Asset Pricing Implications of Firms' Financing Constraints

João F. Gomes
University of Pennsylvania and CEPR

Amir Yaron
University of Pennsylvania and NBER

Lu Zhang
University of Rochester and NBER

We use a production-based asset pricing model to investigate whether financing constraints are quantitatively important for the cross-section of returns. Specifically, we use GMM to explore the stochastic Euler equation imposed on returns by optimal investment. Our methods can identify the impact of financial frictions on the stochastic discount factor with cyclical variations in cost of external funds. We find that financing frictions provide a common factor that improves the pricing of cross-sectional returns. Moreover, the shadow cost of external funds exhibits strong procyclical variation, so that financial frictions are more important in relatively good economic conditions. (*JEL* E22, E44, G12)

We investigate whether financial frictions are quantitatively important in determining the cross-section of expected stock returns. Specifically, we construct a production-based asset pricing framework in the presence of financial market imperfections and use GMM to explore the stochastic Euler equation restrictions imposed on asset returns by the optimal investment decisions of firms.

Our results suggest that financial frictions provide an important common factor that can improve the pricing of the cross-section of expected returns. In addition, we find that the shadow price of external funds is strongly procyclical, that is, financial market imperfections are more important when economic conditions are relatively good. These results are generally robust to the use of alternative measures of fundamentals

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such as profits and investment, alternative assumptions about the forms of the stochastic discount factor, and alternative measures of the shadow price of external funds.

The intuition behind our results is simple. The empirical success of production-based asset pricing models lies in the alignment between the theoretical returns on capital investment and stocks returns. Given the forward-looking nature of the firms' dynamic optimization decisions, the returns to capital accumulation will be positively correlated with expected future profitability. Accordingly, the model generates a series of investment returns that is procyclical and leads the business cycle. This pattern accords well with the observed cyclical behavior of stock returns documented by Fama (1981) and Fama and Gibbons (1982).

Financial frictions create an important additional source of variation in investment returns. Specifically, financial market imperfections introduce a wedge, driven by the shadow cost on external funds, between investment returns and fundamentals such as profitability. All else equal, a countercyclical wedge generally lowers the correlation between the theoretical investment returns and the observed stock returns. This will weaken the performance of the standard production-based asset pricing model. Conversely, a procyclical shadow cost of external funds will strengthen the empirical success of the model.

Our work has important connections to the existing literature on empirical asset pricing. Our findings that financing frictions provide an important risk factor for the cross-section of expected returns are consistent with recent research by Lamont, Polk, and Saa-Requejo (2001) and Whited and Wu (2004). However, by explicitly modeling the effect of financial market imperfections on optimal investment and returns, our structural approach helps to shed light on the precise nature of the underlying financial market imperfections.

By identifying the role of cyclical fluctuations in the shadow price of external funds, our results also have important implications for the corporate finance literature. In particular, our basic finding that financing frictions are more important when economic conditions are relatively good can be used to distinguish across the various existing theories of financial market imperfections.[1]

Our research builds on Cochrane (1991, 1996) who first explores the asset pricing implications of optimal production and investment decisions by firms. Our work is also closely related to recent research by Li (2003) and Whited and Wu (2006). Li (2003) builds directly on our approach to investigate implications of financial frictions at the firm level. Whited and Wu (2006) adopt a similar framework to estimate the shadow price

[1] For example, Dow, Gorton, and Krishnamurthy (2004) agency model also has the feature that financing frictions become more important when economic conditions are relatively good.

of external funds using firm-level data and then construct return factors on the estimated shadow price. Their work provides an important empirical link between the shadow price of external funds and firm-specific variables. Both sets of authors find that financing frictions are particularly important for the subset of firms a priori classified as financially constrained.[2]

Finally, our work complements Gomes, Yaron, and Zhang (2003a) who study asset pricing implications of a very stylized model of costly finance in an asset pricing setting. Specifically, they use a fully specified general equilibrium model to show that to match the equity premium and typical business cycle facts, the model must imply a procyclical variation in the cost of external funds. In contrast to the very simple example in Gomes, Yaron, and Zhang (2003a), our article allows a much more general characterization of the role of financial market imperfections—thus providing a more suitable framework for empirical analysis.

The remainder of this article is organized as follows. Section 1 shows how financial market imperfections affect firm investment and asset prices under fairly general conditions. This section derives the expression for returns to physical investment, the key ingredient in the stochastic discount factor in this economy. Section 2 describes our empirical methodology while Section 3 discusses the results of our GMM estimation and tests. Finally, Section 4 offers some concluding remarks.

1. Production-Based Asset Pricing with Financial Frictions

In this section, we incorporate financial frictions in a production-based asset pricing framework in the tradition of Cochrane (1991, 1996) and derive the expression for the behavior of investment returns, the key ingredient in our stochastic discount factor.

1.1 Modeling financial frictions

Several theoretical foundations of financial market imperfections are available in the literature. Rather than offering another rationalization for their existence, we seek instead to summarize the common ground across the existing literature with a representation of financial constraints that is both parsimonious and empirically useful.

While exact assumptions and modeling strategies often differ quite significantly across authors, the key feature of this literature is the simple idea that external funds (new equity or debt) are not perfect substitutes for internal cash flows. It is this crucial property that we

[2] Other papers in this area include Whited (1992), Bond and Meghir (1994), Restoy and Rockinger (1994), and Li, Vassalou and Xing (2004).

explore in our analysis below by assuming that any form of financial market imperfection can be usefully summarized by adding a distortion to the relative price between internal and external funds.

Consider the case of new equity finance. Suppose that a firm issues N_t dollars in new equity, and let W_t denote the reduction on the claim of existing shareholders per dollar of new equity issued. In a frictionless world, it must be the case that $W_t = 1$, since the value of the firm is not affected by financing decisions. However, the presence of any financial market imperfections such as transaction costs, agency problems, or market timing issues will cause W_t to differ from 1. Characterizing the exact form of W_t requires a detailed model of the precise nature of the distortion, but it is not necessary to derive the key asset pricing restrictions below. Similarly we need not take a stand about whether new issues add or lower firm value.

Suppose now that the firm also uses debt financing, B_t, and let R_t denote the gross (interest plus principal) repayment per dollar of debt raised. As before, without any financial frictions, the cost of this debt will be equal to the return on savings and the opportunity cost of internal funds, say R_{ft}. The presence of any form of imperfection, such as asymmetric information or moral hazard problems, will again distort relative prices and will cause R_t to differ from R_{ft}, at least when $B_t > 0$. As in the case of equity issues, this basic idea is sufficient to derive the asset pricing results below.

1.2 Investment returns

Consider the problem of a firm seeking to maximize the value to existing shareholders, denoted V_t. The firm makes investment decisions by choosing the optimal amount of capital at the beginning of the next period, K_{t+1}. Investment, I_t, and dividends, D_t, can be financed by internal cash flows Π_t, new equity issues, N_t, or new one-period debt B_{t+1}. Assuming one-period debt simplifies the notation significantly but does not change the basic results.

The value-maximization problem of the firm can then be summarized as follows:

$$V(K_t, B_t, S_t) = \max_{\substack{D_t, B_{t+1}, \\ K_{t+1}, N_t}} \{D_t - W_t N_t + \mathrm{E}_t[M_{t+1} V(K_{t+1}, B_{t+1}, S_{t+1})]\} \quad (1)$$

subject to

$$D_t = \Pi(K_t, S_t) - I_t - \frac{a}{2}\left(\frac{I_t}{K_t}\right)^2 K_t + N_t + B_{t+1} - R_t B_t \quad (2)$$

$$I_t = K_{t+1} - (1-\delta)K_t \tag{3}$$

$$D_t \geq \overline{D}, \quad N_t \geq 0, \tag{4}$$

where S_t summarizes all sources of uncertainty, M_{t+1} is the stochastic discount factor (of the owners of the firm) between t to $t+1$, and $\overline{D}$ is the firm's minimum, possibly zero, dividend payment. Note that we allow a firm to accumulate financial assets, in which case debt, B_t, is negative. We also assume that investment is subject to convex (quadratic) adjustment costs, the magnitude of which is governed by the parameter a. Without adjustment costs, the price of capital is always one and the capital-gain component of returns is always zero and is clearly counterfactual. The form of the internal cash flow function $\Pi(\cdot)$ is not important. For simplicity, we assume only that it exhibits constant return to scale.

Equation (2) is the resource constraint for the firm. It implies that dividends must equal internal funds $\Pi(K_t, S_t)$, net of investment spending I_t, plus new external funds $N_t + B_{t+1}$, net of debt repayments $R_t B_t$. Equation (3) is the standard capital accumulation equation, relating current investment spending, I_t, to future capital, K_{t+1}. We assume that old capital depreciates at the rate δ.

Letting μ_t denote the Lagrange multiplier associated with the inequality constraint on dividends, the optimal first-order condition with respect to K_{t+1} (derived in Appendix A.1) implies that

$$\mathrm{E}_t(M_{t+1} R^I_{t+1}) = 1, \tag{5}$$

where R^I_{t+1} denotes the returns to investment in physical capital and is given by

$$R^I_{t+1} = R^I_{t+1}(\pi, i, \mu) \equiv \frac{(1+\mu_{t+1})[\pi_{t+1} + \frac{a}{2} i^2_{t+1} + (1 + a i_{t+1})(1-\delta)]}{(1+\mu_t)(1 + a i_t)}. \tag{6}$$

And $i \equiv I/K$ is the investment-to-capital ratio and $\pi \equiv \Pi/K$ is the profits-to-capital ratio.[3]

To gain some intuition on the role of the financial frictions, we can decompose Equation (6) into

$$R^I_{t+1}(\pi, i, \mu) = \frac{1+\mu_{t+1}}{1+\mu_t} \tilde{R}^I_{t+1} \quad \text{and}$$

$$\tilde{R}^I_{t+1}(\pi, i) \equiv \frac{\pi_{t+1} + \frac{a}{2} i^2_{t+1} + (1 + a i_{t+1})(1-\delta)}{1 + a i_t}, \tag{7}$$

[3] Equation (6) is general and holds in the presence of quantity constraints such as those created by credit rationing (Whited 1992; Whited and Wu 2005). For details see Gomes, Yaron, and Zhang (2003b).

where $\widetilde{R}^I_{t+1}$ denotes the investment return with no financial constraints, that is, $\mu_{t+1} = \mu_t = 0$, which is entirely driven by fundamentals, i and π. The role of the financial market imperfections is completely captured by the term $(1 + \mu_{t+1})/(1 + \mu_t)$, which depends only on the shadow price of external funds.

The decomposition in Equation (7) provides important intuition on the effects of financial market imperfections on returns. This result shows that if $\mu_t = \mu_{t+1}$, financing frictions do not affect returns at all. They will simply have a permanent effect on the value of the firm without producing time series variation in returns. This highlights the crucial role of cyclical variation in the shadow price of external funds. From the standpoint of asset returns, however, the exact level of μ is irrelevant.

2. Empirical Methodology

2.1 Testing framework

The essence of our empirical strategy is to use the information contained in asset prices to formally evaluate the effects of financial constraints. Specifically, we test

$$\mathrm{E}_t(M_{t+1}\mathbf{R}_{t+1}) = 1, \tag{8}$$

where $\mathbf{R}_{t+1}$ is a vector of returns that may include stocks and bonds as well as the returns to physical investment from Equation (5).

Following Cochrane (1996), we ask whether investment returns are factors for asset returns. Formally, we parameterize the stochastic discount factor as a linear function of the returns to physical investment:

$$M_{t+1} = l_0 + l_1 R^I_{t+1}. \tag{9}$$

The role of financial frictions in explaining the cross-section of expected returns as a factor is captured by their impact on R^I in the pricing kernel (9). Thus, financial frictions will be relevant for the pricing of expected returns only to the extent that they provide a common factor or a source of systematic risk, which influences the stochastic discount factor. In this sense, our formulation is essentially a structural version of an arbitrage pricing theory (APT)-type framework such as those proposed in Fama and French (1993, 1996) and Lamont, Polk, and Saá-Requejo (2001), in which one of the factors proxies for aggregate financial conditions.

