Motivating Factors

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Abstract

Factor models are not immune to p-hacking. In spanning regressions, the q-factor model dominates the Fama-French (2015, 2017) five- and six-factor models, and the Q5 model subsumes the Stambaugh-Yuan (2017) "mispricing" factor model. The "mispricing" factors are sensitive to their construction, and once replicated via the standard approach, are close to the q-factors, with correlations of 0.8 and 0.84. The Fama-French five-factor model does not follow from valuation theory, which predicts a positive relation between the expected investment and the expected return. Finally, the investment CAPM provides a "factors" perspective that differs fundamentally from the consumption CAPM.

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1 Introduction

A new generation of factor pricing models has emerged in the cross section of expected returns, including the Hou-Xue-Zhang (2015) q-factor model and the Hou-Mo-Xue-Zhang (2017) Q5 model, the Fama-French (2015, 2017) five- and six-factor models, as well as the Stambaugh-Yuan (2017) "mispricing" four-factor model. In this paper, we compare the new factor models on both empirical and conceptual grounds. Specifically, we make three contributions to the asset pricing literature.

Our first contribution is to show that the seemingly different factor models are in fact closely related. In factor spanning tests, the q-factor and Q5 models largely explain the Fama-French five- and six-factor returns. From January 1967 to December 2016, the average premiums of the value, investment, profitability, and momentum factors (HML, CMA, RMW, and UMD) in the Fama-French five- and six-factor models are 0.37%, 0.33%, 0.26%, and 0.64% per month ($t = 2.72$, 3.51, 2.53, and 3.6), respectively. However, their q-factor alphas are tiny, only 0.07%, −0.00%, 0.01%, and 0.11% ($t = 0.63, -0.13$, 0.11, and 0.49), and Q5 alphas 0.02%, −0.06%, −0.05%, and −0.12% ($t = 0.2, -1.45, -0.62$, and −0.54), respectively. Their cash-based profitability factor, RMWc, earns on average 0.33% ($t = 4.16$), and survives the q-factor model with an alpha of 0.25% ($t = 3.83$). However, the Q5 model reduces RMWc to insignificance, with an alpha of 0.1% ($t = 1.64$).

Conversely, the Fama-French five- and six-factor models cannot explain the q- and Q5-factor premiums. The investment, return on equity, expected growth factors in the q-factor and Q5 models are on average 0.41%, 0.55%, and 0.56% per month ($t = 4.92$, 5.25, and 6.66), their five-factor alphas 0.12%, 0.47%, and 0.56% ($t = 3.48$, 5.91, and 7.55), the six-factor alphas 0.11%, 0.3%, and 0.5% ($t = 3.15$, 4.5, and 7.63), and the alphas from the alternative six-factor model with RMWc are 0.11%, 0.23%, and 0.38% ($t = 2.84$, 2.79, and 5.35), respectively. As such, for the most part, the Fama-French five- and six-factors are just noisy versions of the q- and Q5 factors.

Deviating from the standard factor construction per Fama and French (1993), Stambaugh and Yuan (2017) use the NYSE, Amex, and NASDAQ breakpoints of the 20th and 80th percentiles when forming the two "mispricing" factors, as opposed to the more standard NYSE breakpoints of the 30th and 70th percentiles. We reproduce their factors following their construction, and also replicate their factors via the standard approach. The performance of the "mispricing" factor model (M4) is sensitive to the factor construction. While the original M4 factors survive the q-factor and Q5 models, the replicated M4 factors are largely absorbed by the Q5 model. Neither the original nor the replicated M4 model can explain the q- and Q5 factors. The replicated "mispricing" factors are close to the q-factors, with correlations of 0.8 and 0.84. As such, the Stambaugh-Yuan statistical cluster analysis essentially rediscovers the q-factors, which are in turn motivated from economic theory.

While Stambaugh and Yuan (2017) use statistical analysis to identify their factors, Fama and French (2015) attempt to motivate their five-factor model from valuation theory. Our second contribution is to show that, conceptually, their five-factor model does not follow from valuation theory. Specifically, we raise four critiques on their motivation. First, Fama and French derive the relations between book-to-market, investment, and profitability only with the internal rate of return. These relations do not necessarily carry over to the one-period-ahead expected return. Estimating the internal rate of returns for their profitability factor, RMW, using accounting-based models, we show that these estimates differ greatly from their one-period-ahead average returns. In particular, the estimates for the internal rate of return for RMW are often significantly negative.

Second, Fama and French (2015) argue that the value factor should be a separate factor based on valuation theory, but find it to be redundant in describing average returns in the data. However, the evidence is consistent with the investment CAPM. Intuitively, the first principle of investment says that the marginal cost of investment, which rises with investment, equals marginal q, which is closely related to market-to-book equity. This tight investment-value linkage implies that the value and investment factors are largely substitutable in factor regressions.

Third, Fama and French (2015) motivate their investment factor, CMA, from the negative relation between the expected investment and the internal rate of return. Reformulating the valuation equation with the one-period-ahead expected return, we show that the theoretical relation between the expected investment and the expected return is positive. In fact, the same mechanism from the investment CAPM motivates the expected growth factor in the Q5 model. As such, in the valuation equation the investment factor can only be motivated from the market-to-book term, augmented with the investment-value linkage, which is in turn a key insight from the investment CAPM.

Fourth, after motivating CMA from the expected investment effect, Fama and French (2015) use past investment as a proxy for the expected investment. This practice is problematic. While past profitability forecasts future profitability, past investment does not forecast future investment. In the annual cross-sectional regressions of future book equity growth on asset growth, the average R^2 starts at 5% in year one, and drops quickly to zero in year four. In contrast, in the annual cross-sectional regressions of future operating profitability on operating profitability, the average R^2 starts at 54% in year one, drops to 27% in year three, and remains above 10% even in year ten.

More broadly, our last contribution is to elaborate how the investment CAPM, on which the q-factor and Q5 models are based, provides a fundamentally different "factors" perspective from the consumption CAPM. Asset pricing is *not* all about the stochastic discount factor. Factor models can be linear approximations to the firm cost of capital from the investment CAPM, not just to the aggregate marginal utility growth from the consumption CAPM. Both covariances and characteristics, not just covariances, matter for the expected return in efficient markets. The consumption CAPM anomalies are largely the investment CAPM regularities, which confirm to standard economic principles. As a supply theory of asset pricing, the investment CAPM expands greatly the scope of asset pricing beyond the consumption CAPM and behavioral finance, both of which are demand theories.

The rest of the paper is organized as follows. Section 2 conducts factor spanning tests. Section 3 summarizes the economic foundation of the q-factor and Q5 models based on the investment CAPM. Section 4 shows that conceptually, the Fama-French (2015) five-factor model does not follow from valuation theory. Section 5 details the investment CAPM perspective of "factors," and shows how it differs from the traditional consumption CAPM perspective. Finally, Section 6 concludes.

2 Factor Spanning Tests

Section 2.1 describes the factors construction, and Section 2.2 reports the spanning regressions.

2.1 Factor Construction

We detail the construction of the q-factors and the expected growth factor in the Q5 model, the Fama-French five- and six-factors, as well as the Stambaugh-Yuan "mispricing" factors. Monthly returns are from Center for Research in Security Prices (CRSP) and accounting variables from Compustat Annual and Quarterly Fundamental Files. We require CRSP share codes to be 10 or 11.

The q-factor Model

Following Hou, Xue, and Zhang (2015), we construct the size, investment, and return on equity (Roe) factors from a triple ($2 \times 3 \times 3$) sort on size, investment-to-assets (I/A), and Roe. Size is the market equity, which is stock price per share times shares outstanding from CRSP. I/A is the annual change in total assets (Compustat annual item AT) divided by one-year-lagged total assets. Roe is income before extraordinary items (Compustat quarterly item IBQ) divided by one-quarter-lagged book equity.[1] We exclude financial firms and firms with negative book equity.

At the end of June of each year t, we use the NYSE median to split stocks into two groups, small and big. Independently, at the end of June of year t, we break stocks into three I/A groups using the NYSE breakpoints for the low 30%, middle 40%, and high 30% of the ranked values of I/A for the fiscal year ending in calendar year $t-1$. Also, independently, at the beginning of each month, we sort all stocks into three groups based on the NYSE breakpoints for the low 30%, middle 40%, and high 30% of Roe. Earnings data in Compustat quarterly files are used in the months immediately after the most recent public quarterly earnings announcement dates (Compustat quarterly item RDQ).

[1] Book equity is shareholders' equity, plus balance sheet deferred taxes and investment tax credit (Compustat quarterly item TXDITCQ) if available, minus the book value of preferred stock (item PSTKQ). Depending on availability, we use stockholders' equity (item SEQQ), or common equity (item CEQQ) plus the carrying value of preferred stock (item PSTKQ), or total assets (item ATQ) minus total liabilities (item LTQ) in that order as shareholders' equity.

For a firm to enter the factor construction, we require the end of the fiscal quarter that corresponds to its announced earnings to be within six months prior to the portfolio formation month.

Taking the intersection of the two size, three I/A, and three Roe groups, we form 18 portfolios. Monthly value-weighted portfolio returns are calculated for the current month, and the portfolios are rebalanced monthly. The size factor, denoted R_{Me}, is the difference (small-minus-big), each month, between the simple average of the returns on the nine small size portfolios and the simple average of the returns on the nine big size portfolios. The investment factor, $R_{\text{I/A}}$, is the difference (low-minus-high), each month, between the simple average of the returns on the six low I/A portfolios and the simple average of the returns on the six high I/A portfolios. Finally, the Roe factor, R_{Roe}, is the difference (high-minus-low), each month, between the simple average of the returns on the six high Roe portfolios and the simple average of the returns on the six low Roe portfolios.

Hou, Xue, and Zhang (2015) start their sample in January 1972, which is restricted by the limited coverage of earnings announcement dates and book equity in Compustat quarterly files. We follow their procedure from January 1972 onward, but extend the starting point of the q-factors backward to January 1967. To overcome the lack of coverage for quarterly earnings announcement dates, we use the most recent quarterly earnings from fiscal quarters ending at least four months prior to the portfolio formation month. To expand the coverage for quarterly book equity, we use book equity from Compustat annual files, and impute quarterly book equity with clean surplus accounting. Whenever available we first use quarterly book equity from Compustat quarterly files. We then supplement the coverage for fiscal quarter four with book equity from Compustat annual files.[2]

If both approaches are unavailable, we apply the clean surplus relation to impute the book equity. If available, we backward impute the beginning-of-quarter book equity as the end-of-quarter

[2]We measure annual book equity per Davis, Fama, and French (2000) as stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (Compustat annual item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if available. Otherwise, we use the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption value (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock.

book equity minus quarterly earnings plus quarterly dividends.[3] Because we impose a four-month lag between earnings and the holding period month (and the book equity in the denominator of Roe is one-quarter-lagged relative to earnings), all the Compustat data in the backward imputation are at least four-month lagged relative to the portfolio formation month.

If data are unavailable for the backward imputation, we impute the book equity for quarter t forward based on book equity from prior quarters. Let $\text{BEQ}_{t-j}, 1 \leq j \leq 4$, denote the latest available quarterly book equity as of quarter t, and $\text{IBQ}_{t-j+1,t}$ and $\text{DVQ}_{t-j+1,t}$ be the sum of quarterly earnings and the sum of quarterly dividends from quarter $t-j+1$ to t, respectively. BEQ_t can be imputed as $\text{BEQ}_{t-j}+\text{IBQ}_{t-j+1,t}-\text{DVQ}_{t-j+1,t}$. We do not use prior book equity from more than four quarters ago ($1 \leq j \leq 4$) to reduce imputation errors. We start the sample in January 1967 to ensure that all the 18 benchmark portfolios from the triple sort on size, I/A, and Roe have at least ten firms.

The Expected Growth Factor in the Q5 Model

Hou, Mo, Xue, and Zhang (2017) augment the q-factor model with an expected growth factor, denoted R_{Eg}, to form the Q5 model. The expected growth factor is constructed from an independent 2×3 sort on size and the expected one-year-ahead investment-to-assets change, $E_t[\text{d}^1\text{I/A}]$. Two instruments, including Tobin's q and operating cash flow-to-assets, are used to form $E_t[\text{d}^1\text{I/A}]$.

In June of year t, we measure Tobin's q as the market equity (price per share times the number of shares outstanding from CRSP) plus long-term debt (Compustat annual item DLTT) and short-term debt (item DLC) scaled by total assets (item AT), all from the fiscal year ending in calendar year $t-1$. For firms with multiple share classes, we merge the market equity for all classes. Following Ball, Gerakos, Linnainmaa, and Nikolaev (2016), we measure operating cash flow-to-assets, denoted Cop, as total revenue (item REVT) minus cost of goods sold (item COGS), minus selling, general,

[3]Quarterly earnings are income before extraordinary items (Compustat quarterly item IBQ). Quarterly dividends are zero if dividends per share (item DVPSXQ) are zero. Otherwise, total dividends are dividends per share times beginning-of-quarter shares outstanding adjusted for stock splits during the quarter. Shares outstanding are from Compustat (quarterly item CSHOQ supplemented with annual item CSHO for fiscal quarter four) or CRSP (item SHROUT), and the share adjustment factor is from Compustat (quarterly item AJEXQ supplemented with annual item AJEX for fiscal quarter four) or CRSP (item CFACSHR).

and administrative expenses (item XSGA), plus research and development expenditures (item XRD, zero if missing), minus change in accounts receivable (item RECT), minus change in inventory (item INVT), minus change in prepaid expenses (item XPP), plus change in deferred revenue (item DRC plus item DRLT), plus change in trade accounts payable (item AP), and plus change in accrued expenses (item XACC), scaled by book assets, all from the fiscal year ending in calendar year $t-1$. As in Ball et al., all changes are annual changes, and the missing changes are set to zero.

