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The investment manifesto

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ABSTRACT

A deep-ingrained doctrine in asset pricing says that if an empirical characteristic-return relation is consistent with investor “rationality,” the relation must be “explained” by a risk (factor) model. The investment approach questions the doctrine. Factors formed on characteristics are not necessarily risk factors; characteristics-based factor models are linear approximations of firm-level investment returns. The evidence that characteristics dominate covariances in horse races does not necessarily mean mispricing; measurement errors in covariances are likely to blame. Most important, risks do not “determine” expected returns; the investment approach is no more and no less “causal” than the consumption approach in “explaining” anomalies.

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1. Introduction

What risks “explain” asset pricing anomalies? The question is at the core of modern asset pricing research. We question the question.

In particular, a deep-ingrained risk doctrine in asset pricing says that if an empirical characteristic-return relation is consistent with “rationality,” the relation must be “explained” by a risk (factor) model. For example, [Fama and French \(1993, 1996\)](#) argue that common factors formed on characteristics such as size and book-to-market are risk factors in the context of [Merton's \(1973\)](#) intertemporal capital asset pricing model or [Ross's \(1976\)](#) arbitrage pricing theory. Building on the same premise, [Daniel and Titman \(1997\)](#) argue that the evidence that characteristics dominate covariances in horse races means mispricing.

The investment approach changes the big picture of asset pricing. Derived from investment first-order condition, the investment approach is equivalent to the weighted average cost of capital approach to capital budgeting in corporate finance. While the consumption approach connects expected stock returns to consumption risks, the investment approach connects stock returns (discount rates) to firm characteristics.

The investment approach questions the risk doctrine in two ways. First, the risk doctrine ignores measurement errors in risk proxies, but measurement errors are potentially responsible for why characteristics often dominate covariances in the [Daniel–Titman \(1997\)](#) test in the data. Our simulation results, which are based on the [Zhang \(2005\)](#) model embedded with a conditional risk structure, show that measurement errors in estimated betas can make betas lose the horse race. The beta estimates from 36-month rolling-window regressions are averaged betas in the past 3-year period, whereas the true beta is time-varying. The time-lag between the estimated betas and portfolio formation reduces the power of covariances to

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predict returns. In addition, using up-to-date instruments in estimating conditional betas hardly alleviates the problem of measurement errors in simulations. The crux is that the theoretical relations between instruments and betas are nonlinear. As such, linear beta models typically used in empirical work contain large specification errors.

As a testimony to the bias against covariances in the Daniel–Titman (1997) test, book-to-market retains significant predictive power for future returns in simulations, *even after controlling for the true beta*. Two sources are quantitatively important for this bias. The true beta and book-to-market are positively correlated, both depending on the model's state variables. Purging the effect of the true beta from book-to-market eliminates the predictive power of book-to-market in simulations. In addition, the Daniel–Titman test implements multivariate sorts sequentially, first on characteristics, then on covariances. Reversing the order weakens substantially the relative predictive power of characteristics over covariances. Overall, our quantitative results show that the Daniel–Titman test is largely uninformative about the driving forces behind the cross section of expected returns.

The investment approach also questions the risk doctrine by showing that risks are *not* “determinants” of expected returns. In a general equilibrium production economy, risks, expected returns, and characteristics are all endogenous variables that are determined simultaneously by a system of equilibrium conditions. No causality runs from covariances to expected returns, from expected returns to characteristics, or vice versa. Risks do not “determine” expected returns, meaning that the investment approach is as “causal” as the consumption approach in “explaining” anomalies. As such, asset pricing is not all about the pricing kernel; the investment approach provides a new basis for asset pricing research.

Our work expands (rather than reduces) the set of restrictions that stock returns data must satisfy. While the investment approach makes it clear that predicting returns with characteristics can be consistent with “rationality,” this approach alone cannot “explain” anomalies, or prove the “rationality” of asset prices. To this end, one must show that both consumption and investment first-order conditions hold in the data. However, this high hurdle also means that the consumption approach (or its behavioral variant) cannot “explain” anomalies either. Remember that characteristics are not even modeled in the consumption approach.

2. The risk doctrine

The risk doctrine permeates critical writings in asset pricing.

Fama and French (1993, 1996) argue for the risk-based interpretation of their common factors formed on size and book-to-market equity. Fama and French (1996, pp. 4 and 5) write: “[I]f assets are priced rationally, variables that are related to average returns, such as size and book-to-market equity, must proxy for sensitivity to common (shared and thus undiversifiable) risk factors in returns. The time-series regressions give direct evidence on this issue. In particular, the slopes and R^2 values show whether mimicking portfolios for risk factors related to size and [book-to-market] capture shared variation in stock and bond returns not explained by other factors.” Fama and French (1996, p. 57) further claim: “[T]he empirical successes of [the three-factor model] suggest that it is an equilibrium pricing model, a three-factor version of Merton's (1973) intertemporal CAPM (ICAPM) or Ross's (1976) arbitrage pricing theory (APT). In this view, SMB and HML mimic combinations of two underlying risk factors or state variables of special hedging concern to investors.”

The risk interpretation of the Fama and French factors is controversial. Disputing this interpretation, Daniel and Titman (1997) study whether portfolios with similar characteristics but different loadings on the Fama–French factors have different expected returns. Finding that expected returns do not correlate positively with the factor loadings after controlling for characteristics, Daniel and Titman (p. 4) write: “In equilibrium asset pricing models the covariance structure of returns determines expected returns. Yet we find that variables that reliably predict the future covariance structure do not predict future returns. Our results indicate that high book-to-market stocks and stocks with low capitalizations have high average returns whether or not they have the return patterns (i.e., covariances) of other small and high book-to-market stocks. Similarly, after controlling for size and book-to-market ratios, a common share that ‘act like’ a bond (i.e., has a low market beta) has the same expected return as other common shares with high market betas.”

The Daniel–Titman (1997) test has become the workhorse for “disentangling” risk versus mispricing in asset pricing.¹ Hirshleifer et al. (2012, p. 322) apply the Daniel–Titman test in the context of the accrual anomaly of Sloan (1996): “Under the rational factor pricing explanation of the accrual anomaly, expected returns are determined by a stock's accrual factor loading, and the accrual characteristic incrementally must have no return predictive power.” “[O]ur tests show that it is the accrual characteristic rather than the accrual factor loading that predicts returns, and therefore reject the rational factor pricing explanation of the accrual anomaly in favor of the characteristics-based behavioral hypothesis (p. 322).” In addition, Hou et al. (2011, p. 2530) use global stock returns to examine: “whether the cross-sectional explanatory power of our global factor-mimicking portfolios is directly related to the firm-level characteristics on which they are based – for such reasons as investor over- or under-reaction or illiquidity – or whether it derives from the covariance structure of returns that is related to these characteristics.”

¹ Davis et al. (2000) conduct the Daniel–Titman test in a longer sample from 1929 to 1997, and find that covariances do better than characteristics in forecasting returns. Daniel et al. (2001) conduct their tests in Japanese stocks, and find that book-to-market is more related to stock returns than HML loadings. After finding that default betas are significantly correlated to average bond returns even after controlling for duration, ratings, and yield-to-maturity, Gebhardt et al. (2005) conclude that systematic risk matters for corporate bonds.

The evidence that characteristics dominate covariances in horse races has often been interpreted as mispricing. Daniel et al. (2002, p. 152) write: “[F]or the factors in these [factor] models to represent risk factors, it would have to be the case that the factor realizations have a strong covariance with investors’ marginal utility across states. For example, the empirical evidence shows that growth (low book-to-market) stocks have had consistently low returns given their CAPM beta. For these low returns to be consistent with a rational asset pricing model, the distribution of returns provided by a portfolio of growth stocks must be reviewed by investors as ‘insurance’; it must provide high returns in bad (high marginal utility) states and low returns in good (low marginal utility) states.” Barberis and Thaler (2003, p. 1091) argue: “One general feature of the rational approach is that it is loadings or betas, and not firm characteristics, that determine average returns.” “[Daneil and Titman’s (1997)] results appear quite damaging to the rational approach.”

The accounting literature has also embraced the risk doctrine. Richardson et al. (2010, p. 430) write: “[I]t is important to empirically distinguish (1) the covariance between stock returns and a given attribute from (2) the returns attributable to the characteristic. Finding evidence in support of (1) is consistent with a risk based explanation for the return relation, whereas finding (2) would suggest mispricing.”

3. What we mean by the investment approach

Consider a two-period general equilibrium economy. The economy, in the spirit of Long and Plosser (1983), has three distinguishing characteristics defining new classical macroeconomics, which emphasizes rigorous foundations based on microeconomics. The three characteristics are: (i) agents have rational expectations, (ii) individuals maximize utility and firms maximize market equity, and (iii) markets clear.

A representative household maximizes expected utility $U(C_0) + \rho E_0[U(C_1)]$, in which ρ is time preference, and C_0 and C_1 are consumption in dates 0 and 1, respectively. There are N firms, indexed by i , producing a single commodity to be consumed or invested in capital. Firm i starts with capital K_{i0} and produces in both dates. The firm exits at the end of date 1 with a liquidation value of zero, i.e., the depreciation rate of capital is 100%.