In the presence of financing frictions, Equation (9) is only an approximation to the exact pricing kernel for this economy, since in general, the pricing kernel will also depend on the corporate bond return, as shown in Gomes, Yaron, and Zhang (2003b). Below we also implement this more general representation of the pricing kernel in Section 3.

2.2 The shadow price of external funds

Empirically, our characterization of investment returns is useful because it requires only data on the two fundamentals, i and π, as well as a measure of the shadow cost of external funds to be implemented. Formally, we parameterize the shadow price of external funds μ_t with the following form:

$$\mu_t = b_0 + b_1 f_t, \tag{10}$$

where b_0 and b_1 are parameters and f_t is some aggregate index of financial frictions.

Recall that the key element for asset returns is the time series variation in μ_t, which is captured by the cyclical properties of the financing factor f_t. Thus, the estimated value of b_1 will summarize all information about the impact of financial market imperfections on returns.[4]

Cyclicality plays an important role in the various theories of financial market imperfections. For example, models emphasizing the importance of agency issues between insiders and outsiders suggest that frictions are more important when economic conditions are good and managers have too many funds available. Conversely, models that focus on costly external finance typically emphasize the role of credit market constraints and rely on the fact that the cost of external funds rises when economic conditions are adverse.[5] By isolating the dynamic properties of the shadow price of external funds, we can do more than just assess the overall impact of financing frictions on asset returns. Our methodology also allows us to distinguish between the various theories of financial market imperfections.

As a first measure of aggregate financial frictions, we use the default premium, defined as the yield spread between Baa and Aaa rated corporate bonds. Stock and Watson (1989, 1999) show that the default premium is one of the most powerful predictors of aggregate economic conditions. The default premium is also a frequent measure of the premium of external funds in the literature (Kashyap, Stein, and Wilcox 1993; Kashyap, Lamont, and Stein 1994; Bernanke and Gertler 1995; and Bernanke, Gertler, and Gilchrist 1996, 1999).

In our tests, we also use two additional measures of the marginal cost of external finance. The first is the aggregate return factor of financial constraints constructed in Lamont, Polk, and Saa-Requejo (2001). The other measure is the aggregate distress likelihood constructed by Vassalou

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[4] Since levels do not affect returns, b_0 is irrelevant. As a practical matter for our empirical estimation we fix b_0. Later we show that our results are not affected by this choice.

[5] Jensen (1986) is an example of the former while Bernanke and Gertler (1989) provide an example of the latter. Stein (2003) offers a detailed survey of this literature.

and Xing (2003). Section 3.4 describes these measures, as well as more elaborate specifications of Equation (10) in detail.

2.3 Implementation

We use GMM to estimate the factor loadings, $\mathbf{l}$, as well as the parameters, a and b_1, by utilizing M as specified in Equation (9) in conjunction with moment conditions (8). Specifically, three alternative sets of moment conditions in implementing Equation (8) are examined (Cochrane 1996). First, we look at the relatively weak restrictions implied by the unconditional moments. We then focus on the conditional moments by scaling returns with instruments, and finally, we look at time variation in the factor loadings by scaling the factors.

For the unconditional factor pricing, we use standard GMM procedures to minimize a weighted average of the sample moments (8). Letting $\sum_T$ denote the sample mean, we rewrite these moments, $\mathbf{g}_T$ as

$$\mathbf{g}_T \equiv \mathbf{g}_T(a, b_0, b_1, \mathbf{l}) \equiv \sum\nolimits_T (M\mathbf{R} - \mathbf{p}),$$

where $\mathbf{R}$ is the menu of asset returns being priced and $\mathbf{p}$ is a vector of prices. We then choose $(a, b_1, \mathbf{l})$ to minimize a weighted sum of squares of the pricing errors across assets:

$$J_T = \mathbf{g}_T'\mathbf{W}\mathbf{g}_T. \tag{11}$$

A convenient feature of our setup is that, given the cost parameters, the criterion function above is linear in $\mathbf{l}$, the factor loading coefficients. Standard χ^2 tests of over-identifying restrictions follow from this procedure. This also provides a natural framework to assess whether the loading factors or technology parameters are important for pricing assets.

It is straightforward to include the effects of conditioning information by scaling the returns by instruments. The essence of this exercise lies in extracting the conditional implications of Equation (8) since, for a time-varying conditional model, these implications may not be well captured by a corresponding set of unconditional moment restrictions as noted by Hansen and Richard (1987).

To test conditional predictions of Equation (8), we expand the set of returns to include returns scaled by instruments to obtain the moment conditions:

$$\mathrm{E}(\mathbf{p}_t \otimes \mathbf{z}_t) = \mathrm{E}[M_{t,t+1}(\mathbf{R}_{t+1} \otimes \mathbf{z}_t)],$$

where z_t is some instrument in the information set at time t and $\otimes$ is Kronecker product.

A more direct way to extract the potential nonlinear restrictions embodied in Equation (8) is to let the stochastic discount factor be a linear combination of factors with weights that vary over time. That is, the vector of factor loadings **l** is a function of instruments **z** that vary over time. With sufficiently many powers of **z**, the linearity of l can actually accommodate nonlinear relationships. Therefore, to estimate and test a model in which factors are expected to price assets only conditionally, we simply expand the set of factors to include factors scaled by instruments. The stochastic discount factor utilized in estimating Equation (8) is then

$$M_{t+1} = \left[l_0 + l_1 R^I_{t+1}\right] \otimes \mathbf{z}_t.$$

3. Findings

Section 3.1 describes our data. Section 3.2 reports the results from GMM estimation and tests for our benchmark specifications while Section 3.3 discusses and interprets our empirical findings regarding the role of financing frictions. Finally, Section 3.4 includes a wide array of robustness checks on our results.

3.1 Data and descriptive statistics

Macroeconomic data comes from National Income and Product Accounts (NIPA) published by the Bureau of Economic Analysis and the Flow of Funds Accounts available from the Federal Reserve System. These data are cross-referenced and mutually consistent, so they form, for practical purposes, a unique source of information. The construction of investment returns requires data on profits, investment, and capital. Capital consumption data are used to compute the time series average of the depreciation rate, δ, the only technology parameter not formally estimated. To avoid measurement problems due to chain weighting in the earlier periods, our sample of macroeconomic data starts in the first quarter of 1954 and ends in the last quarter of 2000. Since models of financing frictions usually apply to nonfinancial firms, we focus mainly on data from the Non-Financial Corporate Sector. However, for comparison purposes, we also report results for the aggregate economy. Appendix B provides a more detailed description of the macroeconomic data.

Information about stock and bond returns comes from Center for Research in Security Prices (CRSP) and Ibbotson, and accounting information is from Compustat. To implement the GMM estimation, we require a reasonable number of moment conditions constructed from stock and bond returns. Our benchmark specification uses the Fama–French 25 size and book-to-market portfolios that are well known to display substantial cross-sectional variation in average returns. The portfolio data are obtained from Kenneth French's website. In all cases, we use real returns, constructed using the Consumer Price Index for all Urban Households reported by the Bureau of Labor Statistics.

Investment data are quarterly averages, while stock returns are from the beginning to the end of the quarter. As a correction, following Cochrane (1996), we average monthly asset returns over the quarter and then adjust them so that they go from approximately the middle of the initial quarter to the middle of the next quarter.[6] Next, the default premium is defined as the difference between the yields on Baa and Aaa corporate bonds, both obtained from the Federal Reserve System. As an alternative measure, we also use the spread between Baa and long-term government bond yields. Finally, conditioning information comes from two sources: the term premium, defined as the yield on 10-year notes minus that on three-month Treasury bills, and the dividend-price ratio of the equally weighted NYSE portfolio.

Besides the size and book-to-market portfolios, we also use portfolios that are expected *ex ante* to display some cross-sectional dispersion in the degree of financing constraints. These portfolios are the NYSE size deciles; ten deciles sorted on the cash flow to assets ratio; ten deciles sorted on interest coverage, defined as the ratio of interest expense to the sum of interest expense and cash flow (earnings plus depreciation); 27 portfolios based on a three-dimensional, independent $3 \times 3 \times 3$ sort on size, book-to-market, and the Kaplan and Zingales (1997, KZ hereafter) index, and finally nine portfolios based on a two-dimensional, independent 3×3 sort on size and the Whited and Wu (2006, hereafter WW) index.[7]

Our sample selection and construction of the KZ portfolios follows closely Lamont, Polk, and Saá-Requejo (2001). We include only data from manufacturing firms that have all the data necessary to construct the KZ index and have a positive sales growth rate deflated by the Consumer Price Index in the prior year. We form portfolios in each June of year t, using accounting data from the firm's fiscal year end in calender year $t - 1$, and using market value in June of year t. And we calculate subsequent value-weighted portfolio returns from July of year t to June of year $t + 1$. Because of data limitations, the sample of monthly returns goes from July 1968 to December 2000. See Appendix B for more details on portfolio construction.

Finally, Whited and Wu (2006) construct an index of financial constraints through structural estimation of an investment Euler equation. They argue convincingly that their index provides a much better way of capturing firm characteristics associated with financial constraints

[6] Lamont (2000) also discusses the importance of aligning investment and asset returns.

[7] Although the size deciles do not display much cross-sectional variation in average returns, we include them because size is a common proxy for financing constraints (Gertler and Gilchrist 1994; Lamont et al. 2001).

than the KZ index.[8] A limitation of using WW index is that we cannot use $3 \times 3 \times 3$ sorts on size, book-to-market, and the WW index as this yields portfolios with almost no observations. Instead, we use only the double-sorted portfolios based on size and the WW index.

Table 1 summarizes descriptive statistics for our test portfolios. Panel A summarizes the statistics for the Fama–French size and book-to-market portfolios, and Panel B summarizes those for the ten size deciles. These results are well known. Panels C and D summarize the results for ten deciles sorted on the ratio of cash flow to assets and interest coverage, respectively. Firms with more cash flow relative to assets and firms with low interest coverage are generally considered to be less financially constrained. From Panel C, firms with high cash flow to assets have generally higher average returns than those with low cash flow to assets. And from Panel D, firms with high interest coverage also earn higher average returns than firms with low interest coverage. But in both cases, the differences in average returns are insignificant.

Panel E of Table 1 summarizes the results of portfolios from independent three-way sorts of the top third, the medium third, and the bottom third of size, of the KZ index and of book-to-market. We classify all firms into one of 27 groups. For example, portfolio p_{123} contains all firms that are in the bottom third sorted by size, in the medium third sorted by the KZ index, and in the top third sorted by book-to-market. We also construct a zero-investment portfolio on the KZ index, denoted p_{KZ}, while controlling for both size and book-to-market. Formally, $p_{KZ} = (p_{131} + p_{132} + p_{133} + p_{231} + p_{232} + p_{233} + p_{331} + p_{332} + p_{333})/9 - (p_{111} + p_{112} + p_{113} + p_{211} + p_{212} + p_{213} + p_{311} + p_{312} + p_{313})/9$. From Panel E, consistent with the evidence in Lamont, Polk, and Saá-Requejo (2001), the financing constraints factor earns an average return of -0.30% per month with a t-statistic of -2.73.

