(g_{xt})/(\widehat{V}_{it} - \widehat{D}_{it})$.

A major challenge in solving and analyzing our general equilibrium model is that the cross-sectional distribution, μ_t, is an endogenous aggregate state variable that affects the pricing kernel, M_{t+1}. We adopt the idea of approximate aggregation from Krusell and Smith (1997, 1998) to make the firms' problem computationally tractable. We guess and verify that the cross-sectional average detrended capital, denoted $\overline{K}_t$, contains all the information of μ_t that is relevant for forecasting the pricing kernel, M_{t+1}. Appendix A details our computational algorithm.

4 Quantitative Results

We calibrate the model, and report its basic moments in Section 4.1. We present key equilibrium properties in Section 4.2. We explain the failure of the CAPM in Section 4.3, the beta "anomaly" in Section 4.4, and the performance of the consumption CAPM in Section 4.5.

4.1 Calibration and Basic Moments

Table 6 reports the parameters in our monthly calibration. For preference parameters, we set the intertemporal elasticity of substitution, ψ, to 1.5, the relative risk aversion, γ, five, and the time

Table 6 : Parameter Values in the Benchmark Monthly Calibration

ι denotes the time discount factor, γ the relative risk aversion, ψ the intertemporal elasticity of substitution, $\bar{g}$ the long-run mean of log aggregate productivity growth, ρ_g the persistence of productivity growth, σ_g the conditional volatility of productivity growth, η the disaster probability, λ_D the disaster size, θ the disaster persistence, λ_R the recovery size, ν the recovery persistence, ξ the curvature of the production function, δ the capital depreciation rate, f the fixed costs of production parameter, $\overline{z}$ the long-run mean of log firm-specific productivity level, ρ_z the persistence of log firm-specific productivity, σ_z the conditional volatility of log firm-specific productivity, a^+ upward nonconvex adjustment costs parameter, a^- downward nonconvex adjustment costs parameter, c^+ upward convex adjustment costs parameter, c^- downward convex adjustment costs parameter, s the liquidation value parameter, κ the reorganization costs parameter, and $\widetilde{R}$ the delisting return.

ι	γ	ψ	$\bar{g}$	ρ_g	σ_g	η	λ_D	θ	λ_R	ν	ξ
0.9945	5	1.5	1.9%/12	0.6	0.003	2%/12	−2.75%	$0.914^{1/3}$	1.5%	0.964	0.65
δ	f	$\overline{z}$	ρ_z	σ_z	a^+	a^-	c^+	c^-	s	κ	$\widetilde{R}$
0.01	0.005	−8.52	0.985	0.5	0.035	0.05	75	150	0	0.25	−12.33%

discount factor, ι, 0.9945. For the parameters that govern the dynamics in normal times, we set the balance growth rate, $\overline{g}$, to 1.9%/12, which matches an annualized growth rate of 1.9% for real per capita consumption (nondurables and services) growth from the second quarter of 1947 to the second quarter of 2017 in National Income and Product Accounts (NIPA) Table 7.1. The persistence of the demeaned aggregate productivity growth, ρ_g, is 0.6, and its conditional volatility, σ_g, 0.003, which yield a reasonable match with consumption growth dynamics in the postwar data (Table 7).

For the parameters that govern the disaster dynamics, we set the disaster persistence, $\theta = 0.914^{1/3}$, which is the probability that the economy remains in the disaster state in the next month conditional on it being in the disaster state in the current month. This monthly persistence accords with a quarterly persistence of 0.914 as in Gourio (2012), and the average duration of disasters is $1/(1-0.914^{1/3}) = 33$ months (roughly three years), consistent with Barro and Ursua (2008). We set the disaster probability, η, to be 2%/12, which implies an annual disaster probability of 2%. This disaster probability is conservative relative to the 2.8% annual probability estimated in Nakamura, Steinsson, Barro and Ursua (2013) and the 0.72% quarterly probability calibrated in Gourio.

Following Gourio (2012), we calibrate the remaining disaster parameters, including the disaster size, λ_D, the recovery size, λ_R, and the recovery persistence, ν, in the demeaned aggregate productivity growth, g_t, to ensure that the impulse response of consumption to a disaster shock in the model's simulations replicates the basic pattern in the data reported in Nakamura, Steinsson, Barro, and Ursua (2013). This procedure yields $\lambda_D = -2.75\%$, $\lambda_R = 1.5\%$, and $\nu = 0.964$.

Panel A of Figure 3 shows that the model's impulse response is conservative relative to that in the data. The average maximum short-term effect of disasters across more than 28,000 disaster episodes simulated from the model is a drop of 13.9% for consumption, and the median maximum short-term effect is a drop of 18.9% of consumption. The average long-term negative effect is about 9% fall, and the median 11% fall in consumption. For comparison, Nakamura, Steinsson, Barro and Ursua (2013) report that the mean maximum short-term effect of disasters is 29% drop in consumption across countries, and the long-term effect is 14% fall. The median maximum short-term effect is 24% drop in consumption, and the median long-term impact is 10% fall.

Panel B shows that the 2.5 and 97.5 percentiles of the consumption impulse responses to a disaster shock are wide in the model's simulations. The large amount of uncertainty at the beginning of a disaster on its impact is also clearly visible in the data, as shown in Nakamura, Steinsson, Barro and Ursua (2013, Figure 3). The large uncertainty is perhaps not surprising. Disasters are rare events. As such, estimating their statistical properties comes with large standard errors.

The remaining parameters govern the various technologies in the economy. We set the curvature parameter in the production function, $\xi = 0.65$, per Hennessy and Whited (2007). The monthly depreciation rate, δ, is 0.01, which implies an annual rate of 12%, as estimated by Cooper and Haltiwanger (2006). The persistence, ρ_z, and conditional volatility, σ_z, of the firm-specific productivity are set to be 0.985 and 0.5, respectively, which are somewhat larger than the values in Zhang (2005) after adjusting for the curvature parameter ξ. We do so to ensure a sufficient amount of the cross-sectional dispersion of firms. The long-run mean of log firm-specific productivity, $\bar{z}$, is -8.52 to scale the long-run average detrended capital around unity in simulations.

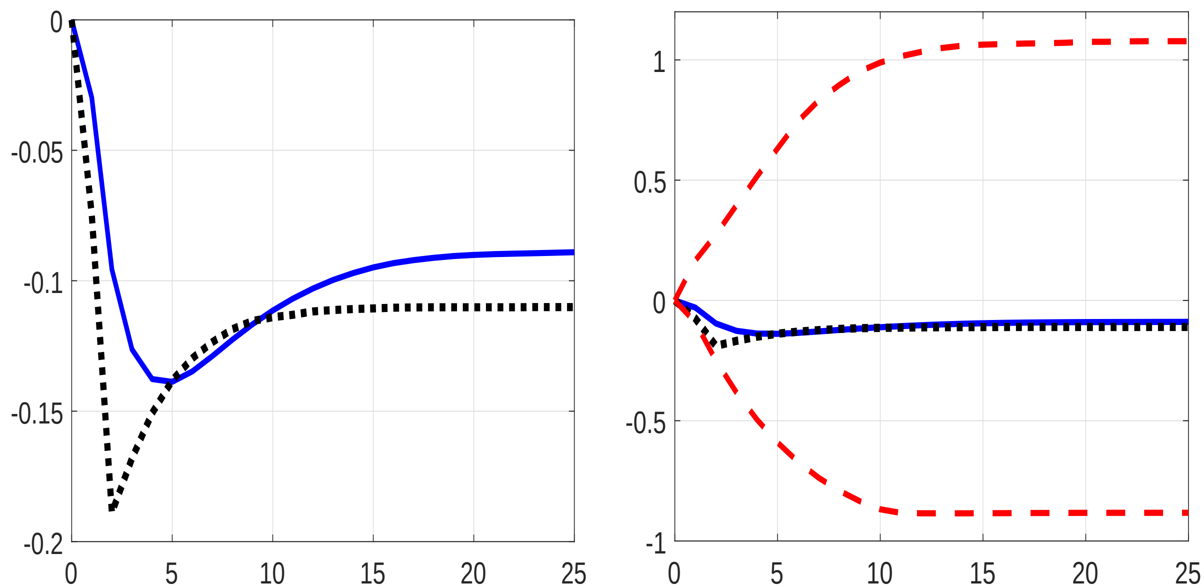
We set the liquidation parameter, $s = 0$, implying that shareholders receive nothing in bankruptcy. We set the reorganizational cost parameter, κ, to 0.25, and the adjustment cost

Figure 3 : The Impulse Response of Consumption to a Disaster Shock in the Model

In simulated data, when the economy enters the disaster state, we calculate the cumulative fractional drop in consumption for 25 years after the impulse. The impulse responses are based on more than 28,000 disaster episodes. Consumption is time-aggregated from the monthly to annual frequency. The blue solid line is the mean impulse response, the black dotted line is the median, and the two red broken lines are the 2.5 and 97.5 percentiles in the simulations.

Panel A: Without 2.5 and 97.5 percentiles

Panel B: With 2.5 and 97.5 percentiles



parameters $a^+ = 0.035$, $a^- = 0.05$, $c^+ = 75$, $c^- = 150$, and the fixed costs parameter, $f = 0.005$. Because of the lack of evidence on their values, we calibrate these parameters to the properties of the book-to-market deciles, and conduct extensive comparative statics to quantify their impact (Section 4.3.3). Finally, Hou, Xue, and Zhang (2017) report that the average delisting return is -12.33% in the CRSP database. Accordingly, we set the delisting return in the model, $\widetilde{R}$, to the same value.

Table 7 reports the basic moments of aggregate output, consumption, and investment growth rates both in the data and in the model. Output in the data is per capita gross domestic product in chained dollars from NIPA Table 7.1, consumption per capita consumption expenditures on nondurables plus services in chained dollars from NIPA Table 7.1, and investment real nonresidential gross private, fixed domestic investment from NIPA Table 1.1.3, scaled by population series from NIPA Table 7.1. The data sample with disasters is annual from 1930 to 2016, and the data sample without disasters is quarterly from the second quarter of 1947 to that of 2017.

To calculate the model moments, we simulate 2,000 artificial samples, each with 30,000 firms and 2,000 months. Because we need to compute consumption moments, we simulate a large number of firms, 30,000, which is necessary to ensure convergence in the laws of motion in the Krusell-Smith algorithm (Appendix A). We start each simulation by setting the initial capital stocks of all firms to unity and the initial log firm-specific productivity levels to its long-run mean, $\bar{z}$. We drop the first 944 months to neutralize the impact of the initial condition. The remaining 1,056 months of simulated data are treated as from the model's stationary distribution. The sample size is comparable with the annual sample from 1929 to 2016 for output, consumption, and investment in the data.

When at least one disaster is realized in an artificial sample, we time-aggregate the 1,056 months into 88 annual observations. Time-aggregation means that we add up 12 months within a given year, and treat the sum as the year's observation. On artificial samples with no disasters, we time-aggregate the initial 846 months into 282 quarters to be consistent with the quarterly sample from the first quarter of 1947 to the second quarter of 2017 in the data. Out of the 2,000 artificial samples, 1,688 have at least one disaster, and the remaining 312 have none. As such, the frequency of having 1,056 months (88 years) with at least one disaster episode is $1,688/2,000 = 84.4\%$.[3]

From Panel A of Table 7, the output volatility in the model is close to that in the data, 4.41% versus 4.79% per annum, with disasters, but lower, 0.5% versus 0.94% per quarter, without disasters. The first-order autocorrelation of output growth is somewhat higher in the model than that in the data, 0.69 versus 0.54, with disasters, and 0.43 versus 0.37, without disasters. The autocorrelations turn negative at the 4- and 5-year horizons in the data, but remain positive in the model.

Panel B shows that the consumption volatility in the model is close to that in the data, 0.46% versus 0.5% per quarter, without disasters, but higher, 4.28% versus 2.13% per annum, with disas-

[3]The relatively high frequency of the disaster samples out of 2,000 artificial samples is consistent with the low disaster probability of only 2% per year. The crux is that we count a (long) sample as a disaster sample if it contains at least one disaster episode. Intuitively, if a disaster occurs with a probability of p in any given period, the chance of observing no disasters in a given sample is $(1-p)^T$, in which T is the sample length. The probability with at least one disaster in the sample is $1-(1-p)^T$. With our monthly calibration, this probably is $1-(1-0.02/12)^{1,056} = 82.8\%$.

