

# Lecture Notes

## Liu, Whited, and Zhang (2009, J. of Political Economy): Investment-Based Expected Stock Returns

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BUSFIN 920: Theory of Finance  
The Ohio State University  
Autumn 2011

# Outline

What and Why

Key Results

Model

Econometric Methods

Matching Expected Stock Returns

Matching Expected Returns and Variances

Summary and Future Work

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# Introduction

## What

We derive and test  $q$ -theory implications for the cross-section of expected stock returns

# Introduction

Motivation: Many characteristics-return relations in capital markets research

$$\underbrace{r_{jt+1}}_{\text{Realized returns}} = \underbrace{E_t[r_{jt+1}]}_{\text{Expected returns}} + \underbrace{\epsilon_{jt+1}}_{\text{Abnormal returns}}$$

Use the  $q$ -theory of investment to link expected returns to firm characteristics

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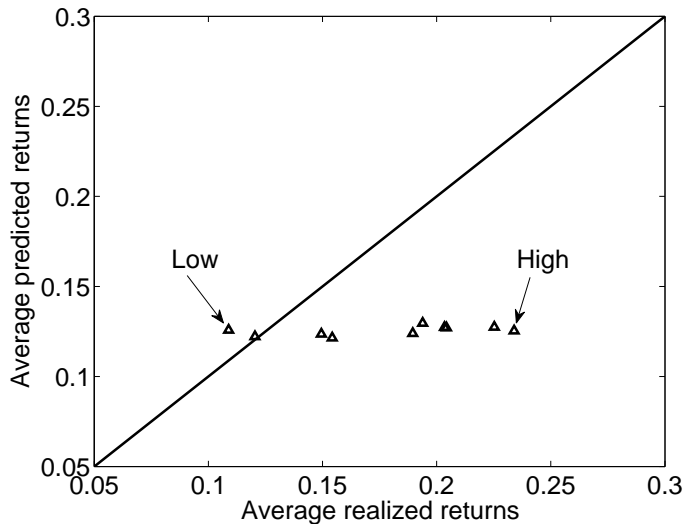
Matching Expected Stock Returns

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Summary and Future Work

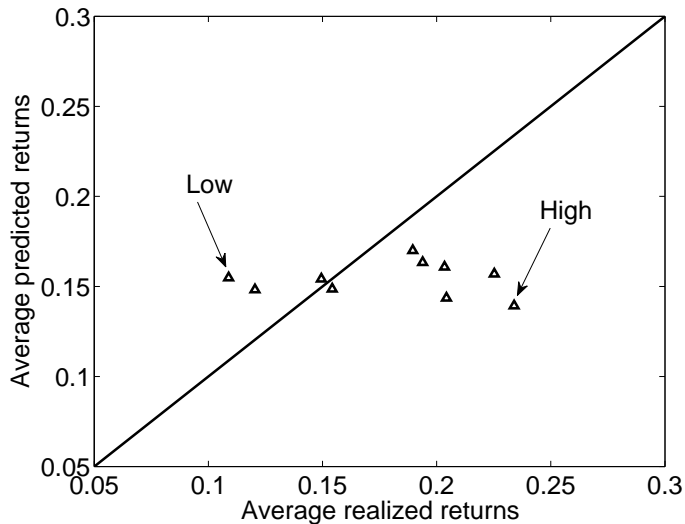
# Key Results

Average predicted vs. realized returns, ten SUE portfolios, the CAPM



# Key Results

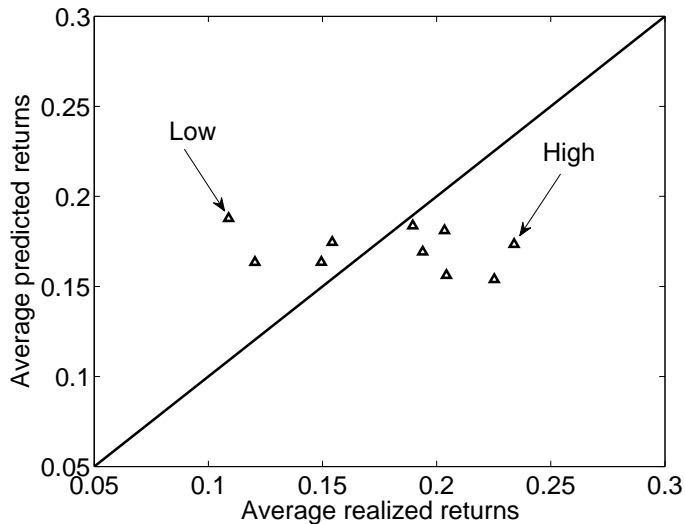
Average predicted vs. realized returns, ten SUE portfolios, the Fama-French model