Panel F of Table 1 is based on the nine Whited and Wu (2006) portfolios from independent two-way sorts of the top third, the medium third, and the bottom third of size and the WW index. All firms are classified into one of nine groups. For example, portfolio SC contains all firms that are both in the bottom one-third sorted by size (S) and the top one-third (constrained) sorted by the WW index. And, portfolio SU contains all firms that are both in the bottom one-third sorted by size and the bottom one-third (Unconstrained) sorted by the WW index. The zero-investment portfolio on financing constraints, denoted p_{WW}, with size controlled, is defined as $p_{WW} = (SC + MC + BC)/3 - (SU + MU + BU)/3$. This portfolio earns an average return of 0.18% per month with a t-statistic of 0.95.

[8] In particular, WW (2005) find that firms deemed constrained by the KZ index are often large, over-invest, and have a higher incidence of bond ratings.

Table 1
Descriptive statistics of test portfolios

	Mean					SD						
	Low	2	3	4	High	Low	2	3	4	High		
Panel A: Fama-French 25 size and book-to-market portfolios (January 1954–December 2000)												
Small	0.79	1.27	1.27	1.51	1.58	7.70	6.66	5.74	5.38	5.64		
2	0.89	1.18	1.36	1.43	1.55	6.95	5.69	5.05	4.90	5.43		
3	1.01	1.26	1.26	1.40	1.48	6.39	5.14	4.74	4.59	5.10		
4	1.08	1.08	1.31	1.40	1.40	5.66	4.90	4.65	4.52	5.26		
Big	1.06	1.05	1.14	1.14	1.23	4.65	4.41	4.18	4.27	4.64		
	Small	2	3	4	5	6	7	8	9	Big	1–10	t_{1-10}
Panel B: ten NYSE portfolios sorted on size (January 1954–December 2000)												
Mean	1.45	1.26	1.24	1.23	1.19	1.20	1.13	1.17	1.10	1.03	0.42	1.64
SD	6.43	5.59	5.34	5.12	4.98	4.87	4.73	4.65	4.41	4.05		
	Low	2	3	4	5	6	7	8	9	High	10–1	t_{10-1}
Panel C: ten portfolios sorted on the ratio of cash flow to assets (July 1968–December 2000)												
Mean	0.13	0.36	0.52	0.53	0.44	0.49	0.50	0.49	0.37	0.52	0.39	1.15
SD	9.03	6.02	4.94	5.06	5.12	5.34	5.33	5.63	5.80	7.73		
	Low	2	3	4	5	6	7	8	9	High	10–1	t_{10-1}
Panel D: ten portfolios sorted on interest coverage (July 1968–December 2000)												
Mean	–0.09	0.46	0.48	0.45	0.62	0.55	0.55	0.45	0.06	0.21	0.30	1.01
SD	7.40	6.68	5.79	5.19	5.25	5.39	5.10	5.52	5.93	6.98		

	p_{111}	p_{113}	p_{131}	p_{133}	p_{222}	p_{311}	p_{313}	p_{331}	p_{333}	p_{KZ}	$t_{p_{KZ}}$
Panel E: portfolios from a $3 \times 3 \times 3$ sort on size, the KZ Index, and BE/ME (July 1968–December 2000)											
Mean	−0.03	0.86	0.20	0.80	0.51	0.56	1.05	−0.07	0.50	−0.30	−2.73
SD	9.11	7.19	9.62	7.13	6.68	4.92	6.54	7.34	6.66		

	SC	SM	SU	MC	MM	MU	BC	BM	BU	p_{WW}	$t_{p_{WW}}$
Panel F: nine portfolios from a 3×3 sort on size and the Whited-Wu Index (October 1975–December 2000)											
Mean	0.83	0.66	0.89	0.75	0.81	0.66	1.23	0.97	0.71	0.18	0.95
SD	6.44	6.45	7.99	6.77	6.11	6.55	7.75	6.16	4.78		

This table summarizes mean and volatility in monthly percent for testing portfolios. The starting sample date in Panels A and B is limited by the availability of macroeconomic series used in the GMM estimation. The starting date in Panels C and D is limited by data availability from Compustat. The portfolios used in Panel F are from Whited and Wu (2006). In Panel E, we form 27 portfolios based on independent sorts of the top third, the medium third, and the bottom third of size, of the Kaplan and Zingales (KZ) index and of book-to-market. For example, portfolio p_{123} contains firms that are in the bottom third sorted by size, in the medium third sorted by KZ, and in the top third sorted by book-to-market. To save space, we only report nine out of 27 portfolios including all eight extreme portfolios in the three dimensions of size, KZ, and book-to-market, and the medium group, portfolio p_{222}. Portfolio p_{KZ} is the zero-investment constrained-minus-unconstrained (high-minus-low KZ index) factor-mimicking portfolio, after controlling for both size and book-to-market, that is, $p_{KZ} = (p_{131} + p_{132} + p_{133} + p_{231} + p_{232} + p_{233} + p_{331} + p_{332} + p_{333})/9 - (p_{111} + p_{112} + p_{113} + p_{211} + p_{212} + p_{213} + p_{311} + p_{312} + p_{313})/9$. In Panel F, the nine portfolios are based on independent sorts of the top third, the medium third, and the bottom third of size and of the Whited–Wu (WW) index are small size/constrained (SC), small size/median WW (SM), small size/unconstrained (SU), medium size/constrained (MC), medium size/medium WW (MM), medium size/unconstrained (MU), big size/constrained (BC), big size/medium WW (BM), and big size/unconstrained (BU). Portfolio p_{WW} is the zero-investment constrained-minus-unconstrained (high-minus-low WW index) factor-mimicking portfolio on the WW index, after controlling for size, that is, $p_{WW} = (SC + MC + BC)/3 - (SU + MU + BU)/3$.

3.2 GMM estimates and tests

Table 2 summarizes iterated GMM estimates and tests for the benchmark specification. The benchmark uses the Fama–French 25 size and book-to-market portfolio returns to form moment conditions. We limit the number of moment conditions by using in the unconditional model excess returns of portfolios 11, 13, 15, 23, 31, 33, 35, 43, 51, 53, and 55, one investment excess return, and the real corporate bond return.[9] The conditional and scaled models use excess returns of portfolios 11, 15, 51, and 55, scaled by instruments, excess investment return, and the real corporate bond return. The subsets of portfolios used to form moment conditions maintain the cross-sectional dispersion of average returns in the original 25 portfolios. Our benchmark estimates use the default premium

Table 2
GMM estimates and tests in the benchmark specification

	Unconditional		Conditional		Scaled factor	
Parameters						
a	6.55	(1.50)	4.81	(0.98)	6.44	(1.22)
b_1	−0.11	(−2.41)	−0.12	(−4.80)	−0.12	(−2.96)
Loadings						
l_0	40.14	(3.31)	50.35	(4.04)	41.93	(3.51)
l_1	−38.57	(−3.24)	−48.60	(−3.98)	−40.27	(−3.42)
l_2					−0.31	(−3.23)
l_3					0.28	(1.80)
J_T test						
χ^2	40.84		25.32		24.03	
p	0.00		0.01		0.01	
Likelihood Ratio Test ($b_1 = 0$)						
$\chi^2_{(1)}$	0.97		22.49		10.74	
p	0.33		0.00		0.00	

This table summarizes GMM estimates and tests for the benchmark specification. The sample is from the second quarter of 1954 to the third quarter of 2000. The shadow price of external funds is $\mu_t = b_0 + b_1 f_t$, where f_t is the default premium, defined as the difference between the yields on Baa and Aaa corporate bonds. We report the estimates for a, b_1, the pricing kernel loadings, ls, the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and the χ^2 statistics and associated p-values of the Wald and Likelihood ratio tests on the null hypothesis that $b_1 = 0$. t-statistics are reported in parentheses to the right of parameter estimates. The unconditional model uses the excess returns of portfolios 11, 13, 15, 23, 31, 33, 35, 43, 51, 53, and 55 of the Fama–French 25 size and book-to-market portfolios, one investment excess return over real corporate bond return, and real corporate bond return. The Fama–French portfolios are labeled such that the first digit denotes the size group and the second digit denotes the book-to-market group, both in ascending order. The conditional and scaled factor estimates use excess returns of the Fama–French portfolios 11, 15, 51, and 55, scaled by instruments, excess investment return, and the real corporate bond return. Instruments are the constant, term premium (tp), and equally weighted dividend-price ratio (dp). The pricing kernel is $M = l_0 + l_1 R^I$ for the unconditional and conditional models and $M = l_0 + l_1 R^I + l_2(R^I tp) + l_3(R^I dp)$ for the scaled factor model. R^I is real investment return and is constructed from the flow-of-fund accounts using data from the nonfinancial corporate sector with before-tax profits.

[9] The first digit denotes the size group, and the second digit denotes the book-to-market group, both in ascending order. For example, portfolio 15 is formed by taking the intersection of smallest size quintile and highest book-to-market quintile.

as the instrument for the shadow price of external funds in Equation (10). In all cases, we report the value of the parameters a and b_1 as well as estimated loadings, **l**, and corresponding t-statistics. Also included are the results of J tests on the model's overall ability to match the data and the corresponding p-values.

The results in Table 2 show a consistently positive estimate for the adjustment cost parameter, a. Although the exact values are relatively large when compared to typical microeconomic estimates, they are probably a result of the smoothness of the aggregate investment data and are consistent with those used in Cochrane (1991).

More importantly, however, we also find a negative, and significant, value for the financing parameter, b_1. Given the strongly countercyclical nature of the default premium, our finding that b_1 is negative also implies that the shadow price of external funds is quite procyclical. Intuitively, financing distortions are more important when aggregate economic conditions are relatively good.

The J_T tests of over-identification show that our benchmark specification is rejected at conventional significance levels. This result is probably not surprising, given our parsimonious model structure and the strong cross-sectional variations in the average Fama–French portfolio returns. Nevertheless, Figure 1 shows that our setting generally improves upon a frictionless model. The pricing errors associated with the model with financing constraints are consistently smaller than those associated with the model when $b_1 = 0$. Specifically, adding financial constraints reduces the pricing error from 2.24% per quarter to 2.10% in the unconditional model, from 1.82% to 1.23% in the conditional model, and from 3.48% to 1.80% in the scaled factor model.

Table 3 departs from our benchmark specification by augmenting the pricing kernel (9) to include the return on corporate bonds, R^B.[10] The results are generally consistent with our findings in Table 2. The adjustment cost parameters are again consistently positive, although generally lower in value. The point estimates for the shadow cost coefficient, b_1, are again consistently negative and significant.

3.3 The effects of financial frictions

The implications of the results in Tables 2 and 3 for the effects of financing frictions on returns can be summarized as follows: (i) financial market imperfections play an important role in pricing the cross-section of expected returns and (ii) the shadow price of external funds seems to exhibits procyclical variation.

[10] Gomes, Yaron, and Zhang (2003b) show that this is the correct form of the pricing kernel in the presence of financing frictions, since the return to physical investment is now a linear combination of stock and bond returns, with the weights given by the leverage ratio.