Table 7 : Basic Moments of Log Output, Consumption, and Investment Growth

The data moments in samples with disasters are based on the annual sample from 1930 to 2016, and those in samples without disasters on the quarterly sample from the second quarter of 1947 to the second quarter of 2017. "Vol" denotes volatility, "Skew" skewness, and "Kurt" kurtosis. The volatilities in samples without disasters are in quarterly percent, and the volatilities in samples with disasters in annual percent. Output in the data is per capita gross domestic product in chained dollars from NIPA Table 7.1, consumption per capita consumption expenditures on nondurables plus services in chained dollars from NIPA Table 7.1, and investment real nonresidential gross private, fixed domestic investment from NIPA Table 1.1.3, scaled by population series from NIPA Table 7.1. ρ_i is the ith-order autocorrelation. The model moments in the columns denoted "mean" are averaged across 2,000 samples, each with 30,000 firms and 2,000 months. Columns denoted "2.5%," "50%," and "97.5" report 2.5, 50, and 97.5 percentiles across the simulations. The p-value (p-val) is the percentage with which a given model moment is larger than its data counterpart.

	Samples with disasters, annual							Samples without disasters, quarterly					
	Data	mean	2.5%	50%	97.5%	p-val		Data	mean	2.5%	50%	97.5%	p-val
Panel A: Output growth													
Vol	4.79	4.41	1.37	4.26	8.50	0.41		0.94	0.50	0.44	0.49	0.65	0.00
Skew	−0.29	−1.89	−4.32	−2.09	2.07	0.15		−0.18	0.02	−0.32	−0.02	1.02	0.88
Kurt	6.14	11.43	2.95	9.54	27.52	0.78		4.51	3.05	2.41	2.90	5.11	0.04
ρ_1	0.54	0.69	0.27	0.73	0.93	0.80	ρ_1	0.37	0.43	0.30	0.42	0.63	0.82
ρ_2	0.19	0.38	−0.15	0.40	0.82	0.74	ρ_4	−0.07	0.11	−0.06	0.09	0.35	0.99
ρ_3	−0.14	0.23	−0.22	0.21	0.72	0.92	ρ_8	−0.02	0.07	−0.09	0.06	0.26	0.82
ρ_4	−0.34	0.14	−0.26	0.12	0.62	0.99	ρ_{12}	−0.12	0.05	−0.10	0.04	0.24	0.99
ρ_5	−0.19	0.09	−0.25	0.07	0.53	0.94	ρ_{20}	0.05	0.02	−0.13	0.02	0.19	0.35
Panel B: Consumption growth													
Vol	2.13	4.28	1.30	4.13	8.28	0.87		0.50	0.46	0.40	0.45	0.60	0.09
Skew	−1.48	−1.93	−4.42	−2.14	2.13	0.32		−0.41	0.02	−0.31	−0.03	1.14	0.99
Kurt	8.09	11.66	2.98	9.63	28.82	0.63		4.17	3.10	2.44	2.93	5.83	0.04
ρ_1	0.48	0.69	0.24	0.74	0.93	0.85	ρ_1	0.31	0.44	0.31	0.44	0.66	0.97
ρ_2	0.18	0.39	−0.15	0.42	0.83	0.75	ρ_4	0.10	0.13	−0.05	0.12	0.39	0.61
ρ_3	−0.05	0.24	−0.22	0.23	0.72	0.86	ρ_8	−0.02	0.08	−0.08	0.08	0.30	0.86
ρ_4	−0.19	0.16	−0.24	0.13	0.63	0.95	ρ_{12}	0.08	0.06	−0.10	0.05	0.28	0.35
ρ_5	0.00	0.10	−0.24	0.08	0.55	0.70	ρ_{20}	−0.04	0.03	−0.13	0.03	0.21	0.83
Panel C: Investment growth													
Vol	13.53	19.56	3.10	12.28	71.84	0.45		2.40	1.09	0.98	1.08	1.33	0.00
Skew	−1.33	−0.17	0.02	−1.56	2.69	0.68		−0.53	−0.20	−0.58	−0.20	0.25	0.96
Kurt	7.07	27.45	6.68	19.50	100.98	0.96		4.73	3.70	2.85	3.41	5.26	0.03
ρ_1	0.41	0.18	0.00	0.23	0.59	0.17	ρ_1	0.46	0.24	0.11	0.24	0.38	0.01
ρ_2	−0.15	−0.06	0.00	0.00	−0.44	0.71	ρ_4	−0.03	−0.00	−0.12	−0.01	0.14	0.63
ρ_3	−0.33	−0.07	0.00	0.00	0.38	0.96	ρ_8	−0.18	−0.01	−0.12	−0.01	0.11	1.00
ρ_4	−0.17	−0.06	−0.00	0.00	−0.07	0.84	ρ_{12}	−0.09	−0.01	−0.13	−0.01	0.11	0.90
ρ_5	−0.05	−0.05	−0.00	−0.05	−0.06	0.57	ρ_{20}	0.03	−0.00	−0.12	0.00	0.11	0.29

ters. The consumption growth is negatively skewed and fat-tailed both in the data and in the model, with disasters. Without disasters, the autocorrelation structure of the consumption growth in the model resembles that in the data. Except for the 1-quarter autocorrelation, which is somewhat higher in the model than in the data, 0.44 versus 0.31, none of the p-values at longer lags indicate incompatibility between the data and model autocorrelations. With disasters, the autocorrelations are somewhat higher in the model than in the data, but none of the p-values indicate incompatibility.

Finally, Panel C shows that the investment volatility in the model is higher than that in the data, 19.6% versus 13.5% per annum, with disasters, but lower, 1.1% versus 2.4% per quarter, without disasters. The aggregate investment growth is more autocorrelated in the data than in the model. The first-lag autocorrelation is 0.41 in the long annual sample, but only 0.18 in the model's disaster samples. The first-lag autocorrelation is 0.46 in the short quarterly sample in the data, but 0.24 in the model's samples without disasters. Investment growth is negatively autocorrelated at longer lags in annual samples with disasters, but largely uncorrelated in quarterly samples without disasters.

For aggregate asset pricing moments, it is customary in the disaster literature to match international data (Barro 2006). In particular, Petrosky-Nadeau, Zhang, and Kuehn (2018) compile a historical cross-country panel of real stock market returns and real interest rates by drawing from Global Financial Data and an updated Dimson-Marsh-Staunton (2002) dataset obtained from Morningstar. Petrosky-Nadeau et al. report that the equity premium is on average 6.6% per annum across countries, ranging from 3.66% in the United Kingdom to 9.66% in Japan. The real interest rate is on average 1%, ranging from -2.44% in Austria to 3.5% in Denmark. The stock market volatility is on average 25.6%, and the real interest rate volatility 12.32%. The high volatilities in historical data are mostly due to sovereign default, which is abstracted from our model.

In simulations, our model implies an average equity premium of 9.6%, with a 95% confidence interval of $[8.5\%, 10.2\%]$, and an average interest rate of 2.6%, with a confidence interval of $[0.15\%, 4.15\%]$. The interest rate volatility is 0.8%, as the intertemporal elasticity of substitution, ψ, is 1.5. More important, the stock market volatility is only 7.7% in the model. This lower volatility than that in the data is in line with Barro (2006, 2009). Introducing the time-varying disaster probability per Gourio (2012) and Wachter (2013) can fix this weakness. Alas, doing so would add one more state, and increase the computational burden exponentially. More important, introducing an extra aggregate state will most likely strengthen the model's ability to explain the failure of the CAPM, which is our main focus. We opt to achieve this goal with a more parsimonious model.

4.2 Key Properties of the Competitive Equilibrium

Before we present detailed quantitative results on the cross section, we characterize key equilibrium properties by presenting key variables on the numerical grid and across the book-to-market deciles.

4.2.1 Optimal Policy Functions

Figure 4 uses the model's solution on the $\widehat{K}_{it}$-z_{it}-g_t-$\overline{K}_t$ grid to plot the optimal investment-to-capital ratio, $\widehat{I}_{it}/\widehat{K}_{it}$, against the detrended capital, $\widehat{K}_{it}$, and the log firm-specific productivity, z_{it}. Panel A makes the plot in the disaster state, with the demeaned aggregate productivity growth, g_t, set to the disaster size, λ_D. To examine the impact of disasters, Panel B plots the difference between $\widehat{I}_{it}/\widehat{K}_{it}$, when $g_t = 0$ (the mean of normal states), and $\widehat{I}_{it}/\widehat{K}_{it}$ when $g_t = \lambda_D$. In both panels, the cross-sectional average detrended capital, $\overline{K}_t$, is set to be the median on its grid.

Panel A shows that the optimal investment-to-capital ratio, $\widehat{I}_{it}/\widehat{K}_{it}$, rises with firms-specific productivity. Intuitively, more productive firms have higher shadow value of capital, and consequently invest more. In addition, $\widehat{I}_{it}/\widehat{K}_{it}$ decreases with capital. This pattern is a result of decreasing returns to scale in the production function in equation (5).

Panel B shows that the disaster risk affects the investment policy the most for firms that are close to the exit boundary. For these firms, the differences in the optimal investment-to-capital ratio between the mean normal state and the disaster state are most visible.

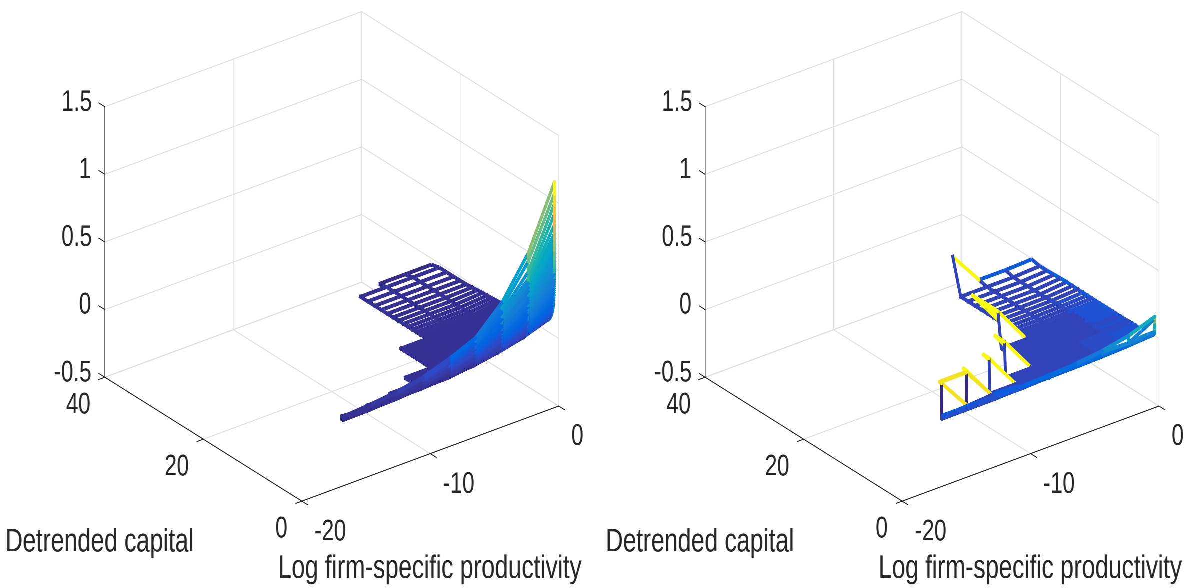
In both panels, only a portion of the $\widehat{K}_{it}$-z_{it} grid is plotted. This missing region is exactly where

Figure 4 : Optimal Policy Functions in the Model

Based on the model's competitive equilibrium, Panel A plots the investment-to-capital ratio, $\widehat{I}_{it}/\widehat{K}_{it}$, on the $\widehat{K}_{it}$-z_{it}-g_t-$\overline{K_t}$ grid, when the demeaned aggregate productivity growth, g_t, is set to be the disaster size, λ_D. $\widehat{K}_{it}$ is the detrended firm-level capital, z_{it} log firm-specific productivity, and $\overline{K_t}$ the cross-sectional average detrended capital (which is set to be the median of its grid). Panel B plots the investment-to-capital ratio when g_t is set to be zero (the mean normal state) minus the investment-to-capital ratio in the disaster state.

Panel A: $\widehat{I}_{it}/\widehat{K}_{it}$ in the disaster state

Panel B: The difference in $\widehat{I}_{it}/\widehat{K}_{it}$ between the mean normal state and disaster state



firms exit the economy. Naturally, firms with low firm-specific productivity are more likely to exit than firms with high firm-specific productivity. In addition, because the fixed costs of production are proportional to capital, firms with more capital have to pay higher costs than firms with less capital to stay in production. As such, high-$\widehat{K}$ firms are more likely to exit than low-$\widehat{K}$ firms.