At the end of June of each year t, we compute $E_t[\mathrm{d}^1\mathrm{I/A}]$ with the $\log(q)$ and Cop values winsorized at the 1–99% level from the fiscal year ending in calendar year $t-1$ and the Fama-MacBeth (1973) cross-sectional predictive regression slopes estimated from the prior ten-year rolling window from year $t-10$ to $t-1$. We require a minimum of five years. At the end of June of each year $s \in [t-10, t-1]$, the left-hand side variable in the regressions is the one-year-ahead investment-to-assets change, $\mathrm{d}^1\mathrm{I/A}$, measured as I/A from the fiscal year ending in calendar year s minus I/A from the fiscal year ending in calendar year $s-1$. The right-hand side variables in the cross-sectional regressions are the log of Tobin's q and Cop from the fiscal year ending in calendar year $s-1$. We winsorize both the left- and right-hand side variables at the firm level in each June at the 1–99% level, and estimate the regressions via weighted least squares with the market equity as weights. We use weighted, not ordinary, least squares to mitigate the impact of microcaps (Fama and French 2008).

At the end of June of each year t, we use the NYSE median to split stocks into two groups, small and big. Independently, we split all stocks into three groups, low, median, and high, based on the NYSE breakpoints for the low 30%, middle 40%, and high 30% of the ranked values of $E_t[\mathrm{d}^1\mathrm{I/A}]$ calculated at the fiscal year ending in calendar year $t-1$. Taking the intersection of the two size and three $E_t[\mathrm{d}^1\mathrm{I/A}]$ groups, we form six benchmark portfolios. Monthly value-weighted portfolio returns are calculated from July of year t to June of year $t+1$, and the portfolios are rebalanced at the end of June of $t+1$. The expected growth factor, R_{Eg}, is the difference (high-minus-low), each month, between the simple average of the returns on the two high $E_t[\mathrm{d}^1\mathrm{I/A}]$ portfolios and the simple average of the returns on the two low $E_t[\mathrm{d}^1\mathrm{I/A}]$ portfolios.

The Fama-French (2015, 2017) Five- and Six-factor Models

Subsequent to Hou, Xue, and Zhang (2015), Fama and French (2015) incorporate two factors that resemble the q-factors into their three-factor model to form a five-factor model.[4] RMW is the difference between the returns on portfolios of stocks with robust and weak profitability, and CMA the difference between the returns on portfolios of low and high investment stocks.

Fama and French (2015) measure operating profitability to equity as total revenue (Compustat annual item REVT) minus cost of goods sold (item COGS, zero if missing), minus selling, general, and administrative expenses (item XSGA, zero if missing), and minus interest expense (item XINT, zero if missing), scaled by book equity (the denominator is current, not lagged, book equity). At least one of the three expense items (COGS, XSGA, and XINT) must be nonmissing. Investment is measured as I/A, the annual change in total assets divided by one-year-lagged total assets.

Fama and French (2015) construct RMW and CMA from independent 2×3 sorts by interacting size with operating profitability, and separately, with investment-to-assets. At the end of June of year t, stocks are split into two groups, small and big, based on the NYSE median size, and independently into three groups, low, median, and high, based on the 30th and 70th NYSE percentiles of operating profitability, and separately, of investment-to-assets. Taking intersections yields six size-profitability portfolios and six size-I/A portfolios. Monthly value-weighted portfolio returns are calculated from July of year t to June of $t+1$, and the portfolios are rebalanced at the June-end of year $t+1$. RMW is the average of the two high profitability portfolio returns minus the average of the two low profitability portfolio returns. Similarly, CMA is the average of the two low I/A portfolio returns minus the average of the two high I/A portfolio returns.

Fama and French (2017) incorporate the momentum factor, UMD, from Jegadeesh and Titman

[4] Hou, Xue, and Zhang (2015) first appear in October 2012 as NBER working paper 18435, which supersedes the previous work with various titles, including "Neoclassical factors" (NBER working paper 13282, July 2007), "An equilibrium three-factor model (January 2009)," "Production-based factors (April 2009)," "A better three-factor model that explains more anomalies (June 2009)," and "An alternative three-factor model (April 2010)." In contrast, the Fama and French (2013, 2015) work is first circulated in June 2013. Their 2013 draft adds only a profitability factor to their three-factor model, and subsequent drafts, starting from November 2013, also add an investment factor.

(1993), into their five-factor model to form a six-factor model. At the beginning of each month t, stocks are split into two groups, small and big, based on the NYSE median size, and independently into three groups, low, median, and high, based on the 30th and 70th NYSE percentiles of prior 11-month returns from month $t-12$ to $t-2$, skipping month $t-1$. Taking intersections yields six size-momentum portfolios. Monthly value-weighted portfolio returns are calculated for the current month, and the portfolios are rebalanced at the beginning of month $t+1$. UMD is the average of the two winner portfolio returns minus the average of the two loser portfolio returns.

Fama and French (2017) also introduce a cash-based profitability factor, denoted RMWc. At the June end of year t, cash-based operating profitability is revenues (Compustat annual item REVT) minus cost of goods sold (item COGS, zero if missing), minus selling, general, and administrative expenses (item XSGA, zero if missing), minus interest expense (item XINT, zero if missing) minus change in accounts receivable (item RECT), minus change in inventory (item INVT), minus change in prepaid expenses (item XPP), plus change in deferred revenue (item DRC plus item DRLT), plus change in trade accounts payable (item AP), and plus change in accrued expenses (item XACC), scaled by book equity, all from the fiscal year ending in calendar year $t-1$. We require at least one of the three expense items (COGS, XSGA, and XINT) to be nonmissing. The numerator of this variable is a variant of that in Ball, Gerakos, Linnainmaa, and Nikolaev (2016), without adding back research and development expenses. The construction of RMWc is analogous to that of RMW.[5]

The Stambaugh-Yuan (2017) "Mispricing" Factor Model

Stambaugh and Yuan (2017) start with 11 anomalies, which are grouped into two clusters based on pairwise cross-sectional correlations. The first cluster, labeled MGMT (management), includes net stock issues, composite issues, accruals, net operating assets, asset growth (from Cooper, Gulen, and Schill 2008, investment-to-assets in Hou, Xue, and Zhang 2015), and the annual change in gross property, plant, and equipment plus the annual change in inventories scaled by lagged book assets.

[5]The sample in Fama and French (2015, 2017) includes financial firms and firms with negative book equity, except that positive book equity is required for constructing HML, RMW, and RMWc.

The second cluster, labeled PERF (performance), includes failure probability (Campbell, Hilscher, and Szilagyi 2008), O-score, momentum, gross profitability, and return on assets. Appendix A details the variable definitions. Conceptually, MGMT contains different investment measures, and PERF different profitability measures. The individual variables in each cluster are realigned to yield positive low-minus-high decile returns. The composite measures, MGMT and PERF, are formed by equal-weighting a stock's rankings with respect to the anomaly variables within a given cluster.

Stambaugh and Yuan (2017) form the MGMT and PERF factors from independent 2×3 sorts on size and MGMT, and on size and PERF. At the beginning of each month t, stocks (excluding those with prices per share less than \$5) are split by the NYSE median size into two groups, small and big. Independently, stocks are split based on MGMT, and separately, on PERF, into three groups, low, median, and high, with breakpoints of the 20th and 80th percentiles of the NYSE, Amex, and NASDAQ universe. Taking intersections yields six size-MGMT portfolios and six size-PERF portfolios. Monthly value-weighted portfolio returns are calculated for the current month t, and the portfolios are rebalanced at the beginning of month $t+1$. The MGMT factor is the average of the returns on the two low MGMT portfolios minus the average of the returns on the two high MGMT portfolios. The PERF factor is the average of the returns on the two low PERF portfolios minus the average of the returns on the two high PERF portfolios. The size factor is the returns of the value-weighted portfolio of stocks in the intersection of the small-cap middle portfolios from the double sorts of size with MGMT and with PERF minus the returns of the value-weighted portfolio of stocks in the intersection of both big-cap middle portfolios from the two double sorts.

Most important, the Stambaugh-Yuan (2017) factor construction deviates from the standard approach in Fama and French (1993, 2015) and Hou, Xue, and Zhang (2015) in several important ways. First, when sorting on MGMT and PERF, the breakpoints of the 20th and 80th percentiles are adopted, as opposed to the standard 30th and 70th percentiles. Second, the NYSE, Amex, and NASDAQ breakpoints are used, instead of the NYSE breakpoints. Finally, the size factor contains stocks only in the middle portfolios of the MGMT and PERF sorts, not stocks from all three portfolios. To evaluate the sensitivity of the Stambaugh-Yuan (2017) model's performance to its factor construction, we present two sets of results. In the first, we use their original factors series, and in the second set, we follow the standard approach to reconstruct their factors.[6]

2.2 Spanning Regressions

The q-factor and Q5 Models versus the Fama-French Five- and Six-factor Models

The upshot is that the q-factor and Q5 models largely explain the five- and six-factor returns, but the five- and six-factor models cannot explain the q- and Q5-factor returns.

In Panel A of Table 1, we regress the q- and Q5-factor returns on the Fama-French five- and six-factor models, as well as their alternative six-factor specification with RMW replaced by RMWc. From January 1967 to December 2016, the size factor, R_{Me}, in the q-factor model earns an average return of 0.31% per month ($t = 2.43$). All three Fama-French specifications account for this size premium, with alphas up to 0.05%, due to the presence of SMB.

The investment factor, $R_{\mathrm{I/A}}$, in the q-factor model earns an average return of 0.41% per month ($t = 4.92$). Despite the presence of CMA, the Fama-French five-factor model reduces the $R_{\mathrm{I/A}}$ premium to an alpha of 0.12%, but still significant ($t = 3.48$). The two specifications of the six-factor model yield largely similar results. $R_{\mathrm{I/A}}$ is stronger than CMA, because $R_{\mathrm{I/A}}$ is based on a joint sort with Roe, whereas the CMA construction does not control for profitability.

The Roe factor, R_{Roe}, earns an average return of 0.55% per month ($t = 5.25$). The Fama-French five-factor model only reduces the Roe premium to an alpha of 0.47% ($t = 5.91$), despite a large RMW loading of 0.7 ($t = 12.8$). Intuitively, the Roe factor is based on monthly sorts on latest announced quarterly earnings data, whereas RMW is based on annual sorts on the more stale operating profitability from the last fiscal year end. As such, the Roe factor is more powerful than RMW. The six-factor model reduces the Roe premium further to an alpha of 0.3% ($t = 4.5$), with the help of an UMD loading of 0.24 ($t = 9.94$). Replacing RMW with RMWc in the six-factor model

[6]Stambaugh and Yuan (2017) include financial firms and firms with negative book equity, but impose a $5 price screen. We show later that this sample difference from Hou, Xue, and Zhang (2015) has little impact on the results.

yields a smaller alpha of 0.23%, due to a higher premium of RMWc than RMW, 0.33% versus 0.26%. However, the alternative six-factor alpha for the Roe factor, 0.23%, is still significant ($t = 2.79$).

The expected growth factor, R_{Eg}, in the Q5 model earns an average return of 0.56% per month ($t = 6.66$). The five-factor model has no effect on the R_{Eg} premium, with an alpha of 0.56% ($t = 7.55$). The six-factor model with RMW reduces the R_{Eg} premium only slightly to an alpha of 0.5% ($t = 7.63$), with the help of a small UMD loading of 0.09 ($t = 4.29$). Finally, replacing RMW with RMWc in the six-factor model reduces the R_{Eg} premium to an alpha of 0.38%, which remains highly significant ($t = 5.35$), helped by a large RMWc loading of 0.41 ($t = 4.72$).

In untabulated results, we have also performed the Gibbons, Ross, and Shanken (1989, GRS) test on the null hypothesis that the alphas of the key q- and Q5 factors in the Fama-French five- and six-factor regressions are jointly zero. For the null that the alphas of the investment and Roe factors are jointly zero, the GRS statistic is 22.78 ($p = 0.00$) in the five-factor model, 14.7 ($p = 0.00$) in the six-factor model with RMW, and 8.45 ($p = 0.00$) in the alternative six-factor model with RMWc. For the nulll that the alphas of the investment, Roe, and expected growth factors are jointly zero, the GRS statistic is 40.69 ($p = 0.00$) in the five-factor model, 34.77 ($p = 0.00$) in the six-factor model with RMW, and 22.16 ($p = 0.00$) in the alternative six-factor model with RMWc.

Panel B of Table 1 shows that the q-factor and Q5 models largely subsume the Fama-French five- and six-factors in spanning regressions, with economically small and mostly insignificant alphas. SMB has an average return of 0.25% per month ($t = 1.92$), and its q- and Q5-alphas are 0.04% and 0.07%, with t-values of 1.32 and 2.13, respectively. HML has an average return of 0.37% ($t = 2.71$), and its q- and Q5-alphas are 0.07% ($t = 0.63$) and 0.02% ($t = 0.2$), respectively. The investment factor, $R_{\mathrm{I/A}}$, delivers the explanatory power. The factor loadings are economically large (close to one), and also highly significant (t-values above 11).

The momentum factor, UMD, is on average 0.64% per month ($t = 3.6$). The q-factor alpha is only 0.11% ($t = 0.49$), with the help of a large Roe factor loading of 0.91 ($t = 5.88$). The

Q5-alpha is weakly negative, -0.12% ($t = -0.54$). In addition to a large Roe factor loading of 0.85 ($t = 5.34$), the expected growth factor loading, 0.44 ($t = 2.25$), also helps. Intuitively, momentum winners have both higher Roe and higher expected growth than momentum losers.

CMA has an average return of 0.33% per month ($t = 3.51$). The q-factor alpha is virtually zero ($t = -0.13$), helped by a large investment factor loading of 0.96 ($t = 34.9$). The Q5-alpha is also tiny, -0.06% ($t = -1.45$), with a similar investment factor loading. RMW has an average return of 0.26% ($t = 2.53$). The q-factor alpha is only 0.01% ($t = 0.11$), with a large Roe factor loading of 0.54 ($t = 8.53$). Similarly, the Q5 alpha is also tiny, -0.05% ($t = -0.62$), with a large Roe factor loading of 0.52 ($t = 7.74$). Finally, RMWc has an average return of 0.33% ($t = 4.16$). RMWc survives the control of the q-factors, with a q-alpha of 0.25% ($t = 3.83$). Although the Roe factor loading is significant ($t = 9.88$), its magnitude is not large, 0.29. The Q5 model reduces RMWc to insignificance, with an alpha of 0.1% ($t = 1.64$), helped by both the Roe and expected growth factors.