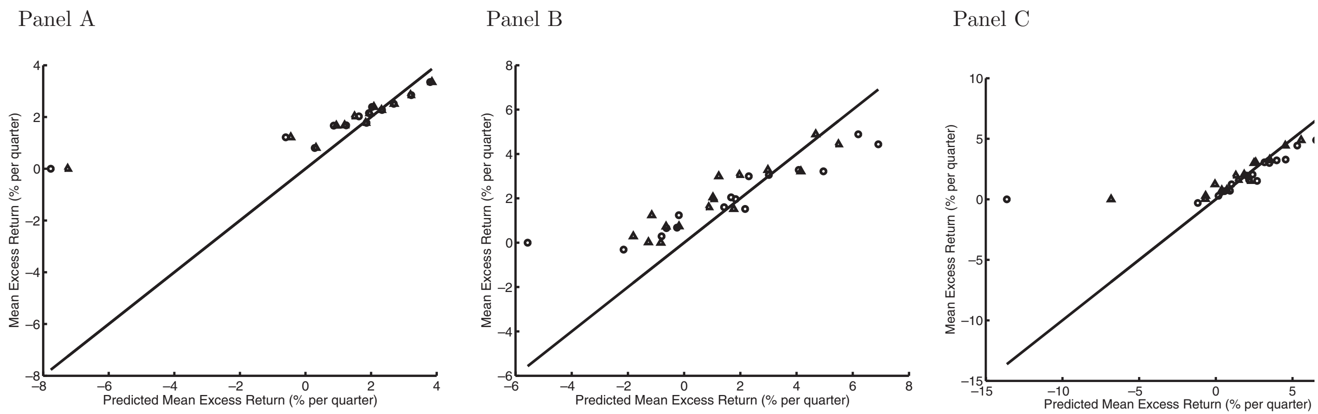

Figure 1
Predicted versus actual mean excess returns
This figure plots the mean excess returns against predicted mean excess returns both in quarterly percent for the unconditional model (Panel A), conditional model (Panel B), and scaled factor model (Panel C). All three plots are from iterated GMM estimates. The triangles represent the benchmark specification with financing constraints, and the circles represent the restricted benchmark specification without financing constraints, that is, $b_1 = 0$. The pricing kernel and the moment conditions are the same as those described in Table 3.

Table 3
GMM estimates and tests with bond returns in the pricing kernel

	Unconditional		Conditional		Scaled factor	
Parameters						
a	10.75	(1.40)	17.43	(2.98)	5.15	(0.68)
b_1	−0.38	(−2.11)	−0.11	(−7.96)	−0.10	(−3.10)
Loadings						
l_0	59.81	(1.33)	49.09	(4.70)	51.06	(2.83)
l_1	−57.38	(−1.28)	−37.70	(−4.32)	−67.77	(−2.70)
l_2	−0.69	(−0.16)	−9.85	(−2.11)	18.98	(1.17)
l_3					6.02	(1.43)
l_4					3.62	(0.68)
l_5					−6.48	(−1.49)
l_6					−3.40	(−0.61)
J_T test						
χ^2	48.27		21.54		14.85	
p	0.00		0.03		0.04	
Likelihood Ratio Test ($b_1 = 0$)						
$\chi^2_{(1)}$	0.64		9.97		4.47	
p	0.42		0.00		0.03	

This table summarizes GMM estimates and tests for the benchmark model using an augmented pricing kernel. The sample is from the second quarter of 1954 to the third quarter of 2000. μ_t is the same as in Table 2. We report the estimates for a, b_1, and the loadings, ls, in the pricing kernel, the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and χ^2 statistic and p-value of the Wald test on the null hypothesis that $b_1 = 0$. t-statistics are reported in parentheses to the right of parameter estimates. The pricing kernel is $M = l_0 + l_1 R^I + l_2 R^B$ for the unconditional and conditional models and $M = l_0 + l_1 R^I + l_2 R^B + l_3(R^I tp) + l_4(R^I dp) + l_5(R^B tp) + l_6(R^B dp)$ for the scaled factor model. R^I and R^B are real investment and bond returns, respectively. Moment conditions, instruments, and data are the same as those reported in Table 2.

What drives these results? Mechanically, our GMM estimation seeks to minimize a weighted average of the price errors associated with Equation (8). Intuitively, this requires aligning the dynamic properties of the stochastic discount factor (essentially driven by investment return) and those of asset returns (basically driven by the large volatility in stock returns). A successful estimation procedure will then choose parameter values for a and b_1 so that the investment return has similar dynamic properties to those of the targeted stock returns.

To gain more intuition on our results, we therefore examine the dynamic properties of the investment returns generated under alternative values of b_1 and compare those with the behavior of stock returns.

3.3.1 Correlation structure. We start by focusing on the correlation structure of stock and investment returns with the two economic fundamentals, aggregate investment/capital ratio i and aggregate profits/capital ratio π. Recall that Equation (6) decomposes investment returns into a frictionless component, $\widetilde{R}^I$, that is driven by the fundamentals i and π, and a financing component, captured by the dynamics in the shadow price of external funds, μ.

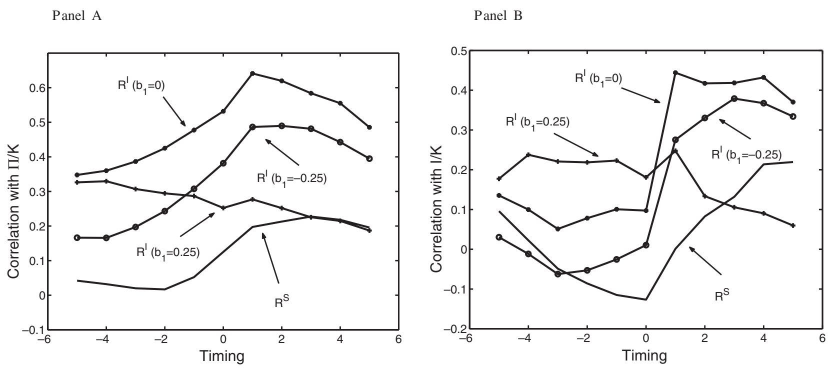

Figure 2
Correlation structure of stock and investment returns with leads and lags of i and π
This figure presents the correlations of investment returns R^I and real value-weighted market returns R^S with the various leads and lags of I/K and Π/K. Panel A plots the correlation structure of the above series with Π/K and Panel B plots that with I/K. In the figures, b_1 is the slope term in the specification of financing premium (10).

Figure 2 displays the correlation structure between returns and various leads and lags of the fundamentals π (Panel A) and i (Panel B). In both panels, the dynamic pattern of the frictionless returns, $\widetilde{R}^I$ $(b_1 = 0)$, is very similar to that of the observed R^S. In particular, both returns *lead* future economic activity, while their contemporaneous correlations with fundamentals are somewhat low. As Cochrane (1991) notes, this is to be expected if firms adjust current investment in response to an anticipated shock to future productivity.

Figure 2 also shows how the effect of financing on investment returns depends on the cyclical nature of the premium on external funds, here measured by the default premium. As the figure shows, financing frictions improve the model's ability to match the underlying pattern of stock returns only when $b_1 < 0$.

The economic intuition is the following. Suppose for a moment that the shadow price of external funds was countercyclical, so that $b_1 > 0$. In this case, a rise in expected future productivity is also associated with an expected decline in the marginal cost of external financing. Productivity and financial constraints provide two competing forces for the response of investment returns to business cycle conditions. An increase in expected future productivity implies that firms should respond by investing immediately. However, since the shock also entails lower marginal cost of external funds in the future, firms prefer to delay investment. Relative to a frictionless world, Equation (6) implies a reduction in investment returns and thus *lower* correlations with future economic activity. Figure 2 shows, however, that this reaction is not consistent with observed asset return data.

Finally, Figure 2 also indicates that there is no obvious phase shift between any of the series, suggesting that our results are not likely to be sensitive to timing issues such as those created by the existence of time-to-plan or perhaps time-to-finance in this context. What seems crucial is the cyclical pattern of the shadow price of external funds.

3.3.2 Properties of the pricing kernel. Further intuition can be obtained by looking directly at the effect of the financing premium on the properties of the pricing kernel. Table 4 describes the effects of imposing $b_1 > 0$ in each set of moment conditions (unconditional, conditional, and scaled factor), while keeping the value of the adjustment cost parameter a at its optimal level reported in Table 3.

The left panel of the table indicates that a countercyclical shadow price of external funds, $b_1 \geq 0$, lowers the absolute magnitude of the correlation between the stochastic discount factor and value-weighted returns (as well as the price of risk $\sigma(M)/E(M)$), thus deteriorating the performance of the stochastic discount factor.

Perhaps, a more direct way to evaluate the effect of a positive b_1 on the pricing kernel is to examine the implied pricing errors. A simple way is to use the beta representation, which is equivalent to the stochastic discount factor representation in Equation (8), (Cochrane 2001):

$$R^p - R^f = \alpha_i + \beta_{1i}(R^I - R^f) + \beta_{2i}(R^B - R^f)$$

Table 4
Properties of pricing kernels, Jensen's α, and investment returns

	Pricing kernel		Jensen's α				Investment return			
b_1	$\sigma[M]/\mathrm{E}[M]$	ρ_{M,R^S}	α^{vw}	t_α^{vw}	α^{d1}	t_α^{d1}	Mean	σ_{R^I}	$\rho(1)$	ρ_{R^I,R^S}
Unconditional model										
0.00	0.82	−0.28	0.26	0.35	1.02	0.78	6.55	0.97	0.76	0.30
0.15	0.57	−0.03	3.03	4.94	5.69	5.45	6.56	1.70	0.38	−0.31
0.30	0.58	−0.07	3.07	6.22	5.58	6.66	6.58	2.98	0.31	−0.41
Conditional model										
0.00	0.75	−0.29	0.16	0.30	0.68	0.77	5.91	2.24	0.09	0.35
0.15	0.37	0.39	1.46	2.70	3.01	3.25	5.92	2.23	0.00	−0.01
0.30	0.79	0.17	2.22	4.51	4.21	5.02	5.93	3.05	0.10	−0.24
Scaled factor model										
0.00	0.81	−0.36	0.03	0.06	0.51	0.55	6.02	1.99	0.14	0.36
0.25	0.67	−0.06	1.63	2.92	3.35	3.48	6.03	2.06	0.06	−0.05
0.50	0.61	0.01	2.38	4.79	4.48	5.30	6.04	2.98	0.15	−0.27

This table summarizes, for each combination of parameters a and b_1, properties of the pricing kernel, including market price of risk ($\sigma[M]/E[M]$), the contemporaneous correlation between pricing kernel and real market return (ρ_{M,R^S}), Jensen's α and its corresponding t-statistic (t_α), summary statistics of investment return, including mean, volatility (σ_{R^I}), first-order autocorrelation [$\rho(1)$], and correlation with the real value-weighted market return (ρ_{R^I,R^S}). Jensen's α is defined from the following regression: $R^p - R^f = \alpha + \beta_1(R^I - R^f) + \beta_2(R^B - R^f)$, where R^p is either the real value-weighted market return (R^{vw}) or the real decile one return (R^1), R^f is the real interest rate proxied by the real treasury-bill rate, R^I is the real investment return, and R^B is the real corporate bond return. In each case, the cost parameters as are held fixed at the GMM estimates.

for any portfolio p. Given the assumed structure of the pricing kernel, this representation exists, with $\alpha_p = 0$. Therefore, large values of α indicate poor performance of the model.

The middle panel of Table 4 summarizes the implied αs for the regressions on both small firms (NYSE decile 1) and value-weighted returns. The panel displays a clear pattern of rising α as we increase the magnitude of b_1. Indeed, while we cannot reject that $\alpha = 0$ when $b_1 = 0$, this is no longer true for most positive values of b_1.

Finally, we report the implications of financing constraints for the moments of investment returns and their correlations with market returns. While both the mean and the variance of investment returns are not really affected when b_1 increases, the correlation with stock returns falls significantly. Indeed, while the correlation between the two returns is about 30% with $b_1 = 0$, the correlation becomes negative with a positive b_1. Since the overall performance of a factor model hinges on its covariance structure with stock returns, it is not surprising that financing constraints are important only if the shadow price of external funds is $b_1 < 0$.

3.3.3 Implications. Our findings on the procyclical properties of the shadow cost of external funds effectively impose a restriction on the nature of these costs. Thus, our results can also be viewed as an important test to the various alternative theories of financial market imperfections.

In this sense, our estimates lend some support to models that emphasize the importance of frictions generated by the presence of agency problems (Dow, Gordon and Krishnamurthy 2004). The reason is that these types of financial imperfections are much more likely to be important when economic conditions are relatively good.