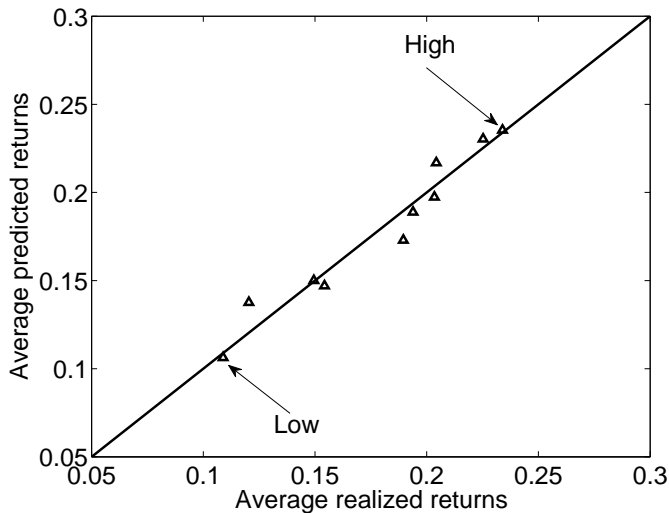
# Key Results

Average predicted vs. realized returns, ten SUE portfolios, the consumption-CAPM



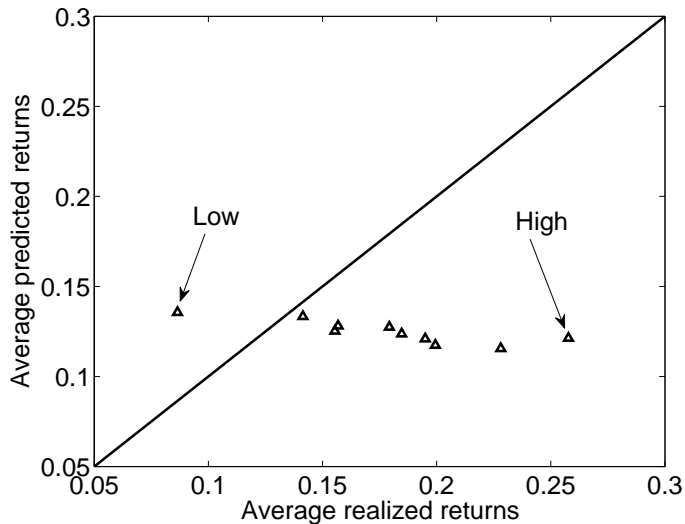
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Average predicted vs. realized returns, ten SUE portfolios, the  $q$ -theory model



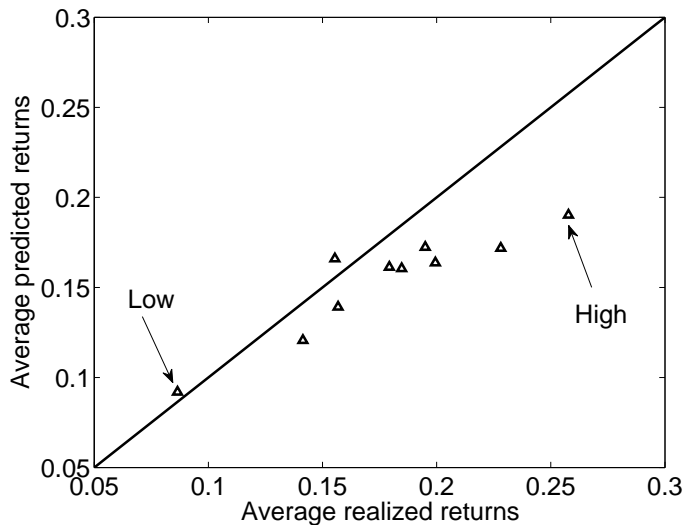
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Average predicted vs. realized returns, ten B/M portfolios, the CAPM



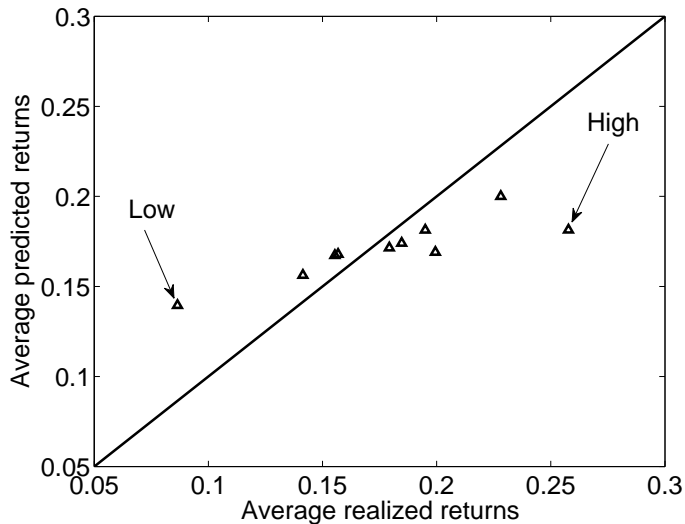
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Average predicted vs. realized returns, ten B/M portfolios, the Fama-French model