The operating cash flow of firm i is $\Pi_{it}K_{it}$, for $t=0,1$, in which Π_{it} is firm i ’s productivity subject to a vector of aggregate shocks affecting all firms and a vector of firm-specific shocks affecting only firm i . Due to constant returns to scale, Π_{it} is both marginal product and average product of capital. Capital at the beginning of date 1 is $K_{i1} = I_{i0}$, in which I_{i0} is investment for date 0. Investing involves adjustment costs, $(a/2)(I_{i0}/K_{i0})^2 K_{i0}$, in which $a > 0$. In the two-period model, firms do not invest in date 1, $I_{i1} = 0$.

Let P_{it} and D_{it} be the ex dividend equity value and dividend of firm i at date $t=0$ or 1, respectively. The consumption first-order condition says that

$$P_{i0} = E_0[M_1(P_{i1} + D_{i1})] \Rightarrow E_0[M_1 r_{i1}^S] = 1, \tag{1}$$

in which the stock return is defined as $r_{i1}^S \equiv (P_{i1} + D_{i1})/P_{i0}$, and $M_1 \equiv \rho U'(C_1)/U'(C_0)$ is the stochastic discount factor. Using the definition of covariance to rewrite $E_0[M_1 r_{i1}^S] = 1$ in the beta-pricing form yields (see, e.g., Cochrane, 2005, p. 16)

$$E_0[r_{i1}^S] - r_f = \beta_i^M \lambda_M, \tag{2}$$

in which $r_f \equiv 1/E_0[M_1]$ is the risk-free rate, $\beta_i^M \equiv -\text{Cov}(r_{i1}^S, M_1)/\text{Var}(M_1)$ is the sensitivity of the stock return with respect to M_1 , and $\lambda_M \equiv \text{Var}(M_1)/E_0[M_1]$ is the price of risk.

In the production side, firm i uses the operating cash flow at date 0 to pay investment, I_{i0} , and adjustment costs, $(a/2)(I_{i0}/K_{i0})^2 K_{i0}$. If the free cash flow, $D_{i0} = \Pi_{i0}K_{i0} - I_{i0} - (a/2)(I_{i0}/K_{i0})^2 K_{i0}$, is positive, firm i distributes it back to the household. A negative D_{i0} means external equity. At date 1, the firm uses capital K_{i1} to obtain the operating profits, $\Pi_{i1}K_{i1}$, which are distributed as dividends, D_{i1} . The ex dividend equity value, P_{i1} , is zero.

Taking M_1 as given, firm i chooses investment to maximize cum dividend equity value

$$P_{i0} + D_{i0} \equiv \max_{(I_{i0})} \left[\Pi_{i0}K_{i0} - I_{i0} - \frac{a}{2} \left(\frac{I_{i0}}{K_{i0}} \right)^2 K_{i0} + E_0[M_1 \Pi_{i1}K_{i1}] \right]. \tag{3}$$

The investment first-order condition is given by

$$1 + a \frac{I_{i0}}{K_{i0}} = E_0[M_1 \Pi_{i1}]. \tag{4}$$

Intuitively, to generate an extra unit of capital at the beginning of date 1, firm i must pay the price of capital (unity), and the marginal adjustment cost, $a(I_{i0}/K_{i0})$. The marginal benefit of this extra unit of capital over period 1 is the marginal product of capital, Π_{i1} . Discounting the marginal benefit at date 1 using the stochastic discount factor yields marginal q .

Because $D_{i0} = \Pi_{i0}K_{i0} - I_{i0} - (a/2)(I_{i0}/K_{i0})^2 K_{i0}$, the cum dividend equity value in Eq. (3) means that the ex dividend equity value is given by $P_{i0} = E_0[M_1 \Pi_{i1}K_{i1}]$, and the stock return, r_{i1}^S , is given by $r_{i1}^S = (P_{i1} + D_{i1})/P_{i0} = \Pi_{i1}K_{i1}/E_0[M_1 \Pi_{i1}K_{i1}]$. As in Cochrane (1991), the investment return is defined as $r_{i1}^I = \Pi_{i1}/[1 + a(I_{i0}/K_{i0})]$, which is the ratio of the date 1’s marginal benefit of investment divided by the date 0’s marginal cost of investment. Eq. (4) implies that $E_0[M_1 r_{i1}^I] = 1$. Dividing both

the numerator and the denominator of r_{i1}^S by K_{i1} and using Eq. (4) yield:

$$r_{i1}^S = \frac{\Pi_{i1}}{1 + a(I_{i0}/K_{i0})} = r_{i1}^I. \quad (5)$$

Intuitively, firm i keeps investing until the marginal cost of investment at date 0, $1 + a(I_{i0}/K_{i0})$, is equated to the marginal benefit of investment at date 1, Π_{i1} , discounted to date 0 with the stock return, r_{i1}^S , as the discount rate.² Alternatively, the discount rate can be inferred from the ratio of the marginal benefit of investment at date 1 divided by the marginal cost of investment at date 0. A purely characteristics-based model, Eq. (5), is the two-period producer-based counterpart to the two-period consumer-based CAPM.

4. Characteristics-based factors \neq risk factors

The investment approach casts doubt on the risk doctrine. First, different from Fama and French (1993, 1996), characteristics-based factors are not necessarily risk factors. Instead, characteristics-based factor models seem to be linear approximations to firm-level investment returns (this section). Second, different from Daniel and Titman (1997), the evidence that characteristics dominate covariances in horse races does not necessarily mean mispricing (Section 5). Instead, measurement errors in covariances are more likely to blame (Section 6).

Brock (1982) derives Merton's (1973) ICAPM and Ross's (1976) APT within a general equilibrium production economy. Firm i faces a vector of F aggregate technological uncertainties, denoted \tilde{X}_t^f , for $f=1, \dots, F$. Brock models technological heterogeneity across firms as firm-specific loadings, denoted L_{it}^f , on the aggregate factors, \tilde{X}_t^f . Although time-varying, the loadings are not stochastic. Specifically, firm i 's marginal product of capital, Π_{it} , is

$$\Pi_{it} \equiv \sum_{f=1}^F L_{it}^f \tilde{X}_t^f. \quad (6)$$

The Brock (1982) economy has no capital adjustment costs, while all the other aspects are similar to our model in Section 3. Eq. (5) implies for the stock return

$$r_{i1}^S = \sum_{f=1}^F L_{i1}^f \tilde{X}_1^f, \quad (7)$$

which inherits the F -factor structure in aggregate technological uncertainties. Capital adjustment costs do not affect the F -factor structure for the stock return. In Eq. (5), the investment-to-capital ratio, I_{i0}/K_{i0} , is decided at date 0 before the realization of \tilde{X}_1^f at the beginning of date 1. As such, a linear F -factor structure continues to hold

$$r_{i1}^S = \sum_{f=1}^F \frac{L_{i1}^f}{1 + a(I_{i0}/K_{i0})} \tilde{X}_1^f. \quad (8)$$

For a factor to be a risk factor, it must be a source of aggregate uncertainty, \tilde{X}_1^f , affecting all firms in the economy. Examples include shocks to total factor productivity, government policy shocks, and aggregate demand shocks driven by changes to preferences. From this economic perspective, characteristics-based factors are not ICAPM or APT risk factors. Firm-specific characteristics, on which the factors are based, have no immediate linkages with aggregate sources of uncertainty affecting the fundamentals of all the firms simultaneously.

The investment approach provides a straightforward interpretation for characteristics-based factors. Because the marginal cost of investment equals marginal q , the denominator of the investment return in Eq. (5) is market-to-book. Size does not appear directly in the investment return equation, beyond being the numerator of market-to-book. In this sense, the size factor is redundant to the value factor in the context of the investment return equation. All in all, it seems more natural to interpret characteristics-based factor models as linear approximations to the nonlinear investment return.

5. Characteristics dominate covariances \neq mispricing

The evidence that characteristics dominate covariances does not necessarily mean mispricing. First, the evidence that characteristics predict stocks returns is consistent with Eq. (5). Investment and profitability from the investment return equation combine to predict, on economic ground, many empirical relations between anomaly variables and subsequent

² Incorporating corporate taxes and debt, Liu et al. (2009) show that the investment return equals the weighted average cost of capital (WACC): $r_{it+1}^I = w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^S$, in which w_{it} is market leverage, and r_{it+1}^{Ba} is the after-tax corporate bond return. Using the definition of the investment return in our two-period economy yields $w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^S = \Pi_{i1} / [1 + a(I_{i0}/K_{i0})]$. This equation provides the microeconomic foundation for the standard WACC approach to capital budgeting. The WACC approach instructs managers to invest in all projects with positive net present values. As such, at the margin the last infinitesimal project has a zero net present value. In our economy, the present value of the marginal project is $\Pi_{i1} / [w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^S]$, which uses the WACC as the discount rate. The cost of the marginal project is $1 + a(I_{i0}/K_{i0})$. Optimal investment requires that $1 + a(I_{i0}/K_{i0}) = \Pi_{i1} / [w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^S]$, meaning that the net present value for the marginal project is zero.

stock returns (e.g., Hou et al., 2012). Although anomalies to the consumption approach, these relations are potentially regularities to the investment approach.