We have also performed the GRS test on the null hypothesis that the alphas of the key Fama-French five- and six-factors are jointly zero in the q-factor and Q5 regressions. For the null that the alphas of HML, CMA, and RMW are jointly zero, the GRS statistic is 0.23 ($p = 0.87$) in the q-factor model, and 1.38 ($p = 0.25$) in the Q5 model. For the null that the alphas of HML, CMA, RMW, and UMD are jointly zero, the GRS statistic is 0.39 ($p = 0.82$) in the q-factor model, and 1.21 ($p = 0.3$) in the Q5 model. For the null that the alphas of HML, CMA, RMWc, and UMD are jointly zero, the GRS statistic is 6.19 ($p = 0.00$) in the q-factor model, and 1.59 ($p = 0.17$) in the Q5 model.

The q-factor and Q5 Models versus the Stambaugh-Yuan "Mispricing" Factor Model

Table 2 reports the factor spanning tests of the q-factor and Q5 models versus the Stambaugh-Yuan "mispricing" factor model, M4. As noted, their factor construction deviates from the standard approach in several important ways. As such, we report two sets of results, using their original factors and our replicated M4 factors constructed via the standard approach. The upshot is that the M4 model's performance is sensitive to the factor construction. While the original M4 factors survive the q-factor and Q5 models, the replicated M4 factors are largely absorbed by the Q5 model. In addition, neither the original nor the replicated M4 model can explain the q- and Q5 factors.

In Panel A, we use the M4 model to explain the q and Q5 factor returns. Consistent with Stambaugh and Yuan (2017), the original M4 model explains the size and investment factors, but not the Roe factor. The M4 alphas of the size and investment factors are -0.04% and 0.08% per month ($t = -0.65$ and 1.26), respectively. However, the M4 alpha of the Roe factor is 0.33% ($t = 3.55$), despite a large PERF factor loading of 0.42 ($t = 11.65$). Furthermore, the expected growth factor also survives the M4 model, with an alpha of 0.38% ($t = 5.84$).

Using the replicated M4 factors yields largely similar results. The M4 alphas of the size and investment factors are 0.01% ($t = 0.18$) and 0.07% ($t = 1.41$), but the M4 alphas of the Roe and expected growth factors are 0.32% ($t = 4.71$) and 0.43% ($t = 6.88$), respectively. For the null hypothesis that the investment and Roe factor alphas are jointly zero, the GRS statistic is 8.16 ($p = 0.00$) in the original M4 model, and 12.12 ($p = 0.00$) in the replicated M4 model (untabulated). For the null that the alphas of the investment, Roe, and expected growth factors are jointly zero, the GRS statistic is 18.58 ($p = 0.00$) in the original M4 model, and 28.49 ($p = 0.00$) in the replicated M4 model.

In Panel B, we use the q-factor and Q5 models to explain the original and replicated M4 factors. The original size factor, denoted SMBm (to be distinct from SMB in the Fama-French models), earns on average 0.44% per month ($t = 3.6$), and the replicated SMBm via the standard construction earns 0.31% ($t = 2.13$). The q-factor and Q5 alphas of the original SMBm are both 0.16%, and the t-values are above three. For the replicated SMBm, the q-factor alpha is 0.06% ($t = 1.13$), and the Q5 alpha is 0.14% ($t = 2.58$). The original MGMT factor earns on average 0.61% per month ($t = 4.72$), with a q-factor alpha of 0.36% ($t = 4.73$) and a Q5 alpha of 0.18% ($t = 2.39$). The replicated MGMT factor earns on average 0.47% ($t = 4.68$). The q-factor model yields an alpha of 0.2%, albeit still significant ($t = 3.59$), with a large investment factor loading of 0.92 ($t = 22.65$). The Q5 model shrinks the alpha further to 0.05% ($t = 0.91$), helped by an expected growth factor loading of 0.29 ($t = 8.45$).

The original PERF factor earns on average 0.68% per month ($t = 4.2$). The q-factor model yields an alpha of 0.34% ($t = 2$), with the help of a large Roe factor loading of 0.95 ($t = 10.42$). The Q5 model yields a tiny alpha of 0.05% ($t = 0.36$), helped by both the Roe and expected growth factor loadings, 0.88 ($t = 10.16$) and 0.54 ($t = 5.27$), respectively. The replicated PERF factor earns on average 0.49% ($t = 3.67$). The q-factor and Q5 alphas are both insignificant, 0.03% ($t = 0.28$) and -0.15% ($t = -1.48$), respectively. The Roe and expected growth factors again pull the weight. Clearly, the standard factor construction weakens the M4 model.

In untabulated results, the null hypothesis that the alphas of the original MGMT and PERF factors are jointly zero has a GRS statistic of 17.16 ($p = 0.00$) in the q-factor model, and 3.61 ($p = 0.03$) in the Q5 model. For comparison, the null that the alphas of the replicated MGMT and PERF factors are jointly zero has a GRS statistic of 7.96 ($p = 0.00$) in the q-factor model, and 2.09 ($p = 0.12$) in the Q5 model. As such, the Q5 model subsumes the replicated M4 factors.

As noted, Stambaugh and Yuan (2017) include financial firms and firms with negative book equity, but impose a $5 price screen in their sample selection, while we exclude financial firms and firms with negative book equity, but without imposing the price screen. Without going through the details, we can report that the remainder of Table 2, which is based on the Stambaugh-Yuan sample, shows that the sample difference has little impact on the spanning regressions between the q-factor and Q5 models and the replicated M4 model.

In addition, for the null that the investment and Roe factor alphas are jointly zero, the GRS statistic is 7.91 ($p = 0.00$) in the replicated M4 model in the Stambaugh-Yuan sample (untabulated). For the null that the investment, Roe, and expected growth factor alphas are jointly zero, the GRS statistic is 21.71 ($p = 0.00$). Conversely, for the null that the alphas of the replicated MGMT and PERF factors in the Stambaugh-Yuan sample are jointly zero, the GRS statistic is 11.09 ($p = 0.00$) in the q-factor model, but 1.01 ($p = 0.37$) in the Q5 model.

Correlation Matrix

To shed further light on the relations between the factors, Table 3 reports their correlation matrix. The size factors in the q-factor model, the Fama-French models, and the replicated M4 model are largely equivalent, with pairwise correlations at least 0.95. The investment factor, $R_{\mathrm{I/A}}$, in the q-factor model has high correlations of 0.67 with HML, 0.91 with CMA, and 0.84 with the replicated MGMT factor (0.81 with the replicated MGMT factor in the Stambaugh-Yuan sample and 0.77 with the original MGMT factor, untabulated). As such, HML contains pricing information of investment-to-assets, and MGMT is just a different version of the investment factor.

The Roe factor, R_{Roe}, in the q-factor model has high correlations of 0.67 with RMW and 0.57 with RMWc. Intuitively, R_{Roe}, RMW, and RMWc are all based on profitability, measured differently. The Roe factor has a high correlation of 0.5 with UMD. As such, momentum contains pricing information of Roe. More important, the Roe factor also has a high correlation of 0.8 with the replicated PERF factor (0.8 with that in the Stambaugh-Yuan sample and 0.64 with the original PERF factor, untabulated). As such, the PERF factor is just a different version of the Roe factor.

The expected growth factor, R_{Eg}, has a high correlation of 0.6 with RMWc. Firms with high expected growth tend to have high cash profits. Cash- and earnings-based profitability measures are related, giving rise to correlations of R_{Eg} with the Roe factor, 0.33, with RMW, 0.35, and with the replicated PERF factor, 0.4 (0.4 with that in the Stambaugh-Yuan sample and 0.41 with the original PERF, untabulated). Cash flows are also related to investment, giving rise to correlations of R_{Eg} with $R_{\mathrm{I/A}}$, 0.32, with CMA, 0.34, and with the replicated MGMT factor, 0.51 (0.51 with that in the Stambaugh-Yuan sample and 0.52 with the original MGMT, untabulated). In all, the Fama-French five- and six-factor models as well as the "mispricing" factor model have much common pricing information with the q-factor and Q5 models.

3 Economic Foundation of the q-factor and Q5 Models

The q-factor and Q5 models are motivated from the investment CAPM (Hou, Mo, Xue, and Zhang 2017). Time is discrete and the horizon infinite. The economy is populated by a representative consumer and heterogeneous firms, indexed by $i = 1, 2, \ldots, N$. The consumer maximizes $\sum_{t=0}^{\infty} \rho^t U(C_t)$, in which ρ is the time discount coefficient, and C_t is consumption in period t. Let P_{it} be the ex-dividend equity, and D_{it} the dividend of firm i at period t. The first principle of consumption says that $E_t[M_{t+1} r^S_{it+1}] = 1$, in which $r^S_{it+1} \equiv (P_{it+1} + D_{it+1})/P_{it}$ is firm i's stock return, and $M_{t+1} \equiv \rho U'(C_{t+1})/U'(C_t)$ is the stochastic discount factor. Equivalently,

$$E_t[r^S_{it+1}] - r_{ft} = \beta^M_{it} \lambda_{Mt}, \tag{1}$$

in which $r_{ft} \equiv 1/E_t[M_{t+1}]$ is the real interest rate, $\beta^M_{it} \equiv -\text{Cov}(r^S_{it+1}, M_{t+1})/\text{Var}(M_{t+1})$ the consumption beta, and $\lambda_{Mt} \equiv \text{Var}(M_{t+1})/E_t[M_{t+1}]$ the price of the consumption risk. Equation (1) is the consumption CAPM, and the Sharpe (1964) and Lintner (1965) CAPM is a special case.

Firms produce a single commodity to be consumed or invested. Firms use capital and costlessly adjustable inputs. These inputs are chosen each period to maximize operating profits, which are defined as revenue minus the costs of these inputs. Taking operating profits as given, firms choose investment to maximize the market equity. Time-t operating profits of firm i are given by $X_{it}A_{it}$, in which A_{it} is productive assets, and X_{it} return on assets (profitability). The next period profitability, X_{it+1}, is stochastic, and is subject to a vector of exogenous aggregate and firm-specific shocks. Let I_{it} denote investment and δ the depreciation rate of assets, then $A_{it+1} = I_{it} + (1-\delta)A_{it}$. Firms incur costs to install new capital or uninstall existing capital. We assume quadratic adjustment costs, $\Phi_{it} \equiv \Phi(I_{it}, A_{it}) = (a/2)(I_{it}/A_{it})^2 A_{it}$, in which $a > 0$ is a constant parameter. This functional form satisfies constant returns to scale, i.e., $\Phi_{it} = I_{it}\partial\Phi_{it}/\partial I_{it} + A_{it}\partial\Phi_{it}/\partial A_{it}$.

At the beginning of time t, firm i issues debt, B_{it+1}, which must be repaid at the beginning of $t+1$. The gross cost of debt on B_{it}, r^B_{it}, varies across firms and over time. Taxable corporate profits equal operating profits less capital depreciation, adjustment costs, and interest expenses, $X_{it}A_{it} - \delta A_{it} - \Phi(I_{it}, A_{it}) - (r^B_{it} - 1)B_{it}$. Let τ be the corporate tax rate. The net payout of firm i equals $D_{it} \equiv (1-\tau)[X_{it}A_{it} - \Phi(I_{it}, A_{it})] - I_{it} + B_{it+1} - r^B_{it}B_{it} + \tau\delta A_{it} + \tau(r^B_{it} - 1)B_{it}$, in which $\tau\delta A_{it}$ is the depreciation tax shield, and $\tau(r^B_{it} - 1)B_{it}$ is the interest tax shield. Let M_{t+1} be the stochastic discount factor, which is correlated with the aggregate component of X_{it+1}. Firm i chooses optimal streams of investment, $\{I_{it+s}\}_{s=0}^{\infty}$, and new debt, $\{B_{it+s+1}\}_{s=0}^{\infty}$, to maximize the cum-dividend market equity, $V_{it} \equiv E_t\left[\sum_{s=0}^{\infty} M_{t+s}D_{it+s}\right]$, subject to $\lim_{T\to\infty} E_t\left[M_{t+T}\, B_{it+T+1}\right] = 0$.

The first principle of investment implies $E_t[M_{t+1}r^I_{it+1}] = 1$, in which the investment return is:

$$r^I_{it+1} \equiv \frac{(1-\tau)\left[X_{it+1} + \frac{a}{2}\left(\frac{I_{it+1}}{A_{it+1}}\right)^2\right] + \tau\delta + (1-\delta)\left[1 + (1-\tau)a\left(\frac{I_{it+1}}{A_{it+1}}\right)\right]}{1 + (1-\tau)a\left(\frac{I_{it}}{A_{it}}\right)}. \tag{2}$$

The first principle says that the marginal cost of investment equals the next period marginal benefit of investment discounted to time t. The investment return is the marginal benefit of investment at time $t+1$ divided by the marginal cost of investment at t. In the numerator, $(1-\tau)X_{it+1}$ is the marginal after-tax profits produced by an extra unit of capital, $(1-\tau)(a/2)(I_{it+1}/A_{it+1})^2$ is the marginal after-tax reduction in adjustment costs, $\tau\delta$ is the marginal depreciation tax shield, and the last term is the marginal continuation value of the extra unit of capital net of depreciation.