4.2.2 Risk and Risk Premiums

Figure 5 plots the true beta, β_{it}^M, and the expected risk premium, $E_t[R_{it+1}] - r_{ft}$, against the detrended capital, $\widehat{K}_{it}$, and the log firm-specific productivity, z_{it}, for two values of the detrended aggregate productivity growth, g_t, the disaster state, λ_D, and the mean normal state (zero). The cross-sectional average detrended capital, $\overline{K_t}$, is set to be the median of its grid.

Panel A shows that in the disaster state, firms that are close to the exit boundary, such as low-z firms, are substantially riskier than firms that are far away from the exit boundary, such as high-z firms. Accordingly, Panel B shows that low-z firms earn substantially higher risk premiums than high-z firms in the disaster state. In sharp contrast, Panels C and D show that risk and risk premiums are largely flat across firms in the mean normal state.

Intuitively, the economic mechanism is similar qualitatively to, but more powerful quantitatively than, the asymmetry mechanism in Zhang (2005). Because of asymmetric adjustment costs, low-z firms are burdened with more unproductive capital, finding it more difficult to downsize than high-z firms. As such, low-z firms are riskier than high-z firms in disasters. In contrast, in normal times, even low-z firms do not have strong incentives to disinvest. As such, the asymmetry mechanism fails to take strong effect, giving rise to weak spreads in risk and risk premiums across firms. While Zhang describes the working of this mechanism in recessions, we turbocharge it via disasters.

This asymmetry mechanism is related to our modeling of disasters as large drops in the aggregate productivity growth. Besides productivity disasters, Gourio (2012) also models disasters via capital destruction, which would seem to weaken the asymmetry mechanism in Figure 5. However, while capital destruction is realistic for wars, it is less obvious for economic disasters. Because we aim to explain the stylized facts (Section 2) that feature the important impact of the Great Depression, which is an economic disaster, we opt not to model capital destruction. More important, Gourio motivates capital destruction in disasters as large, negative shocks on the "quality" of capital: "Perhaps it is not the physical capital but the intangible capital (customer and employee value) that is destroyed during prolonged economic depressions (p. 2740)." The accumulation of a large quantity of capital with deteriorating quality in disasters likely strengthens our mechanism.

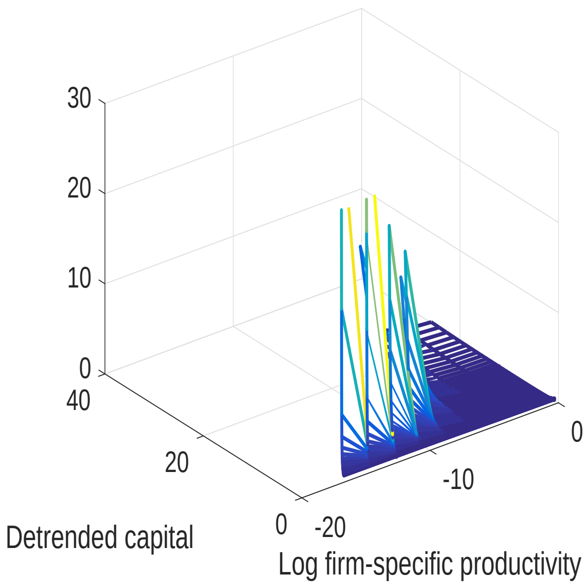
4.2.3 Value versus Growth

To shed light on the key properties of the book-to-market deciles, we simulate 2,000 artificial samples, each with 5,000 firms and $2,000$ months. We start each simulation by setting the initial capital

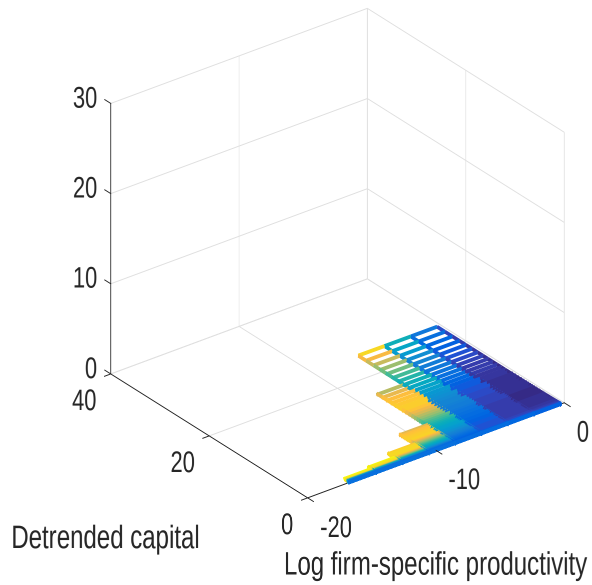
Figure 5 : Disaster Risk and Risk Premiums in the Model

This figure plots the true beta, β_{it}^M, and the expected risk premium, $E_t[R_{it+1}] - r_{ft}$, on the $\widehat{K}_{it}$-z_{it}-g_t-$\overline{K_t}$ grid. $\widehat{K}_{it}$ is the detrended firm-level capital, z_{it} log firm-specific productivity, g_t the demeaned aggregate productivity growth, and $\overline{K_t}$ the cross-sectional average detrended capital. We set g_t to be the disaster size, λ_D, in Panels A and C, and set g_t to be zero, which is the mean of the normal states, in Panels B and D. $\overline{K_t}$ is set to be the median of its grid in all panels.

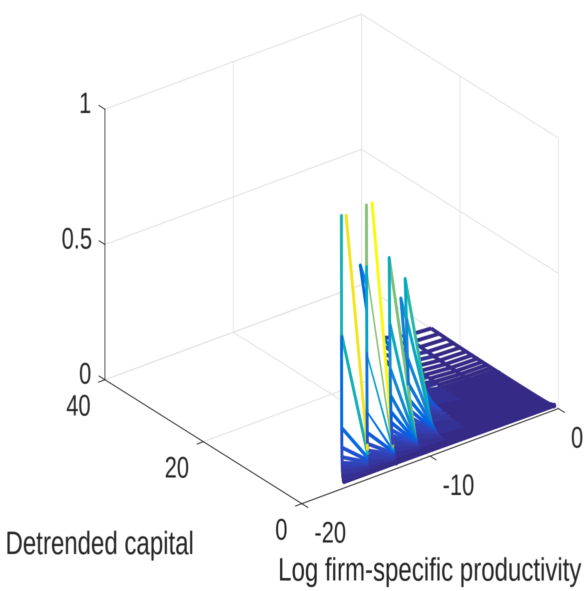
Panel A: True beta, β_{it}^M, the disaster state



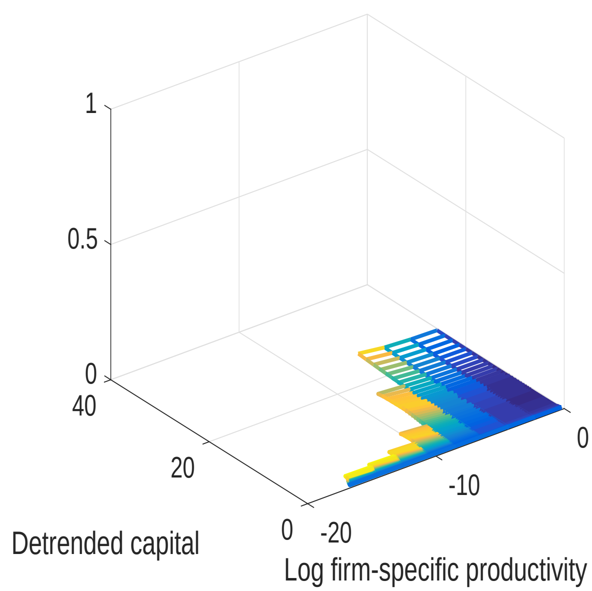
Panel B: True beta, β_{it}^M, the mean normal state



Panel C: Expected risk premium, $E_t[R_{it+1}] - r_{ft}$, the disaster state



Panel D: Expected risk premium, $E_t[R_{it+1}] - r_{ft}$, the mean normal state



stocks of all firms at unity and the initial log firm-specific productivity to its long-run mean, $\bar{z}$. We drop the first 908 months to neutralize the impact of the initial condition, and treat the remaining $1,092$ months as from the economy's stationary distribution. The sample size is comparable to the period from July 1926 to June 2017 in the data. We calculate the model moments on each artificial sample, and report cross-simulation averaged results. To demonstrate the impact of disasters, we calculate cross-simulation averages separately on samples with and without disasters.

Figure 6 reports the results based on 2,000 artificial samples. From Panel A, value firms with high book-to-market have about 4.5 times more capital than growth firms with low book-to-market. All firms have slightly more capital in the disaster samples than in the no-disaster samples, but the basic pattern across value and growth holds with and without disasters.

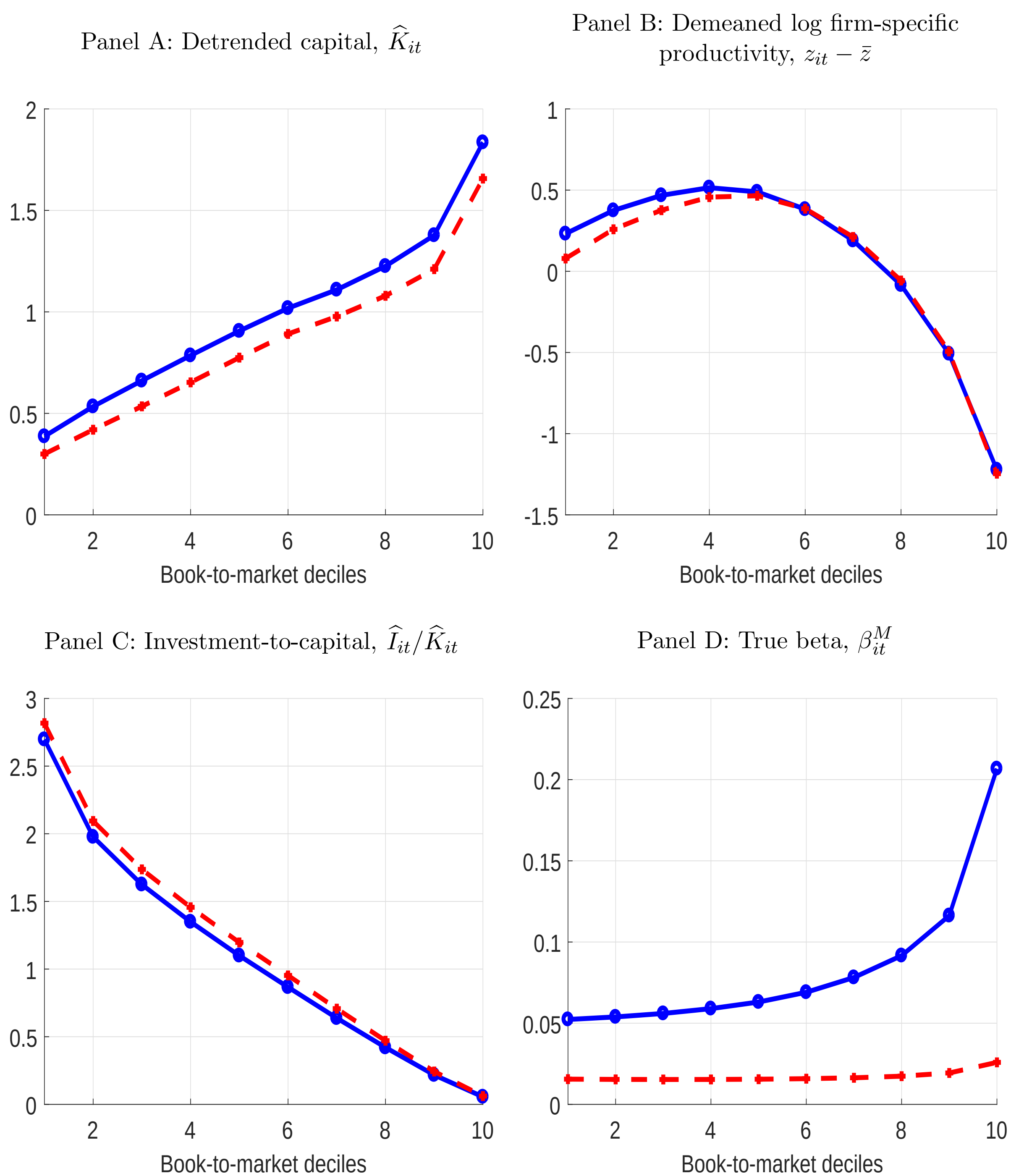
Moving to the log firm-specific productivity, z_{it}, which is the other firm-specific state variable besides the detrended capital, $\widehat{K}_{it}$, Panel B shows that value firms have much lower firm-specific productivity than growth firms. The conditional volatility of z_{it} is 0.5. As such, the average z_{it} of the value decile is almost 2.5 conditional volatilities below its unconditional mean of $\bar{z}$. Depending on whether disasters are realized in a given sample or not, the average z_{it} of the growth decile can be above $\bar{z}$ by up to one half of the conditional volatility. In total, the difference in the average z_{it} between the extreme deciles is about three conditional volatilities of z_{it} in the disaster samples.