Second, the covariances-based consumption model and the characteristics-based investment model of expected returns are theoretically equivalent. Far from being mutually exclusive (as maintained in the existing literature, see Section 2), covariances and characteristics are the two sides of the same coin in general equilibrium, delivering identical expected stock returns. Their relation is neatly complementary. Combining Eqs. (2) and (5) yields

$$r_f + \beta_i^M \lambda_M = E_0[r_{it}^S] = \frac{E_0[II_{it}]}{1 + a(I_{i0}/K_{i0})}. \tag{9}$$

Solving for β_i^M provides an intrinsic link between covariances and characteristics

$$\beta_i^M = \left[\frac{E_0[II_{it}]}{1 + a(I_{i0}/K_{i0})} - r_f \right] / \lambda_M. \tag{10}$$

Intuitively, the risk-based model, $E_0[r_{it}^S] = r_f + \beta_i^M \lambda_M$, derived from the consumption first-order condition, connects risk premiums to covariances. It says that covariances are sufficient statistics of expected returns. Once covariances are controlled for, characteristics should not affect the cross section of expected returns. This prediction is the basic premise underlying the risk doctrine, which is the organizing framework for the bulk of empirical asset pricing.

In contrast, the characteristics-based model, $E_0[r_{it}^S] = E_0[II_{it}]/[1 + a(I_{i0}/K_{i0})]$, derived from the investment first-order condition, connects expected returns to characteristics. It says that characteristics are sufficient statistics of expected returns. Once characteristics are controlled for, covariances should not affect expected returns. This “outrageous” characteristics-centered prediction from the investment approach is in effect the dual statement to the equally “outrageous” risks-centered prediction from the consumption approach. In general equilibrium, Eq. (9) shows that both approaches give the identical answer.

Third, if covariances and characteristics are intrinsically linked per Eq. (10), why do characteristics often dominate covariances in the Daniel–Titman (1997) test? Mispricing is not the only possibility. Another possibility is that characteristics are more precisely measured than covariances in the data (e.g., Berk, 1995; Gomes et al., 2003; Carlson et al., 2004). Horse races between covariances and characteristics only test whether covariances or characteristics are more correlated with future returns in the data. However, the correlations depend on the magnitude of measurement (and specification) errors in the “explanatory” variables. The next section shows quantitatively how measurement errors in risk proxies can make covariances fail the covariances versus characteristics test, even in a model embedded with a conditional covariance structure.

6. The role of measurement errors in risk proxies

A simplified version of the Zhang (2005) economy can shed light on the theoretical properties of the Daniel–Titman (1997) test. The key insight is that even though the model admits a covariance structure, characteristics dominate covariances in simulated data.

6.1. The model economy

The production function is given by

$$\Pi_{it} = X_t Z_{it} K_{it}^\alpha - f, \tag{11}$$

in which Π_{it} is firm i 's operating profits, K_{it} is capital, $0 < \alpha < 1$ is the curvature parameter, and $f > 0$ is the fixed cost of production. The aggregate productivity, X_t , has a stationary Markov transition function. Let $x_t \equiv \log X_t$, the transition function follows:

$$x_{t+1} = \bar{x}(1 - \rho_x) + \rho_x x_t + \sigma_x \mu_{t+1}, \tag{12}$$

in which μ_{t+1} is an independent and identically distributed (i.i.d.) standard normal shock. The firm-specific productivity for firm i , Z_{it} , has a transition function given by

$$z_{it+1} = \rho_z z_{it} + \sigma_z v_{it+1}, \tag{13}$$

in which $z_{it+1} \equiv \log Z_{it+1}$, and v_{it+1} is an i.i.d. standard normal shock. In addition, v_{it+1} and v_{jt+1} are uncorrelated for any $i \neq j$, and μ_{t+1} and v_{it+1} are uncorrelated for any i .

Capital accumulates as

$$K_{it+1} = I_{it} + (1 - \delta)K_{it}, \tag{14}$$

in which δ is the rate of depreciation. Capital investment entails adjustment costs

$$\Phi(I_{it}, K_{it}) = \begin{cases} a^+ K_{it} + \frac{c^+}{2} \left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & \text{for } I_{it} > 0, \\ 0 & \text{for } I_{it} = 0, \\ a^- K_{it} + \frac{c^-}{2} \left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & \text{for } I_{it} < 0, \end{cases} \quad (15)$$

in which $a^- > a^+ > 0$, and $c^- > c^+ > 0$ capture nonconvex adjustment costs.

The stochastic discount factor, denoted M_{t+1} , follows:

$$M_{t+1} = \eta \exp[\gamma_0 + \gamma_1(x_t - \bar{x})(x_t - x_{t+1})], \quad (16)$$

in which $0 < \eta < 1$, $\gamma_0 > 0$, and $\gamma_1 < 0$ are constant parameters. Upon observing X_t and Z_{it} , firm i chooses optimal investment, I_{it} , to maximize its market value of equity, given by

$$V_{it} \equiv V(K_{it}, X_t, Z_{it}) = \max_{\{I_{it}\}} [\Pi_{it} - I_{it} - \Phi(I_{it}, K_{it}) + E_t[M_{t+1} V(K_{it+1}, X_{t+1}, Z_{it+1})]], \quad (17)$$

subject to Eq. (14). At the optimum, $V_{it} = D_{it} + E_t[M_{t+1} V_{it+1}]$, with $D_{it} \equiv \Pi_{it} - I_{it} - \Phi(I_{it}, K_{it})$. Equivalently, $E_t[M_{t+1} r_{it+1}^S] = 1$, in which $r_{it+1}^S \equiv V_{it+1}/(V_{it} - D_{it})$ is the stock return. Rewriting in the beta pricing form yields $E_t[r_{it+1}^S] = r_{ft} + \beta_{it}^M \lambda_{Mt}$, in which $r_{ft} = 1/E_t[M_{t+1}]$ is the real interest rate, $\beta_{it}^M \equiv -\text{Cov}_t[r_{it+1}^S, M_{t+1}]/\text{Var}_t[M_{t+1}]$ is the true conditional beta, and $\lambda_{Mt} \equiv \text{Var}_t[M_{t+1}]/E_t[M_{t+1}]$ is the price of risk.

The model is calibrated in monthly frequency. The parameters, $\eta = 0.9999$, $\gamma_0 = 17$, and $\gamma_1 = -1000$ are set to match the average real interest rate, 2.72% per annum; the annualized volatility of the real interest rate, 2.02%; and the average Sharpe ratio, 0.37 per annum. The persistence of aggregate productivity ρ_x is set to be $0.95^{1/3} = 0.983$ and conditional volatility $\sigma_x = 0.007/\sqrt{1 + 0.983^2 + 0.983^4} = 0.0041$.³ For the adjustment cost parameters: $a^+ = 0.01$, $a^- = 0.03$, $c^+ = 20$, and $c^- = 200$; for the remaining parameters, $\bar{x} = -3.65$, $\rho_z = 0.97$, $\sigma_z = 0.10$, $\alpha = 0.70$, $\delta = 0.01$, and $f = 0.0032$.

The model is solved with value function iteration on discrete state space.⁴ In total 1000 artificial samples are simulated from the model, each with 5000 firms and 1000 month. The initial condition for the simulations consists of capital stocks of all firms at their long-run average level and firm-specific productivity of all firms drawn from the unconditional distribution of Z_{it} . The first 400 months are dropped to neutralize the impact of the initial condition. The remaining 600 months of simulated data are treated as from the model's stationary distribution. The sample size is largely comparable to the merged CRSP/Compustat dataset. Empirical tests are performed on each artificial sample and cross-simulation averaged results are reported as model moments to compare with those in the real data. With the calibrated parameters, the model produces a value premium of 6.0% per annum, which is the average return of the high-minus-low book-to-market decile constructed from applying the standard (Fama and French, 1993) portfolio approach on the simulated data.

6.2. Covariances versus characteristics in the data

To study the properties of the Daniel–Titman (1997) test, the first step is to replicate their empirical results using an updated sample. Monthly stock returns are from the Center for Research in Security Prices (CRSP) and accounting information from the CRSP/Compustat Merged Annual Industrial Files. The sample is from July 1973 to June 2010. The starting point is the same as in Daniel and Titman. Firm-year observations for which book equity is either zero or negative are excluded. So are firms that do not have ordinary common equity as classified by CRSP. Firms whose primary SIC classification is between 4900 and 4999 (regulated firms) or between 6000 and 6999 (financial firms) are also omitted. Finally, a firm must have at least two years of data in Compustat to be included in the sample.

As in Daniel and Titman (1997), in June of year t , all NYSE, Amex, and Nasdaq stocks are sorted into quintiles based on the NYSE breakpoints of book-to-market at the last fiscal year end. Book-to-market is calculated with the book equity at the end of the fiscal year ending in calendar year $t-1$ and the market equity at the end of December of year $t-1$. Firms remain in these quintiles from July of year t to June of year $t+1$. The individual firms in each quintile are further sorted into one of five subportfolios based on their preformation HML loadings. The HML loadings are estimated from the Fama and French (1993) three-factor regressions performed from month -42 to -7 relative to the formation date (June of year t). The value-weighted returns are calculated for the resulting 25 portfolios for each month, and their average returns in excess of the risk-free rate are reported.