Let the after-tax cost of debt be $r^{Ba}_{it+1} \equiv r^B_{it+1} - (r^B_{it+1} - 1)\tau$, then $E_t[M_{t+1}r^{Ba}_{it+1}] = 1$. Let $w_{it} \equiv B_{it+1}/(P_{it} + B_{it+1})$ be the market leverage. It follows that the investment return is the weighted average of the stock return and the after-tax cost of debt (Liu, Whited, and Zhang 2009):

$$r^I_{it+1} = w_{it}\, r^{Ba}_{it+1} + (1 - w_{it})\, r^S_{it+1}, \tag{3}$$

which is exactly the weighted average cost of capital in the Modigliani-Miller (1958) Proposition II. Finally, solving for the stock return, r^S_{it+1}, from equation (3) yields the investment CAPM:

$$r^S_{it+1} = r^I_{it+1} + \frac{w_{it}}{1 - w_{it}}\left(r^I_{it+1} - r^{Ba}_{it+1}\right), \tag{4}$$

in which $w_{it}/(1-w_{it}) = B_{it+1}/P_{it}$ is debt-to-equity. The investment CAPM connects the stock return to firm variables, and provides economic relations missing from the consumption CAPM.

Hou, Xue, and Zhang (2015) work with a simplified, two-period model with no taxes, no debt, and full depreciation. Equation (4) becomes $E_t[r^S_{it+1}] = E_t[X_{it+1}]/[1 + a\,(I_{it}/A_{it})]$. All else equal, low investment and high expected profitability stocks should earn higher expected returns than high investment and low expected profitability stocks, respectively. Intuitively, investment predicts returns because given expected profitability, high costs of capital imply low net present values of new capital and low investment, and low costs of capital imply high net present values of new capital and high investment. Profitability predicts returns because high expected profitability, combined with low investment, must mean high discount rates. The discount rates must be high to counteract the high expected profitability to induce low net present values of new capital and low investment.

Hou, Mo, Xue, and Zhang (2017) bring in the expected investment-to-assets growth as a component of the expected return. The expected investment return from equation (2) can be decomposed into the expected "dividend yield" and the expected "capital gain." The first component is given by $((1-\tau)[E_t[X_{it+1}] + (a/2)E_t[(I_{it+1}/A_{it+1})^2]] + \tau\delta)/(1 + a(1-\tau)(I_{it}/A_{it}))$, which largely conforms to the two-period model, since the squared term, $(I_{it+1}/A_{it+1})^2$, is small. The second term, $(1-\delta)[1 + a(1-\tau)E_t[I_{it+1}/A_{it+1}]]/(1 + a(1-\tau)(I_{it}/A_{it}))$, is analogous to the expected "capital gain," which is the expected growth of marginal q. Although the marginal q growth involves an unobservable parameter, a, it is roughly proportional to the investment-to-assets growth, $(I_{it+1}/A_{it+1})/(I_{it}/A_{it})$. As such, the expected investment-to-assets growth is the third determinant of the expected return (Cochrane 1991). Empirically, since investment-to-assets can be frequently negative in the data, Hou et al. opt to forecast future investment-to-assets changes, and form the expected growth factor based on the expected one-year-ahead investment-to-assets change.

The intuition behind the positive relation between the expected investment change and expected return is analogous to the intuition behind the positive relation between the expected profitability and expected return. Relative to current investment, the expected investment is part of the expected marginal benefit of investment. In equation (2), $(1-\tau)(a/2)(I_{it+1}/A_{it+1})^2$ is the marginal after-tax reduction in adjustment costs, and $(1-\delta)[1+(1-\tau)a(I_{it+1}/A_{it+1})]$ is the marginal continuation value of the extra unit of capital net of depreciation. As such, high $E_t[I_{it+1}/A_{it+1}]$ relative to current investment, I_{it}/A_{it}, must imply high discount rates to offset the high expected benefit of investment.

4 The Fama-French (2015) Five-factor Model Does Not Follow from Valuation Theory

Fama and French (2015) attempt to provide a theoretical foundation for their five-factor model based on the residual income model (Preinreich 1938, Miller and Modigliani 1961, Ohlson 1995). In this section, we show in depth that their economic arguments are conceptually flawed.

In the dividend discounting model, a firm's market equity is the present value of its dividends:

$$P_{it} = \sum_{\tau=1}^{\infty} \frac{E[D_{it+\tau}]}{(1+r_i)^{\tau}}, \tag{5}$$

in which P_{it} is the market equity, D_{it} dividends, and r_i the long-term average expected return, or the internal rate of return (Williams 1938). The clean surplus relation says that dividends equal earnings minus the change in book equity, $D_{it+\tau} = Y_{it+\tau} - \triangle Be_{it+\tau}$, in which $Y_{it+\tau}$ is earnings, and $\triangle Be_{it+\tau} \equiv Be_{it+\tau} - Be_{it+\tau-1}$ the change in book equity. The dividend discounting model becomes:

$$\frac{P_{it}}{Be_{it}} = \frac{\sum_{\tau=1}^{\infty} E[Y_{it+\tau} - \triangle Be_{it+\tau}]/(1+r_i)^{\tau}}{Be_{it}}. \tag{6}$$

Fama and French (2006, 2015) make three predictions based on equation (6). First, fixing everything except the current market value, P_{it}, and the expected stock return, r_i, a low P_{it}, or a high book-to-market equity, Be_{it}/P_{it}, implies a high expected return. Second, fixing everything except the expected profitability and the expected stock return, high expected profitability implies a high expected return. Finally, fixing everything except the expected book equity growth (expected investment) and the expected return, high expected book equity growth implies a low expected return.

Crucially, equation (6) connects book-to-market, investment, and profitability to the internal rate of return. Fama and French (2015) argue that the difference between the one-period-ahead expected return and the internal rate of return is unimportant: "Most asset pricing research focuses on short-horizon returns—we use a one-month horizon in our tests. If each stock's short-horizon expected return is positively related to its internal rate of return—if, for example, the expected return is the same for all horizons—the valuation equation implies that the cross-section of expected returns is determined by the combination of current prices and expectations of future dividends. The decomposition of cash flows then implies that each stock's relevant expected return is determined by its price-to-book ratio and expectations of its future profitability and investment (p. 2)."

Empirically, Fama and French (2015) use profitability as a proxy for the expected profitability to form RMW, and asset growth as a proxy for the expected investment to form CMA. Earlier on, Fama and French (2006) construct the expected profitability and expected investment (the growth in book equity or total assets) as the fitted components from first-stage cross-sectional regressions of future profitability and future investment on current variables. In second-stage cross-sectional regressions of returns on these proxies, Fama and French find some evidence on the expected profitability effect, but the relation between the expected investment and expected returns is weakly *positive*.

We show that the Fama-French (2015) economic logic is flawed. First, the internal rate of return can differ drastically from, and can even correlate negatively with the one-period-ahead expected return (Section 4.1). Second, HML is a separate factor from CMA in the Fama-French conceptual analysis, but is redundant in describing average returns in the data. In contrast, the tight investment-value linkage is predicted by the investment CAPM (Section 4.2). Third, CMA can only arise from the market-to-book term, P_{it}/Be_{it}, in equation (6). In contrast, the expected book equity growth is positively correlated with the one-period-ahead expected return (Section 4.3). Finally, past investment is a poor proxy for the expected investment (Section 4.4).

4.1 The Internal Rate of Return Is Not the One-period-ahead Expected Return

The Fama-French (2015) assumption that the expected return is the same for all horizons contradicts the notion of time-varying expected returns. The internal rate of return (IRR) can differ greatly from the one-period ahead expected return. The difference is most striking in the context of price and earnings momentum. Chan, Jegadeesh, and Lakonishok (1996) show that momentum profits are short-lived, large, and positive for up to 12 months, but negative afterward. In contrast, Tang, Wu, and Zhang (2014) estimate price and earnings momentum to be significantly negative, once measured as the internal rate of return per Gebhardt, Lee, and Swaminathan (2001).

To quantify how the IRRs deviate from one-period-ahead average returns, we estimate the IRRs for the Fama-French (2015) SMB, HML, RMW, and CMA per Claus and Thomas (2001), Gebhardt, Lee, and Swaminathan (2001), Easton (2004), and Ohlson and Juettner-Nauroth (2005). Although differing in implementation details, these methods all share the basic idea of backing out the IRRs from different versions of the valuation equation (6). Appendix B details the estimation procedures.

The baseline versions of these accounting methods all use analysts' earnings forecasts to predict future profitability. Because analysts' forecasts are limited to a relatively small sample and are likely even biased, we also implement two sets of modified procedures. The Hou-van Dijk-Zhang (2012) modification uses pooled cross-sectional regressions to forecast future earnings, and the Tang-Wu-Zhang (2014) modification uses annual cross-sectional regressions to forecast future profitability.

Empirically, we measure one period as one year, and compare the average factor IRRs at the June end of each year t with the annual average factor returns from the July of year t to the June of year $t+1$. Panel A of Table 4 reports that the IRRs estimated with analysts' earnings forecasts for RMW differ significantly from their one-period-ahead average returns. The differences for RMW are significant in 12 out of the 12 experiments from intersecting the three expected Roe estimation procedures with the four accounting models. The IRRs of RMW are even significantly negative in eight experiments, in contrast to the average returns that are significantly positive in all 12.

Averaging across the four IRR models implemented with analysts' earnings forecasts, the IRR for the RMW is −1.58% per annum ($t = -9.66$), whereas its one-period-ahead average return is 4.52% ($t = 2.88$). The contrast from implementing the accounting models with cross-sectional earnings forecasts is largely similar, −1.84% ($t = -9.41$) versus 3.61% ($t = 2.66$). With cross-sectional Roe forecasts, the contrast is between −2.47% ($t = -21.47$) versus 3.14% ($t = 2.54$).

Table 4 also reports important IRR-average-return differences for CMA, although not as drastic as the differences for RMW. The differences for CMA are significant for six out of 12 experiments. Finally, without going through the details, we can report that, consistent with Tang, Wu, and Zhang (2014), the IRR-average-return differences for SMB and HML are mostly insignificant.

4.2 The Relation between Investment and Book-to-market Equity

Fama and French (2015) argue that market-to-book, expected profitability, and expected investment give rise to three separate factors in equation (6). However, empirically, once RMW and CMA are added to their three-factor model, Fama and French find that HML becomes redundant in describing average returns. This evidence contradicts their conceptual argument.

However, the evidence is predicted by the investment CAPM. The denominator of equation (2) is the marginal cost of investment (an increasing function of investment-to-assets), which equals marginal q (the value of an extra unit of capital). With constant returns to scale, marginal q equals average q (Hayashi 1982), which is in turn highly correlated with market-to-book equity. This tight economic relation between investment and value implies that HML should be highly correlated with the investment factor. From January 1967 to December 2016, the correlation between HML and CMA is 0.7, and the correlation between HML and the investment factor in the q-factor model is 0.67 (Table 3). The investment-value linkage in the investment CAPM also means that CMA can only be motivated from the market-to-book term in the valuation equation (6).

4.3 The Relations between Past Investment, the Expected Investment, and the Expected Return

Fama and French (2015) argue that equation (6) predicts a negative relation between the expected investment and the internal rate of return. However, this negative relation does not apply to the one-period-ahead expected return, $E_t[r_{it+1}]$. From the definition of return, $P_{it} = (E_t[D_{it+1}] + E_t[P_{it+1}])/(1 + E_t[r_{it+1}])$, and the clean surplus relation, we reformulate the valuation equation (6) in terms of the one-period-ahead expected return as:

$$P_{it} = \frac{E_t[Y_{it+1} - \triangle Be_{it+1}] + E_t[P_{it+1}]}{1 + E_t[r_{it+1}]}. \tag{7}$$

Dividing both sides of equation (7) by Be_{it} and rearranging, we obtain:

$$\frac{P_{it}}{Be_{it}} = \frac{E_t\left[\frac{Y_{it+1}}{Be_{it}}\right] - E_t\left[\frac{\triangle Be_{it+1}}{Be_{it}}\right] + E_t\left[\frac{P_{it+1}}{Be_{it+1}}\left(1 + \frac{\triangle Be_{it+1}}{Be_{it}}\right)\right]}{1 + E_t[r_{it+1}]}, \tag{8}$$

$$\frac{P_{it}}{Be_{it}} = \frac{E_t\left[\frac{Y_{it+1}}{Be_{it}}\right] + E_t\left[\frac{\triangle Be_{it+1}}{Be_{it}}\left(\frac{P_{it+1}}{Be_{it+1}} - 1\right)\right] + E_t\left[\frac{P_{it+1}}{Be_{it+1}}\right]}{1 + E_t[r_{it+1}]}. \tag{9}$$

Fixing everything except $E_t\left[\triangle Be_{it+1}/Be_{it}\right]$ and $E_t[r_{it+1}]$, high $E_t\left[\triangle Be_{it+1}/Be_{it}\right]$ implies high $E_t[r_{it+1}]$, because $P_{it+1}/Be_{it+1} - 1$ is more likely to be positive in the data. This prediction is consistent with the weakly positive $E_t\left[\triangle Be_{it+1}/Be_{it}\right]$-$E_t[r_{it+1}]$ relation in Fama and French (2006). As noted, the relation between the expected investment and the expected return is also positive in the investment CAPM (Section 3), and is the foundation behind the expected growth factor. As such, the prediction from the valuation equation (6) is consistent with the investment CAPM.

4.4 Past Investment Is a Poor Proxy for the Expected Investment

After motivating CMA from the expected investment effect, Fama and French (2015) use past investment as a proxy for the expected investment. This practice is problematic. Whereas past profitability is a good proxy for the expected profitability, past investment is a poor proxy for the expected investment. A large economics literature on lumpy investment emphasizes the lack of persistence of micro-level investment data (Dixit and Pindyck 1994, Doms and Dunne 1998, Whited 1998).

To show the poor quality of past investment as a proxy for the expected investment, we perform annual cross-sectional regressions of future book equity growth rates, $\triangle Be_{it+\tau}/Be_{it+\tau-1} \equiv (Be_{it+\tau} - Be_{it+\tau-1})/Be_{it+\tau-1}$, for $\tau = 1, 2, \ldots, 10$, on the current asset growth, $\triangle A_{it}/A_{it-1} = (A_{it} - A_{it-1})/A_{it-1}$, and, separately, on book equity growth, $\triangle Be_{it}/Be_{it-1}$. For comparison, we also report annual cross-sectional regressions of future operating profitability, $Op_{it+\tau}$, on operating profitability, Op_{it}. As in Fama and French (2006), the sample contains all common stocks traded on NYSE, Amex, and NASDAQ from 1963 to 2016, including financial firms.