Panel B also shows that the relation between z_{it} and book-to-market is not monotonic: z_{it} rises from the growth decile to decile four, and then drops at an increasing rate from decile four to the value decile. The crux is that, as noted, the detrended capital, $\widehat{K}_{it}$, is another firm-specific state variable. The growth decile contains firms that have the lowest $\widehat{K}_{it}$ but relatively high z_{it} levels. At the other extreme, the value decile contains firms that have the highest $\widehat{K}_{it}$ but the lowest z_{it} levels.

From Panel C, growth firms have higher investment-to-capital ratios, $\widehat{I}_{it}/\widehat{K}_{it}$, than value firms. With disasters, the average $\widehat{I}_{it}/\widehat{K}_{it}$ of the value decile is only 0.06% per month, whereas the average $\widehat{I}_{it}/\widehat{K}_{it}$ of the growth decile is 2.7% per month. The relation between $\widehat{I}_{it}/\widehat{K}_{it}$ and book-to-market

Figure 6 : Properties of the Book-to-market Deciles in the Model

Results are based on 2,000 simulated economies, each with 5,000 firms and 2,000 months. We drop the first 908 months, and treat the remaining 1,092 months as from the model's stationary distribution. On each artificial sample, we form the book-to-market deciles. The growth decile is denoted "1," and the value decile "10." In each panel, the blue solid line with circles is averaged across samples with disasters, and the red broken line with pluses across samples without disasters. The investment-to-capital ratio, $\widehat{I}_{it}/\widehat{K}_{it}$, is in monthly percent.



is strictly monotonic. Firms invest more in the no-disaster samples than the disaster samples, but the difference is small, relative to the cross-sectional dispersion across the book-to-market deciles.

Most important, Panel D shows that risk dynamics differ drastically across the disaster and no-disaster samples. Without disasters, the red broken line shows that the true beta, β_{it}^M, is largely flat across the book-to-market deciles. In sharp contrast, with at least one disaster episode, the true beta rises monotonically, with an increasing speed, with book-to-market. The true beta starts at 0.05 for the growth decile, increases to 0.06 for decile five, to 0.12 for decile nine, and then drastically to 0.21 for the value decile. As such, the relation between the true beta and book-to-market is convex.

4.3 Explaining the Performance of the CAPM

Based on 2,000 artificial samples, Table 8 reports the quantitative results on the CAPM regressions under the benchmark calibration. Panel A shows the results in the samples with at least one disaster episode. The value premium is on average 0.46% per month, which is close to 0.48% in the data (Table 1). However, its t-value in the model is 4.92, which is large relative to 2.63 in the data. Similarly, the t-values for the deciles are often more than three times larger than those in the data, consistent with lower return volatilities in the model than those in the data.

In samples with disasters, Panel A shows that the market beta of the high-minus-low decile is high, 1.01 ($t = 7.85$). The increasing relation between the market beta and book-to-market is largely monotonic, rising from 0.83 for the low decile to 1.84 for the high decile. The market beta spread is large enough to make the CAPM alpha of the high-minus-low decile negative, -0.35% per month ($t = -2.44$). However, the alpha estimate of 0.19% in the data lies outside the model's 95% confidence interval, and so is its t-value of 0.99 in the data.

More important, Panel B shows that the model is capable of explaining the failure of the CAPM in accounting for the value premium in samples without disasters. Averaged across samples without disasters, the high-minus-low decile earns on average 0.4% per month, which is not far from 0.47% in the 1963–2017 sample. In addition, the CAPM fails in the no-disaster samples. The CAPM

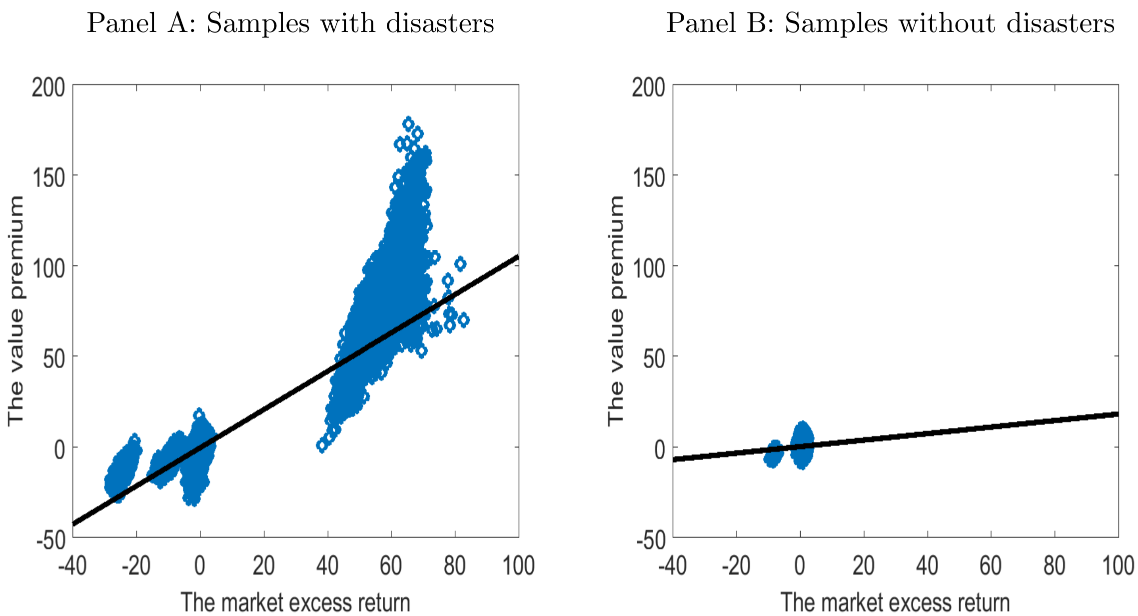
Table 8 : The CAPM Regressions for the Book-to-market Deciles in the Model

Results are based on 2,000 simulated economies, each with 5,000 firms and 2,000 months. We drop the first 908 months, and treat the remaining 1,092 months as from the model's stationary distribution. The mean excess returns, denoted m, and the CAPM alphas are in monthly percent. We report the cross-simulation averaged results, as well as the 2.5 and 97.5 percentiles for the alphas, betas, and their t-statistics that are adjusted for heteroscedasticity and autocorrelations.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: Samples with disasters										
m	0.75	0.74	0.74	0.74	0.75	0.77	0.81	0.86	0.96	1.20	0.46
t_m	11.17	10.95	10.73	10.50	10.29	10.02	9.83	9.59	9.29	8.94	4.92
α	0.08	0.06	0.04	0.03	0.01	−0.02	−0.05	−0.09	−0.15	−0.27	−0.35
t_α	1.75	1.55	1.22	0.74	0.18	−0.54	−1.10	−1.60	−2.05	−2.32	−2.44
$\alpha, 2.5\%$	−0.03	−0.03	−0.04	−0.05	−0.08	−0.12	−0.19	−0.29	−0.42	−0.70	−0.86
$\alpha, 97.5\%$	0.21	0.16	0.13	0.10	0.08	0.06	0.05	0.04	0.02	0.00	0.00
$t_\alpha, 2.5\%$	−0.84	−0.91	−1.03	−1.53	−2.11	−3.01	−3.63	−4.19	−4.16	−4.33	−4.53
$t_\alpha, 97.5\%$	4.43	3.99	3.56	2.92	2.29	1.90	1.39	1.07	0.57	0.05	0.05
β	0.83	0.85	0.87	0.89	0.93	0.99	1.07	1.19	1.40	1.84	1.01
t_β	35.57	42.36	51.84	69.25	74.28	65.01	53.50	38.76	25.28	18.49	7.85
$\beta, 2.5\%$	0.66	0.73	0.79	0.84	0.87	0.90	0.94	1.00	1.13	1.47	0.52
$\beta, 97.5\%$	0.98	0.96	0.94	0.96	1.04	1.18	1.33	1.57	1.85	2.32	1.61
$t_\beta, 2.5\%$	8.64	12.52	17.67	22.78	18.11	12.68	10.11	8.22	7.10	7.45	3.47
$t_\beta, 97.5\%$	132.89	133.16	145.36	174.58	184.14	169.89	166.37	139.47	77.09	42.45	17.28
R^2	0.77	0.78	0.79	0.79	0.80	0.81	0.83	0.85	0.86	0.87	0.57
	Panel B: Samples without disasters										
m	0.77	0.76	0.75	0.74	0.75	0.76	0.78	0.82	0.91	1.16	0.40
t_m	23.37	23.02	22.48	22.05	22.08	21.79	22.75	23.93	25.51	28.69	7.72
α	0.10	0.04	−0.02	−0.07	−0.10	−0.13	−0.07	0.02	0.13	0.35	0.25
t_α	1.46	0.57	−0.22	−0.99	−1.37	−1.80	−0.93	0.32	1.83	4.25	2.26
$\alpha, 2.5\%$	−0.04	−0.09	−0.16	−0.20	−0.24	−0.26	−0.20	−0.12	−0.00	0.17	0.02
$\alpha, 97.5\%$	0.25	0.18	0.12	0.08	0.05	0.00	0.06	0.16	0.27	0.51	0.49
$t_\alpha, 2.5\%$	−0.55	−1.21	−2.21	−2.82	−3.24	−3.62	−2.78	−1.63	−0.01	1.77	0.18
$t_\alpha, 97.5\%$	3.61	2.68	1.68	1.16	0.88	0.02	0.88	2.46	3.87	6.61	4.37
β	0.83	0.90	0.96	1.02	1.06	1.10	1.06	1.00	0.97	1.01	0.18
t_β	11.04	11.91	12.60	13.23	13.69	14.06	13.58	12.94	11.89	10.64	1.44
$\beta, 2.5\%$	0.67	0.74	0.81	0.86	0.89	0.97	0.90	0.84	0.80	0.80	−0.09
$\beta, 97.5\%$	0.98	1.05	1.11	1.18	1.23	1.24	1.22	1.15	1.13	1.20	0.47
$t_\beta, 2.5\%$	8.56	9.05	10.40	10.63	11.07	11.51	10.92	10.29	9.55	7.66	−0.70
$t_\beta, 97.5\%$	14.75	16.68	15.84	16.53	16.60	17.57	17.53	17.10	15.10	13.49	3.59
R^2	0.10	0.12	0.13	0.14	0.15	0.16	0.15	0.13	0.12	0.10	0.00

Figure 7 : The CAPM Regressions of the Value Premium in the Model

The value premium is the value-minus-growth decile return. The market excess return is the market portfolio return value-weighted from all the firms minus the interest rate. Based on 2,000 simulations from the model, this figure reports the scatter plot and the fitted line from regressing the value premium on the market excess return. The fitted line in Panel A are averaged across the samples with disasters, and that in Panel B across the samples without disasters. Both the value premium and market excess return are in monthly percent.



regression of the high-minus-low decile yields an alpha of 0.25% ($t = 2.26$). The 95% confidence interval for the alpha spans from 0.02% to 0.49%, and the interval for its t-value from 0.18 to 4.37. As such, the alpha estimate of 0.43% ($t = 1.89$) in the data lies well within the model's distribution. Also, the market beta for the high-minus-low decile is small, 0.18 ($t = 1.44$), which is not far from 0.07 ($t = 0.86$) in the data (Table 1). Finally, the R^2 is in effect zero.

4.3.1 Nonlinearity in the CAPM Regressions

To shed light on the driving force behind our key results in Table 8, Figure 7 reports the scatter plots of the CAPM regressions of the value-minus-growth decile in the model. Panel A is the scatter plot from stacking the disaster samples that underlie Panel A in Table 8, and Panel B the scatter plot from stacking the no-disaster samples that underlie Panel B in Table 8.