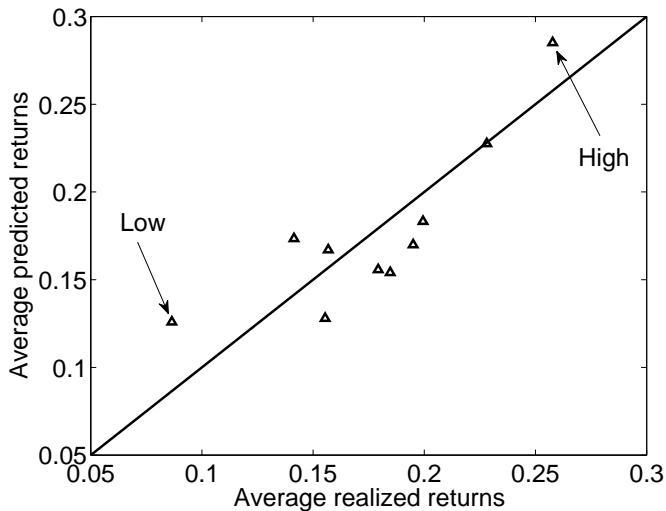
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Average predicted vs. realized returns, ten B/M portfolios, the consumption-CAPM



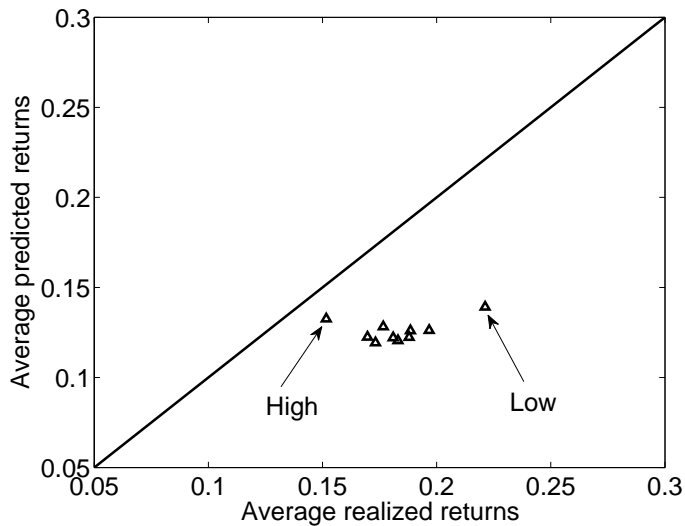
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Average predicted vs. realized returns, ten B/M portfolios, the  $q$ -theory model



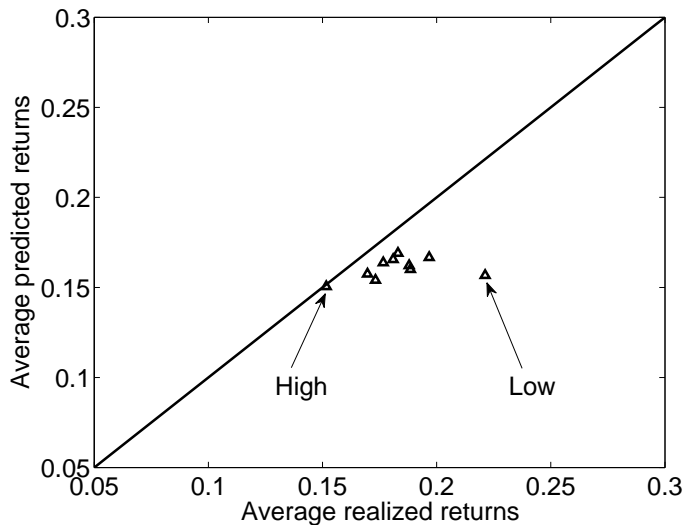
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Average predicted vs. realized returns, ten CI portfolios, the CAPM