From Panel A of Table 1, book-to-market dominates the HML loading empirically in predicting future returns. The last row shows that after controlling for book-to-market, the average return spread between the low- and the high-HML loading quintiles is 0.24% per month. In contrast, the last column of Panel A shows that after controlling for the HML

³ These values correspond to the quarterly values of 0.95 and 0.007, respectively, as in Cooley and Prescott (1995). The formulas that convert the quarterly values to monthly values are from Heer and Maussner (2009).

⁴ The Matlab programs used to solve and simulate the model are available on our Web sites.

Table 1

Mean monthly percentage excess returns of the 25 portfolios formed on book-to-market and HML loadings (in the data and in the model) and the 25 portfolios formed on book-to-market and true conditional betas (in the model).

	Panel A: data, HML loadings						Panel B: model, HML loadings						Panel C: model, true betas					
	Low	2	3	4	High	All	Low	2	3	4	High	All	Low	2	3	4	High	All
Low	0.28	0.29	0.37	0.48	0.42	0.37	0.65	0.64	0.64	0.65	0.66	0.65	0.58	0.63	0.66	0.70	0.78	0.67
2	0.58	0.40	0.73	0.66	0.92	0.66	0.71	0.70	0.69	0.70	0.72	0.70	0.63	0.68	0.71	0.75	0.86	0.72
3	0.84	0.58	0.57	0.84	0.93	0.75	0.76	0.76	0.76	0.76	0.77	0.76	0.67	0.74	0.77	0.82	0.90	0.78
4	0.50	0.54	0.70	0.74	0.83	0.66	0.82	0.81	0.81	0.82	0.85	0.82	0.70	0.79	0.85	0.91	1.04	0.86
High	1.05	1.04	0.99	0.90	1.34	1.07	0.96	0.93	0.94	0.96	1.01	0.96	0.78	0.89	0.98	1.11	1.40	1.03
All	0.65	0.57	0.67	0.72	0.89		0.78	0.77	0.77	0.78	0.80		0.67	0.74	0.80	0.86	1.00	

Note: Monthly returns are from the Center for Research in Security Prices (CRSP) and accounting information is from the CRSP/Compustat Merged Annual Industrial Files. The sample is from July 1973 to June 2010. Book equity is the Compustat book value of stockholders' equity, plus balance-sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, the value of preferred stock is the redemption, liquidation, or par value (in that order). Firm-year observations for which book value of equity is either zero or negative are excluded. Firms whose primary SIC classification is between 4900 and 4999 (regulated firms) or between 6000 and 6999 (financial firms) are omitted. Only firms with ordinary common equity are included, meaning that ADRs, REITs, and units of beneficial interest are not. Finally, a firm must have at least 2 years of data in Compustat before being included in the sample. All NYSE firms are ranked on their book-to-market at the end of year $t-1$ to form 20%, 40%, 60% and 80% breakpoints based on these rankings. Starting in July of year t , all NYSE, Amex, and Nasdaq firms are sorted into the five book-to-market groups based on these breakpoints. The firms remain in these portfolios from July of year t to June of year $t+1$. The individual firms in each of these five portfolios are further sorted into five subportfolios based on their HML loadings. The HML loadings are from the Fama–French three-factor regressions performed between 42 months and 6 months prior to the formation date. The data for the three Fama–French factors and the risk-free rate are from Kenneth French's Web site. The value-weighted returns for the 25 book-to-market and HML loadings portfolios are calculated for each month, and the average excess returns are reported in Panel A. In Panel B, 1000 artificial panels are simulated from the model economy in Section 6.1. Each panel contains 5000 firms and 1000 monthly observations. The first 400 months are dropped to neutralize the impact of initial conditions in the simulations. The empirical procedures in Panel A are implemented on each of the artificial panels, and the cross-sample averaged results are reported in Panel B. Panel C reports the simulated results from the empirical procedures similar to those in Panel B, except that HML loadings are replaced with the true conditional betas, β_i^M .

loading, the average return spread between the low and the high book-to-market quintiles is 0.70%. As such, consistent with Daniel and Titman (1997), the characteristic rather than the covariance structure of returns appears to “explain” the cross-section of average returns.

6.3. Covariances versus characteristics in the model

To see whether the Daniel and Titman's (1997) evidence is as “disturbing” as claimed, the next step is to implement the same procedure on simulated data from the model in Section 6.1. Panel B of Table 1 reports the quantitative results averaged across 1000 artificial samples. The pattern is largely consistent with the pattern in the data. After controlling for book-to-market, the HML loading generates a tiny average return spread of 0.02% per month between the extreme quintiles. In contrast, after controlling for the HML loading, book-to-market produces a higher average return spread, 0.31%, across its extreme quintiles.

Because the HML loadings are *estimated* on each artificial sample, these loadings are noisy proxies for the true beta, β_i^M , in the model. To evaluate the impact of measurement errors in the estimated loadings, the tests from Panel B are repeated but with the HML loadings replaced by β_i^M . The true beta can be calculated accurately within the model.⁵ From the last row in Panel C, after controlling for book-to-market, the true beta produces an average return spread of 0.33% per month across the extreme quintiles. As such, measurement errors in the empirical proxies of covariances make characteristics win the Daniel–Titman (1997) test, even in a model embedded with a dynamic covariance structure.

6.3.1. Why book-to-market predicts returns even after controlling for the true beta?

As a testimony to the bias in favor of characteristics in the Daniel–Titman (1997) test, Panel C of Table 1 shows that book-to-market retains strong predictive power in the model, *even after controlling for the true beta!* In particular, the last column of the panel shows that book-to-market still generates an average return spread of 0.36% per month across the extreme quintiles. This spread is even slightly higher than the average return spread across the extreme true beta quintiles, 0.33%.

One source underlying this bias is that the true beta and book-to-market are tightly linked within the model (and likely so in the data). If the true beta is known and book-to-market does not add any predictive information on returns, a large-sample bivariate regression of returns on the true beta and book-to-market should yield a zero slope on book-to-market. However, even if the true beta is known (in the model, but not in the data), book-to-market might still obtain a positive

⁵ Specifically, β_{it}^M is calculated as $(E_t[r_{it+1}^S] - r_{it}) / \lambda_{Mt}$ on the (K_{it}, Z_{it}, X_{it}) grid, in which $E_t[r_{it+1}^S] = E_t[V_{it+1}] / (V_{it} - D_{it})$. The solution of β_{it}^M on the grid is then used in simulations. See Zhang (2005, Appendix B) for additional technical details.

slope through its correlation with the true beta. In addition, the Daniel–Titman multivariate sort is different from the bivariate regression, and can fail to disentangle the effect of the true beta on expected returns from that of book-to-market.

To show how the true beta and book-to-market are connected in the model, Fig. 1 plots the two variables on the grid of capital, K_{it} , and firm-specific productivity, Z_{it} . The aggregate productivity, X_t , is fixed at its long-run average (other X_t values yield similar results). Both the true beta and book-to-market decrease with the firm-specific productivity, consistent with prior studies (e.g., Zhang, 2005). In addition, in the region of capital that is most relevant for simulations (the average capital in simulations is around 0.45), both the true beta and book-to-market also decrease with capital. Using simulations, one can calculate monthly cross-sectional correlations between the true beta and book-to-market, and compute the time series average of the correlations. Averaged across 1000 simulations, the average correlation is 0.66, with a cross-simulation standard deviation of 0.09.

To see how the correlation between book-to-market and the true beta affects the Daniel–Titman test, our next experiment purges the effect of the true beta from book-to-market by calculating risk-adjusted book-to-market. In simulations, book-to-market is defined as K_{it}/P_{it} . To purge the effect of the true beta, for each month t , the following cross-sectional regression is performed across all firms indexed by i , $K_{it}/P_{it} = a_t + b_t \beta_{it}^M + \varepsilon_{it}$, in which β_{it}^M is the true beta. The risk-adjusted book-to-market is measured as $a_t + \varepsilon_{it}$, which has purged away the (first-order) effect of the true beta. The tests in Panels B and C of Table 1 are repeated, but with book-to-market replaced by the risk-adjusted book-to-market.

From Panels A and B of Table 2, when used either with HML loading or the true beta, the risk-adjusted book-to-market shows essentially no forecasting power for returns. The average return spreads across the risk-adjusted book-to-market quintiles are only 0.03% per month. A comparison of these tiny spreads with the large spreads (more than 0.30%) across the book-to-market quintiles from Panels B and C of Table 1 reveals that the predictive power of book-to-market for returns is almost entirely driven by its correlation with the true beta. This result is not surprising because the model admits a covariance structure by construction.

6.3.2. The order of multivariate sorts

Another reason why the Daniel–Titman (1997) test is biased against covariances is the order of multivariate sorts. Daniel and Titman implement the multivariate sorts sequentially, first on characteristics, then on covariances. This order can exaggerate the relative predictive power of characteristics over covariances.

To quantify the impact of the sorting order, Panels C and D of Table 2 redo the tests in Panels B and C of Table 1, respectively, but with the order of the sequential sorts reversed. The results show that the relative predictive power of covariances is substantially strengthened, whereas that of characteristics is weakened. Specifically, from Panel C of Table 2, sorting on the HML loading first produces an average return spread of 0.31% per month, which is identical to the spread from the second sort on book-to-market. From Panel D, sorting on the true beta first produces an average return spread of 0.45%, which is substantially larger than the 0.06% from the second sort on book-to-market.