The basic patterns in Figure 7 resemble those in Figure 1 in the U.S. sample. From Panel A of Figure 7, the value-minus-growth return covaries strongly with the market excess return in the disaster samples. Both returns are large and negative in disasters, and large and positive in the subsequent recoveries. As a result, the market beta for the value-minus-growth decile is 1.06, which is a population moment because of the large number of simulations. However, the CAPM alpha is -0.39% per month, implying that the unconditional CAPM does not hold approximately in our dynamic single-factor model. In contrast, Panel B shows that the value-minus-growth return does not covary much with the market excess return in the no-disaster samples. Without the large swings in the same direction in the value-minus-growth return and market excess return during disasters and subsequent recoveries, the CAPM regression line is largely flat, resembling the 1963–2017 U.S. evidence (Figure 1). The market beta is only 0.18, and the CAPM alpha is 0.25% per month.[4]

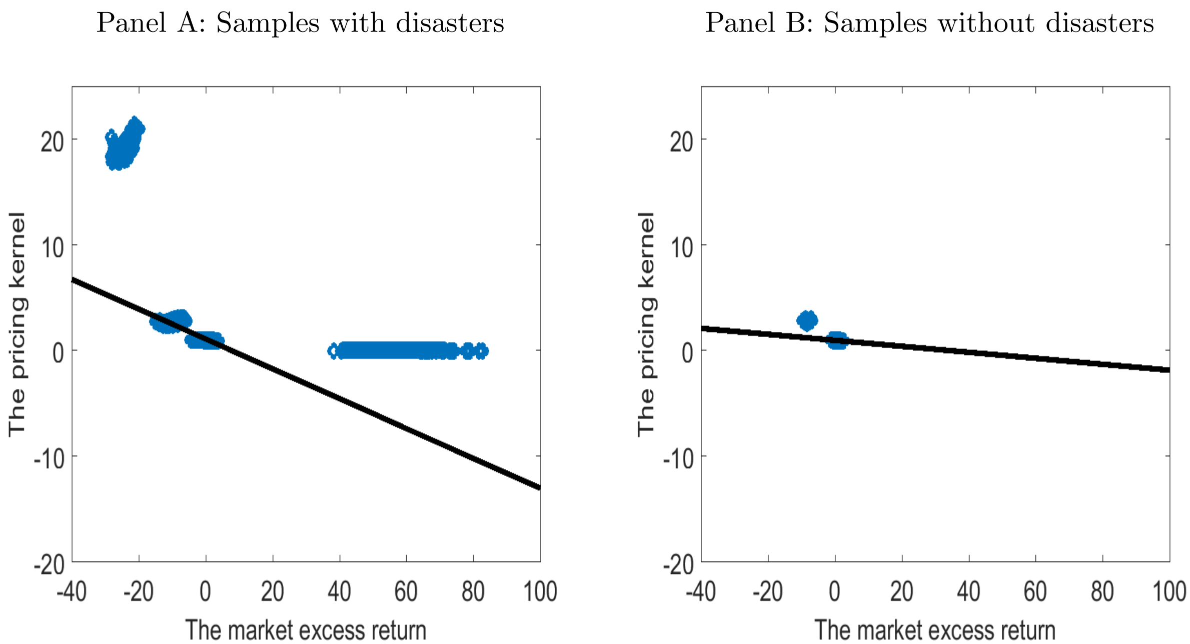
4.3.2 Nonlinearity in the Pricing Kernel

The disaster risk induces strong nonlinearity in the pricing kernel, making the CAPM a poor proxy of the pricing kernel. If the CAPM holds exactly, the pricing kernel can be expressed as a linear function of the market excess return, R_{Mt+1}, i.e., $M_{t+1} = l_0 + l_1 R_{Mt+1}$, in which l_0 and l_1 are constants (Cochrane 2005). Figure 8 shows that the pricing kernel in the model is far from a linear function of the market excess return. Panel A reports the scatter plot for regressing the pricing kernel on the market excess return based on the disaster samples. The regression yields an intercept of 1.22, a slope of -0.14, but an R^2 of only 21%, despite the model's single-factor structure. The linear CAPM fits poorly the observations from the disaster state, with high realizations of the pricing kernel with marginal utilities, and the observations from the recovery state, with low realizations of the pricing kernel. From Panel B, the CAPM is an even worse proxy for the pricing kernel in the no-disaster samples. The regression slope is only -0.03, although the R^2 is 23%

[4] The scatter plot in Panel A of Figure 7 shows three large blocks, with the left, middle, and right block from disasters, normal times, and recoveries, respectively. The discreteness arises because we simulate from the discrete Markov chain with the transition probabilities in equation (13). If we use a sufficiently large number of grid points for the normal states, and also simulate the economy for a sufficiently long period to ensure remote grid points are visited, the discreteness would disappear. However, because the persistence in the aggregate demeaned productivity growth, ρ_g, is relatively low, 0.6, a five-point grid is sufficient to ensure accuracy for simulated moments.

Figure 8 : The Pricing Kernel versus the CAPM in the Model

The pricing kernel is given by $M_{t+1} = \iota \exp\left[-\left(\widehat{g}_{ct+1} + \overline{g} + g_t\right)/\psi\right]\left[\frac{\exp[(1-\gamma)(\widehat{u}_{t+1}+\widehat{g}_{ct+1})]}{E_t[\exp[(1-\gamma)(\widehat{u}_{t+1}+\widehat{g}_{ct+1})]]}\right]^{\frac{1/\psi-\gamma}{1-\gamma}}$, in which $\widehat{g}_{ct+1}$ is the detrended consumption growth, $\overline{g}$ is the balanced growth rate, g_t the demeaned aggregate productivity growth, and $\widehat{u}_t$ the log utility-to-consumption ratio (equation 27). The market excess return in monthly percent is the value-weighted market return minus the interest rate. Based on 2,000 simulations, this figure reports the scatter plot and the fitted line from regressing the pricing kernel on the market excess return. The fitted line in Panel A are averaged across the samples with disasters, and that in Panel B across the samples without disasters.



(because of missing large outliers). As such, the CAPM fails badly in the no-disaster samples.

4.3.3 Comparative Statics

To gain further insights into the economic mechanism, we conduct comparative statics on a wide array of parameters. We group the parameters into three categories: (i) disaster dynamics: the disaster size, λ_D, the disaster persistence, θ, the disaster probability, η, the recovery persistence, ν, and the recovery size, ν; (ii) technology: the adjustment costs parameters, a^+, a^-, c^+, and c^-, the curvature in production, ξ, the fixed costs parameter, f, the liquidation parameter, s, the reorganization costs, κ, and the delisting return, $\widetilde{R}$; as well as (iii) preferences: the risk aversion, γ, and the intertemporal elasticity of substitution, ψ. In each experiment, we only vary one parameter, while keeping all the others unchanged from the benchmark calibration. Finally, for each parameter, we consider two values, with one above and the other below the benchmark value. The only exception is s, for which we consider two values, 0.15 and 0.3. Both are higher than the benchmark value of zero.

From the first four columns in the upper panel of Table 9, increasing the disaster size and persistence raises the average value premium, and exacerbates the failure of the CAPM in samples without disasters. Intuitively, a larger disaster, or a more persistent disaster, strengthens nonlinear disaster dynamics, making the linear CAPM a poorer proxy for the pricing kernel, especially in normal times. Raising the disaster probability, η, goes in the same direction, but its quantitative impact is small. Intuitively, η mainly determines the percentage of samples with at least one disaster out of 2,000 simulations. However, conditioning on at least one disaster appearing in a given sample, the nonlinear dynamics are mostly governed by the disaster size and persistence.

The recovery size and persistence have little impact on the magnitude of the average value premium and the performance of the CAPM. Intuitively, risk and risk premiums are mostly determined by the dynamics in bad times, particularly disasters, in which the representative agent's marginal utility is the highest. In contrast, the marginal utility is the lowest in the recovery state, giving rise to small spreads in risk and risk premium between value and growth firms.

The upward nonconvex costs parameter, a^+, and its downward counterpart, a^-, work in the opposite direction. While increasing a^+ reduces the average value premium and its CAPM alpha in normal times, increasing a^- does the opposite. Intuitively, the a^- effect works through the asymmetry mechanism. A high value of a^- means that value firms face a higher hurdle in reducing their unproductive capital in the disaster state, giving rise to higher risk and risk premiums.

Why does the upward nonconvex costs parameter, a^+, work differently? Intuitively, firms disinvest very infrequently. Across simulations, on average only 0.6% of the firm-month observations have negative investment. Such a low disinvestment frequency means that a^+ is the main parameter that determines the magnitude of nonconvex adjustment costs, a^+K_{it}. A lower a^+ means that firms

Table 9 : Comparative Statics

Results are averaged across 2,000 simulations. Each column shows results from one experiment. In each column, we vary only one parameter, while keeping the others unchanged from the benchmark calibration. Table 6 describes the symbols. m is the average return, α the CAPM alpha, and β the market beta of the high-minus-low book-to-market decile. m and α are in monthly percent. The t-values are adjusted for heteroscedasticity and autocorrelations.

	λ_D		θ		η		ν		λ_R		a^+		a^-		c^+	
	−2.25%	−3.25%	0.955	0.985	$\frac{1\%}{12}$	$\frac{3\%}{12}$	0.95	0.98	1%	2%	2.5%	4.5%	3.5%	6.5%	50	100
Samples with disasters																
m	0.27	0.75	0.32	0.41	0.41	0.49	0.46	0.45	0.46	0.45	0.43	0.30	0.22	0.53	0.46	0.44
t_m	3.42	6.67	3.87	4.29	4.46	4.87	4.91	4.54	4.86	4.75	4.77	3.68	3.20	5.17	4.86	4.72
α	−0.20	−0.46	−0.20	−0.63	−0.39	−0.32	−0.34	−0.37	−0.34	−0.35	−0.43	−0.25	−0.24	−0.34	−0.36	−0.34
t_α	−2.03	−2.47	−2.06	−2.96	−2.66	−2.34	−2.33	−2.62	−2.33	−2.49	−2.86	−2.10	−2.07	−2.37	−2.26	−2.58
β	0.90	1.09	0.93	1.08	1.00	1.03	1.01	1.00	1.02	0.99	1.11	0.69	0.63	1.08	0.96	1.05
t_β	7.86	8.04	8.43	8.40	8.32	8.08	7.77	8.07	7.72	7.91	7.72	6.85	5.87	7.87	8.05	7.97
R^2	0.52	0.60	0.56	0.56	0.57	0.59	0.56	0.59	0.56	0.49	0.62	0.39	0.36	0.60	0.53	0.60
Samples without disasters																
m	0.22	0.62	0.26	0.53	0.35	0.42	0.39	0.39	0.39	0.39	0.37	0.21	0.16	0.41	0.42	0.37
t_m	4.68	10.84	5.53	9.53	6.93	7.95	7.51	7.52	7.57	7.55	8.41	3.94	3.40	7.95	7.57	7.63
α	0.06	0.51	0.09	0.44	0.20	0.31	0.26	0.26	0.26	0.25	0.36	−0.06	0.03	0.28	0.16	0.31
t_α	0.76	3.26	1.13	3.07	1.78	2.71	2.30	2.28	2.31	2.22	3.38	−0.61	0.33	2.47	1.29	2.98
β	0.31	0.09	0.31	0.09	0.19	0.14	0.16	0.17	0.16	0.17	0.01	0.34	0.17	0.16	0.31	0.08
t_β	2.56	0.68	2.60	0.65	1.54	1.12	1.31	1.35	1.28	1.41	0.08	2.94	1.38	1.23	2.40	0.59
R^2	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.01	0.00

	c^-		ξ		f		s		κ		$\widetilde{R}$		γ		ψ	
	100	200	0.6	0.7	0	0.01	0.15	0.3	0.15	0.35	−16%	−8%	4	6	1	2
Samples with disasters																
m	0.43	0.43	0.39	0.38	0.44	0.51	0.33	0.22	0.51	0.39	0.44	0.46	0.30	0.55	0.40	0.52
t_m	4.80	4.53	3.71	5.09	4.74	5.09	4.41	3.60	5.23	4.31	4.72	4.83	2.88	6.35	6.44	4.55
α	−0.31	−0.39	−0.41	−0.52	−0.38	−0.29	−0.24	−0.19	−0.31	−0.39	−0.36	−0.34	−0.28	−0.40	−0.04	−0.44
t_α	−2.22	−2.65	−2.84	−2.88	−2.66	−2.11	−2.25	−2.08	−2.14	−2.69	−2.49	−2.40	−2.36	−2.41	−0.28	−2.65
β	0.93	1.04	1.00	1.11	1.05	1.00	0.74	0.54	1.03	0.98	1.01	1.01	1.06	0.95	1.87	0.98
t_β	7.54	8.11	7.22	7.66	7.93	7.85	8.07	7.86	7.73	8.10	8.01	7.91	8.21	7.66	7.71	7.89
R^2	0.53	0.58	0.52	0.62	0.59	0.56	0.46	0.33	0.57	0.55	0.57	0.57	0.65	0.48	0.43	0.64
Samples without disasters																
m	0.36	0.40	0.30	0.40	0.38	0.41	0.29	0.22	0.41	0.37	0.39	0.39	0.23	0.52	0.37	0.43
t_m	6.94	7.69	5.07	10.40	7.82	7.46	5.84	4.45	7.90	7.14	7.51	7.59	4.88	9.36	8.68	8.07
α	0.20	0.26	0.14	0.22	0.28	0.23	0.19	0.15	0.27	0.22	0.25	0.27	0.08	0.40	0.21	0.34
t_α	1.81	2.34	1.22	2.08	2.58	1.93	1.73	1.37	2.40	1.98	2.22	2.38	1.02	2.79	4.23	2.75
β	0.19	0.17	0.20	0.21	0.12	0.23	0.13	0.09	0.17	0.18	0.18	0.16	0.29	0.11	0.67	0.10
t_β	1.57	1.37	1.59	1.74	0.98	1.79	1.07	0.76	1.41	1.48	1.40	1.29	2.41	0.86	5.22	0.85
R^2	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.01	0.00	0.03	0.00

would in general have higher capital, especially value firms. When a disaster hits, value firms are burdened with more unproductive capital, reinforcing the asymmetry mechanism. As such, a lower a^+ value increases the average value premium and its CAPM alpha in the no-disaster samples.