# Key Results

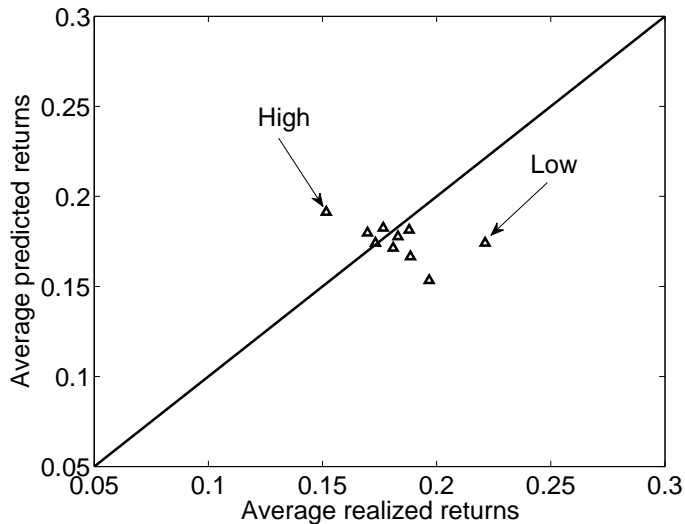
Average predicted vs. realized returns, ten CI portfolios, the Fama-French model





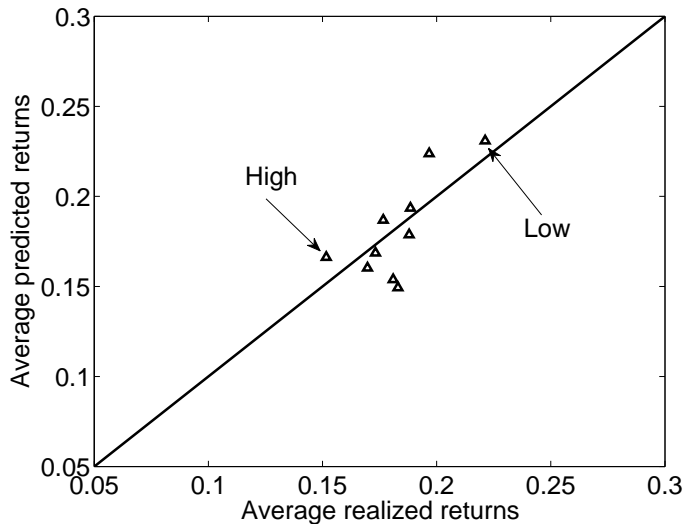
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Average predicted vs. realized returns, ten CI portfolios, the consumption-CAPM



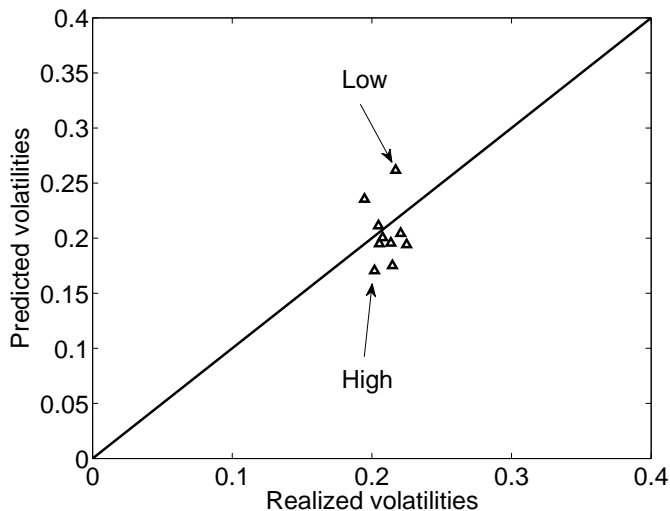
# Key Results

Average predicted vs. realized returns, ten CI portfolios, the  $q$ -theory model



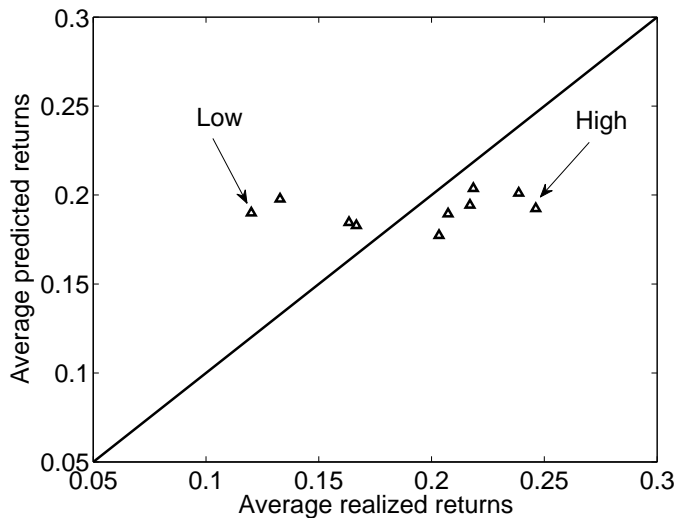
# Key Results

Predicted vs. realized stock return volatilities, joint estimation of mean and variance, the  $q$ -theory model



# Key Results

Average predicted vs. realized returns, joint estimation of mean and variance, the  $q$ -theory model



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# Model

The neoclassical  $q$ -theory framework à la Cochrane (1991). Firms use capital and costlessly adjustable inputs such as labor