Conversely, our results seem less supportive of costly external finance theories, where adverse liquidity shocks are magnified by a rising cost of external funds. As we have seen, this interpretation of the data significantly worsens the ability of investment returns to match the observed data on asset returns.[11]

Finally, it is tempting to interpret our findings that $b_1 < 0$ as evidence that external funds are less expensive than internal cash flows. This interpretation, however, is incorrect since our tests cannot identify the overall level of the constant term, b_0, in Equation (10).

3.4 Robustness

We now examine the robustness of our basic results by exploring several alternatives to the benchmark test specification.

[11] Gomes, Yaron, and Zhang (2003a) study a general equilibrium version of one model of costly external finance and show the potentially counterfactual implications for asset prices.

3.4.1 Alternative sets of moment conditions. Table 5 summarizes GMM estimates and testing results using moment conditions derived from the various alternative portfolios discussed in Section 3.1. Panel A of Table 5 forms moment conditions using the ten size portfolios; these portfolios are interesting because size is a common proxy for financing constraints (Gertler and Gilchrist 1994; Lamont, Polk, and Saá-Requejo 2001). The model is able to price this set of moment conditions much better, and it cannot be rejected using the over-identification test. The estimated b_1 coefficients are also all negative and significant, a result again reinforced by the reported likelihood ratio tests. The shadow price of external funds therefore continues to display procyclical variations using this alternative set of moment conditions.

Panels B and C of Table 5 summarize that the evidence is somewhat more mixed when we use as testing portfolios ten deciles sorted on the ratio of cash flow to assets and deciles sorted on interest coverage. Although the estimated b_1 coefficients are mostly negative, they are often insignificant. Overall, the evidence seems to lean toward a procyclical shadow price of external funds. And from the over-identification tests, the model is again reasonably successful in pricing these returns. Similar evidence about the role of financial frictions comes from the triple-sorted portfolios on size, the KZ index, and book-to-market, as well as the double-sorted portfolios on size and the WW index, as reported by Panels D and E of Table 5.

3.4.2 Alternative specifications for the shadow cost of external funds. Table 6 investigates whether our results are sensitive to the use of our benchmark specification for the shadow cost of external funds in Equation (10).

To construct the estimates in Panel A of Table 6, we follow Whited (1992) and Love (2003) and directly parameterize the ratio

$$\frac{(1+\mu_{t+1})}{(1+\mu_t)} = b_0 + b_1 f_t,$$

where we again choose the default premium to be the common financing factor, f_t. While this specification does not allow us to identify the shadow cost directly, it has the benefit of allowing us to identify the properties of the wedge between investment returns with and without financing frictions. Our estimates of a negative value for the slope parameter, b_1, illustrate again the procyclical nature of this wedge.

Although the default premium is a good predictor of aggregate economic activity (Stock and Watson 1989), Panels B and C investigate the results of using two other proxies for the financing factor. The first is the aggregate default likelihood measure constructed in Vassalou and Xing (2003), who construct an aggregate measure of financial distress by

Table 5
GMM estimates and tests with alternative moment conditions

	Unconditional		Conditional		Scaled Factor	
Panel A: size deciles						
Parameters						
a	2.80	(0.84)	9.35	(1.78)	8.56	(1.54)
b_1	−0.36	(−3.14)	−0.08	(−5.07)	−0.08	(−4.12)
Loadings in the pricing kernel						
l_0	111.81	(2.51)	47.72	(3.94)	51.53	(2.85)
l_1	−110.43	(−2.52)	−43.85	(−3.32)	−57.57	(−2.89)
l_2	1.43	(0.27)	−2.19	(−0.54)	7.88	(0.55)
l_3					3.69	(1.08)
l_4					2.46	(0.54)
l_5					−3.79	(−1.05)
l_6					−2.37	(−0.50)
J_T	6.44		8.27		6.75	
P	0.49		0.69		0.46	
Likelihood Ratio Test on $b_1 = 1$						
χ_1^2	0.89		9.18		5.64	
P	0.35		0.00		0.02	
Panel B: deciles on cash flow/assets						
Parameters						
a	17.72	(0.54)	53.19	(0.32)	3.07	(0.15)
b_1	0.16	(0.76)	−0.18	(−0.43)	−0.14	(−1.81)
Loadings in the pricing kernel						
l_0	42.32	(0.79)	−1.86	(−0.26)	−13.11	(−0.75)
l_1	−33.84	(−0.61)	1.43	(0.43)	9.42	(0.79)
l_2	−7.01	(−0.94)	1.39	(0.31)	3.66	(0.41)
l_3					0.57	(0.31)
l_4					−0.25	(−0.04)
l_5					−0.58	(−0.30)
l_6					1.11	(0.17)
J_T	4.29		19.47		12.59	
p	0.75		0.05		0.08	
Likelihood Ratio Test on $b_1 = 1$						
χ_1^2	0.58		1.18		3.29	
P	0.45		0.28		0.07	
Panel C: deciles on interest coverage						
Parameters						
a	2.50	(0.32)	34.83	(0.39)	1.80	(0.27)
b_1	0.06	(0.65)	−0.17	(−1.23)	−0.27	(−5.57)
Loadings in the pricing kernel						
l_0	43.51	(0.76)	−6.11	(−1.01)	26.84	(1.90)
l_1	−41.21	(−0.72)	4.61	(1.24)	−28.38	(−2.16)
l_2	−0.67	(−0.14)	2.40	(0.78)	2.17	(0.25)
l_3					2.43	(1.03)
l_4					1.88	(0.59)
l_5					−1.89	(−0.76)
l_6					−1.95	(−0.58)
J_T	12.61		20.21		12.64	
p	0.08		0.04		0.08	
Likelihood Ratio Test on $b_1 = 1$						
χ_1^2	0.43		4.20		3.18	
p	0.51		0.04		0.07	

Table 5
(continued)

	Unconditional		Conditional		Scaled Factor	
Panel D: 27 portfolios on size, KZ, and b/m						
Parameters						
a	33.98	(0.78)	35.00	(1.33)	31.45	(0.92)
b_1	0.11	(1.43)	−0.13	(−2.60)	−0.13	(−0.83)
Loadings in the pricing kernel						
l_0	53.88	(1.58)	13.04	(1.80)	5.69	(0.44)
l_1	−41.86	(−1.17)	−7.81	(−1.31)	−9.84	(−0.81)
l_2	−10.55	(−0.77)	−4.13	(−1.12)	6.94	(0.34)
l_3					1.29	(0.40)
l_4					2.47	(0.37)
l_5					−2.05	(−0.60)
l_6					−3.05	(−0.43)
J_T	14.56		40.60		15.95	
p	0.02		0.00		0.10	
Likelihood Ratio Test on $b_1 = 1$						
χ_1^2	2.52		11.02		0.45	
p	0.11		0.00		0.50	
Panel E: nine portfolios on size and WW						
Parameters						
a	1.75	(0.18)	1.79	(0.23)	13.67	(0.53)
b_1	−0.13	(−2.37)	−0.09	(−1.81)	0.08	(0.95)
Loadings in the pricing kernel						
l_0	−23.04	(−1.12)	−23.41	(−1.30)	71.74	(0.39)
l_1	21.21	(1.09)	22.73	(0.61)	−79.47	(−0.43)
l_2	2.40	(0.66)	1.27	(0.61)	8.81	(0.69)
l_3					8.87	(2.32)
l_4					8.04	(1.01)
l_5					−8.19	(−2.18)
l_6					−8.32	(−0.91)
J_T	7.66		18.36		11.78	
p	0.26		0.07		0.11	
Likelihood Ratio Test on $b_1 = 1$						
χ_1^2	5.59		3.27		0.89	
p	0.02		0.07		0.33	

This table summarizes GMM estimates and tests using alternative moment conditions constructed from ten size deciles of NYSE stocks (Panel A), ten deciles sorted on cash flow to assets ratio (Panel B), ten deciles sorted on interest coverage (Panel C), 27 portfolios sorted on size, the KZ index, and book-to-market (Panel D), and from nine portfolios sorted on size and the WW index (Panel E). The sample of Panel A is from the second quarter of 1954 to the third quarter of 2000. Because of data restriction from Compustat, the sample of Panels B–D is from the fourth quarter of 1968 to the third quarter of 2000. And the sample in Panel E goes from the first quarter of 1976 to the third quarter of 2000. In Panels A to C, the unconditional models use as moment conditions the excess returns of the respective ten deciles and one investment excess return (all over the real corporate bond returns) and the real corporate bond returns. The conditional and the scaled factor models use the excess returns of decile one, four, seven, and ten, investment excess returns, all scaled by instruments, and the real corporate bond returns. In Panel D, the unconditional model uses as moment conditions investment excess return, the real corporate bond returns, and the excess returns of portfolios p_{111}, p_{113}, p_{131}, p_{133}, p_{222}, p_{311}, p_{313}, p_{331}, and p_{333} from the 27 portfolios based on a triple-sort on size, the KZ index, and book-to-market. The portfolio classification is the same as that in Panel E in Table 1. The conditional and the scaled factor models in Panel D use the excess returns of portfolios p_{111}, p_{131}, p_{222}, p_{313}, and p_{333}, investment excess returns, all scaled by instruments, and the real corporate bond returns. Instruments include a constant, term premium, and equally weighted dividend-price ratio. In Panel E, the unconditional model uses as moment conditions the excess returns of all nine portfolios from a double sort on size and the WW index, one investment excess return, and the real corporate bond returns. The conditional and the scaled factor models use the excess returns of portfolios SU, SC, BU, and BC, investment excess returns, all scaled by instruments, and the real corporate bond returns. In all cases, we report the estimates for a and b_1, the factor loadings l, the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and χ^2 statistic and p-value of the Wald test on the null hypothesis that $b_1 = 0$. t-statistics are reported in parentheses to the right of parameter estimates. The pricing kernel and the specification of the shadow price of external funds are the same as those reported in Table 3.

Table 6
GMM estimates and tests with alternative instruments in the shadow price of external funds

	Panel A: $(1+\mu_{t+1})/(1+\mu_t) = b_0 + b_1 f_{t+1}$			Panel B: aggregate default likelihood		
	Unconditional	Conditional	Scaled factor	Unconditional	Conditional	Scaled factor
Parameters						
a	14.77 (2.73)	23.02 (1.71)	19.71 (3.27)	0.00 (0.00)	9.50 (0.80)	28.17 (1.57)
b_1	−0.11 (−1.04)	−0.36 (−8.47)	−0.40 (−4.68)	0.00 (0.83)	−0.01 (−2.47)	0.03 (0.86)
J_T test						
χ^2	56.28	36.83	25.33	54.11	34.56	21.93
p	0.00	0.00	0.00	0.00	0.00	0.00
Likelihood Ratio Test (b_1=0)						
$\chi^2_{(1)}$	0.16	2.84	3.31	1.33	4.79	2.58
p	0.69	0.09	0.07	0.25	0.03	0.11

	Panel C: LPS's financing constraints factor			Panel D: $\mu = b_0 + b_1(CF/K) + b_2 f(CF/K)$		
	Unconditional	Conditional	Scaled factor	Unconditional	Conditional	Scaled factor
Parameters						
a	0.00 (0.00)	5.50 (1.97)	1.40 (0.41)	12.92 (1.80)	5.66 (0.42)	19.85 (2.20)
b_1	−0.10 (−1.12)	0.15 (1.69)	0.14 (1.32)	−1.57 (−0.12)	28.78 (1.05)	−2.75 (−0.47)
b_2				−0.52 (−0.04)	−16.49 (−1.76)	−1.36 (−0.28)
J_T test						
χ^2	10.61	18.98	9.72	46.98	23.29	17.23
p	0.22	0.06	0.21	0.00	0.01	0.01
Likelihood Ratio Test (b_1= 0 or b_1= b_2= 0)						
χ^2	0.58	2.74	1.69	0.55	12.85	1.26
p	0.44	0.10	0.19	0.76	0.00	0.53

This table summarizes GMM estimates and tests using alternative specifications of the shadow price of external funds. Panel A specifies the shadow price as a linear function of the default premium measured as the difference between yields of Baa and long-term government bonds, as opposed to that between yields of Baa and Aaa corporate bonds in Table 3. Panel B specifies the shadow price as a linear function of the aggregate default likelihood indicator constructed by Vassalou and Xing (2003). Panel C specifies the shadow price as a linear function of the common factor of financing constraints constructed by Lamont, Polk, and Saá-Requejo (2001, LPS). We report the estimates for a and b_1 (as well as b_2 in Panel D), the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and χ^2 statistic and p-value of the Wald test on the null hypothesis that $b_1 = 0$ or $b_1 = b_2 = 0$. t-statistics are reported in parentheses to the right of parameter estimates. The pricing kernel and moment conditions are the same as those reported in Table 3.

aggregating over estimated firm-level default likelihood indicators. This measure of financial distress increases substantially during recessions. Data for this indicator is available at monthly frequency between January of 1971 and December of 1999. The second measure is the common factor of financing constraints measured by the KZ index after controlling for size constructed by Lamont, Polk, and Saá-Requejo (2001).[12]

[12] This common factor is based on the portfolios from a double sort of the top third, medium third, and the bottom third of size (B, M, and S) and the KZ index (H, M, and L). All firms are then classified into nine groups. For example, portfolio LS contains all firms both in the bottom third of size and the KZ index. The common factor is then defined as $(HS + HM + HB)/3 - (LS + LM + LB)/3$.