Similarly, the upward and downward convex costs parameters, c^+ and c^-, respectively, work in the opposite direction, but their impact is small. A higher c^- works through the asymmetry mechanism by restricting the flexibility of value firms in downsizing in disasters, giving rise to higher risk and risk premiums. However, because of the vast majority of positive investment, the upward parameter, c^+, mainly determines the magnitude of convex costs. A lower c^+ implies that firms have more capital in general, especially value firms, reinforcing the asymmetry mechanism.

Increasing the curvature parameter, ξ, from 0.6 to 0.7 increases the value premium in the no-disaster samples. Increasing the fixed costs parameter, f, raises the value premium, but decreases its CAPM alpha in the no-disaster samples. A higher f means a higher operating leverage for value firms, increasing the value premium (Carlson, Fisher, and Giammarino 2004). However, a higher f also means higher market beta for the value premium, decreasing its CAPM alpha.

The next three technological parameters involve entry and exit, including the liquidation value, s, the reorganization costs, κ, and the delisting return, $\widetilde{R}$. Increasing s reduces the average value premium and its CAPM alpha. Intuitively, with a higher s, in the event of exit, shareholders get to extract a higher liquidation value of sK_{it}, which is in effect a free abandonment option. This option acts as an insurance against the disaster risk. The abandonment option is especially attractive for shareholders of value firms, which tend to have more unproductive capital than growth firms. Consequently, instead of facing asymmetric adjustment costs in disasters, the shareholders opt to exit, thereby reducing the risk for value firms relative to growth firms. A lower reorganization costs parameter, κ, raises the value premium and its CAPM alpha, although its impact is small in the no-disaster samples. The impact of the delisting return, $\widetilde{R}$, is negligible. Finally, increasing either the risk aversion, γ, or the intertemporal elasticity of substitution, ψ, strengthens the nonlinear dynamics, raising the average value premium and its CAPM alpha in the no-disaster samples.

4.4 Explaining the Beta "Anomaly"

Our model also explains the flat beta-return relation. Applying the empirical procedure in Table 3 on artificial samples, we sort stocks at the end of each June based on pre-ranking market betas from prior 60-month rolling windows, calculate monthly value-weighted decile returns for the subsequent year, and rebalance the deciles in June. Panel A of Table 10 shows that in artificial samples with disasters, the high-minus-low decile on the market beta earns on average only 0.06% per month ($t = 0.85$). The pre-ranking market beta sorts also yield a spread in the post-ranking betas, although its magnitude, 0.37 ($t = 2.57$), is smaller than that in the data. The CAPM alpha of the high-minus-low decile is -0.24%, albeit insignificant ($t = -1.74$). From Panel B, the results from the no-disaster samples are quantitatively similar. The high-minus-low decile on the market beta earns on average only -0.02% ($t = -0.48$). Sorting on the pre-ranking beta continues to yield a significant spread in the post-ranking beta, 0.23 ($t = 1.98$). As a result, the CAPM alpha for the high-minus-low beta decile is significantly negative, -0.21% ($t = -1.96$).

It is perhaps surprising that our risk-based model can reproduce the flat beta-return relation in simulations. The crux is that the rolling market beta contains a great deal of measurement errors, and is, consequently, a poor proxy for the true market beta. Because of our single-factor structure, all aggregate variables are conditionally perfectly correlated, including the pricing kernel and the expected market risk premium, $E_t[R_{Mt+1}] - r_{ft}$. As such, the conditional CAPM holds approximately in theory (but not the unconditional CAPM), meaning that the true market beta can be backed out as $(E_t[R_{it+1}] - r_{ft})/(E_t[R_{Mt+1}] - r_{ft})$. The true market beta differs from the true beta, β_{it}^M, which is calculated as $(E_t[R_{it+1}] - r_{ft})/\phi_{Mt}$, with ϕ_{Mt} being the price of risk.

In untabulated results, we show that, not surprisingly, sorting on the true market beta yields large average return spreads across extreme deciles in the model, with and without disasters. In samples with disasters, the average return spread is 1% per month ($t = 5.99$). The unconditional

Table 10 : The CAPM Regressions for the Rolling Market Beta Deciles in the Model

Results are based on 2,000 simulations, each with 5,000 firms and 2,000 months. We drop the first 908 months, and treat the remaining 1,092 months as from the model's stationary distribution. The mean excess returns, denoted m, and the CAPM alphas are in monthly percent. We report the cross-simulation averaged results, as well as the 2.5 and 97.5 percentiles for the alphas, betas, and their t-statistics that are adjusted for heteroscedasticity and autocorrelations.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: Samples with disasters										
m	0.77	0.79	0.81	0.83	0.82	0.85	0.85	0.85	0.85	0.83	0.06
t_m	10.48	10.68	10.54	10.26	9.78	9.83	9.57	9.27	8.69	8.31	0.85
α	0.03	0.05	0.04	0.02	−0.02	−0.03	−0.05	−0.09	−0.16	−0.21	−0.24
t_α	0.70	1.36	1.17	0.46	−0.49	−0.53	−0.91	−1.23	−1.64	−2.15	−1.74
$\alpha, 2.5\%$	−0.12	−0.04	−0.04	−0.06	−0.13	−0.15	−0.22	−0.29	−0.47	−0.55	−0.67
$\alpha, 97.5\%$	0.16	0.15	0.12	0.11	0.09	0.09	0.09	0.08	0.07	0.04	0.11
$t_\alpha, 2.5\%$	−2.92	−1.17	−1.09	−1.75	−3.10	−3.33	−3.80	−4.06	−4.39	−4.78	−4.52
$t_\alpha, 97.5\%$	3.66	3.77	3.26	2.84	2.29	2.33	2.33	2.17	1.84	1.05	1.86
β	0.92	0.92	0.96	1.01	1.05	1.09	1.12	1.16	1.25	1.28	0.37
t_β	35.79	48.38	62.91	74.19	61.90	48.67	41.79	36.71	28.14	20.98	2.57
$\beta, 2.5\%$	0.78	0.84	0.90	0.93	0.94	0.96	0.95	0.95	0.94	0.92	−0.09
$\beta, 97.5\%$	1.12	1.03	1.04	1.08	1.17	1.24	1.31	1.41	1.64	1.72	0.93
$t_\beta, 2.5\%$	9.71	15.76	21.14	21.10	17.95	13.39	9.96	7.94	5.54	5.57	−2.85
$t_\beta, 97.5\%$	134.73	167.31	168.44	192.12	192.23	156.44	157.07	154.49	140.85	79.15	7.00
R^2	0.81	0.81	0.82	0.82	0.82	0.84	0.84	0.84	0.85	0.85	0.21
	Panel B: Samples without disasters										
m	0.78	0.81	0.82	0.83	0.81	0.84	0.83	0.81	0.79	0.76	−0.02
t_m	23.48	23.66	23.62	23.53	21.72	23.38	23.13	22.88	22.50	21.87	−0.48
α	−0.05	0.07	0.11	0.13	0.01	0.13	0.10	0.04	−0.05	−0.25	−0.21
t_α	−0.69	0.99	1.58	1.82	0.16	1.77	1.27	0.53	−0.67	−3.67	−1.96
$\alpha, 2.5\%$	−0.17	−0.06	−0.04	−0.01	−0.14	−0.01	−0.04	−0.14	−0.20	−0.37	−0.39
$\alpha, 97.5\%$	0.07	0.22	0.27	0.29	0.16	0.29	0.25	0.17	0.08	−0.12	−0.02
$t_\alpha, 2.5\%$	−2.43	−0.84	−0.47	−0.13	−1.77	−0.16	−0.55	−1.80	−2.73	−5.62	−3.91
$t_\alpha, 97.5\%$	1.01	3.24	3.75	4.14	2.08	3.96	3.26	2.39	1.36	−1.91	−0.15
β	1.03	0.92	0.88	0.87	0.99	0.88	0.91	0.96	1.05	1.26	0.23
t_β	14.06	12.43	11.32	11.06	11.88	11.17	11.18	12.21	13.84	16.87	1.98
$\beta, 2.5\%$	0.89	0.75	0.72	0.68	0.83	0.69	0.74	0.81	0.88	1.10	−0.00
$\beta, 97.5\%$	1.17	1.08	1.04	1.02	1.15	1.04	1.08	1.15	1.20	1.41	0.46
$t_\beta, 2.5\%$	11.01	9.35	8.88	8.28	9.19	8.06	8.43	9.32	10.32	13.45	−0.01
$t_\beta, 97.5\%$	17.00	16.61	14.24	15.16	15.76	13.54	14.07	14.84	16.66	21.02	4.19
R^2	0.16	0.12	0.11	0.10	0.12	0.10	0.11	0.13	0.15	0.22	0.00

CAPM fails to price these deciles, as the post-ranking beta overshoots, giving rise to a negative CAPM alpha of -0.69% ($t = -2.49$). In samples without disasters, the high-minus-low decile on the true market beta earns on average 0.93%, which is highly significant. The post-ranking beta moves in the opposite direction as the true market beta, with a spread of -0.83. Accordingly, the CAPM alpha is 1.6%, which is substantially higher than the average return spread.

As a proxy for the true market beta, the rolling market beta contains a great deal of measurement errors. Across the pre-ranking market beta deciles, the correlation between the true and rolling market betas is weakly positive, 2.84%, in the disaster samples, but weakly negative, -5.43%, in the no-disaster samples. Intuitively, based on 60-month rolling windows, the estimated rolling beta is basically the prior five-year averaged beta. In contrast, true market beta accurately and immediately reflects changes in aggregate and firm-specific conditions. Within a given rolling window, the true market beta can even mean-revert, giving rise to opposite rankings in rolling betas.

Our quantitative results in the context of the beta "anomaly" add to a substantial body of simulation evidence on the importance of beta measurement errors in asset pricing tests. For instance, Miller and Scholes (1972) simulate random returns from the CAPM, and find that test results on simulated data are consistent with those from the real data.[5] Gomes, Kogan, and Zhang (2003) and Carlson, Fisher, and Giammarino (2004) show that how size and book-to-market, and Li, Livdan, and Zhang (2009) show how capital investment and new equity issues can dominate rolling betas in cross-sectional regressions in simulations. Lin and Zhang (2013) show how characteristics can dominate covariances in predicting returns in the Daniel and Titman (1997) tests. In all, we suggest that the evidence on the beta "anomaly" in the data should be interpreted with extreme caution.

[5]In particular, Miller and Scholes (1972) conclude: "We have shown that much of the seeming conflict between [the empirical] results and the almost exactly contrary predictions of the underlying economic theory may simply be artifacts of the testing procedures used. The variable that measures the systematic covariance risk of a particular share is obtained from a first-pass regression of the individual company returns on a market index. Hence it can be regarded at best as an approximation to the perceived systematic risk, subject to the margin of error inevitable in any sampling process, if to nothing else. The presence of such errors of approximation will inevitably weaken the apparent association between mean returns and measured systematic risk in the critical second-pass tests."