Operating profits,  $\Pi(K_{it}, X_{it})$ , with

$$\frac{\partial \Pi(K_{it}, X_{it})}{\partial K_{it}} = \alpha \frac{Y_{it}}{K_{it}} \quad \text{with } Y_{it} = \text{Sales}$$

Capital evolves as:

$$K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$$

Convex adjustment costs:

$$\Phi(I_{it}, K_{it}) = \frac{a}{2} \left( \frac{I_{it}}{K_{it}} \right)^2 K_{it}$$

# Model

## Equity-value maximization

One-period debt,  $B_{it+1}$ , with corporate bond return  $r_{it+1}^B$

Payout,  $D_{it}$ , defined as:

$$(1-\tau_t)[\Pi(K_{it}, X_{it})-\Phi(I_{it}, K_{it})]-I_{it}+B_{it+1}-r_{it}^B B_{it}+\tau_t \delta_{it} K_{it}+\tau_t (r_{it}^B-1)B_{it}$$

The cum-dividend market value of the equity:

$$V_{it} \equiv \max_{\{I_{it+s}, K_{it+s+1}, B_{it+s+1}\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} M_{t+s} D_{it+s} \right]$$

in which  $M_{t+1}$  is the stochastic discount factor, correlated with  $X_{it+1}$

# Model

## The investment return

$E_t[M_{t+1}r'_{it+1}] = 1$ , in which  $r'_{it+1}$  is the investment return:

$$r'_{it+1} \equiv \frac{\overbrace{\left[ (1 - \tau_{t+1}) \left[ \alpha \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2} \left( \frac{I_{it+1}}{K_{it+1}} \right)^2 \right] \right]}^{\text{Marginal benefit of investment at time } t+1}}{\underbrace{1 + (1 - \tau_t) \left( \frac{I_{it}}{K_{it}} \right)}_{\text{Marginal cost of investment at time } t}} + \underbrace{\tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1}) a \left( \frac{I_{it+1}}{K_{it+1}} \right) \right]}_{\text{Expected continuation value}}$$



# Model

## The WACC Proposition

Define  $r_{it+1}^{Ba} = (1 - \tau_{t+1})r_{it+1}^B + \tau_{t+1}$ , then  $E_t [M_{t+1}r_{it+1}^{Ba}] = 1$

Define  $P_{it} \equiv V_{it} - D_{it}$  and the stock return

$$r_{it+1}^S \equiv (P_{it+1} + D_{it+1})/P_{it}$$

Under constant returns to scale, the investment return is the weighted average of stock and after-tax bond returns:

$$r_{it+1}^I = w_{it}r_{it+1}^{Ba} + (1 - w_{it})r_{it+1}^S \Rightarrow r_{it+1}^S = r_{it+1}^{Iw} \equiv \frac{r_{it+1}^I - w_{it}r_{it+1}^{Ba}}{1 - w_{it}}$$

in which  $w_{it}$  is market leverage,  $w_{it} \equiv B_{it+1}/(P_{it} + B_{it+1})$

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# Econometric Methods

## GMM

Do expected stock returns equal expected levered investment returns?

$$E \left[ r_{it+1}^S - r_{it+1}^{lw} \right] = 0$$

Do stock return variances equal levered investment return variances?

$$E \left[ \left( r_{it+1}^S - E \left[ r_{it+1}^S \right] \right)^2 - \left( r_{it+1}^{lw} - E \left[ r_{it+1}^{lw} \right] \right)^2 \right] = 0$$

# Econometric Methods

## Testing portfolios

### Three sets of testing portfolios

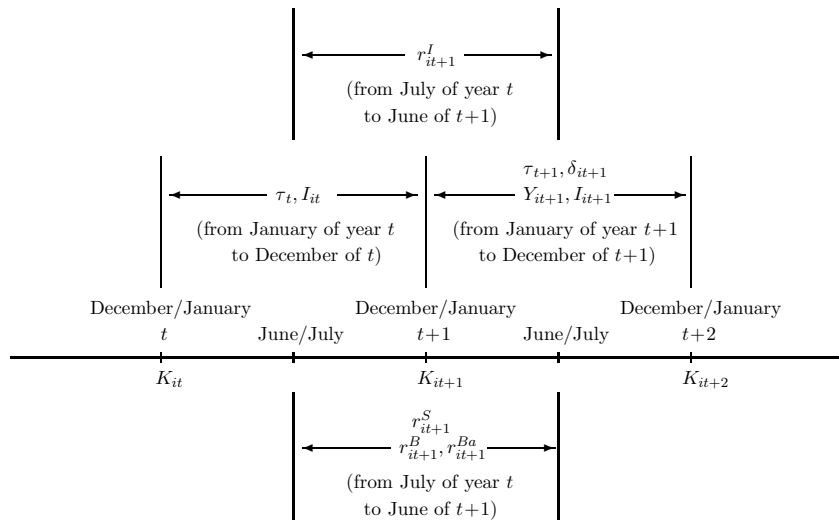
- ▶ Ten Standardized Unexpected Earnings (SUE) portfolios of Chan, Jegadeesh, and Lakonishok (1996)
- ▶ Ten book-to-market portfolios as in Fama and French (1993)
- ▶ Ten “abnormal” investment portfolios of Titman, Wei, and Xie (2004)