Panel B of Table 6 summarizes the GMM results when the aggregate default likelihood is used to model the shadow price of external funds, while Panel C summarizes the effects of using p_{kz} instead. In both cases, we obtain estimates of b_1 that are not significantly different from zero. This is perhaps due to the fact that the common factor of financial constraint does not covary much with business cycle conditions, as shown in Lamont, Polk, and Saá-Requejo (2001).

Finally, we also investigate a more elaborate parametrization of the shadow price of external funds often used in the microeconomic literature (Hubbard, Kashyap, and Whited 1995)

$$\mu_t = b_0 + b_1\pi_t + b_2 f_t \times \pi_t.$$

Our results in Panel D of Table 6 show that this specification works less well at the aggregate level as neither financing factor is generally significant. Similar results are also obtained when using cash flows alone as a factor.

This finding that corporate cash flows do not seem to be an important component of our financing factor is difficult to reconcile with a strict interpretation of popular agency theories (Jensen 1986; Dow, Gorton, and Krishnamurthy 2004) since these typically imply that distortions are directly linked to available cash. Popular versions of models of costly external finance usually also predict that cash flow is (inversely) related to the marginal cost of funds.

Thus, the lack of significance of cash flow does not shed much light on these alternative views on the source of financial market imperfections. The reason is probably the relatively low time series variation in aggregate cash-flows, at least when compared to our other financing factors such as the default premium.

3.4.3 Alternative macroeconomic series. Table 7 documents the effects of using alternative macroeconomic data in the construction of the investment returns in Equation (6). Specifically, in Panel A, we use after tax profit data, while Panel B is based on data for the entire economy and not just the nonfinancial corporate sector.

Panels C considers the case when investment is divided into equipment and structures. To do this we modify our original setup and assume that firms accumulate two forms of capital with potentially different adjustment cost technologies. Note that we now obtain separate moment conditions for equipment and structures. Our results conform with the intuition that adjustment costs are much larger for structures than equipment. Although the model performs generally better than in our benchmark specification, the effects of this disaggregation on our estimates of b_1 are fairly small.

Table 7
GMM estimates and tests with alternative macroeconomic data

	Unconditional		Conditional		Scaled factor	
Panel A: nonfinancial after tax						
Parameters						
a	0.43	(0.11)	1.52	(0.28)	2.45	(0.31)
b_1	−0.05	(−1.53)	−0.12	(−4.33)	−0.09	(−3.59)
J_T test						
χ^2	36.61		21.28		14.35	
p	0.00		0.03		0.05	
Likelihood Ratio Test ($b_1 = 0$)						
$\chi^2_{(1)}$	3.07		15.40		2.26	
p	0.08		0.00		0.13	
Panel B: aggregate profits						
Parameters						
a	1.01	(0.10)	19.53	(1.92)	0.00	(0.00)
b_1	−0.08	(−1.88)	−0.10	(−2.26)	−0.09	(−4.66)
J_T test						
χ^2	39.33		27.06		19.36	
p	0.00		0.00		0.01	
Likelihood Ratio Test ($b_1 = 0$)						
$\chi^2_{(1)}$	4.76		10.65		3.16	
p	0.03		0.00		0.08	
Panel C: disaggregate investment						
Parameters						
a_{equ}	10.79	(2.34)	6.72	(2.55)	16.10	(2.24)
a_{str}	47.93	(2.70)	55.04	(5.36)	90.87	(1.56)
b_1	−0.11	(−1.05)	−0.12	(−3.26)	−0.13	(−1.27)
J_T test						
χ^2	26.15		48.91		12.77	
p	0.00		0.00		0.17	
Likelihood Ratio Test ($b_1 = 0$)						
$\chi^2_{(1)}$	6.19		36.35		8.62	
p	0.01		0.00		0.00	
Panel D: sales						
Parameters						
a_{equ}	2.50	(0.36)	23.92	(1.53)	0.00	(0.00)
a_{str}	0.08	(0.93)	0.17	(1.82)	0.11	(2.10)
b_1	−0.40	(−6.30)	−0.14	(−7.61)	−0.10	(−3.40)
J_T test						
χ^2	37.79		21.77		14.28	
p	0.00		0.02		0.31	
Likelihood Ratio Test ($b_1 = 0$)						
$\chi^2_{(1)}$	2.58		9.69		3.40	
p	0.11		0.00		0.07	

This table summarizes GMM estimates and tests using alternative macroeconomic data. Panel A measures profits Π as nonfinancial profits after tax, and Panel B measures Π as the profits of the aggregate economy (not just the nonfinancial corporate sector). In Panel C, we allow two investment returns instead of one aggregate investment return as in Table 3. R^I_{equ} is the return on equipment investment and R^I_{str} is the return on structure investment. Data on investment and capital on equipment and structure are constructed from the flow-of-fund accounts. Panel C also allows the adjustment cost parameter to vary across the two sectors; a_{equ} is the adjustment cost parameter for equipment investment and a_{str} is that for structure investment. Panel D measure profits Π as $\gamma \times$ Sales, where γ is an additional parameter to be estimated, as opposed to nonfinancial profits before tax in the benchmark specification. We report the estimates for a and b_1, the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and χ^2 statistic and p-value of the Wald test on the null hypothesis that $b_1 = 0$. t-statistics are reported in parentheses to the right of parameter estimates. The moment conditions are the same as those reported in Table 3.

Finally, Panel D summarizes the results of relaxing our assumption of constant returns to scale of cash flows. Specifically, Love (2003) shows that, under fairly general assumptions about technology, marginal profits are proportional to the sales-to-capital ratio. Thus, we replace average profits in Equation (6) with $\gamma \times Y/K$, where Y is the gross product of the nonfinancial corporate sector. The estimates of γ are in the empirically plausible range, between 0.1 and 0.15, and the overall goodness-of-fit of the model is also significantly improved. Moreover the estimated coefficients for the parameter b_1 are always negative and generally quite significant.

3.4.4 Alternative pricing kernels. We also consider two perturbations on the pricing kernels. First, we relax the linear factor representation of the pricing kernels. Several alternative approaches to modeling nonlinear pricing kernels have been advanced in the literature (Bansal and Vishwanathan 1993). Here, we explore this possibility by re-estimating the moment conditions using some nonlinear pricing kernels. Panels A and B in Table 8 summarize that our results are not much affected by assuming that the pricing kernel is quadratic in either R^I alone or in both R^I and R^B.

Alternatively, we also examine the effects of using a more general form for investment returns that allows for the fact that the required rate on debt, R_t, is a (stochastic) function of the leverage ratio, that is, $R_t = R(B_t/K_t, S_t)$. As shown in Appendix A.2, the investment return in this case depends on the first-derivative of the interest rate with respect to the debt-to-capital ratio. Specifically,

$$r^I_{t+1} \equiv \frac{(1+\mu_{t+1})\left[\pi_{t+1}+\frac{a}{2}i^2_{t+1}+R_1\left(\frac{B_{t+1}}{K_{t+1}},S_{t+1}\right)\left(\frac{B_{t+1}}{K_{t+1}}\right)^2+(1-\delta)(1+ai_{t+1})\right]}{(1+\mu_t)(1+ai_t)}. \quad (12)$$

Following Bond and Meghir (1994), we parameterize R as a quadratic function of B_t/K_t:

$$R\left(\frac{B_t}{K_t}, S_t\right) = r_0 + r_1\left(\frac{B_t}{K_t}\right) + r_2\left(\frac{B_t}{K_t}\right)^2, \quad (13)$$

which implies that $R_1(B_t/K_t, S_t) = r_1 + 2r_2(B_t/K_t)$. We then estimate the parameters r_1 and r_2 along with other parameters in the investment return.

Panel C of Table 8 summarizes our findings. As before, the b_1 estimates are negative and often significant, suggesting that our basic conclusion is robust to the alternative specification of investment return in Equation (12). The estimated values for the parameters r_1 and r_2 are generally not statistically significant, although the point estimates have the expected signs.

Table 8
GMM estimates and tests with alternative pricing kernels

	Panel A: $M = l_0 + l_1 R^I + l_2 (R^I)^2$						Panel B: $M = l_0 + l_1 R^I + l_2 R^B + l_3 (R^I)^2 + l_4 (R^B)^2$						Panel C: alternative investment return in M					
	Unconditional		Conditional		Scaled factor		Unconditional		Conditional		Scaled factor		Unconditional		Conditional		Scaled factor	
Parameters																		
a	8.42	(1.98)	9.79	(2.56)	14.97	(1.94)	8.47	(1.30)	5.00	(0.39)	17.50	(0.56)	12.60	(0.31)	46.62	(1.52)	56.24	(0.81)
b_1	−0.09	(−1.95)	−0.10	(−2.56)	−0.09	(−4.49)	−0.40	(−7.83)	−0.13	(−5.41)	−0.16	(−1.90)	−0.38	(−2.92)	−0.11	(−4.59)	−0.10	(−0.65)
r_1													2.85	(0.30)	8.80	(1.64)	4.62	(0.48)
r_2													−4.41	(−0.31)	−14.00	(−1.63)	−6.59	(−0.46)
J_T test																		
χ^2	42.40		27.14		15.25		39.62		21.78		3.61		31.08		15.75		7.98	
p	0.00		0.00		0.03		0.00		0.01		0.06		0.00		0.07		0.16	
Likelihood Ratio Test ($b_1 = 0$)																		
$\chi^2_{(1)}$	1.24		20.40		8.17		0.83		9.92		0.48		0.68		21.07		1.69	
p	0.26		0.00		0.00		0.36		0.00		0.49		0.41		0.00		0.21	

This table summarizes GMM estimates and tests using alternative pricing kernels. Panel A uses the pricing kernel: $M = l_0 + l_1 R^I + l_2 (R^I)^2$ for the unconditional and conditional model and $M = l_0 + l_1 R^I + l_2 (R^I)^2 + l_3 (R^I \cdot tp) + l_4 (R^I \cdot dp) + l_5 [(R^I)^2 \cdot tp] + l_6 [(R^I)^2 \cdot dp]$ for the scaled factor model. Panel B uses the pricing kernel: $M = l_0 + l_1 R^I + l_2 R^B + l_3 (R^I)^2 + l_4 (R^B)^2$ for the unconditional and conditional model and $M = l_0 + l_1 R^I + l_2 R^B + l_3 (R^I)^2 + l_4 (R^B)^2 + l_5 (R^I \cdot tp) + l_6 (R^I \cdot dp) + l_7 (R^B \cdot tp) + l_8 (R^B \cdot dp) + l_9 [(R^I)^2 \cdot tp] + l_{10} [(R^I)^2 \cdot dp] + l_{11} [(R^B)^2 \cdot tp] + l_{12} [(R^B)^2 \cdot dp]$ for the scaled factor model. Panel C uses the same pricing kernel as that used in Table 3, except that the investment return is given by Equation (12) in Appendix A.2. Following Bond and Meghir (1994), this alternative investment return allows the interest rate on one-period debt to depend on the debt-to-capital ratio, that is, $R(B_t/K_t, S_t) = r_0 + r_1 (B_t/K_t) + r_2 (B_t/K_t)^2$. R^I and R^B are the real investment and corporate bond returns, respectively. We report the estimates for a and b_1, the χ^2 statistic and corresponding p-value for the J_T test on over-identification, and χ^2 statistic and p-value of the Wald test on the null hypothesis that $b_1 = 0$. In addition, we report the estimates of r_1 and r_2 in Panel C. t-statistics are reported in parentheses to the right of parameter estimates. The moment conditions are the same as those reported in Table 3.