4.5 Explaining the Performance of the Consumption CAPM

Based on 2,000 artificial samples, Table 11 reports the average excess returns and consumption betas of the 25 size and book-to-market portfolios. The three panels use annual samples with disasters, quarterly samples without disasters, and annual samples of the fourth-quarter consumption growth without disasters, respectively, from the model's simulations. Their sample lengths match those in the corresponding panels in the data (Table 4). We again time-aggregate simulated monthly data to quarterly and annual data, using the same procedure as in Table 7.

4.5.1 Explaining the Higher Average Value Premium in Small Firms

The model succeeds in reproducing a higher average value premium in small firms than in big firms. From Panel A, which is based on annual samples with disasters, the value premium is on average 9.68% per annum ($t = 6.67$) in the smallest quintile, but only 2.18% ($t = 2.37$) in the biggest quintile. Panel B shows that in quarterly samples without disasters, the average value premium is 2.01% per quarter ($t = 11.29$) in the smallest quintile, but only 0.49% ($t = 2.29$) in the biggest quintile. The results from the annual samples without disasters are largely similar (Panel C).

The key mechanism underlying this result is decreasing returns to scale. The curvature parameter, ξ, in the production function in equation (5) is less than one (0.65 in the benchmark calibration). As a result, the detrended capital, $\widehat{K}_{it}$, is a firm-specific state variable in addition to the firm-specific productivity, z_{it}. With constant returns to scale, $\xi = 1$, the investment-to-capital ratio, $\widehat{I}_{it}/\widehat{K}_{it}$, is independent of capital, meaning that $\widehat{K}_{it}$ is not a separate state variable. When $\xi < 1$, $\widehat{I}_{it}/\widehat{K}_{it}$ clearly depends on $\widehat{K}_{it}$, with small firms investing faster than big firms (Figure 4). Figure 5 shows further that big spikes in risk and risk premiums in the disaster states accrue to firms with small capital stock and low firm-specific productivity. This pattern implies that the expected return spread between the low- and high-z_{it} firms is higher in small-$\widehat{K}_{it}$ firms than in big-$\widehat{K}_{it}$ firms.

The disaster risk also plays a role in reproducing the higher value premium in small firms. The presence of the disaster state and the subsequent recovery state enlarges the cross-sectional

Table 11 : Average Excess Returns and Consumption Betas for the 25 Size and Book-to-market Portfolios in the Model

Results are based on 2,000 simulations, each with 5,000 firms and 2,000 months. We drop the first 908 months, and treat the remaining $1,092$ months as from the model's stationary distribution. For each portfolio, we report its average excess return, m, and its consumption beta, β^C, as well as their t-values adjusted for heteroscedasticity and autocorrelations, t_m and t_{β^C}, respectively. Returns in Panels A and C are in annual percent, and those in Panel B in quarterly percent.

	L	2	3	4	H	L	2	3	4	H
	Panel A: Annual samples with disasters									
	m					t_m				
Small	13.69	14.54	15.95	17.90	23.37	12.34	11.38	10.77	10.43	10.26
2	12.33	13.29	14.21	15.45	18.90	12.00	11.71	11.43	11.25	10.78
3	12.05	12.17	12.42	12.95	14.62	10.00	12.01	11.91	11.33	10.33
4	10.57	10.40	10.42	10.85	13.84	12.01	11.89	11.47	10.59	10.41
Big	7.96	7.92	8.18	8.86	10.14	10.16	9.96	9.80	9.67	9.23
	β^C					t_{β^C}				
Small	−0.64	−0.77	−0.93	−1.15	−1.28	−0.61	−0.68	−0.72	−0.74	−0.47
2	−0.49	−0.59	−0.72	−0.89	−1.34	−0.55	−0.62	−0.69	−0.81	−0.97
3	−0.43	−0.47	−0.53	−0.64	−0.74	−0.50	−0.56	−0.63	−0.72	−0.70
4	−0.32	−0.33	−0.36	−0.46	−0.69	−0.41	−0.46	−0.52	−0.64	−0.79
Big	−0.07	−0.08	−0.10	−0.22	−0.23	−0.01	−0.04	−0.08	−0.28	−0.09
	Panel B: Quarterly samples without disasters									
	m					t_m				
Small	3.16	3.31	3.56	3.92	5.17	45.90	34.22	31.77	32.39	29.73
2	2.89	3.08	3.24	3.45	4.09	29.73	31.09	31.73	32.26	28.72
3	2.84	2.85	2.88	2.96	3.33	16.28	29.04	30.16	28.11	19.54
4	2.53	2.48	2.47	2.53	3.19	23.35	23.40	21.99	18.86	18.87
Big	1.93	1.91	1.96	2.07	2.42	13.66	13.75	14.14	15.06	14.71
	β^C					t_{β^C}				
Small	0.11	0.12	0.12	0.13	0.27	1.45	1.13	0.96	0.95	1.39
2	0.12	0.12	0.13	0.13	0.18	1.10	1.04	1.10	1.09	1.14
3	0.16	0.13	0.14	0.16	0.25	0.83	1.19	1.30	1.36	1.30
4	0.16	0.18	0.22	0.24	0.24	1.33	1.54	1.71	1.56	1.37
Big	0.74	0.93	1.08	0.94	0.85	4.83	6.32	7.60	6.51	4.74
	Panel C: Annual samples with fourth-quarter consumption growth without disasters									
	m					t_m				
Small	13.54	14.20	15.34	17.00	22.80	43.01	32.13	30.08	30.64	28.12
2	12.35	13.21	13.91	14.86	17.76	28.37	29.75	30.32	30.72	27.52
3	12.13	12.15	12.28	12.63	14.31	15.79	27.81	28.90	26.74	18.91
4	10.75	10.53	10.48	10.72	13.66	22.29	22.50	21.19	18.31	18.16
Big	8.13	8.04	8.25	8.75	10.28	13.21	13.29	13.76	14.64	14.31
	β^C					t_{β^C}				
Small	0.21	0.23	0.22	0.24	0.50	1.55	1.23	0.99	1.01	1.42
2	0.24	0.22	0.24	0.25	0.31	1.27	1.17	1.23	1.18	1.11
3	0.27	0.22	0.24	0.26	0.35	0.83	1.17	1.30	1.28	1.07
4	0.26	0.28	0.30	0.32	0.36	1.21	1.38	1.38	1.27	1.09
Big	0.82	0.97	1.08	0.95	0.84	3.22	4.03	4.62	4.02	2.82

dispersion in the detrended capital, making firms more heterogeneous. As a result, we can perform independent sorts on size and book-to-market to form the 25 portfolios in simulated samples. Without showing the details, we can report that such independent sorts are infeasible in the Lin and Zhang (2013) model, which is in turn a simplified version of the Zhang (2005) model. Because size and book-to-market are negatively correlated in the cross section, several portfolios contain no firms in simulated samples, including the small-growth and the big-value portfolios. The aggregate shock follows the normal distribution in the prior models, which fail to generate a sufficient amount of firm heterogeneity to allow for the five-by-five independent sorts on size and book-to-market.

4.5.2 Explaining the Performance of the Consumption CAPM

More important, our model largely replicates the poor performance of the consumption CAPM in the data (Table 4). Table 11 shows that the consumption betas are mostly insignificant, and are all negative in the annual samples with disasters. In the second-stage cross-sectional regressions, Table 12 shows that the intercept estimates are all significantly positive. The estimates of the price of consumption risk are all significantly negative, although its 95% confidence intervals are wide in the simulations. In the annual samples with disasters, the cross-sectional R^2 is 63%, but its 95% confidence interval ranges from 0% to 94%. As such, the cross-sectional R^2 seems largely uninformative. The R^2 is 29% in the quarterly samples without disasters, and its 95% confidence interval varies from 12% to 49%. Finally, the model cannot replicate the success of the Jagannathan-Wang (2007) fourth-quarter consumption growth model (Panel C). The performance is largely similar to the standard consumption CAPM tests in the model. Investors make the consumption and portfolio choice decision every period in the model, and the fourth-quarter does not stand out as special.

We emphasize that in our model, a nonlinear consumption CAPM holds exactly by construction, i.e., $E_t[M_{t+1}R_{it+1}] = 1$, in which M_{t+1} is the true pricing kernel given by equation (27). However, in the standard implementation of the consumption CAPM, the pricing kernel is specified as a linear function of the aggregate consumption growth. With recursive utility, the pricing kernel

Table 12 : Cross-sectional Regression Tests of the Consumption CAPM in the Model

Results are based on 2,000 simulations, each with 5,000 firms and 2,000 months. We report cross-sectional tests of the consumption CAPM. Testing assets are the 25 Fama-French size and book-to-market portfolios. Consumption betas are estimated from time-series regressions of portfolio excess returns on the aggregate consumption growth. Panel A uses annual consumption growth on the disaster samples, Panel B quarterly consumption growth on the no-disaster samples, and Panel C the fourth-quarter consumption growth on the no-disaster samples. ϕ_0 is the intercept, ϕ_1 the slope, t_{FM} the Fama-MacBeth t-statistics, and t_{S} the Shanken-adjusted t-statistics. The estimates of ϕ_0 and ϕ_1 are annual percent in Panels A and C, and in quarterly percent in Panel B. We report the cross-simulation averaged results, as well as the 2.5 and 97.5 percentiles.

	Panel A: Annual, with disasters		Panel B: Quarterly, without disasters		Panel C: Fourth-quarter, without disasters	
	ϕ_0	ϕ_1	ϕ_0	ϕ_1	ϕ_0	ϕ_1
Estimates	9.02	−6.59	3.34	−1.19	14.03	−3.40
2.5%	5.24	−13.52	3.16	−1.73	12.80	−6.46
97.5%	13.41	1.09	3.53	−0.76	15.28	−0.47
t_{FM}	15.26	−6.41	73.52	−13.68	63.77	−8.67
2.5%	6.50	−13.36	50.28	−18.35	45.15	−15.51
97.5%	50.48	1.00	83.57	−8.61	80.52	−1.26
t_{S}	7.97	−3.34	44.02	−9.16	36.50	−5.34
2.5%	3.84	−5.82	27.09	−10.89	20.11	−7.82
97.5%	23.10	0.99	57.68	−6.85	62.69	−1.24
R^2		0.63		0.29		0.16
2.5%		0.00		0.12		0.00
97.5%		0.94		0.49		0.44

depends not only on contemporaneous consumption growth, but also on (a nonlinear function of) the continuation value of future utility. To quantify the impact of the specification error of the pricing kernel in the context of our model, we repeat the consumption CAPM tests, but with the aggregate consumption growth replaced by the true pricing kernel, which we can compute in simulations.

Table 13 details the two-stage tests. Panel A shows that the estimated beta, $\hat{\beta}^M$, from regressing returns on the true pricing kernel is generally higher for value firms than for growth firms, going in the right direction as the average returns. The $\hat{\beta}^M$ estimates are also all significantly positive, both in annual samples with disasters and in quarterly samples without disasters. The magnitude of the regression-based estimates of $\hat{\beta}^M$ is largely in line with that of the true beta calculated on the grid (Panel D of Figure 6). Also, the magnitude of $\hat{\beta}^M$ in samples without disasters is roughly three times of that in samples with disasters. Intuitively, the average returns are comparable in magnitude across the two types of samples. However, the pricing kernel's volatility is higher in samples with disasters than without disasters, meaning that the realized pricing of risk, ϕ_{Mt}, is lower in samples without disasters. Accordingly, the $\hat{\beta}^M$ estimates must be higher in samples without disasters to match the average returns that are comparable to those with disasters.

In second-stage cross-sectional regressions, Panel B shows that with disasters, the intercept, $\hat{\phi}_0$, is economically small, only 1% per annum. Although its Fama-MacBeth t-value is significant, 2.44, the Shanken-adjusted t-value is not, only 0.91. The price of consumption risk, $\hat{\phi}_M$, is 5.18, which is highly significant. In addition, the cross-sectional R^2 is high, 89%, and its 95% confidence interval spans from 52% to 97%. Interestingly, even the true pricing kernel does not perform perfectly in the standard empirical design for testing the consumption CAPM. The culprit is the test's unconditional form. The regression-based beta, $\hat{\beta}^M$, is estimated on the full sample, and is assumed to be constant. In contrast, the true beta, β_{it}^M, is time-varying, as shown in Figure 5.