What happens if one repeats the Daniel–Titman (1997) test in Panel A of Table 1 on the real data, but by sorting on the HML loading first and then on book-to-market? Reversing the order of the sequential sorts increases the average return spread across the HML loading quintile from 0.24% per month to 0.40%, and reduces the spread across the book-to-market quintiles from 0.70% to 0.63% (untabulated). As such, reversing the order of the sequential sorts has an important impact on the Daniel–Titman test.

6.3.3. Sources of measurement errors in risk proxies

Estimated risk proxies have severe measurement errors. One specific source of measurement errors is the time-variation in betas in regression-based proxies for β_{it}^M . First, the beta estimates from 36-month rolling-window regressions in Daniel and Titman (1997) are unconditional betas in the past 3-year period. This time-lag between the estimated betas

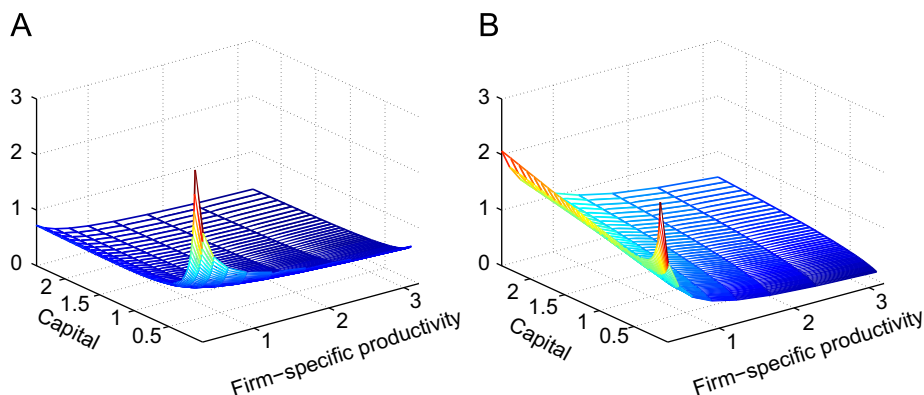


Fig. 1. The true beta and book-to-market in the model. Note: Panel A of this figure plots the true beta, β_{it}^M , and Panel B plots book-to-market, K_{it}/P_{it} , on the grid of capital, K_{it} , and firm-specific productivity, Z_{it} . The aggregate productivity, X_t , is fixed at its mean.

Table 2
Mean monthly percentage excess returns of the 25 portfolios formed on book-to-market and different estimates of covariances in the model.

	Low	2	3	4	High	All	Low	2	3	4	High	All	Low	2	3	4	High	All
Panel A: risk-adjusted book-to-market, HML loadings							Panel B: risk-adjusted book-to-market, true betas						Panel C: HML loadings, book-to-market					
Low	0.77	0.76	0.77	0.77	0.80	0.77	0.64	0.73	0.80	0.89	1.13	0.84	0.65	0.64	0.64	0.65	0.67	0.65
2	0.73	0.72	0.72	0.73	0.74	0.73	0.62	0.69	0.75	0.82	1.00	0.78	0.71	0.69	0.69	0.71	0.73	0.71
3	0.72	0.71	0.72	0.72	0.74	0.72	0.62	0.69	0.75	0.82	0.99	0.77	0.76	0.75	0.75	0.76	0.79	0.76
4	0.74	0.72	0.73	0.74	0.76	0.74	0.63	0.70	0.77	0.84	1.01	0.79	0.82	0.79	0.80	0.82	0.87	0.82
High	0.80	0.78	0.79	0.80	0.84	0.80	0.67	0.76	0.83	0.93	1.16	0.87	0.96	0.91	0.92	0.95	1.04	0.96
All	0.75	0.74	0.74	0.75	0.78		0.64	0.71	0.78	0.86	1.06		0.65	0.70	0.76	0.82	0.96	
Panel D: true betas, book-to-market							Panel E: book-to-market, rolling SDF betas						Panel F: book-to-market, conditional SDF betas					
Low	0.60	0.70	0.77	0.85	1.02	0.79	0.62	0.62	0.65	0.68	0.69	0.65	0.63	0.64	0.66	0.67	0.69	0.66
2	0.62	0.70	0.77	0.85	1.02	0.79	0.67	0.68	0.70	0.73	0.76	0.71	0.68	0.70	0.71	0.72	0.75	0.71
3	0.62	0.70	0.78	0.86	1.06	0.80	0.73	0.74	0.77	0.78	0.80	0.76	0.74	0.76	0.77	0.78	0.80	0.77
4	0.65	0.71	0.77	0.86	1.09	0.82	0.76	0.79	0.84	0.87	0.89	0.83	0.78	0.81	0.84	0.86	0.89	0.84
High	0.65	0.71	0.78	0.86	1.23	0.85	0.87	0.90	0.96	1.03	1.13	0.98	0.89	0.93	0.98	1.03	1.10	0.98
All	0.63	0.71	0.77	0.86	1.08		0.73	0.75	0.78	0.82	0.85		0.74	0.77	0.79	0.81	0.85	

Note: In total 1000 panels are simulated from the model in Section 6.1, each with 5000 firms and 1000 months. The first 400 months are dropped to neutralize the impact of initial conditions. Panels A and B repeat the tests in Panels B and C of Table 1, respectively, but with book-to-market replaced by risk-adjusted book-to-market. Book-to-market is defined as $K_{it}/P_{it} = K_{it}/(V_{it} - D_{it})$ in the model. To purge the impact of the true beta on book-to-market, for each month, the following cross-sectional regression is performed: $K_{it}/P_{it} = a_t + b_t \beta_{it}^M + \epsilon_{it}$. The regression is run for each month t across all the firms indexed by i . The risk-adjusted book-to-market is defined as $a_t + \epsilon_{it}$, which has purged away the (first-order) effect of risk. Panels C and D repeat the tests in Panels B and C of Table 1, respectively, but with the order of sequential sorts reversed (covariances first, book-to-market second). Panel E repeats the tests in Panel C of Table 1, but with the true betas replaced by rolling SDF betas. The same 36-month rolling windows are used as in Daniel and Titman (1997), but the betas are estimated by regressing stock returns on the pricing kernel, M_{t+1} . Finally, Panel F repeats the tests in Panel C of Table 1, but with the true betas replaced by conditional SDF betas. The aggregate dividend-to-price ratio $d_t \equiv \sum_{i=1}^{5000} D_{it} / \sum_{i=1}^{5000} P_{it}$ is the single instrument. The conditional beta regression is: $r_{it+1}^S = \alpha_{it} + (\beta_{it}^0 + \beta_{it}^1 d_t) M_{t+1} + \epsilon_{it+1}$, in which the regression coefficients are time-varying with expanding-window regressions. The estimated conditional SDF betas are calculated as $\beta_{it}^0 + \beta_{it}^1 d_t$, in which d_t is the aggregate dividend-to-price ratio at the portfolio formation date.

and portfolio formation date reduces the ability of the estimated betas to predict returns. To show this point quantitatively, the tests in Panel C of Table 1 are repeated, but with the true beta replaced by rolling-window regression-based betas. The betas are estimated by regressing stock returns on the pricing kernel, M_{t+1} , using the same 36-month window.

Panel E of Table 2 reports the results. With the rolling betas in the Daniel–Titman (1997) test along with book-to-market, the average return spread is only 0.12% per month across the rolling beta quintiles. This spread is substantially lower than the spread of 0.33% across the true beta quintiles, as in Panel C of Table 1. In contrast, the predictive power of book-to-market is largely unaffected. As such, the bias from using rolling-window regressions to estimate the (unobservable) true beta can be quantitatively large.

Second, another common approach to estimating time-varying betas in empirical asset pricing is the conditioning approach of Ferson and Harvey (1991) and Jagannathan and Wang (1996). This approach uses up-to-date information in estimating conditional betas. However, in theory both aggregate and firm-specific variables enter the beta specification nonlinearly. Alas, theory rarely provides functional forms that can be estimated in the data. As such, linear beta models contain large specification errors (e.g., Ghysels, 1998).

To quantify the impact of this specification error on the Daniel–Titman (1997) test, the tests in Panel C of Table 1 are repeated, but with the true beta replaced by the estimated conditional beta. The aggregate dividend-to-price ratio, $d_t \equiv \sum_{i=1}^{5000} D_{it} / \sum_{i=1}^{5000} P_{it}$, serves as the single instrument. The conditional regression is specified as

$$r_{it+1}^S = \alpha_{it} + (\beta_{it}^0 + \beta_{it}^1 d_t) M_{t+1} + \epsilon_{it+1}. \tag{18}$$

The regressions are performed with expanding windows. For the time series regression (18) of firm i at each month t , instead of starting from month $t - 42$ as in rolling-window regressions, one takes observations starting from the beginning month of the whole sample. With the conditional regression (18) estimated, the conditional beta is calculated as $\hat{\beta}_{it}^0 + \hat{\beta}_{it}^1 d_t$, in which $\hat{\beta}_{it}^0$ and $\hat{\beta}_{it}^1$ are estimated coefficients, and d_t is the aggregate dividend-to-price ratio at month t .