### Why portfolios?

- ▶ Larger and more reliable expected return spreads across portfolios than across individual stocks
- ▶ Smoothing lumpy investment as in Thomas (2002)

# Econometric Methods

## Timing



# Econometric Methods

## Measurement

- ▶  $K_{it}$ : gross property, plant, and equipment
- ▶  $I_{it}$ : capital expenditure minus sales of property, plant, and equipment
- ▶  $Y_{it}$ : sales
- ▶  $B_{it}$ : total long-term debt
- ▶  $P_{it}$ : market value of common equity
- ▶  $\delta_{it}$ : the amount of depreciation divided by capital
- ▶  $r_{it+1}^B$ : impute bond ratings, assign corporate bond returns of a given rating to all firms with the same rating
- ▶  $\tau_t$ : statutory tax rate of corporate income

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# Expected Stock Returns

Point estimates and tests of overidentification

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	SUE	B/M	CI
$a$	7.68	22.34	0.97
[ste]	[1.72]	[25.47]	[0.29]
$\alpha$	0.32	0.50	0.21
[ste]	[0.03]	[0.31]	[0.02]
$\chi^2$	4.37	5.99	6.52
d.f.	8	8	8
$p$	0.82	0.65	0.59
m.p.e.	0.74	2.32	1.51

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# Expected Stock Returns

Euler equation errors, ten SUE portfolios

	Low	5	High	H-L	$[t_{H-L}]$
Panel A: Ten SUE portfolios					
$e_i$	-1.69	6.56	10.86	12.55	[5.53]
$e_i^{FF}$	-4.59	1.96	9.47	14.06	[5.31]
$e_i^C$	-8.07	-0.04	5.31	13.38	[1.35]
$e_i^q$	0.26	1.66	-0.15	-0.40	[-0.41]

# Expected Stock Returns

Euler equation errors, ten B/M portfolios

	Low	5	High	H-L	$[t_{H-L}]$
Panel B: Ten B/M portfolios					
$e_i$	-4.91	5.19	13.65	18.56	[2.51]
$e_i^{FF}$	-0.54	1.80	6.76	7.30	[3.25]
$e_i^C$	-5.43	0.27	6.88	12.31	[0.26]
$e_i^q$	-3.94	2.35	-2.73	1.21	[0.79]

# Expected Stock Returns

Euler equation errors, ten CI portfolios

	Low	5	High	H-L	$[t_{H-L}]$
Panel C: Ten CI portfolios					
$e_i$	8.21	5.89	1.91	-6.30	[-3.88]
$e_i^{FF}$	6.45	1.54	0.11	-6.34	[-3.99]
$e_i^C$	4.03	0.46	-4.35	-8.38	[-1.35]
$e_i^q$	-0.97	2.72	-1.45	-0.49	[-0.41]

# Expected Stock Returns

Economic determinants of expected stock returns

$$r_{it+1}^I \equiv \frac{(1 - \tau_{t+1}) \left[ \alpha \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2} \left( \frac{l_{it+1}}{K_{it+1}} \right)^2 \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1}) a \left( \frac{l_{it+1}}{K_{it+1}} \right) \right]}{1 + (1 - \tau_t) a \left( \frac{l_{it}}{K_{it}} \right)}$$

$$r_{it+1}^{Iw} \equiv \frac{r_{it+1}^I - w_{it} r_{it+1}^{Ba}}{1 - w_{it}}$$

Determinants:  $Y_{it+1}/K_{it+1}$ ,  $l_{it+1}/l_{it}$ ,  $\delta_{it+1}$ , and  $l_{it}/K_{it}$ , also  $w_{it}$  and  $r_{it+1}^B$

# Expected Stock Returns

Characteristics, ten SUE portfolios

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	Low	5	High	H-L	$[t_{H-L}]$
$I_{it}/K_{it}$	0.12	0.11	0.12	0.00	[0.70]
$(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$	0.89	1.00	1.06	0.17	[4.06]
$Y_{it+1}/K_{it+1}$	1.52	1.50	1.83	0.31	[5.16]
$\delta_{it+1}$	0.08	0.08	0.08	0.00	[0.63]
$w_{it}$	0.30	0.28	0.21	-0.10	[-5.83]
$r_{it+1}^B$	9.44	9.76	9.38	-0.06	[-0.27]