In quarterly samples without disasters, the true model's performance deteriorates. The intercept is 2% per quarter, which is also significant per both Fama-MacBeth and Shanken t-values. The price of consumption risk is only 0.11, but highly significant. In addition, the cross-sectional R^2 is lower, only 43%, and its 95% confidence interval ranges from 13% to 79%. Intuitively, without the extreme observations from disasters and subsequent recoveries, the regression-based beta, $\hat{\beta}^M$, from projecting returns on the true pricing kernel is a poor proxy for the true beta.

5 Conclusion

Rare disasters help explain the value premium puzzle that value stocks earn higher average returns than growth stocks, despite their similar market betas. In a general equilibrium economy with disasters and heterogenous firms, value stocks are more exposed to the disaster risk than growth stocks. More important, the disaster risk induces strong nonlinearity in the pricing kernel. In finite samples, in which disasters are materialized, the CAPM often does an adequate job in accounting for the value premium. However, in finite samples without disasters, the estimated market beta

Table 13 : Two-stage Cross-sectional Regression Tests of the Consumption CAPM with the True Pricing Kernel in the Model

Results are based on 2,000 simulations, each with 5,000 firms and 2,000 months. For each of the 25 size and book-to-market portfolios, we report the consumption beta, $\hat{\beta}^M$, estimated from regressing excess returns on the true pricing kernel, M_{t+1}, as well as the t-value adjusted for heteroscedasticity and autocorrelations, $t_{\hat{\beta}^M}$. We also report the second-stage cross-sectional regressions, including the intercept, $\hat{\phi}_0$, the slope, $\hat{\phi}_M$, the Fama-MacBeth t-value, t_{FM}, and the Shanken-adjusted t-value, t_{S}. We report the cross-simulation averaged results, as well as the 2.5 and 97.5 percentiles.

Panel A: First-stage time-series regressions

	L	2	3	4	H	L	2	3	4	H
	Annual samples with disasters									
	$\hat{\beta}^M$					$t_{\hat{\beta}^M}$				
Small	0.04	0.04	0.04	0.05	0.07	8.26	7.87	7.58	7.20	7.08
2	0.03	0.04	0.04	0.04	0.05	8.51	8.25	8.04	7.85	7.71
3	0.03	0.03	0.03	0.04	0.04	8.26	8.53	8.34	8.03	7.49
4	0.03	0.03	0.03	0.03	0.04	8.94	8.79	8.63	8.16	8.47
Big	0.02	0.02	0.02	0.02	0.03	8.79	8.53	8.26	7.76	7.49
	Quarterly samples without disasters									
	$\hat{\beta}^M$					$t_{\hat{\beta}^M}$				
Small	0.12	0.13	0.14	0.15	0.25	7.78	5.74	5.20	5.32	6.11
2	0.12	0.12	0.13	0.14	0.17	5.09	5.26	5.36	5.49	5.17
3	0.12	0.12	0.12	0.12	0.15	2.99	5.10	5.32	4.95	3.70
4	0.11	0.11	0.11	0.11	0.15	4.29	4.27	4.08	3.53	3.57
Big	0.09	0.10	0.10	0.10	0.12	2.66	2.81	2.91	2.99	2.90

Panel B: Second-stage cross-sectional regressions

	Annual, with disasters		Quarterly, without disasters	
	$\hat{\phi}_0$	$\hat{\phi}_M$	$\hat{\phi}_0$	$\hat{\phi}_M$
Estimates	0.01	5.18	0.02	0.11
2.5%	−0.01	0.35	0.01	0.06
97.5%	0.06	7.66	0.02	0.26
t_{FM}	2.44	8.36	19.22	15.47
2.5%	−1.48	3.17	7.33	8.77
97.5%	17.72	18.90	30.19	20.62
t_{S}	0.91	3.56	6.80	5.42
2.5%	−0.61	1.54	1.94	3.78
97.5%	5.74	7.09	14.16	8.11
R^2		0.89		0.43
2.5%		0.52		0.13
97.5%		0.97		0.79

fails to measure the higher exposure of value stocks to disasters than growth stocks. This strong nonlinearity allows the model to explain the failure of the CAPM in the post-1963 sample. In addition, due to severe beta measurement errors, the relation between the pre-ranking market beta and the average return is flat in the model's simulations, despite a strong positive relation between the true beta and the expected return. As such, the model also explains the beta "anomaly."

A fundamental innovation of our work relative to prior theoretical models on the cross section is general equilibrium, in which consumption and the pricing kernel are endogenous. Endogenous consumption makes it feasible for us to quantify the performance of the consumption CAPM within our model. Despite a nonlinear consumption CAPM structure, our model largely replicates the poor performance of the standard consumption CAPM, in which the pricing kernel is severely misspecified as a linear function of the aggregate consumption growth. Our extensive simulation evidence suggests that the poor performance of the (consumption) CAPM in the data should be interpreted with caution. The widely documented empirical failures might have more to do with the deficiencies of standard empirical tests, rather than the deficiencies of standard economic theory.

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A Computation

A.1 Solving the Firms' Problem

As an intermediate step for solving the detrended value function in equation (29), we solve for the log utility-to-consumption ratio, $\widehat{u}_t$, by iterating on equation (26), and calculate M_{t+1} from equation (27), which only depends on g_t, g_{t+1}, and $\overline{K}_t$. We then solve firms' problem by iterating on:

$$\widehat{V}(\widehat{K}_{it}, Z_{it}, g_t, \overline{K}_t) = \max_{\{\chi_{it}\}} \left[\max_{\{\widehat{K}_{it+1}\}} \widehat{D}_{it} + E_t \left[M_{t+1} \widehat{V}(\widehat{K}_{it+1}, Z_{it+1}, g_{t+1}, \overline{K}_{t+1}) \right] \exp(g_{xt}),\ s\widehat{K}_{it} \right]. \quad \text{(A1)}$$

We use 100 grid points for the detrended capital, $\widehat{K}_{it}$. The lower bound of the $\widehat{K}_{it}$ grid is 0.01, and the upper bound 25. The $\widehat{K}_{it}$ grid is formed recursively as in McGrattan (1999), with $\widehat{K}_j = \widehat{K}_{j-1} + c_1 \exp(c_2(j-2))$, in which $j = 2, ..., 100$ is the index of grid points, and c_1 and c_2 are two constant parameters chosen to provide the desired number of grid points and the grid's upper bound, given a predetermined lower bound of $\widehat{K}_1 = 0.01$. A seven-point grid for the aggregate productivity growth, g_t, is constructed as in Section 3.3, and a nine-point grid for the log firm-specific productivity, z_{it}, is formed via the Rouwenhorst (1995) procedure. To form the $\overline{K}_t$ grid, we use 15 even-spaced points from 0.25 to seven. The boundaries are chosen judiciously via trial and error to be never binding in simulations. We work directly with the discrete state spaces of g_t and z_{it}, both in solving and simulating the model. For the continuous state spaces of $\widehat{K}_{it}$ and $\overline{K}_t$, we use the piecewise linear interpolation extensively to obtain the model's key moments corresponding to the $\widehat{K}_{it}$ and $\overline{K}_t$ values that lie between the grid points on their respective grid. We use a simple (but robust) global search routine to maximize the right-hand side of equation (A1). We construct a dense grid for the next period detrended capital, $\widehat{K}_{it+1}$ (the control variable), by assigning 100 even-spaced points between any two adjacent points on the grid of $\widehat{K}_{it}$ (the state variable). We compute the objective function on each point in the $\widehat{K}_{it+1}$ grid, and take the maximum.

A.2 Approximate Aggregation

We solve the general equilibrium model with an approximate aggregation algorithm. Starting with an initial guess on the equilibrium laws of motion for the average detrended capital, $\overline{K}_{t+1}$, and the detrended consumption, $\widehat{C}_t$, we solve individual firms' problem. Based on the resulting optimal policy functions, we simulate the economy for a large number of firms, and use the simulated data to update the guess for the equilibrium laws of motion. We continue the iteration process until the laws of motion converge. We then check the accuracy of the laws of motion by comparing the implied $\overline{K}_{t+1}$ and $\widehat{C}_t$ values with their actual, realized values in simulations. If the accuracy is high, we stop. Otherwise, we specify different functional forms for the laws of motion, and repeat the process.

Specifically, suppose at the jth iteration, the current guess for the laws of motion is given by:

$$\log \widehat{C}_t^{(j)}(g_t = g_i) = a_{0i}^{(j)} + a_{1i}^{(j)} \log \overline{K}_t + a_{2i}^{(j)} \left(\log \overline{K}_t\right)^2, \quad \text{(A2)}$$

$$\log \overline{K}_{t+1}^{(j)}(g_t = g_i) = b_{0i}^{(j)} + b_{1i}^{(j)} \log \overline{K}_t + b_{2i}^{(j)} \left(\log \overline{K}_t\right)^2, \quad \text{(A3)}$$

in which $i \in [1, 7]$, and "$(g_t = g_i)$" indicates the values of $\log \widehat{C}_t^{(j)}$ and $\log \overline{K}_{t+1}^{(j)}$ conditional on $g_t = g_i$. We adopt the quadratic functional form in logs, and allow the coefficients to depend on the aggregate state, g_t, to accommodate the strong nonlinearity of the model.

Under the approximate laws of motion, we solve firms' problem by iterating on the value function in equation (A1), and obtain optimal policy functions, $\widehat{K}_{it+1}^{(j)}\left(\widehat{K}_{it}, Z_{it}, g_t, \overline{K}_t\right)$ and $\chi_{it+1}^{(j)}\left(\widehat{K}_{it}, Z_{it}, g_t, \overline{K}_t\right)$. Based on the optimal policy functions, we simulate a long series of aggregate productivity growth, $\{g_t\}_{t=1}^T$, starting from $g_1 = \bar{g}$, with $T = 55,000$ monthly periods, and a panel of $N = 30,000$ firms over the T periods. The initial detrended capital, $\widehat{K}_{it}$, is set to be one, and the initial log firm-specific productivity, z_{it}, set to be the long-run mean, $\bar{z}$, across all firms. Based on the simulated data, we compute the cross-sectional average detrended capital, $\overline{K}_t$, and detrended consumption $\widehat{C}$, as aggregate detrended output minus aggregate detrended investment. We discard the first $5,000$ periods to ensure that the economy has reached its stationary distribution.

On the remaining 50,000 periods, we pick out the observations when $g_t = g_i$ for each value of $i \in [1, 7]$, and then fit the following two regressions on these observations:

$$\begin{aligned}
\log \widehat{C}_t^{(j+1)}(g_t = g_i) &= a_{0i}^{(j+1)} + a_{1i}^{(j+1)} \log \overline{K_t} + a_{2i}^{(j+1)} \left(\log \overline{K_t}\right)^2 + e_t^C, && \text{(A4)} \\
\log \overline{K}_{t+1}^{(j+1)}(g_t = g_i) &= b_{0i}^{(j+1)} + b_{1i}^{(j+1)} \log \overline{K_t} + b_{2i}^{(j+1)} \left(\log \overline{K_t}\right)^2 + e_t^K. && \text{(A5)}
\end{aligned}$$

We next check the convergence for the coefficients, for $l = \{0, 1, 2\}$:

$$\max_{i \in [1,7]} |a_{li}^{(j+1)} - a_{li}^{(j)}| < 10^{-2}, \text{ and } \max_{i \in [1,7]} |b_{li}^{(j+1)} - b_{li}^{(j)}| < 10^{-3}. \qquad \text{(A6)}$$

If not, we update the coefficients as follows:

$$\begin{aligned}
a_{li}^{(j+1)} &= a_{li}^{(j+1)}\omega + a_{li}^{(j)}(1 - \omega), && \text{(A7)} \\
b_{li}^{(j+1)} &= b_{li}^{(j+1)}\omega + b_{li}^{(j)}(1 - \omega), && \text{(A8)}
\end{aligned}$$

for $l = \{0, 1, 2\}$, in which ω is the dampening parameter. In practice, we set $\omega = 0.8$.

The large number of firms, $N = 30,000$, is necessary to ensure that the coefficients converge to an acceptable degree. More important, once the coefficients have converged, we use the simulated 50,000 periods to check the time series R^2 from regressing the actual realized values of the average detrended capital on those values predicted from its approximate law of motion, as well as the R^2 from regressing the actual realized values of the aggregate detrended consumption on those values predicted from its approximate law of motion. In practice, the former R^2 is 0.9999983, and the latter R^2 is 0.99494656. Both are largely comparable with those reported in Krusell and Smith (1997, 1998), Favilukis and Lin (2016), and Favilukis, Ludvigson, and Van Nieuwerburgh (2017).