Comparing Panel F of Table 2 with Panel C of Table 1 shows that the bias from using the conditional beta to estimate the true beta is again quantitatively large. With the conditional beta in the Daniel–Titman (1997) test along with book-to-market, the average return spread is only 0.11% per month across the conditional beta quintiles. This spread is substantially lower than the spread of 0.33% across the true beta quintiles. The predictive power of book-to-market is again largely unaffected.

Because of these measurement errors, the cost of capital estimates from risk-based models are extremely unreliable. Accounting academics, who face the daily task of teaching students to value real companies, have long been disillusioned.

For example, [Lundholm and Sloan \(2007, p. 193\)](#) lament: “None of the standard finance models provide estimates that describe the actual data very well. The discount rate that you use in your valuation has a large impact on the result, yet you will rarely feel very confident that the rate you have assumed is the right one. The best we can hope for is a good understanding of what the cost of capital represents and some ballpark range for what a reasonable estimate might be.” [Penman \(2010, p. 666\)](#) also expresses extreme pessimism: “Compound the error in beta and the error in the risk premium and you have a considerable problem. The CAPM, even if true, is quite imprecise when applied. Let us be honest with ourselves: no one knows what the market risk premium is. And adopting multifactor pricing models adds more risk premiums and betas to estimate. These models contain a strong element of smoke and mirrors.”

7. Reply to critics

This section addresses several critiques on the investment approach. The critiques are largely misplaced, arising from treating risks as “determinants” of expected returns.⁶

7.1. Does the investment approach “explain” anomalies?

According to the risk doctrine, risk premiums are “determined” by exposures to risk factors times the factor risk premiums. As such, critics argue that the investment approach does not “explain” anomalies for several reasons. First, a rational “explanation” for anomalies should account for why there seems to be a common factor related to a given anomaly variable, suggesting that extreme portfolios have different exposures to unknown sources of systematic risk. Second, predictability means time-varying risk premiums in a “rational” model. Because the investment approach does not model risks, it has nothing to say about predictability. Third, a rational “explanation” for anomalies should account for why extreme portfolios have similar market (and consumption) betas, suggesting that the CAPM and the consumption CAPM fail to “explain” the average return spread across the extreme portfolios. Each of these points is addressed in what follows.

7.1.1. Debunking the myth of common factors

There is nothing mysterious about common variations in characteristics-based factors. The common variations are not some inexplicable sources of systematic risk. As shown in [Section 4](#), a characteristics-based factor model can be interpreted as a linear approximation to the nonlinear investment return equation. As components of firm-level investment returns, characteristics are *supposed* to predict subsequent returns in cross-sectional regressions.

Time series and cross-sectional regressions are largely equivalent ways of summarizing correlations in the data. If a characteristic shows up significant in cross-sectional regressions, its factor mimicking portfolio is likely to show “explanatory” power in time series factor regressions. If a factor loading shows up significant in time series regressions, its underlying characteristic is likely significant in cross-sectional regressions. Factor loadings (risks) are no more primitive than characteristics, and characteristics are no more primitive than risks in “explaining” expected returns. To the extent that characteristics are more precisely measured than factor loadings, characteristics *should* be more useful in forecasting returns in practice.

7.1.2. Predicting risk premiums \approx predicting discount rates

Because the risk-free rate is not very predictable, predictability is largely identical to time-varying risk premiums. However, the fact that the risk-free rate is roughly constant also means that time-varying risk premiums are roughly equivalent to time-varying discount rates. In the time series, [Cochrane \(1991\)](#) is the classical work that shows aggregate investment forecasts aggregate discount rates (stock market returns) with a negative slope. In the cross section, because the risk-free rate does not vary across firms, the cross-sectional variation of expected returns is the cross-sectional variation of risk premiums. The high-minus-low expected return is the high-minus-low risk premium. Modeling the cross section of expected returns via the investment approach is modeling the cross section of risk premiums!

7.1.3. Understanding the failure of the CAPM

Critics emphasize the importance of matching the stylized fact that extreme anomalies-based testing portfolios have similar market betas and consumption betas. This critique speaks directly to [Zhang \(2005, Table III\)](#), who reports a low market beta of 0.14 for the Fama–French’s HML factor in the 1927–2001 sample, but a high market beta of 0.43 for the HML factor in the model’s simulations.⁷ This critique is valid and important.

⁶ Critiques are not unexpected. [Kuhn \(1962, p. 109\)](#) says: “In learning a paradigm the scientist acquires theory, methods, and standards together, usually in an inextricable mixture. Therefore, when paradigms change, there are usually significant shifts in the criteria determining the legitimacy both of problems and of proposed solutions.” However, Kuhn also warns that: “To the extent, as significant as it is incomplete, that two scientific schools disagree about what is a problem and what a solution, they will inevitably talk through each other when debating the relative merits of their respective paradigms.”

⁷ [Belo and Lin \(2012\)](#) raise a similar point that the [Zhang \(2005\)](#) class of models fails to reproduce quantitatively the failure of the CAPM, and more so the failure of the Fama–French model in factor regressions.

Table 3
Properties of the book-to-market deciles in the data.

	Low	2	3	4	5	6	7	8	9	High	H–L
Panel A: January 1965–December 2010											
Mean	0.33	0.44	0.48	0.48	0.46	0.55	0.60	0.65	0.74	0.88	0.55
Std	5.3	4.9	4.8	4.9	4.6	4.6	4.5	4.7	4.9	6.0	4.8
α	-0.13	0.01	0.06	0.06	0.08	0.16	0.23	0.27	0.35	0.43	0.56
t_α	-1.2	0.1	1.0	0.5	0.7	1.6	2.0	2.1	3.2	2.9	2.4
β	1.07	1.01	0.98	0.99	0.91	0.93	0.87	0.88	0.93	1.06	0.00
t_β	33.4	35.0	25.7	24.8	24.4	26.5	19.6	15.3	17.4	12.6	-0.03
Adjusted R^2	0.86	0.92	0.90	0.87	0.83	0.85	0.77	0.76	0.75	0.67	0.00
Panel B: January 1927–December 2010											
Mean	0.55	0.65	0.64	0.63	0.71	0.74	0.75	0.91	0.97	1.08	0.53
Std	5.8	5.5	5.4	6.1	5.7	6.2	6.7	7.0	7.6	9.5	6.7
α	-0.07	0.05	0.05	-0.03	0.10	0.08	0.05	0.19	0.20	0.18	0.25
t_α	-1.0	0.9	1.1	-0.4	1.3	0.9	0.6	1.8	1.9	1.1	1.2
β	1.00	0.98	0.94	1.06	0.98	1.07	1.12	1.16	1.24	1.45	0.45
t_β	37.5	35.1	29.1	18.9	21.1	15.0	12.3	10.6	14.1	11.9	3.1
Adjusted R^2	0.90	0.93	0.92	0.90	0.88	0.88	0.83	0.81	0.79	0.71	0.14

Note: The returns data on the book-to-market deciles, the market factor, and the risk-free rate are from Kenneth French's Web site. For each decile and the high-minus-low decile, H–L, the table reports the mean percentage excess returns (mean), stock return volatility in monthly percent (Std), as well as the CAPM regressions including α, β , their t -statistics adjusted for heteroscedasticity and autocorrelations, and adjusted R^2 .

Table 4
Properties of the book-to-market deciles in the Zhang (2005) model.

	Low	2	3	4	5	6	7	8	9	High	H–L
Mean	0.62	0.66	0.69	0.70	0.77	0.76	0.81	0.86	0.92	1.12	0.50
Std	5.9	6.3	6.5	6.6	7.1	7.0	7.4	7.8	8.2	9.5	3.9
α	-0.02	-0.01	-0.01	-0.01	0.00	0.00	0.01	0.02	0.04	0.10	0.11
t_α	-0.8	-0.6	-0.5	-0.4	0.0	-0.1	0.5	0.6	1.0	1.5	1.4
α , 2.5%	-0.09	-0.08	-0.06	-0.06	-0.03	-0.03	-0.03	-0.03	-0.03	-0.04	-0.05
α , 97.5%	0.02	0.02	0.02	0.02	0.04	0.03	0.07	0.12	0.21	0.50	0.59
β	0.86	0.91	0.95	0.96	1.03	1.02	1.07	1.13	1.17	1.36	0.50
t_β	123.2	164.4	219.8	162.5	123.9	227.4	127.3	112.2	76.9	42.0	12.4
β , 2.5%	0.83	0.87	0.93	0.93	1.00	1.00	1.05	1.07	1.10	1.18	0.27
β , 97.5%	0.91	0.94	0.96	0.99	1.07	1.06	1.12	1.18	1.29	1.52	0.68
Adjusted R^2	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.98	0.83

Note: In total 1000 artificial panels are simulated from the model in Section 6.1. Each panel contains 5000 firms and 1000 months. The first 400 months are dropped to neutralize the impact of initial conditions in the simulations. Ten book-to-market deciles are constructed on each of the artificial panels, the CAPM regressions are performed, and the cross-sample averaged results are reported. The table also reports the 2.5 and 97.5 percentiles (%) of α and β across the 1000 simulations.