Following Fama and French (2015), we measure operating profitability as total revenue (Compustat annual item REVT) minus cost of goods sold (item COGS, zero if missing), minus selling, general, and administrative expenses (item XSGA, zero if missing), and minus interest expense (item XINT, zero if missing), scaled by book equity (the denominator is current, not lagged, book equity). We require at least one of the three expense items (COGS, XSGA, and XINT) to be non-missing. Book equity is measured per Davis, Fama, and French (2000) (footnote 2). Variables dated t are from the fiscal year ending in calendar year t. Firms with total assets (Compustat annual item AT) below \$5 million or book equity below \$2.5 million in year t are excluded in Panel A of Table 5. The cutoffs are \$25 million and \$12.5 million, respectively, in Panel B. The right- and left-hand side variables in the regressions are winsorized each year at the 1–99% level.

Asset growth does not predict future book equity growth. In Panel A in Table 5, the slope starts at 0.22 at the one-year horizon, drops to 0.06 in year three and to 0.04 in year five. The average R^2 of the cross-sectional regressions starts at 5% in year one, drops to zero in year four, and stays at zero for the remaining years. Past book equity growth does not predict future book equity growth either. The slope starts at 0.2 at the one-year horizon, drops to 0.06 in year three and to 0.02 in year five. The average R^2 of the cross-sectional regressions starts at 6% in year one, drops to zero in year four, and stays at zero for the remaining years. The results with the more stringent sample criterion in Panel B are largely similar. The evidence casts doubt on the motivation of CMA from the expected investment effect, but lends support to the investment CAPM reinterpretation of CMA via the market-to-book term in the valuation equation (6).

The last five columns in Table 5 show that operating profitability forecasts future operating profitability. In Panel A, the slope in the annual cross-sectional regressions starts with 0.8 in year one, drops to 0.59 in year three and 0.49 in year five, and remains at 0.38 even in year ten. The average R^2 starts at 54% in year one, drops to 27% in year three and 19% in year five, and remains above 10% in year ten. The evidence with the more stringent sample criterion in Panel B is largely similar. As such, using past profitability as a proxy for the expected profitability is sensible, but using past investment as a proxy for the expected investment is not.

5 The Evolving Nature of "Factors"

In this section, we take a step back, and ask: What are "factors?" We summarize the traditional covariance view of "factors" from the consumption CAPM in Section 5.1, and then elaborate how the investment CAPM provides a richer perspective of "factors" in Section 5.2.

5.1 The Traditional Covariance View of "Factors"

We summarize the traditional view based on the consumption CAPM in four central tenets.

Asset Pricing Is All About the Stochastic Discount Factor

In arguably the most prominent asset pricing textbook in the past two decades, Cochrane (2005) writes: "This book advocates a discount factor/generalized method of moments view of asset pricing theory and associated empirical procedures. I summarize asset pricing by two equations:

$$
\begin{aligned}
p_t &= E(m_{t+1}x_{t+1}), \\
m_{t+1} &= f(\text{data}, \text{parameters}),
\end{aligned}
$$

where p_t = asset price, x_{t+1} = asset payoff, m_{t+1} = stochastic discount factor. The major advantages of the discount factor/moment condition approach are its simplicity and universality (p. xv)." "Again, my organizing principle is that everything can be traced back to specializations of the basic pricing equation $p = E(mx)$. Therefore, after reading the first chapter, one can pretty much skip around and read topics in as much depth or order as one likes. Each major subject always starts back at the same pricing equation (p. xvi–xvii)." "*All* asset pricing models amount to alternative ways of connecting the stochastic discount factor to data (original emphasis, p. 7)."

Factor Models Are Linear Approximations to the Stochastic Discount Factor

With $p = E(mx)$ as the organizing framework, it is no surprise that factor models are just linear approximations to the stochastic discount factor. Cochrane (2005) writes: "In my opinion, the best hope for finding pricing factors that are robust out of sample and across different markets, is to try to understand the fundamental macroeconomic sources of risk. By this I mean, tying asset prices to macroeconomic events, in the way the ill-fated consumption-based model does via $m_{t+1} = \beta u'(c_{t+1})/u'(c_t)$. The difficulties of the consumption-based model have made this approach lose favor in recent years. However, the alternative approach is also running into trouble in that the number and identity of empirically determined risk factors do not seem stable (p. 124–125)."

"The big question is, what should one use for factors f_{t+1}? Factor pricing models look for variables that are good proxies for aggregate marginal utility growth, i.e., variables for which

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} \approx a + b' f_{t+1}$$

is a sensible and economically interpretable approximation (p. 149)." "This is a point worth remembering: *all factor models are derived as specializations of the consumption-based model.* Many authors of factor model papers disparage the consumption-based model, forgetting that their factor model *is* the consumption-based model plus extra assumptions that allow one to proxy for marginal utility growth from some other variables (original emphasis, p. 151)."

Cochrane (2005) further states that "clear economic foundation was important for factor models, since it helps us to guard against fishing. Alas, the current state of factor pricing models is not a particularly good guard. One can call for better theories or derivations, more carefully aimed at limiting the list of potential factors and describing the fundamental macroeconomic sources of risk, and thus providing more discipline for empirical work. But the best minds in finance have been working on this problem for 40 years, so a ready solution is not immediately in sight (p. 151)."

Only Risks Matter for the Expected Return in Efficient Markets: After Controlling for Risks, Characteristics Should Not Predict Returns

Again with $p = E(mx)$ as the organizing framework, it is natural to think that only risks (covariances with the stochastic discount factor) matter for the expected return in efficient markets. Over the years, researchers have had to finesse the tension between this covariance view and the evidence that characteristics outside the $p = E(mx)$ framework forecast returns.

Most notably, even though their size and book-to-market factors are formed on characteristics, Fama and French (1993) insist on the risk factors interpretation: "One of our central themes is that if assets are priced rationally, variables that are related to average returns, such as size and book-to-market equity, must proxy for sensitivity to common (shared and thus undiversifiable) risk factors in returns. The time-series regressions give direct evidence on this issue. In particular, the slopes and R^2 values show whether mimicking portfolios for risk factors related to size and [book-to-market] capture shared variation in stock and bond returns not explained by other factors (p. 4–5)."

The tension between theory and empirics, and the required finesse, reach a boiling point in Fama and French (2017): "We include momentum factors (somewhat reluctantly) now to satisfy insistent popular demand. We worry, however, that opening the game to factors that seem empirically robust but lack theoretical motivation has a destructive downside — the end of discipline that produces parsimonious models and the beginning of a dark age of data dredging that produces a long list of factors with little hope of sifting through them in a statistically reliable way (p. 7)."

"There is an obvious danger that, in the absence of discipline from theory, factor models degenerate into long lists of factors that come close to spanning the *ex post* mean-variance-efficient (MVE) tangency portfolio of a particular period — in other words empty data dredging exercises (p. 24)."

The Failure of the Consumption CAPM to Explain Anomalies Must be Due to Behavioral Mispricing and Limits to Arbitrage

Many researchers interpret the failure of the consumption CAPM as indicative of behavioral finance. Daniel and Titman (1997) write: "In equilibrium asset pricing models the covariance structure of returns determines expected returns. Yet we find that variables that reliably predict the future covariance structure do not predict future returns. Our results indicate that high book-to-market stocks and stocks with low capitalizations have high average returns whether or not they have the return patterns (i.e., covariances) of other small and high book-to-market stocks (p. 4)."

Barberis and Thaler (2003, p. 1091) echo: "One general feature of the rational approach is that it is loadings or betas, and not firm characteristics, that determine average returns. For example, a risk-based approach would argue that value stocks earn high returns not because they have high book-to-market ratios, but because such stocks happen to have a high loading on the book-to-market factor. Daniel and Titman (1997) cast doubt on this specific prediction by performing double sorts of stocks on both book-to-market ratios and loadings on book-to-market factors, and showing that stocks with different loadings but the same book-to-market ratio do *not* differ in their average returns. These results appear quite damaging to the rational approach."

Even in the context of aggregate asset pricing, in which the consumption models have done well, Cochrane (2011) starts to blur the line between rational and behavioral finance: "It is therefore pointless to argue 'rational' versus 'behavioral' in the abstract. There is a discount rate and equivalent distorted probability that can rationalize any (arbitrage-free) data. 'The market went up, risk aversion must have declined' is as vacuous as 'the market went up, sentiment must have increased.' Any model only gets its bite by restricting discount rates or distorted expectations, ideally tying them to other data. The only thing worth arguing about is how persuasive those ties are in a given model and data set, and whether it would have been easy for the theory to 'predict' the opposite sign if the facts had come out that way. And the line between recent 'exotic preferences' and 'behavioral finance' is so blurred that it describes academic politics better than anything substantive (p. 1067–1068)." "Changing expectations of consumption 10 years from now (long-run risks) or changing probabilities of a big crash are hard to distinguish from changing 'sentiment' (p. 1068)."

Taking the stock of the failure of the consumption CAPM in the cross section, Cochrane (2017) further concedes: "It is curious that macro-finance has spent quite so much effort on a tenuous new fact, the term structure of equity premiums, and so little on the much more extensively documented finance factors. That may be a selection bias that nobody has gotten a positive result so far (p. 974)." "But it is also possible that most of the above macro-finance approaches will not be useful to understand the zoo of cross-sectional premiums, and they will be the province of institutional or frictions finance (p. 974)." "So we are likely to end up with an economic picture of asset markets that, in the end, unites two or more of these fundamental approaches. A representative-consumer model such as habits may well describe large movements in widely-available securities and funds, leaving near-arbitrages and premiums of high-frequency trading strategies to the economics of institutional finance and mechanics of information trading (p. 974–975)."

From the other direction, recent behavioral studies break the covariances versus characteristics dichotomy (Daniel and Titman 1997), and argue that covariances should explain returns even when markets are inefficient. Stambaugh and Yuan (2017) argue: "Factor models can be useful whether expected returns reflect risk or mispricing. Factors can capture systematic risks for which investors require compensation, or they can capture common sources of mispricing, such as market-wide investor sentiment (p. 1272)." Also, "there need not be a clean distinction between mispricing and risk compensation as alternative motivations for factor models of expected return. For example, DeLong et al. (1990) explain how fluctuations in market-wide 'noise-trader' sentiment create an additional source of systematic risk for which rational traders require compensation (p. 1273)."

However, Stambaugh and Yuan (2017) acknowledge: "When expected returns reflect mispricing and not just compensation for systematic risks, some of the mispricing may not be driven by pervasive sentiment factors but may instead be asset specific, as discussed for example by Daniel and Titman (1997). In that sense the concept of 'mispricing factors' potentially embeds some inconsistency (p. 1273)." More important, decades of research have not provided concrete linkage between investor psychology and specific anomalies. In a recent survey from the behavioral perspective, Lee and So (2015) admit: "Be forwarned: none of these [behavioral] studies will provide a clean one-to-one mapping between the investor psychology literature and specific market anomalies. Rather, their goal is to simply set out the experimental evidence from psychology, sociology, and anthropology. The hope is that, thus armed, financial economists would be more attuned to, and more readily recognize, certain market phenomena as manifestations of these enduing human foibles (p. 69)."

5.2 The Investment CAPM Perspective of "Factors"

The investment CAPM provides a richer "factors" perspective, which differs fundamentally from both the consumption CAPM and behavioral finance. We describe this perspective also in four central tenets, which contrast those based on the traditional covariance view (Section 5.1).

Asset Pricing Is *Not* All About the Stochastic Discount Factor, M: Derived from the Optimal Supply of Risky Assets, the Investment CAPM Abstracts Away from M

Cochrane's (2005) unification of asset pricing under the $p = E(mx)$ framework, as ambitious and admirable as it is, is incomplete. In the general equilibrium framework laid out in Section 3, combining the consumption CAPM in equation (1) and the investment CAPM in equation (4) yields:

$$r_{ft} + \beta_{it}^{M} \lambda_{Mt} = E_t[r_{it+1}^{S}] = E_t[r_{it+1}^{I}] + \frac{w_{it}}{1 - w_{it}}(E_t[r_{it+1}^{I}] - E_t[r_{it+1}^{Ba}]). \tag{10}$$

As such, asset prices are equilibrated by both demand and supply of risky assets in general equilibrium (Lin and Zhang 2013, Zhang 2017). Derived from the first principle of consumption, the consumption CAPM predicts that the consumption betas "determine" the expected return. In contrast, derived from the first principle of investment, the investment CAPM predicts that firm characteristics "determine" the expected return. The consumption CAPM is a demand theory of asset pricing, and the investment CAPM a supply theory of asset pricing outside the scope of $p = E(mx)$. In equilibrium, equation (10) says that both models deliver the identical expected return.

Although equivalent in theory, the aggregation problem renders the consumption CAPM largely untestable (Zhang 2017). Perhaps due to limitations of disaggregated consumption data, most consumption CAPM studies use aggregate consumption data, assuming the existence of the representative consumer. However, the Sonnenschein-Mantel-Debreu theorem in general equilibrium states that individual rationality does not impose any restrictions on the aggregate excess demand function (Sonnenschein 1973, Debreu 1974, Mantel 1974).

Kirman (1992) raises four objections to the representative consumer. "First, whatever the objective of the modeler, there is no plausible formal justification for the assumption that the aggregate of individuals, even maximizers, acts itself like an individual maximizer. Individual maximization does *not* engender collective rationality, nor does the fact that the collectivity exhibits a certain rationality necessarily imply that individuals act rationally. There is simply no direct relation between individual and collective behavior (original emphasis, p. 118)." Second, "[the] reaction of the representative to some change in a parameter of the original model — a change in government policy for example — may not be the same as the aggregate reaction of the individuals he 'represents.' Hence using such a model to analyze the consequences of policy changes may not be valid (p. 118)."