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# Expected Stock Returns

Expected returns accounting, ten SUE portfolios

	Low	5	High	H-L
$\overline{l_{it}/K_{it}}$	-2.48	4.45	-4.26	-1.78
$\overline{q_{it+1}/q_{it}}$	-5.23	1.76	3.62	8.85
$\overline{Y_{it+1}/K_{it+1}}$	-0.78	0.39	3.53	4.31
$\overline{w_{it}}$	0.13	1.89	-1.46	-1.58

# Expected Stock Returns

Characteristics, ten B/M portfolios

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	Low	5	High	H-L	$[t_{H-L}]$
$I_{it}/K_{it}$	0.18	0.11	0.08	-0.10	$[-7.95]$
$(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$	0.98	1.00	1.02	0.04	$[0.68]$
$Y_{it+1}/K_{it+1}$	1.95	1.45	1.38	-0.57	$[-6.77]$
$\delta_{it+1}$	0.10	0.07	0.07	-0.03	$[-5.01]$
$w_{it}$	0.08	0.27	0.53	0.44	$[12.44]$
$r_{it+1}^B$	8.17	8.09	8.52	0.35	$[1.05]$

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# Expected Stock Returns

Expected returns accounting, ten B/M portfolios

	Low	5	High	H-L
$\overline{l_{it}/K_{it}}$	-42.06	4.69	48.17	90.23
$\overline{q_{it+1}/q_{it}}$	-1.92	2.11	-4.06	-2.14
$\overline{Y_{it+1}/K_{it+1}}$	0.16	0.92	-6.33	-6.49
$\overline{w_{it}}$	-6.00	2.19	5.58	11.58



# Expected Stock Returns

Characteristics, ten CI portfolios

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	Low	5	High	H-L	$[t_{H-L}]$
$I_{it}/K_{it}$	0.09	0.11	0.16	0.07	[11.06]
$(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$	1.25	1.04	0.81	-0.44	[-7.23]
$Y_{it+1}/K_{it+1}$	1.84	1.58	1.89	0.05	[0.38]
$\delta_{it+1}$	0.08	0.07	0.08	0.00	[-0.46]
$w_{it}$	0.35	0.25	0.28	-0.07	[-2.59]
$r_{it+1}^B$	8.47	8.27	8.44	-0.03	[-0.15]

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# Expected Stock Returns

Expected returns accounting, ten CI portfolios

	Low	5	High	H-L
$\overline{l_{it}/K_{it}}$	2.86	3.50	-5.67	-8.53
$\overline{q_{it+1}/q_{it}}$	0.73	2.97	-3.87	-4.60
$\overline{Y_{it+1}/K_{it+1}}$	0.57	-0.44	0.09	-0.48
$\overline{w_{it}}$	1.80	2.61	-0.91	-2.71

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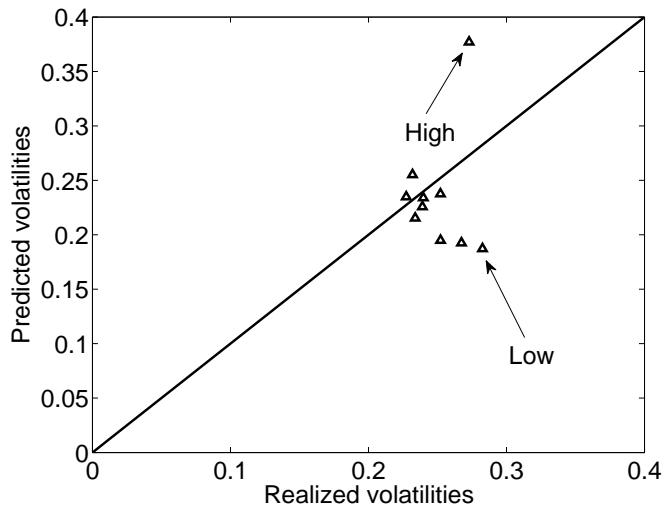
# Joint Estimation

## Point estimates and tests of overidentification

	SUE	B/M	CI
$a$	28.88	11.48	16.23
[ste]	[16.25]	[4.75]	[5.53]
$\alpha$	0.61	0.35	0.36
[ste]	[0.27]	[0.07]	[0.08]
$\chi^2_{(2)}$	5.14	6.18	6.05
d.f.(2)	8	8	8
$p(2)$	0.74	0.63	0.64
m.p.e.(2)	0.03	0.04	0.02
$\chi^2_{(1)}$	5.22	4.38	4.81
d.f.(1)	8	8	8
$p(1)$	0.73	0.82	0.78
m.p.e.(1)	3.45	2.58	2.22
$\chi^2$	5.45	6.17	6.62
d.f.	18	18	18
$p$	1.00	1.00	0.99