Panel A of Table 3 reports descriptive statistics of 10 book-to-market deciles from 1965 to 2010. The data are from Kenneth French's Web site. The value premium (the average return of the high-minus-low decile) is 0.55% per month, which is more than 2.7 standard errors from zero. The CAPM alpha of the high-minus-low decile is 0.56% ($t=2.36$), but its market beta is zero. The extreme deciles have "similar" market betas around 1.06.

While the evidence is important, we caution against taking it literally and ignoring large sampling variations in realized returns. Panel B of Table 3 repeats the same analysis as in Panel A but for the full sample from January 1927 to December 2010. With 38 years of more data, the value premium remains at 0.53% per month, which is close to 0.55% in the shorter sample. However, the CAPM alpha for the high-minus-low decile is insignificant, 0.25% ($t=1.20$), and its market beta is 0.45, which is more than three standard errors from zero (e.g., Ang and Chen, 2007). The adjusted R^2 of the CAPM regression of the high-minus-low decile goes up from zero in the post-Compustat sample to 0.14 in the full sample.

To see how the Zhang (2005) model matches the properties of ten book-to-market deciles, the empirical analysis is performed on 1000 artificial samples simulated from the model in Section 6.1. On each artificial sample, the exact timing convention of Fama and French (1993) is used to sort stocks into deciles in June of each year t on book-to-market at the end of December of year $t-1$. The value-weighted portfolio returns are from July of year t to June of year $t+1$, and the portfolios are rebalanced in June. The CAPM regressions are performed on each artificial sample, and the cross-simulation averaged returns are reported.

Table 4 shows that the model matches the average value premium at 0.50% per month. The CAPM alpha of the high-minus-low decile is 0.11% ($t=1.4$), which is not far from the 0.25% estimate from the 1927 to 2010 sample. The CAPM beta is 0.50, which is also close to 0.45 in the longer sample. However, the model fails in two aspects. First, although the CAPM

alpha of 0.56% for the high-minus-low decile from the 1965 to 2010 sample lies within the 95% confidence interval of alpha from the model $[-0.05\%, 0.59\%]$, the market beta of zero does not. The 2.5% percentile for the market beta is 0.27 in the model and is still substantially higher than zero. Second, the adjusted R^2 from the CAPM regression of the high-minus-low decile is too high in the model, 83%. In contrast, the adjusted R^2 is 14% in the 1927–2010 sample and is zero in the 1965–2010 sample.

To us, these failures are important but not insurmountable. Future work can try to explain the failure of the CAPM in simulations via, for example, disasters (e.g., Bai et al., 2012) or additional aggregate shocks (e.g., Bazzdresch et al., 2012). However, the critique is valid precisely because the model parameterizes M with a dynamic single factor structure. The failures mean that M is more complex (nonlinear) in the data. However, this critique does not apply to the investment return framework in Eq. (5). The investment return does not take a stand on M . The framework is general because it is immune to specification errors in M and measurement errors in consumption data.⁸

7.2. Do covariances “determine” expected returns?

Asset pricing often works with $E[Mr^S] = 1$. If one takes the view that asset pricing is all about M , the investment approach cannot be as “causal” as the consumption approach in “explaining” expected returns. However, both M and r^S are endogenous in general equilibrium. The statement that risks “determine” expected returns is valid only in an endowment economy, but not in a production economy. In an endowment economy, the dividends of firms (trees) are exogenous across dates and states of nature. As such, $E[r^S] - r_f = -r_f \text{Cov}(M, r^S)$, which is equivalent to $E[Mr^S] = 1$, implies that covariances “determine” expected returns. Once the dividends of all the firms are given, M is pinned down by a utility function and aggregate dividends. The covariance of a firm’s dividends (and stock returns) with M is pinned down as well. As such, causality runs from covariances to expected returns. This causality is presumably why Daniel and Titman (1997, p. 4) would write: “In equilibrium asset pricing models, the covariance structure of returns determines expected returns.”

A linear technologies economy (e.g., Cox et al., 1985), in which investment can be used freely to smooth consumption, is the polar extreme to an endowment economy. The dividends of firms arise endogenously as the outcome of firms’ optimal production and investment decisions in maximizing the market value of equity. However, firms do not incur adjustment costs when investing. Eq. (5) then says that the stock return is given by $r_{i1}^S = \Pi_{i1}$, in which Π_{i1} is the stochastic productivity. Once Π_{i1} is specified exogenously, r_{i1}^S is pinned down. As such, causality runs from characteristics (profitability) to expected returns, and then to covariances via $E[r^S] - r_f = -r_f \text{Cov}(M, r^S)$, contrary to the risk doctrine!

An adjustment costs economy, in which firms incur adjustment costs when investing, lies somewhere in between the endowment economy and the linear technologies economy, and paints a more realistic picture of the world. The model economy in Section 3 is such an example. The consumption first-order condition $E_0[M_1 r_{i1}^S] = 1$ and the investment first-order condition $r_{i1}^S = \Pi_{i1} / [1 + a(i_{i0}/K_{i0})]$ are two key equilibrium conditions. The stock return is endogenous, and is no longer determined exogenously by the marginal product of capital. As such, consumption (covariances), expected returns, and investment (characteristics) are all endogenous variables determined by a system of simultaneous optimality conditions in general equilibrium. No causality runs across these endogenous variables. Neither consumption nor investment says anything about causal forces driving expected returns. Neither covariances nor characteristics are more primitive in “explaining” expected returns.

As such, we do not claim that the investment approach “explains” asset pricing anomalies, or that characteristics cause expected returns to vary across firms. The investment approach is based on the investment first-order condition, which does not establish causality from investment to expected returns. The logic is as consistent with the view that investment “explains” expected returns as with the view that expected returns “explain” investment.

However, the investment approach is no more and no less “causal” than the consumption approach in “explaining” anomalies. Suppose a utility function and some consumption data (or several risk factors as a linear specification of M) were found such that $E[Mr^S] = 1$ holds across portfolios sorted on an anomaly variable. A researcher would likely be tempted to claim that the M model “explains” the anomaly in question. Alas, this would-be finding does not support such a claim. The consumption first-order condition says that consumers adjust consumption correctly in response to stock price movements. If equilibrium stock prices move arbitrarily with Mars synodic cycles, $E[Mr^S] = 1$ only aligns consumption with the stock prices accordingly. Consumption is as endogenous to the consumption first-order condition as investment is to the investment first-order condition in general equilibrium.

The investment approach thinks about asset pricing very differently from the consumption approach, but in a complementary way. The consumption approach connects unobservable and hard-to-measure expected returns to equally unobservable and hard-to-measure covariances. In contrast, the investment approach connects expected returns to observable and easier-to-measure characteristics. The standard practice in finance is to estimate the discount rate from the

⁸ This generality cannot be overemphasized. Many investment-based studies construct fully specified and explicitly solved models. However, the models are close to impossible to estimate because their computational complexity makes direct estimation infeasible. Moreover, an ancillary yet wrong specification can easily render a model rejected, even if the model delivers economically interesting and empirically relevant insights. To make progress, it is critical to break down an intractable problem into several tractable subproblems. While being clear about the limitations from the absence of M , the investment approach provides an empirically tractable way to tackle many important issues on the cross section of returns. No one has ever said M is not important, but M does not have to be imposed, especially mechanically, on every paper.

CAPM or multifactor models, and subsequently use the estimated discount rate in capital budgeting. The investment approach turns finance on its head. Because the investment return equals the discount rate, one can back out the cost of equity from the investment return estimated from observable characteristics. As such, risks constitute only one half of asset pricing, with characteristics being the other half. The investment approach completes the consumption approach in general equilibrium.

7.3. The investment approach: rational or irrational?

Asset pricing anomalies are often equated as investor “irrationality.”⁹ The investment approach offers an alternative to “irrationality” as a way to interpret anomalies. Critics argue that because investors are abstracted away, the investment approach has nothing to say about whether anomalies are driven by rational or irrational forces.

Not true. By spelling out technological underpinnings, the investment approach has some implications for the “rationality” of asset prices. In a linear technologies economy, irrationality must only impact on quantities via the optimal investment of rational firms, leaving no effect on asset prices. In an endowment economy, quantities cannot be adjusted, and irrationality must impact fully on asset prices. The real world is an adjustment costs economy that lies somewhere in between. Irrationality could deliver a short-term effect on prices, but rational firms eventually come in, overcome adjustment costs, and flood any “fire” of bubble with the “water” of investment, so as to extinguish any long-term impact on asset prices.

More directly, the investment approach shows that firms’ investment decisions are aligned correctly with the cost of capital. Firms invest more when their costs of capital are low, and vice versa. To the extent that this alignment manifests itself as many empirical relations between characteristics and average returns, these relations in the anomalies literature per se say *nothing* about investor rationality or irrationality. A low cost of capital could result from the sentiment of irrationally optimistic investors or the low market prices of risk demanded by rational investors. The investment first-order condition then correctly connects the low cost of capital with, for example, high asset growth and low profitability.