Third, "[the] 'representative individual' whose choices coincide with the aggregate choices of the individuals in the economy is a utility maximizer. however, it may well be the case that in two situations of which the representative prefers the first to the second, every individual prefers the second to the first. Thus the preferences of the representative individual cannot legitimately be used to decide whether one economic situation is 'better' than another (p. 118)."

"Lastly, when used as a model for empirical testing, the representative agent presents a peculiar disadvantage. Trying to explain the behavior of a group by that of one individual is constraining. The sum of the behavior of simple economically plausible individuals may generate complicated dynamics, whereas constructing one individual whose behavior has these dynamics may lead to that individual having very unnatural characteristics. Furthermore, if one rejects a particular behavioral hypothesis, it is not clear whether one is really rejecting the hypothesis in question, or rejecting the additional hypothesis that there is only one individual (p. 118)." Given these arguments, Kirman (1992) suggests "it is clear that the 'representative' agent deserves a decent burial, as an approach to economic analysis that is not only primitive, but fundamentally erroneous (p. 119)."

We view aggregation as a fatal weakness of the consumption CAPM. The fatal weakness might explain why the existing consumption CAPM studies focus almost exclusively on aggregate asset pricing. Multiple models have developed to explain aggregate stock market in the endowment economy (Campbell and Cochrane 1999, Bansal and Yaron 2004, Barro 2006). It is possible that the damage of aggregation is not too severe in the aggregate economy without production. Yet, the aggregate literature still lacks a benchmark stock market model with production. More important, anomalies in the cross section are micro-level phenomena involving firms, and the representative-consumer consumption CAPM might be easily blown out of water by aggregation.

Our aggregation critique on the consumption CAPM echoes Campbell (2018): "[One] cannot understand asset prices only from a macroeconomic or general equilibrium point of view. One also has to understand the decisions that individual market participants make, taking prices as given in partial equilibrium. This is particularly important in models with heterogeneous investors, which are increasingly central to the field (p. xviii)." Accordingly, the aggregation critique does not apply to portfolio theory, which describes the optimal consumption and portfolio choice behavior of a small investor. The diversification due to Markowitz (1952) works without a representative consumer.

We also interpret the recent intermediary asset pricing literature as an attempt to address the aggregation problem in the demand theory of asset pricing. The basic idea is that the marginal investor is more likely to be financial intermediaries, not the representative consumer. Intermediaries trade in many asset classes, and use sophisticated models and extensive data to guide their investment decisions. As such, the growth of the marginal value of wealth of intermediaries should be the relevant stochastic discount factor (Brunnermeier and Pedersen 2009, He and Krishnamurthy 2013).

Adrian, Etula, and Muir (2014) use the leverage of security broker-dealers to proxy for the marginal value of wealth of intermediaries, and suggest that their single factor model prices size, book-to-market, momentum, and bond portfolios as well as standard multfactor models. However, this conclusion might be premature. First, the leverage beta is not priced in the cross section, suggesting that similar to the consumption CAPM, the intermediary theories are also subject to beta measurement errors. Second, more important, Adrian et al. form their factor mimicking portfolio (used to explain the size, value, and momentum premiums) by projecting intermediary leverage series on the size, book-to-market, and momentum portfolio returns. As such, their model's empirical performance seems mechanical, likely hinging on the choice of basis assets. In contrast, size, book-to-market, and momentum arise naturally from the supply-based investment CAPM.

Factor Models Are Linear Approximations to the Firm Cost of Capital from the Investment CAPM

As noted, in the consumption CAPM, factor models are linear approximations to the marginal utility growth of the representative consumer. Other aggregate variables such as the growth rate of industrial production, inflation rate, and the default premium can be used to substitute out consumption, giving rise to, for example, the classic macroeconomic factor model of Chen, Roll, and Ross (1986). To the extent that firm size, book-to-market, and momentum are not modeled in the consumption CAPM, these factors are perceived as ad hoc, arising from "fishing" exercises.

The investment CAPM provides a more microeconomic perspective of "factors." In particular, factor models are linear approximations to the firm cost of capital given by equation (2). The equation implies that the expected return varies cross-sectionally, depending on firms' investment, expected profitability, and expected investment growth. Motivated by this equation, Hou, Xue, and Zhang (2015) form the investment and Roe factors in the q-factor model, and Hou, Mo, Xue, and Zhang (2017) construct the expected growth factor in the Q5 model.

Under this perspective, factor regressions and cross-sectional regressions of returns on characteristics are two different ways of summarizing correlations between the expected return and characteristics. Factor regressions and cross-sectional regressions are largely equivalent on economic grounds. While the consumption CAPM "factors" are mostly aggregate variables (because of its aggregation assumption), the investment CAPM "factors" are built directly on firm variables (no aggregation assumption is ever needed). The statistical covariations among stocks with similar investment, profitability, and expected growth, in the sense of Ross's (1976) Arbitrage Pricing Theory (APT), arise not from some mysterious, aggregate sources of risks, but from the comovement among stocks with similar current investment, future profitability, and future investment per equation (2).

Both Risks and Characteristics Matter in Efficient Markets: In the Investment CAPM, After Characteristics Are Controlled for, Risks Should Not Matter

The demand side of equation (10), which is the consumption CAPM, says that only risks matter for the expected return in efficient markets. However, the supply side of equation (10), which is the investment CAPM, says that only characteristics matter for the expected return in efficient markets. After characteristics are fully controlled for, risks should not matter. In equilibrium, risks, the expected return, and characteristics are all determined simultaneously. No causality runs from risks to the expected return, and then to characteristics, or vice versa (Lin and Zhang 2013).

From this perspective, there is really no need for Fama and French (1993, 1996, 2017) to finesse their characteristics-based factors into the traditional covariance view of "factors." The investment CAPM provides economic foundation for all the Fama-French factors. Characteristics, such as value and momentum, are just different combinations of investment, return on equity, and the expected growth, which should forecast returns in theory. Characteristics-based factors are not ad hoc. Characteristics are on as solid theoretical ground in the supply theory of asset pricing as aggregate consumption growth in the demand theory of asset pricing (if one ignores aggregation). Taking aggregation seriously, aggregate consumption growth is not even a factor. Characteristics-based factors appear ad hoc, only because the consumption CAPM is incomplete. When theory and empirics are in conflict for almost 50 years, time to move on from (outdated) theory.

We refute the notion that the consumption CAPM is theoretically more pure than the investment CAPM. Both models are based on the first principles in equilibrium theory. One is about the optimal demand of risky assets, and the other the optimal supply. The consumption CAPM is a mathematical restatement of the permanent income theory of consumption (Friedman 1957), and the investment CAPM a mathematical restatement of the neoclassical q-theory of investment (Keynes 1936, Tobin 1969). The consumption and the investment CAPMs have implications on both time series and cross-sectional predictability. However, the key insight of the consumption CAPM is time-varying expected returns, which explain stock market predictability. The key insight of the investment CAPM is cross-sectionally varying expected returns, which explain cross-sectional predictability. In all, the investment CAPM is as fundamental as the consumption CAPM.

We also refute the notion that the investment CAPM is not even a Capital Asset Pricing Model. There is Capital in the framework in Section 3, the framework is about Asset Pricing, and it is a Model. The notion arises likely because risks are not modeled. However, as shown in equation (10), risks and characteristics both matter in equilibrium theory. Dismissing the investment CAPM not as a Capital Asset Pricing Model seems to carry the covariance view a bit too far.

The Consumption CAPM Anomalies Are the Investment CAPM Regularities

The singular, most important contribution of the investment CAPM literature is to show that the consumption CAPM anomalies are the investment CAPM regularities. In particular, Hou, Xue, and Zhang (2017) start with 447 anomaly variables, and show that 161 earn significant high-minus-low average returns ($t \geq 1.96$) with NYSE breakpoints and value-weighted returns. Out of the 161, the

q-factor model reduces 115 to insignificance, and only 46 anomalies have significant q-factor alphas, including 17 with $t \geq 3$. Out of the 46, Hou, Mo, Xue, and Zhang (2017) show that the Q5 model reduces the number of significant high-minus-low alphas further to 19, including only four with $t \geq 3$.

In view of such performance, we propose to modify Cochrane's (2017) grand vision of a unified asset pricing theory as a representative consumer model for aggregate asset pricing and institutional or frictions finance for cross-sectional anomalies. In our vision, we retain the representative consumer model for aggregate asset pricing, but opt for the investment CAPM to make sense of the cross section. After all, intermediary asset pricing has barely begun to take its predictions to the data.

There is no obvious place for behavioral finance in our vision of equilibrium asset pricing. Like portfolio theory, studying the behavior of individual investors makes total sense, but getting to equilibrium asset prices requires a leap of faith on aggregation, again. The historical rise of behavioral finance is fuelled by the anomalies literature, which demonstrates, time after time, the failure of the consumption CAPM. Within the demand side of asset pricing, it might not be unreasonable to question the rationality of investors. Indeed, doing so is standard practice in behavioral finance.[7]

At a minimum, the investment CAPM shows that anomalies do not necessarily prove mispricing. On the contrary, anomalies conform to standard economic principles: The consumption CAPM anomalies are the investment CAPM regularities. This feat is achieved with the single equation (2).

Contrary to Cochrane (2011) on aggregate asset pricing, at least in the cross section, the dichotomy between the investment CAPM and behavioral finance cannot be starker. Future realized

[7]We illustrate this point with quotes from three highly influential papers. "Research in experimental psychology suggests that, in violation of Bayes' rule, most people tend to 'overreact' to unexpected and dramatic news events. This study of market efficiency investigates whether such behavior affects stock prices. The empirical evidence, based on CRSP monthly return data, is consistent with the overreaction hypothesis. Substantial weak form market inefficiencies are discovered (De Bondt and Thaler 1985, p. 793)." "[It] is possible that the market underreacts to information about their short-term prospects of firms but overreacts to information about their long-term prospects. This is plausible given that the nature of the information available about a firm's short-term prospects, such as earnings forecasts, is different from the nature of the more ambiguous information that is used by investors to assess a firm's longer-term prospects (Jegadeesh and Titman 1993, p. 90)." "Individual investors might focus on glamour strategies for a variety of reasons. First, they may make judgement errors and extrapolate past growth rates of glamour stocks, such as Walmart or Microsoft, even when such growth rates are highly unlikely to persist in the future. Putting excessive weight on recent past history, as opposed to a rational prior, is a common judgement error in psychological experiments and not just in the stock market. Alternatively, individuals might just equate well-run firms with good investments, regardless of price (Lakonishok, Shleifer, and Vishny 1994, p. 1575)."

returns equal expected returns plus realized abnormal returns. As such, tautologically, cross-sectional predictability with any firm variable has two parallel interpretations. First, the variable forecasts future abnormal returns (forecasting errors are forecastable), violating efficient markets as in behavioral finance. Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998) work within this framework, and use psychological biases to explain expectation errors (predictable abnormal returns). Second, the firm variable is connected with expected returns (forecasting errors are not forecastable), retaining efficient markets as in the investment CAPM.

Some colleagues argue that because of no investors, the investment CAPM has no implications on efficient markets. For instance, if irrational investors bid up a firm's stock too high, its manager simply lines up real investment with excessively high Tobin's q. This argument, again, reflects the demand-centric view, pretends that demand alone "determines" asset prices, and ignores the full effect of general equilibrium, in which the supply side plays an equally important role. In particular, if Tobin's q is too high from the firm's perspective, the manager will flood the market with more supply of risky assets, and consequently, the price will drop. What we observe is equilibrium prices. The investment CAPM shows that the consumption CAPM anomalies largely conform to the first principle of real investment, which implies cross-sectionally varying expected returns. As such, we question whether predictable abnormal returns (pricing errors) exist at all, and whether one needs psychological biases and limits to arbitrage to explain them.

Going one step further, our aggregation critique questions whether or not the demand approach, which encompasses both the consumption CAPM and behavioral finance, is valid for equilibrium asset pricing to begin with. For the consumption CAPM, individual rationality does not mean aggregate optimization. For behavioral finance, imposing on one individual whose behavior inherits complicated group dynamics can lead to that individual having unnatural characteristics (Kirman 1992). One needs a heterogeneous investors model to sort out the group dynamics, and for the cross section, one also needs to add heterogeneous firms (a modeler's worst nightmare). In contrast, the investment CAPM has no such aggregation difficulty, and offers a simple solution to equilibrium pricing.

The investment CAPM versus behavioral finance debate is scientific, not (just) religious. The debate has far-reaching policy implications. Behavioral finance asserts that capital markets are dysfunctional and inefficient, infested with widespread, psychological biases and limits to arbitrage. Mispricing is systematic, giving rise to undiversifiable sources of comovement (Stambaugh and Yuan 2017). In contrast, the investment CAPM contends that capital markets are well functioning and efficient. And anomalies are in fact equilibrium phenomena that arise from the optimal investment behavior of firms. The behavioral worldview calls for extensive government interventions in capital markets. In contrast, the investment CAPM worldview calls for none, at least to the first order.

Which worldview is closer to the truth? Consider the recent evidence from the U.S. and China, which are the world's two largest economies. Hou, Xue, and Zhang (2017) replicate 447 anomalies in the U.S. markets, and find that 286 (64%) are unreliable in value-weighted returns. In particular, 95 anomaly variables out of 102 (93%) in the trading frictions category are insignificant, while many variables in the value, momentum, investment, and profitability categories are reliable. As such, limits to arbitrage are minimal, yet the value, momentum, investment, and profitability premiums subsist with value-weights. The evidence lends support to the investment CAPM, which derives these premiums as equilibrium phenomena in well functioning, efficient markets, in which managers are more likely to maximize the market value of equity when making real investment decisions.