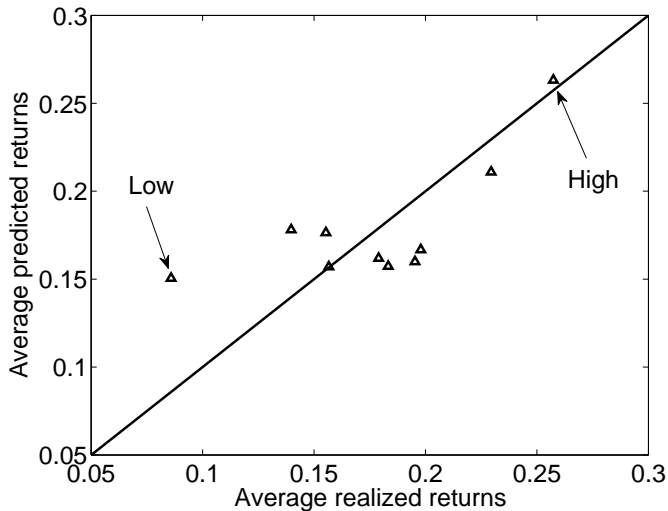
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Predicted vs. realized stock return volatilities, ten B/M portfolios, the  $q$ -theory model



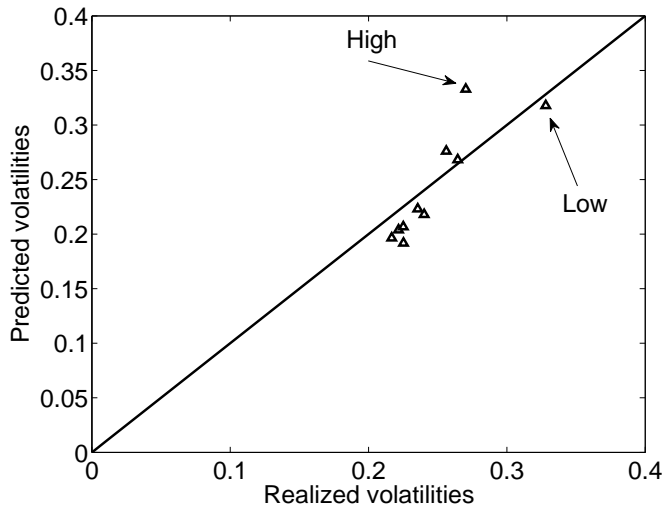
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Average predicted vs. realized returns, ten B/M portfolios, the  $q$ -theory model



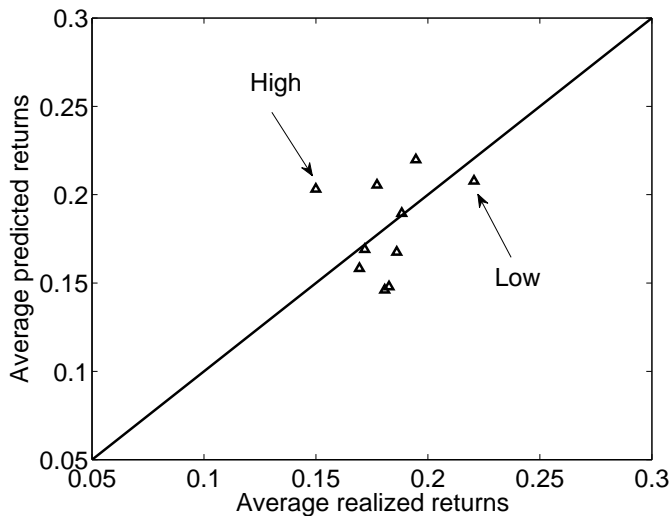
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Predicted vs. realized stock return volatilities, ten CI portfolios, the  $q$ -theory model



# Joint Estimation

Average predicted vs. realized returns, ten CI portfolios, the  $q$ -theory model





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# Conclusion

## Summary and interpretation

Summary: Derive and test the  $q$ -theory model for cross-sectional returns

Interpretation: Portfolios of firms do a good job in aligning their investment policies with costs of equity capital, and this alignment drives many characteristics-return relations

# Conclusion

## Future Work

More realistic ingredients (decreasing returns, investment lags, financing constraints, labor, organizational capital):

- ▶ Balance realism and analytical tractability, empirical challenges (data limitations)

More puzzles in cross-sectional returns (momentum, asset growth, accruals, distress, M&As, net equity issues, governance)

An investment-based theory of corporate bond returns

Belo, Xue, and Zhang (2010): Cross-sectional Tobin's  $Q$

Methodology applicable in dynamic corporate finance