As such, connecting the cost of capital with characteristics via the investment approach does not “explain” anomalies, or prove the “rationality” of asset prices. To do so, one must meet the high hurdle of showing that both $E[Mr^S] = 1$ and the investment return equation hold in the data. However, our big point is that connecting the cost of capital with characteristics per se does not prove “irrationality” either, in contrast to the standard practice in the anomalies literature (footnote 9). Behaviorists are likely to say that $E[Mr^S] = 1$ fails to “explain” the connection. However, the failure can be due to specification errors of M and measurement errors in risk proxies (Section 6). Also, the failure speaks only to the consumption approach, and should not be treated as a failure of the entire “rational” paradigm.¹⁰

7.4. Is the investment return test a weak consistency test?

Cochrane (1991) notes that Eq. (5), taken literally, means that the investment return equals the stock return for every stock, every period, and every state of the world. Because no choice of parameters can satisfy this 100% R^2 equation, it is formally rejected at any level of significance. One has to use some common sense to decide how to take this equation to the data, while being artful about when the equation illuminates key aspects of the data and when it is being pushed too far. Liu et al. (2009), for example, test the ex-ante restriction that the expected (levered) investment return equals the expected stock return, interpreting this restriction as a weaker condition than the ex-post restriction in Eq. (5).

Critics argue that the ex-ante restriction is not literally a model of expected return “determination,” but rather an ancillary implication of the neoclassical investment theory. This assertion is incorrect. First, the restriction is derived from the investment first-order condition, which is as primitive as $E[Mr^S] = 1$. As equilibrium conditions, first-order conditions are not “ancillary.” Second, testing the ex-ante restriction captures the essence of the economic question, i.e., why do testing portfolios formed on characteristics earn different subsequent stock returns *on average*? The cross section of returns means the cross section of *expected* returns!

⁹ De Bondt and Thaler (1985, p. 793) write: “Research in experimental psychology suggests that, in violation of Bayes’ rule, most people tend to ‘overreact’ to unexpected and dramatic news events. This study of market efficiency investigates whether such behavior affects stock prices. The empirical evidence, based on CRSP monthly return data, is consistent with the overreaction hypothesis. Substantial weak form market inefficiencies are discovered.” Jegadeesh and Titman (1993, p. 90) write: “The market underreacts to information about the short-term prospects of firms but overreacts to information about their long-term prospects,” and that “investor expectations are systematically biased.” Ritter (1991, p. 3) interprets the long-run performance of initial public offerings as “consistent with an IPO market in which (1) investors are periodically overoptimistic about the earnings potential of young growth companies, and (2) firms take advantage of these ‘window of opportunity.’” Sloan (1996, p. 289) write: “[S]tock prices are found to act as if investors ‘fixate’ on earnings, failing to reflect fully information contained in the accrual and cash flow components of current earnings until that information impacts future earnings.”

¹⁰ Working with the investment first-order condition, Li and Zhang (2010) show that investment frictions should steepen the theoretical relation between expected returns and investment. However, they find no evidence that investment frictions proxies affect the investment growth, net stock issues, investment-to-assets, or net operating assets anomalies. In addition, limits-to-arbitrage proxies dominate investment frictions proxies in direct comparisons. Lam and Wei (2011) expand Li and Zhang’s analysis by using a broader set of limits-to-arbitrage proxies. They find that limits-to-arbitrage proxies and investment frictions proxies are often highly correlated, making the task of disentangling their effects difficult. Also, the evidence based on equal-weighted returns shows significant support for both the limits-to-arbitrage and the investment frictions hypotheses, but the evidence from value-weighted returns is weaker. Finally, in direct comparisons, each hypothesis is supported by a fair and similar amount of evidence.

Third, unlike $E[Mr^S] = 1$, which is only about expected returns, the investment return speaks also to ex-post returns. In fact, this ex-post nature helps to interpret the puzzling pattern of earnings announcement returns in the data.¹¹ Because $E[Mr^S] = 1$ has nothing to say about ex-post returns, this evidence has been interpreted as investors' expectational errors.

However, the investment approach predicts the realizations of returns around earnings announcements. The logic is as follows. Without adjustment costs, the ex-post investment return Eq. (5) reduces to profitability, Π_{i1} . The expected return should be realized *only* around earnings announcement dates, when news about firm *i*'s profitability is released to the market. In a dynamic world with adjustment costs, in addition to ex-post Π_{i1} , the ex-post investment return is also affected by the ex-post growth rate of investment-to-capital. As such, only a portion of the expected return is realized around earnings announcement dates.¹²

Critics also argue that the expected investment return should be used to forecast stock returns ex-ante. In cross-sectional regressions of future stock returns, the expected investment return should have a positive slope, and even drive out common predictors. In portfolio sorts, one should see a large average return spread across portfolios formed on the expected investment return. Although theoretically sound, this idea is not practical. It amounts to pushing the 100% R^2 equation too far. There are measurement errors in components of the investment return (such as marginal product of capital), and specification errors in the functional forms of the production and capital adjustment technologies. While these errors might be averaged out in matching average ex-post returns, the errors can destroy ex-ante predictive power of the expected investment return for future returns. A more productive strategy would be to pick components of the investment return that are (relatively) immune to measurement and specification errors, and use them to forecast returns. This strategy is exactly what Hou et al. (2012) pursue by using investment-to-capital and profitability to forecast returns.

8. Conclusion

What risks "explain" asset pricing anomalies? This question has been at the core of modern asset pricing research. We question the question. That no solution has emerged from four decades of search should at least hint at the possibility that the profession might have been barking up the wrong tree.¹³ To us, the search for risk factors is baseless. The identity of risk factors is not a clear testable implication from equilibrium theory. First-order conditions are! The search also is hopeless. Measurement errors forever doom covariances in horse races against characteristics. Because expected returns can be inferred directly from characteristics, it is pointless to keep fixating on risk factors. This atheoretical, ad hoc, and mechanical approach, due to Fama and French (1993, 1996), has been asking the wrong questions, dividing the field into two warring schools based on a false premise with no economic basis, and depressing our science into an abyss with insurmountable measurement difficulties.

At the other extreme of the anomalies debate, the Daniel–Titman (1997) test is virtually useless. First, the investment approach predicts that characteristics are correlated with expected returns. Second, there are severe measurement errors in estimated betas. Third, risks and characteristics are closely connected in theory, making the task of disentangling their empirical effects practically impossible. More generally, the anomalies literature is built on the risk doctrine, which is a faulty premise. The investment approach changes the big picture. Factors formed on characteristics are not necessarily risk factors; characteristics-based factor models are linear approximations of firm-level investment returns. The evidence that characteristics dominate covariances in horse races does not necessarily mean mispricing; measurement errors in covariances are likely to blame. Above all, asset pricing is not all about M ; without being burdened by the specification errors of M and the measurement errors in consumption data, the investment approach provides a new basis for asset pricing research.¹⁴

It is important to point out that the investment approach is not a complete approach for asset pricing either. For example, the investment approach is silent about sources of risks, i.e., what constitutes "bad times" and why investors

¹¹ Jegadeesh and Titman (1993, Table IX) document that the average 3-day returns (from day -2 to 0) around quarterly earnings announcement dates represent about 25% of momentum for the first 6-month holding period, and the announcement date returns also display reversal in long horizons. Sloan (1996, p. 312) documents that "over 40% of the predictable stock returns [related to accruals] are concentrated around the subsequent quarterly earnings announcements, even though the announcement period contains less than five percent of the total trading days." La Porta et al. (1997, p. 859) study "stock price reactions around earnings announcements for value and glamour stocks over a 5-year period after portfolio formation" and find that "a significant portion of the return difference between value and glamour stocks is attributable to earnings surprises that are systematically more positive for value stocks."

¹² For further elaboration on this point, see Wu et al. (2010, p. 217) in the context of the accrual anomaly and Liu and Zhang (2011) in the context of momentum profits.

¹³ In particular, Kuhn (1962, pp. 36 and 37) argues: "[T]he really pressing problems, e.g., a cure for cancer and the design of a lasting peace, are often not puzzles at all, largely because they may not have any solution. Consider the jigsaw puzzle whose pieces are selected at random from each of two different puzzle boxes. Since that problem is likely to defy (though it might not) even the most ingenious of men, it cannot serve as a test of skill in solution. In any usual sense, it is not a puzzle at all. Though intrinsic value is no criterion for a puzzle, the assured existence of a solution is."

¹⁴ Kuhn (1962, pp. 5 and 6) says: "Sometimes a normal problem, one that ought to be solvable by known rules and procedures, resists the reiterated onslaught of the ablest members of the group within whose competence it falls. On other occasions a piece of equipment designed and constructed for the purpose of normal research fails to perform in the anticipated manner, revealing an anomaly that cannot, despite repeated effort, be aligned with professional expectation. In these and other ways besides, normal science repeatedly goes astray. And when it does – when, that is, the profession can no longer evade anomalies that subvert the existing tradition of scientific practice – then begin the extraordinary investigations that lead the profession at last to a new set of commitments, a new basis for the practice of science."

are particularly afraid of one group of stocks over other groups. Answering these questions requires explicit utility functions. However, the consumption approach alone cannot “explain” anomalies either. For example, suppose a specification of M were identified, such that $E[Mr^S] = 1$ holds across portfolios formed on asset growth. But then why would asset growth covary with expected returns? Remember that characteristics are not even modeled in the consumption (or behavioral) framework. Finally, the investment approach cannot be used to price derivatives. However, exciting progress has been made along the lines of purely production-based pricing kernels (see Cochrane, 1993; Belo, 2010; Jermann, 2010, 2011). More work is clearly needed.

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