Qiao (2017) replicates 57 anomalies in China's A-share markets, and find that only ten are significant at the 5% threshold in value-weighted returns from 1997 to 2017. Curiously, most of these significant variables are frictions variables, including size, total volatility, turnover, volatility of turnover, dollar trading volume, volatility of dollar volume, the number of zero trading days, and absolute return-to-volume. Yet, the value, momentum, investment, and profitability premiums are mostly absent. Clearly, the Chinese markets are more frictional than the U.S. markets. Clearly, since the A-share markets are largely closed to foreign investors, there are armies of naive, retail investors running around with plenty of sentiments in China. While further studies are certainly called for, the absence of the value, momentum, investment, and profitability premiums casts serious doubt on behavioral view of these anomalies based on mispricing and limits to arbitrage.

The current (simple) investment theories also fail to explain the Chinese evidence. Casual observations suggest that managers in public companies in China (often appointed by the government) are more likely to pursue a multitude of social objectives set by the government, such as meeting the growth target and steadily growing employment. Maximizing the market value of equity might not be the first priority. Future work can study to what extent introducing such governance frictions into the investment CAPM can bring the model's predictions more aligned with the Chinese evidence.

More generally, two pervasive problems plague behavioral finance, p-hacking and HARKing (hypothesizing after the results are known) (Kerr 1998). Empirically, perhaps due to publication biases, the existing literature has exaggerated the economic importance of anomalies by ignoring multiple testing (Harvey, Liu, and Zhu 2016) and by overweighting microcaps that only account for 3% of total market capitalization (Chordia, Goyal, and Saretto 2017, Hou, Xue, and Zhang 2017). After microcaps are controlled for, a few key factors are sufficient to describe the entire cross section.

Kerr (1998) defines HARKing as presenting a post hoc hypothesis based on or informed by one's evidence in the introduction of a research article as if it were an a priori hypothesis. Kerr argues that HARKing is hazardous for scientific progress: (i) it translates false positive findings into theories; (ii) it promotes theories that are more context-specific and ad hoc, less useful, and less refutable; (iii) it breeds statistical abuses and questionable practices in ethically ambiguous areas; and (iv) it discourages the identification of more general theories and plausible alternative hypotheses.

We argue that behavioral theories such as Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998) are subject to HARKing. Both models are context-specific, ad hoc, and close to impossible to refute empirically or to extend theoretically. Both are subject to the aggregation critique. In contrast, theories based on first principles, such as the investment CAPM, are more general, less ad hoc, more refutable, and more amenable to theoretical extensions.

The investment CAPM expands greatly the scope of asset pricing beyond the consumption

CAPM and behavioral finance. In the immensely important area of valuation, $p = E(mx)$ links current price, p, to next period payoff, x, which includes next period price. Determining today's price by guessing tomorrow's price is not particularly enlightening. The accounting literature goes further by implementing a multiperiod valuation model up to 12 years, and then assume a terminal value afterward (Frankel and Lee 1998). In contrast, the investment CAPM expresses valuation in terms of current period investment or the discounted one-period-ahead marginal benefit of investment (Hou, Mo, Xue, and Zhang 2017). Many practical applications await.

Penman (1992) laments: "Up to 25 years ago, fundamental analysis was the primary focus of research in investment analysis (p. 465)." But since then "research in academia has been otherwise directed. Both traditional fundamental analysis and accounting measurement theory have been judged as ad hoc and lacking the theoretical foundations required of rigorous economic analysis. 'Modern Finance' established those foundations but has not brought the theory to the question of fundamental analysis (p. 465)." The investment CAPM provides such an economic foundation for fundamental analysis and capital markets research in accounting (Hou, Mo, Xue, and Zhang 2017).

Finally, the investment CAPM provides an ideal bridge between asset pricing and corporate finance. Most corporate finance models assume risk neutrality, and are silent about the interaction between discount rates and corporate policies. The investment CAPM in equation (4) abstracts from many frictions that are central to corporate finance, such as corporate governance. The investment CAPM provides an apparatus to quantify their impact on discount rates and valuation.

6 Conclusion

This paper compares on both empirical and conceptual grounds a new generation of factor pricing models, including the Hou-Xue-Zhang (2015) q-factor model and the Hou-Mo-Xue-Zhang (2017) Q5 model, the Fama-French (2015, 2017) five- and six-factor models, as well as the Stambaugh-Yuan (2017) "mispricing" factor model. In factor spanning tests, the q-factor and Q5 models dominate the Fama-French five- and six-factor models. The "mispricing" factors are sensitive to their construction, and once replicated via the standard approach, are close to the q-factors, with correlations of 0.8 and 0.84. Neither the original nor the replicated "mispricing" factor model can explain the q- and Q5 factors, but the Q5 model can largely absorb the replicated "mispricing" factors.

Conceptually, the Fama-French (2015) five-factor model does not follow from valuation theory, as originally advertised. First, the internal rate of return implied from valuation theory differs from the one-period-ahead expected return. Despite its significant positive premium, RMW has internal rate of return estimates that are significantly negative. Second, the value and investment factors are two separate factors in valuation theory, but are largely substitutable in the investment CAPM, consistent with their high correlation in the data around 0.7. Third, once reformulated with the one-period-ahead expected return, valuation theory also implies a positive relation between the expected investment and the expected return, consistent with the investment CAPM. Finally, past investment is a poor proxy for the expected investment, as micro-level investment is lumpy.

More broadly, the investment CAPM provides a fundamentally different "factors" perspective from the consumption CAPM. Asset pricing is not all about the stochastic discount factor, which only summarizes the demand of risky assets. The investment CAPM is derived from the optimal supply. Factor models can be linear approximations to the firm cost of capital from the supply side. Both covariances and characteristics matter in efficient markets. Finally, the investment CAPM expands greatly the scope of asset pricing beyond the consumption CAPM and behavioral finance.

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A Variable Definition

We describe the 11 anomaly variables used to construct the Stambaugh-Yuan (2017) "mispricing" factors. At the beginning of each month, we rank stocks into percentiles (1 to 100) based on each anomaly. The rankings are created such that high rankings are associated with lower future average returns. The first composite measure, MGMT (management), is the average of the six percentile rankings in net stock issues, composite equity issuance, accruals, net operating assets, asset growth, and investment-to-assets. The second composite measure, PERF (performance), is the average of the five percentile rankings in failure probability, O-score, momentum, gross profitability, and return on assets. In any given month, an anomaly variable needs at least 30 stocks with non-missing values in order to be included in the composite measure. In addition, we compute a composite measure for a stock only if it has non-missing values for at least three of the (six or five) component anomalies.

Net stock issues. Net stock issues is the annual change in the log of the split-adjusted shares outstanding. The split-adjusted shares outstanding is shares outstanding (Compustat annual item CSHO) times the adjustment factor (item AJEX). At the beginning of each month, we use the latest net stock issues from fiscal year ending at least four months ago.

Composite equity issuance. Following Stambaugh and Yuan (2017), at the beginning of month t, we measure composite equity issuance as the growth rate in market equity minus the cumulative stock return from month $t-16$ to $t-5$ (skipping month $t-4$ to $t-1$).

Accruals. Following Sloan (1996), we measure accruals as changes in noncash working capital minus depreciation, in which the noncash working capital is changes in noncash current assets minus changes in current liabilities less short-term debt and taxes payable. In particular, accruals equals (dCA − dCASH) − (dCL − dSTD − dTP) − DP, in which dCA is the change in current assets (Compustat annual item ACT), dCASH is the change in cash or cash equivalents (item CHE), dCL is the change in current liabilities (item LCT), dSTD is the change in debt included in current liabilities (item DLC), dTP is the change in income taxes payable (item TXP), and DP is depreciation and amortization (item DP). Missing changes in income taxes payable are set to zero. We scale accruals by average total assets from the previous and current years. At the beginning of each month, we use the latest accruals from fiscal year ending at least four months ago.

Net operating assets. We measure net operating assets as operating assets minus operating liabilities. Operating assets are total assets (Compustat annual item AT) minus cash and short-term investment (item CHE). Operating liabilities are total assets minus debt included in current liabilities (item DLC, zero if missing), minus long-term debt (item DLTT, zero if missing), minus minority interests (item MIB, zero if missing), minus preferred stocks (item PSTK, zero if missing), and minus common equity (item CEQ). We scale net operating assets by one-year-lagged total assets. At the beginning of each month, we use the latest net operating assets from fiscal year ending at least four months ago.

Asset growth. Asset growth is the annual change in total assets (Compustat annual item AT) scaled by one-year-lagged total assets. At the beginning of each month, we use the latest asset growth from fiscal year ending at least four months ago.

Changes in PPE and Inventory-to-assets are measured as the annual change in gross property, plant, and equipment (Compustat annual item PPEGT) plus the annual change in inventory (item INVT) scaled by one-year-lagged total assets (item AT). At the beginning of each month, we use the latest investment-to-assets from fiscal year ending at least four months ago.

Failure Probability. At the beginning of month t, we follow Campbell, Hilscher, and Szilagyi (2008, Table IV, Column 3) to construct failure probability:

$$\begin{aligned}\mathrm{Fp}_t \equiv & -9.164 - 20.264\mathrm{NIMTAAVG}_t + 1.416\mathrm{TLMTA}_t - 7.129\mathrm{EXRETAVG}_t \\ & + 1.411\mathrm{SIGMA}_t - 0.045\mathrm{RSIZE}_t - 2.132\mathrm{CASHMTA}_t + 0.075\mathrm{MB}_t - 0.058\mathrm{PRICE}_t \end{aligned} \tag{A1}$$

in which

$$\mathrm{NIMTAAVG}_{t-1,t-12} \equiv \frac{1-\phi^3}{1-\phi^{12}}\left(\mathrm{NIMTA}_{t-1,t-3} + \cdots + \phi^9\mathrm{NIMTA}_{t-10,t-12}\right) \tag{A2}$$

$$\mathrm{EXRETAVG}_{t-1,t-12} \equiv \frac{1-\phi}{1-\phi^{12}}\left(\mathrm{EXRET}_{t-1} + \cdots + \phi^{11}\mathrm{EXRET}_{t-12}\right), \tag{A3}$$

and $\phi = 2^{-1/3}$. NIMTA is net income (Compustat quarterly item NIQ) divided by the sum of market equity (share price times the number of shares outstanding from CRSP) and total liabilities (item LTQ). The moving average NIMTAAVG captures the idea that a long history of losses is a better predictor of bankruptcy than one large quarterly loss in a single month. EXRET $\equiv \log(1+R_{it}) - \log(1+R_{\mathrm{S\&P500},t})$ is the monthly log excess return on each firm's equity relative to the S&P 500 index. The moving average EXRETAVG captures the idea that a sustained decline in stock market value is a better predictor of bankruptcy than a sudden stock price decline in a single month.

TLMTA is total liabilities divided by the sum of market equity and total liabilities. SIGMA is the annualized three-month rolling sample standard deviation: $\sqrt{\frac{252}{N-1}\sum_{k\in\{t-1,t-2,t-3\}} r_k^2}$, in which k is the index of trading days in months $t-1, t-2$, and $t-3$, r_k is the firm-level daily return, and N is the total number of trading days in the three-month period. SIGMA is treated as missing if there are less than five nonzero observations over the three months in the rolling window. RSIZE is the relative size of each firm measured as the log ratio of its market equity to that of the S&P 500 index. CASHMTA, aimed to capture the liquidity position of the firm, is cash and short-term investments (Compustat quarterly item CHEQ) divided by the sum of market equity and total liabilities (item LTQ). MB is the market-to-book equity, in which we add 10% of the difference between the market equity and the book equity to the book equity to alleviate measurement issues for extremely small book equity values (Campbell, Hilscher, and Szilagyi 2008). For firm-month observations that still have negative book equity after this adjustment, we replace these negative values with $1 to ensure that the market-to-book ratios for these firms are in the right tail of the distribution. PRICE is each firm's log price per share, truncated above at $15. We further eliminate stocks with prices less than $1 at the portfolio formation date. Variables requiring quarterly accounting data are from fiscal quarter ending at least four months ago to ensure the availability of balance sheet items. We winsorize the variables on the right-hand side of equation (A1) at the 1th and 99th percentiles of their distributions each month.

Ohlson's O-score. We follow Ohlson (1980, Model One in Table 4) to construct O-score:

$$\begin{aligned} O \equiv & -1.32 - 0.407\log(\text{TA}) + 6.03\text{TLTA} - 1.43\text{WCTA} + 0.076\text{CLCA} \\ & - 1.72\text{OENEG} - 2.37\text{NITA} - 1.83\text{FUTL} + 0.285\text{INTWO} - 0.521\text{CHIN}, \end{aligned} \quad \text{(A4)}$$

in which TA is total assets (Compustat annual item AT). TLTA is the leverage ratio defined as total debt (item DLC plus item DLTT) divided by total assets. WCTA is working capital (item ACT minus item LCT) divided by total assets. CLCA is current liability (item LCT) divided by current assets (item ACT). OENEG is one if total liabilities (item LT) exceeds total assets and zero otherwise. NITA is net income (item NI) divided by total assets. FUTL is the fund provided by operations (item PI plus item DP) divided by total liabilities. INTWO is equal to one if net income is negative for the last two years and zero otherwise. CHIN is $(\text{NI}_s - \text{NI}_{s-1})/(|\text{NI}_s| + |\text{NI}_{s-1}|)$, in which NI_s and NI_{s-1} are the net income for the current and prior years. We winsorize all non-dummy variables on the right-hand side of equation (A4) at the 1th and 99th percentiles of their distributions each year. At the beginning of each month, we use the latest O-score from fiscal year ending at least four months ago.

Momentum. At the beginning of each month t, we measure momentum as the 11-month cumulative return from month $t-12$ to $t-2$ (skipping month $t-1$).

Gross Profitability. Gross profitability is total revenue (Compustat annual item REVT) minus cost of goods sold (item COGS) divided by total assets (item AT, the denominator is current, not lagged, total assets). At the beginning of each month, we use the latest gross profitability from fiscal year ending at least four months ago.

Return on Assets is income before extraordinary items (Compustat quarterly item IBQ) divided by one-quarter-lagged total assets (item ATQ). At the beginning of each month, we use return on assets computed with quarterly earnings from the most recent earnings announcement dates (item RDQ). For a firm to enter our sample, we require the end of the fiscal quarter that corresponds to its most recent return on assets to be within six months prior to the portfolio formation. This restriction is imposed to exclude stale earnings information. To avoid potentially erroneous records, we also require the earnings announcement date to be after the corresponding fiscal quarter end.