3.5 Cross-sectional variations in factor sensitivity

This section provides further information on financial frictions as a common factor in the cross-section of returns by examining the variation in return sensitivity to the constrained aggregate investment returns across different assets. Intuitively, if the economically motivated characteristics used to construct the test assets are good indicators of financial frictions, they should forecast cross-sectional variation in sensitivity to aggregate financial frictions.

Loadings on R^I_{t+1} mask exposures to both the unconstrained investment returns, $\widetilde{R}^I_{t+1}$, and the financing factor $(1+\mu_{t+1})/(1+\mu_t)$. To isolate these two effects, we use Equation (6) to decompose R^I_{t+1} as follows: $\log(R^I_{t+1}) = \log(\widetilde{R}^I_{t+1}) + \log((1+\mu_{t+1})/(1+\mu_t))$. We then use each of these terms as a pricing factor to calculate the return sensitivities across the various portfolios.

Table 9 summarizes the results. Panel A looks at the popular 25 size and book-to-market portfolios and shows that, controlling for size, growth firms have higher loadings on both aggregate investment and financing factors than value firms. Controlling for book-to-market, however, we find that small firms have only slightly lower loadings than big firms.

This evidence contrasts with the findings of one-way sorts on size alone, reported in Panel B, which show that small firms have generally higher loadings than big firms. Together, these results suggest that the conventional wisdom that small firms are more financially constrained merely reflects the fact that they are often also growth firms.

Panels C and D summarize that the loadings on financial factor are generally higher for portfolios of firms that are likely to face more financial frictions—firms with either low ratios of cash flow to assets or high interest coverage.

Panel E summarizes that the same pattern holds for the triple-sorted portfolios on size, the KZ index, and book-to-market. Controlling for size and book-to-market, a higher value for the KZ index is associated with higher loadings in the financing factor. With the exception of large firms, this pattern is weaker when we use double-sorted portfolios on size and the WW index. Nevertheless, the portfolio p_{WW} has a positive loading of 0.24 on the financing factor, nearly identical to the 0.27 loading that we find for the p_{KZ} portfolio.

The last panel in Table 9 summarizes the variation in factor loadings across the 30 industry portfolios constructed in Fama and French (1997). Of these tobacco, oil, and utilities show significantly lower loadings on aggregate financial frictions captured by $\log((1+\mu_{t+1})/(1+\mu_t))$. Since these three industries are often identified as having both relatively high free cash flows and low growth opportunities, they are also the least likely to be dependent on external funds and thus credit market conditions.

Table 9
Cross-sectional variation in return sensitivity on the constrained aggregate investment factor and its financial-constraint component

Panel A: Fama–French 25 size and book-to-market portfolios (January 1954–December 2000)

	Low	2	3	4	High		Low	2	3	4	High
b^I, loadings on R^I_{t+1}						t_{b^I}, t-stats of R^I_{t+1} loadings					
Small	2.44	2.25	2.08	1.79	1.55	Small	4.80	5.45	6.20	6.02	7.50
2	2.51	2.08	2.08	1.76	1.29	2	6.02	6.29	7.18	6.30	5.19
3	2.41	2.10	2.01	1.85	1.20	3	6.34	6.89	7.28	6.03	4.80
4	2.30	1.97	1.89	1.77	1.36	4	6.65	6.83	6.26	6.85	5.18
Big	2.52	2.18	2.07	2.03	1.89	Big	6.39	6.01	5.57	5.92	5.43
b_μ, loadings on $\log((1+\mu_{t+1})/(1+\mu_t))$						t_{b_μ}, t-stats of $\log((1+\mu_{t+1})/(1+\mu_t))$ loadings					
Small	2.40	2.28	2.26	1.73	1.46	Small	3.28	3.85	4.38	3.89	4.22
2	2.40	2.33	2.47	1.86	1.38	2	4.16	4.97	5.82	4.80	3.51
3	2.64	2.42	2.19	2.06	1.16	3	4.90	5.66	6.21	5.43	3.54
4	2.40	2.16	2.10	1.90	1.33	4	5.01	5.55	5.28	5.97	4.65
Big	2.68	2.44	2.26	2.27	1.95	Big	5.18	5.43	5.42	6.25	4.56

Panel B: ten NYSE deciles sorted on size (January 1954–December 2000)

	Small	2	3	4	5	6	7	8	9	Big	1–10
b^I	2.44	2.20	2.09	1.96	1.97	1.86	1.82	1.68	1.48	1.36	1.07
t_{b^I}	6.53	7.03	7.00	7.52	8.00	7.26	7.82	7.42	7.09	8.06	3.38
b_μ	3.97	3.66	3.48	3.29	3.36	3.23	2.92	2.61	2.42	1.91	2.05
t_{b_μ}	5.02	5.32	5.49	5.80	6.43	5.73	5.85	5.76	5.31	4.54	3.66

Panel E: portfolios from a 3 × 3 × 3 sort on size, the KZ Index, and book-to-market (July 1968–December 2000)

	p_{111}	p_{131}	p_{113}	p_{133}	p_{311}	p_{331}	p_{313}	p_{333}
b^I	1.26	1.53	1.01	1.23	0.63	0.87	0.81	0.76
t_{b^I}	4.23	5.30	4.49	5.26	7.00	4.20	3.60	5.67
b_μ	2.04	2.30	1.63	2.64	1.31	1.75	2.08	2.43
t_{b_μ}	2.61	2.30	2.82	3.86	4.20	2.64	4.40	3.90

Panel F: nine portfolios from a 3 × 3 sort on size and the Whited-Wu Index (October 1975–December 2000)

	SC	*SM*	*SU*	*MC*	*MM*	*MU*	*BC*	*BM*	*BU*
b^I	3.41	3.41	3.33	3.27	3.07	3.54	2.51	2.95	1.58
t_{b^I}	3.85	3.98	2.30	3.32	3.90	4.55	2.16	3.66	2.64
b_μ	3.59	4.07	3.81	3.73	3.41	3.98	2.64	3.36	1.53
t_{b_μ}	3.48	4.12	2.51	3.27	4.02	4.35	2.20	3.95	2.40

Panel G: 30 Fama-French (1997) industry portfolios (January 1954–December 2000)

	Food	Beer	Smoke	Games	Books	Hshld	Clths	Hlth	Chems	Txtls	Mean
b^I	0.96	1.46	0.68	2.10	1.83	1.54	2.33	1.11	1.68	2.13	1.73
t_{b^I}	5.26	2.98	1.89	5.77	6.53	5.97	4.91	5.27	6.03	5.07	5.58
b_μ	1.49	1.81	0.91	2.00	2.10	1.63	2.61	1.21	1.93	2.46	1.83
t_{b_μ}	5.48	3.28	1.67	3.97	5.11	4.30	3.83	3.19	4.52	4.00	4.22

Table 9
(continued)

	Cnstr	Steel	FabPr	ElcEq	Autos	Carry	Mines	Coal	Oil	Util
b^I	1.88	2.24	2.19	2.02	2.34	1.91	1.50	1.46	0.84	0.71
t_{b^I}	6.70	5.11	6.16	6.88	9.39	4.17	3.17	2.37	2.11	4.25
b_μ	2.08	2.16	2.02	2.06	2.46	2.01	1.39	1.22	0.54	0.97
t_{b_μ}	4.82	4.19	3.77	4.75	6.60	3.16	2.24	2.11	1.06	5.25
	Telcm	**Servs**	**BusEq**	**Paper**	**Trans**	**Whlsl**	**Rtail**	**Meals**	**Fin**	**Other**
b^I	0.93	1.87	2.57	1.83	2.09	1.72	1.99	2.43	1.74	1.76
t_{b^I}	5.04	7.81	6.89	9.37	5.15	5.80	7.42	6.85	5.62	7.53
b_μ	1.21	1.79	2.23	1.95	2.16	1.58	2.31	2.48	2.12	1.92
t_{b_μ}	5.55	4.96	3.63	6.06	4.05	4.10	5.19	5.26	5.30	5.22

This table summarizes cross-sectional variation in return sensitivity with respect to the constrained aggregate investment return, R^I_{t+1}, and its component capturing aggregate financial frictions, $(1+\mu_{t+1})/(1+\mu_t)$. Note from Equation (6), $R^I_{t+1} = ((1+\mu_{t+1})/(1+\mu_t))\widetilde{R}^I_{t+1}$, where $\widetilde{R}^I_{t+1}$ captures the unconstrained aggregate investment return and $(1+\mu_{t+1})/(1+\mu_t)$ captures the component of financial constraints. We report slope coefficients from univariate regressions of portfolio returns onto R^I_{t+1}, denoted b^I, and their t-statistics, denoted t_{b^I}. We also report slope coefficients (denoted b_μ, and their corresponding t-statistics, denoted t_{b_μ}) of $\log[(1+\mu_{t+1})/(1+\mu_t)]$ from bivariate regressions of portfolio returns onto $\log(\widetilde{R}^I_{t+1})$ and $\log[(1+\mu_{t+1})/(1+\mu_t)]$. All the t-statistics are adjusted for heteroscedasticity and autocorrelations of up to 12 lags. Panels A–F use testing portfolios in the left-hand side of the regressions including Fama–French 25 portfolios (Panel A), ten NYSE size deciles (Panel B), ten deciles sorted on the ratio of cash flow to assets (Panel C), ten deciles sorted on interest coverage (Panel D), selective portfolios from a triple sort on size, the KZ index, and book-to-market (Panel E), and nine portfolios from a double sort on size and the Whited-Wu index (Panel F). When testing portfolios are used, R^I_{t+1} and $\log[(1+\mu_{t+1})/(1+\mu_t)]$ are constructed using their corresponding estimates of adjustment-cost parameter a and financing cost parameter b_1 reported in Tables 2 and 5. Finally, Panel G reports the results from the 30 Fama–French (1997) industry portfolios. In this case, R^I_{t+1} and $\log[(1+\mu_{t+1})/(1+\mu_t)]$ are constructed using the benchmark estimates of adjustment-cost parameter a and financing cost parameter b_1 reported in Tables 2. The 30 industries are Food (food products), Beer (beer and liquor), Smoke (tobacco products), Games (recreation), Books (printing and publishing), Hshld (consumer goods), Clths (apparel), Hlth (healthcare, medical equipment, pharmaceutical products), Chems (chemicals), Txtls (textiles), Cnstr (construction and construction materials), Steel (steel works), FabPr (fabricated products and machinery), ElcEq (electrical equipment), Autos (automobiles and trucks), Carry (aircraft, ships, and railroad equipments), Mines (precious metals, nonmetallic, and industrial metal mining), Coal (coal), Oil (petroleum and natural gas), Util (utilities), Telcm (communication), Servs (personal and business services), BusEq (business equipment), Paper (business supplies and shipping containers), Trans (transportation), Whlsl (wholesale), Rtail (retail), Meals (restaurants, hotels, motels), Fin (banking, insurance, real estate, trading), and Others.

Finally, we also investigate the cyclical properties of the loadings on the financing factor for each of these industries. In particular, we examine both the volatility (coefficient of variation) of the loading on the financing factor and its correlation with the underlying return on investment.

Table 10 documents the time-series behavior of estimated industry loadings for oil and tobacco as well as more "typical" industries such as retail and transportation. These results are interesting because tobacco and oil industries have often been identified as particularly prone to agency problems (Jensen 1989). Relative to our benchmark sectors, we find that loadings of tobacco and oil returns are both significantly more volatile. Moreover in the case of tobacco, the sensitivity on the financing factor actually tends to increase when economic conditions are relatively

Table 10
Time series properties of conditional loadings on financial constraints for tobacco, oil, transportation, and retail industries

	Tobacco	Oil	Transportation	Retail
Mean	1.72	1.17	3.41	2.87
Volatility	1.18	1.41	0.58	0.52
Correlation with R^I	0.14	−0.11	−0.27	−0.43

This table summarizes mean and volatility of conditional loadings on financial constraints captured by $\log[(1+\mu_{t+1})/(1+\mu_t)]$ for four industries including tobacco, oil, transportation, and retail. Conditional loadings are estimated using rolling window regressions. The length of the window is 32 quarters (varying the length in the range of 24–36 quarters yields quantitatively similar results).

good—a feature that is consistent with the idea that agency costs may matter in this sector. Conversely in retail, transportation and also oil, these sensitivities tend to fall in good times, suggesting that agency costs may not be as important for these industries. Although somewhat less structural, the evidence in this section sheds additional light and complements our earlier findings.

4. Conclusion

By concentrating on optimal firm behavior, the investment-based asset pricing model (see Cochrane (1991, 1996)) provides a natural way of integrating new developments in the theory of corporate finance into an asset pricing framework. We pursue this line of research by incorporating financial frictions into a production-based asset pricing model and ask whether they help in pricing the cross-section of expected returns. Our methodology allows us to identify the impact of financial frictions on the stochastic discount factor with cyclical variations in cost of external funds. We find evidence that financing frictions may provide an important common factor for the cross-section of stock returns. Moreover, we also find that if financial market imperfections are important, then the shadow price of external funds must exhibit a strong procyclical variation, so that financial frictions are more important when economic conditions are relatively good. Conversely, a countercyclical shadow cost of external funds worsens the ability of our model to price the cross-section of expected returns.

Appendix A: Derivation of Investment Returns

A.1 The benchmark model

We start by rewriting the firms' value-maximization problem as

$$\max V(K_0, B_0, S_0) \equiv \mathrm{E}_0\left[\sum_{t=0}^{\infty} M_{0t}(D_t - W_t N_t)\right],$$

where M_{0t} is the common stochastic discount factor from time 0 to time t. Dividend is

$$D_t = \Pi(K_t, S_t) - I_t - \frac{a}{2}\left(\frac{I_t}{K_t}\right)^2 K_t + N_t + B_{t+1} - R_t B_t.$$

Capital accumulation is

$$K_{t+1} = I_t + (1-\delta)K_t$$

and the dividend constraint is

$$D_t \geq \overline{D}.$$

Letting μ_t denote the Lagrange multiplier associated with the dividend constraint, the Lagrange function conditional on the information set at time t is

$$\mathcal{L}_t = \ldots + M_{0t}(1+\mu_t)\left\{\Pi(K_t, S_t) - \frac{a}{2}\left[\frac{K_{t+1}}{K_t} - (1-\delta)\right]^2 K_t - K_{t+1} + (1-\delta)K_t + N_t + B_{t+1} - R_t B_t\right\} +$$
$$\mathrm{E}_t\left\{M_{0t+1}(1+\mu_{t+1})\left[\Pi(K_{t+1}, S_{t+1}) - \frac{a}{2}\left[\frac{K_{t+2}}{K_{t+1}} - (1-\delta)\right]^2 K_{t+1} - K_{t+2} + (1-\delta)K_{t+1} + N_{t+1} + B_{t+2} - R_{t+1}B_{t+1}\right]\right\}$$

The first-order condition with respect to K_{t+1}:

$$0 = \frac{\partial \mathcal{L}_t}{\partial K_{t+1}} = M_{0t}(1+\mu_t)(1+ai_t) + \mathrm{E}_t\left\{M_{0t+1}(1+\mu_{t+1})[\Pi_1(K_{t+1}, S_{t+1}) + \frac{a}{2}i_{t+1}^2 + (1+ai_{t+1})(1-\delta)]\right\},$$

where $i_t \equiv I_t/K_t$ and Π_1 is the first-order derivative of $\Pi(K_{t+1}, S_{t+1})$ with respect to K_{t+1}. Equations (5) and (6) then follow by noting that $M_{t+1} = M_{0t+1}/M_{0t}$ and $\Pi_1 = \Pi/K$ with constant return to scale.

A.2 Investment return with alternative financial constraints

In the benchmark model, the interest rate on debt, B_t, is independent of firm characteristics. We now assume that the interest rate on B_t is given by $R(B_t/K_t, S_t)$. Following Bond and Meghir (1994), we model the interest rate as depending on the amount of debt, B_t, and the physical size of the firm, K_t, only through the rate B_t/K_t. Moreover, the interest rate is a function of the exogenous state variable, S_t, and is stochastic. Finally, firms still face the dividend non-negativity constraint and its multiplier is denoted as μ_t.

The value of the firm is now

$$V(K_0, B_0, S_0) = \max_{\{I_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_t\left(\sum_{t=0}^{\infty} M_{0t}\left\{(1+\mu_t)\left[\Pi(K_t, S_t) - I_t - \frac{a}{2}\left(\frac{I_t}{K_t}\right)^2 K_t + B_{t+1} - R\left(\frac{B_t}{K_t}, S_t\right)B_t\right]\right.\right.$$
$$\left.\left. -q_t(K_{t+1} - (1-\delta)K_t - I_t)\right\}\right)$$

The first-order conditions with respect to I_t, K_{t+1}, and B_{t+1} are, respectively,

$$q_t = (1+\mu_t)\left(1 + a\frac{I_t}{K_t}\right) \quad \text{(A1)}$$

$$q_t = \mathrm{E}_t\left\{M_{t+1}(1+\mu_{t+1})\left[\Pi_1(K_{t+1}, S_{t+1}) + \frac{a}{2}\left(\frac{I_t}{K_t}\right)^2 + R_1\left(\frac{B_{t+1}}{K_{t+1}}, S_{t+1}\right)\left(\frac{B_{t+1}}{K_{t+1}}\right)^2 + (1-\delta)q_{t+1}\right]\right\} \quad \text{(A2)}$$

$$1 + \mu_t = \mathrm{E}_t\left\{M_{t+1}(1+\mu_{t+1})\left[R\left(\frac{B_{t+1}}{K_{t+1}}, S_{t+1}\right) + R_1\left(\frac{B_{t+1}}{K_{t+1}}, S_{t+1}\right)\frac{B_{t+1}}{K_{t+1}}\right]\right\}. \quad \text{(A3)}$$

Combining Equations (A1) and (A2) yields the investment return in Equation (12). And the corporate bond return is given by Equation (A3):

$$r^B_{t+1} \equiv \frac{1+\mu_{t+1}}{1+\mu_t}\left[R\left(\frac{B_{t+1}}{K_{t+1}}, S_{t+1}\right) + R_1\left(\frac{B_{t+1}}{K_{t+1}}, S_{t+1}\right)\frac{B_{t+1}}{K_{t+1}}\right]. \tag{A4}$$

Note that the investment and the bond returns satisfy that

$$\mathrm{E}_t(M_{t+1}r^I_{t+1}) = 1 \quad \text{and} \quad \mathrm{E}_t(M_{t+1}r^B_{t+1}) = 1. \tag{A5}$$

Appendix B: Data

Macroeconomic data comes from NIPA published by the Bureau of Economic Analysis and the Flow of Funds Accounts available from the Federal Reserve System. These data are cross-referenced and mutually consistent, so they form, for practical purposes, a unique source of information. Most of our experiments use data for the Nonfinancial Corporate Sector. Specifically, Table F102 from the Flow of Funds Accounts is used to construct measures of profits before (item FA106060005) and after tax accruals (item FA106231005). To these measures, we add both consumption of capital goods (item FA106300015) and inventory valuation adjustments (item FA106020601) to obtain a better indicator of actual cash flows. Investment spending is gross investment (item 105090005). The capital stock comes from Table B102 (item FL102010005). Since stock valuations include cash flows from operations abroad, we also include in our measures of profits the value of foreign earnings abroad (item FA266006003) and that of net foreign holdings to the capital stock (items FL103092005 minus FL103192005, from Table L230) and investment (the change in net holdings). Financial liabilities come from Table B102. They are constructed by subtracting financial assets, including trade receivables, (item FL104090005) from liabilities in credit market instruments (item FL104104005) plus trade payables (item FL103170005). Interest payments come from NIPA Table 1.16, line 35. All these are available at quarterly frequency and require no further adjustments. All data for the aggregate economy come from NIPA.

We obtain Fama–French size and book-to-market portfolios from Kenneth French's website. We also employ ten size deciles of NYSE stocks from CRSP to facilitate comparison of our work to Cochrane (1996). Corporate bond data comes from Ibbotson's index of Long Term Corporate Bonds. The default premium is defined as the difference between the yields on Aaa and Baa corporate bonds, from CRSP. Term premium, defined as the yield on 10 year notes minus that on three-month Treasury bills, and the dividend-price ratio of the equally weighted NYSE portfolio from CRSP. Dividend-price ratios are also normalized so that scaled and nonscaled returns are comparable.

We follow closely Lamont, Polk, and Saá-Requejo (2001) to construct portfolios related to financing constraints. We obtain firm-level accounting information from the annual Compustat file. Our return series start in July 1968, based on accounting data from December 1967. To enter our sample, a firm must have all the data necessary to construct the KZ index, have an SIC code between 2000 and 3999 (manufacturing firms only), and have positive real sales growth deflated by the Consumer Price Index in the prior year.

The KZ index is based on Table 9 in Lamont, Polk, and Saá-Requejo (2001) and is equal to -1.001909 [(item 18, income before extraordinary items + item 14, depreciation and amortization)/(item 8, net property, plant, and equipment)] + 0.2826389 [(item 6, total liabilities and stockholders' equity + CRSP December market equity − item 60, total common equity − item 74, deferred taxes)/(item 6, total liabilities and stockholders' equity)] + 3.139193 [(item 9, long-term debt + item 34, debt in current liabilities)/(item 9, long-term debt + item 34, debt in current liabilities + item 216, stockholders' equity)] -39.3678 [(item 21, common dividends + item 19, preferred dividends)/(item 8, net property, plant, and equipment)] -1.314759 [(item 1, cash and short-term investments)/(item 8, net property, plant, and equipment)], where item 8 is lagged. The item numbers refer to Compustat annual data items. Firms with high KZ index are more constrained financially than firms with low KZ index.

Finally, we measure cash flow to assets ratio by (item 18, income before extraordinary items + item 14, depreciation and amortization)/(item 8, net property, plant, and equipment). And we measure interest coverage by (item 15, interest expense)/(item 15, interest expense + item 18, income before extraordinary items + item 14, depreciation and amortization).

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