B Estimating the Internal Rate of Return

The Gebhardt, Lee, and Swaminathan (2001, GLS) Procedure. At the end of June in each year t, we estimate the IRR from the following nonlinear equation:

$$P_t = Be_t + \sum_{\tau=1}^{11} \frac{(E_t[\text{Roe}_{t+\tau}] - \text{IRR}) \times Be_{t+\tau-1}}{(1+\text{IRR})^{\tau}} + \frac{(E_t[\text{Roe}_{t+12}] - \text{IRR}) \times Be_{t+11}}{\text{IRR} \times (1+\text{IRR})^{11}}, \quad \text{(B1)}$$

in which P_t is the market equity in year t, $Be_{t+\tau}$ is the book equity, and $E_t[\text{Roe}_{t+\tau}]$ is the expected return on equity for year $t+\tau$ based on information available in year t.

We measure current book equity, Be_t, using the latest accounting data from the fiscal year ending between March of year $t-1$ to February of t. This practice implies that for the IRR estimates at the end of June in t, we impose at least a four-month lag to ensure that the accounting information is released to the public. The definition of book equity follows Davis, Fama, and French (2000). We apply clean surplus accounting to construct future book equity as $Be_{t+\tau} = Be_{t+\tau-1} + Be_{t+\tau-1}E_t[\text{Roe}_{t+\tau}](1-k), 1 \leq \tau \leq 11$, in which k is the dividend payout ratio in year t. Dividend payout ratio is dividends (Compustat annual item DVC) divided by earnings (item IB) for profitable firms, or dividends divided by 6% of total assets (item AT) for firms with zero or negative earnings. We drop a firm if its book equity is zero or negative in any year.

We construct the expected Roe for the first three years ahead, using analyst earnings forecasts from the Institutional Brokers' Estimated System (IBES) or forecasts from cross-sectional regressions. After year $t+3$, we assume that the expected firm-level Roe mean-reverts linearly to the historical industry median Roe by year $t+12$, and becomes a perpetuity afterwards. We use the Fama-French (1997) 48 industry classification. We use at least five and up to ten years of past Roe data from non-loss firms to compute the industry median Roe.

Following GLS (2001), we implement the GLS model on a per share basis with analysts' earnings forecasts. P_t is the June-end share price from CRSP. Be_t is book equity per share calculated as book equity divided by the number of shares outstanding reported in June from IBES (unadjusted file, item SHOUT). When IBES shares are not available, we use shares from CRSP (daily item SHROUT) on the IBES pricing date (item PRDAYS) that corresponds to the IBES report.

At the end of June in each year t, we construct the expected Roe for year $t+1$ to $t+3$ as $E_t[\text{Roe}_{t+\tau}] = \text{FEPS}_{t+\tau}/Be_{t+\tau-1}$, in which $\text{FEPS}_{t+\tau}$ is the consensus mean forecast of earnings per share from IBES (unadjusted file, item MEANEST) for year $t+\tau$ (fiscal period indicator $= \tau$) reported in June of t. We require the availability of earnings forecast for years $t+1$ and $t+2$. When the forecast for year $t+3$ is not available, we use the long-term growth rate (item LTG) to compute a three-year-ahead forecast: $\text{FEPS}_{t+3} = \text{FEPS}_{t+2} \times (1 + \text{LTG})$. If the long-term growth rate is missing, we replace it with the growth rate implied by the first two forecasts: $\text{FEPS}_{t+3} = \text{FEPS}_{t+2} \times (\text{FEPS}_{t+2}/\text{FEPS}_{t+1})$, when FEPS_{t+1} and FEPS_{t+2} are both positive.

As noted, we measure current book equity Be_t based on the latest accounting data from the fiscal year ending between March of year $t-1$ and February of t. However, firms with fiscal years ending between March of t and May of t can announce their latest earnings before the IBES report in June of t. In response to earnings announcement for the current fiscal year, the analyst forecasts would "roll forward" to the next year. As such, we also need to roll forward book equity by one year for these firms to match with the updated analyst forecasts. In particular, we roll forward their book equity using clean surplus accounting as: $Be_{t-1} + Y_t - D_t$, in which Be_{t-1} is the lagged book equity (relative to the announced earnings), Y_t is the earnings announced after February of t but before the IBES report in June of t, and D_t is dividends.

In the first modified procedure, we follow Hou, van Dijk, and Zhang (2012) to estimate the IRRs at the firm level (not the per share basis), whenever regression-based earnings forecasts (not analysts' earnings forecasts) are used. We use pooled cross-sectional regressions to forecast future earnings for up to three years ahead:

$$Y_{is+\tau} = a + b_1 A_{is} + b_2 D_{is} + b_3 DD_{is} + b_4 Y_{is} + b_5 Y_{is}^{-} + b_6 AC_{is} + \epsilon_{is+\tau}, \tag{B2}$$

for $1 \leq \tau \leq 3$, in which Y_{is} is earnings (Compustat annual item IB) of firm i for fiscal year s, A_{is} is total assets (item AT), D_{is} is dividends (item DVC), and DD_{is} is a dummy variable that equals one for dividend payers, and zero otherwise. Y_{is}^{-} is a dummy variable that equals one for negative earnings, and zero otherwise, and AC_{is} is operating accruals.

Prior to 1988, we use the balance-sheet approach of Sloan (1996) to measure operating accruals as changes in noncash working capital minus depreciation, in which the noncash working capital is changes in noncash current assets minus changes in current liabilities less short-term debt and taxes payable. In particular, $AC = (\triangle CA - \triangle CASH) - (\triangle CL - \triangle STD - \triangle TP) - DP$, in which $\triangle CA$ is the change in current assets (Compustat annual item ACT), $\triangle CASH$ is the change in cash or cash equivalents (item CHE), $\triangle CL$ is the change in current liabilities (item LCT), $\triangle STD$ is the change in debt included in current liabilities (item DLC, zero if missing), $\triangle TP$ is the change in income taxes payable (item TXP, zero if missing), and DP is depreciation and amortization (item DP, zero if missing). Starting from 1988, we follow Hribar and Collins (2002) to measure AC using the statement of cash flows as net income (item NI) minus net cash flow from operations (item OANCF).

In equation (B2), regressors with time subscript s are from the fiscal year ending between March of year s and February of $s+1$. Following Hou, van Dijk, and Zhang (2012), we winsorize all the level variables in equation (B2) at the 1st and 99th percentiles of their cross-sectional distributions each year. In June of each year t, we estimate the regressions using the pooled panel data from the previous ten years. With a minimum four-month lag, the accounting data are from fiscal years ending between March of $t-10$ and February of t. Differing from the baseline GLS procedure, we forecast the expected earnings as the estimated regression coefficients times the latest values of the (unwinsorized) predictors from the fiscal year ending between March of $t-1$ and February of t.

In the second modified procedure, we use annual cross-sectional regressions per Tang, Wu, and Zhang (2014) to forecast the future ROE for up to three years, $\text{Roe}_{is+\tau} \equiv Y_{is+\tau}/Be_{is+\tau-1}$:

$$\text{Roe}_{is+\tau} = a + b_1 \log\left(\frac{Be_{is}}{P_{is}}\right) + b_2 \log(P_{is}) + b_3 Y_{is}^{-} + b_4 \text{Roe}_{is} + b_5 \frac{A_{is} - A_{is-1}}{A_{is-1}} + \epsilon_{is+\tau}, \quad \text{(B3)}$$

for $1 \leq \tau \leq 3$, in which Roe_{is} is return on equity of firm i for fiscal year s, Y_{is} is earnings (Compustat annual item IB), Be_{is} is the book equity, P_{is} is the market equity at the fiscal year end from Compustat or CRSP, Y_{is}^{-} is a dummy variable that equals one for negative earnings and zero otherwise, and A_{is} is total assets (item AT). Regression variables with time subscript s are from the fiscal year ending between March of year s and February of $s+1$. Extremely small firms tend to have extreme regression variables which can affect the Roe regression estimates significantly. To alleviate this problem, we exclude firm-years with total assets less than \$5 million or book equity less than \$2.5 million. Fama and French (2006, p. 496) require firms to have at least \$25 million total assets and \$12.5 million book equity, but state that their results are robust to using the \$5 million total assets and \$2.5 million book equity cutoff. We choose the less restrictive cutoff to enlarge the sample coverage. We also winsorize each variable (except for Y_{it}^{-}) at the 1st and 99th percentiles of its cross-sectional distribution each year to further alleviate the impact of outliers.

In June of each year t, we run the regression (B3) using the previous ten years of data. With a minimum four-month information lag, the accounting data are from fiscal years ending between March of $t-10$ and February of t. Differing from the baseline GLS procedure, we directly forecast the expected Roe, $E_t[\text{Roe}_{t+\tau}]$, as the average cross-sectional regression coefficients times the latest values of the predictors from fiscal years ending between March of $t-1$ and February of t. We implement this modified GLS procedure at the firm level.

The Easton (2004) Procedure. At the end of June in each year t, we estimate the IRR from:

$$P_t = \frac{E_t[Y_{t+2}] + \text{IRR} \times E_t[D_{t+1}] - E_t[Y_{t+1}]}{\text{IRR}^2}, \quad \text{(B4)}$$

in which P_t is the market equity in year t, $E_t[Y_{t+\tau}]$ is the expected earnings for year $t+\tau$ based on information available in year t, and $E_t[D_{t+1}]$ is the expected dividends for year $t+1$.

Expected earnings are based on analyst forecasts from IBES or forecasts from regression models. Expected dividends are expected earnings times the current dividend payout ratio, which is computed as dividends (Compustat annual item DVC) divided by earnings (item IB) for profitable firms, or dividends divided by 6% of total assets (item AT) for firms with zero or negative earnings. When equation (B4) has two positive roots (in very few cases), we use the average as the IRR estimate.

Following Easton (2004), we implement the model on the per share basis with analysts' earnings forecasts. We measure P_t as the June-end share price from CRSP. At the end of June in year t, the expected earnings per share for year $t+\tau$ is the consensus mean forecast from IBES (unadjusted file, item MEANEST) for year $t+\tau$ (fiscal period indicator $=\tau$) reported in June of t.

Instead of analysts' earnings forecasts, we also use pooled cross-sectional regressions in equation (B2) to forecast future earnings for up to two years ahead. In June of each year t, we estimate the regression using the pooled panel data from the previous ten years. With a four-month information lag, the accounting data are from fiscal years ending between March of $t-10$ and February of t. We construct the expected earnings as the estimated regression coefficients times the latest values of the (unwinsorized) predictors from the fiscal year ending between March of $t-1$ and February of t. We implement the modified procedure at the firm level.

Finally, we also use annual cross-sectional regressions in equation (B3) to forecast future ROE for up to two years ahead. In June of each year t, we estimate the regression using the previous ten years of data. With a four-month information lag, the accounting data are from fiscal years ending between March of $t-10$ and February of t. We forecast the expected Roe, $E_t[\text{Roe}_{t+\tau}]$, as the average regression coefficients times the latest values of the predictors from fiscal years ending between March of $t-1$ and February of t. Expected earnings are then constructed as: $E_t[Y_{t+\tau}] = E_t[Roe_{t+\tau}] \times Be_{t+\tau-1}$, in which $Be_{t+\tau-1}$ is the book equity in year $t+\tau-1$. We measure current book equity Be_t based on the latest accounting data from the fiscal year ending in March of $t-1$ to February of t, and impute future book equity by applying clean surplus accounting recursively. We implement the modified procedure at the firm level.

The Claus and Thomas (2001, CT) Procedure. At the end of June in each year t, we estimate the IRR from:

$$P_t = Be_t + \sum_{\tau=1}^{5} \frac{(E_t[\text{Roe}_{t+\tau}] - \text{IRR}) \times Be_{t+\tau-1}}{(1+\text{IRR})^\tau} + \frac{(E_t[\text{Roe}_{t+5}] - \text{IRR}) \times Be_{t+4} \times (1+g)}{(\text{IRR} - g) \times (1+\text{IRR})^5}, \quad \text{(B5)}$$

in which P_t is the market equity in year t, $Be_{t+\tau}$ is the book equity, $E_t[\text{Roe}_{t+\tau}]$ is the expected Roe for year $t+\tau$ based on information available in year t, and g is the long-term growth rate of abnormal earnings. Abnormal earnings are defined as $(E_t[\text{Roe}_{t+\tau}] - \text{IRR}) \times Be_{t+\tau-1}$.

We measure book equity using the latest accounting data from the fiscal year ending between March of year $t-1$ and February of t. The definition follows Davis, Fama, and French (2000). We apply clean surplus accounting to construct future book equity as $Be_{t+\tau} = Be_{t+\tau-1} + Be_{t+\tau-1}E_t[\text{Roe}_{t+\tau}](1-k), 1 \leq \tau \leq 4$, in which k is the dividend payout ratio in year t. Dividend payout ratio is dividends (Compustat annual item DVC) divided by earnings (item IB) for profitable firms, or dividends divided by 6% of total assets (item AT) for firms with zero or negative earnings. We drop a firm if its book equity is zero or negative in any year. We construct the expected Roe,

$E_t[\text{Roe}_{t+\tau}]$, for up to five years ahead, using analysts' earnings forecasts from IBES or regression-based forecasts. Following CT (2001), we set g to the ten-year Treasury bond rate minus 3%.

Following CT (2001), we implement the CT model on the per share basis when using analysts' earnings forecasts. We measure P_t as the June-end share price from CRSP. Book equity per share, B_t, is book equity divided by the number of shares outstanding reported in June from IBES (unadjusted file, item SHOUT). When IBES shares are not available, we use shares from CRSP (daily item SHROUT) on the IBES pricing date (item PRDAYS) that corresponds to the IBES report. As noted, current book equity B_t is based on the latest accounting data from the fiscal year ending between March of $t-1$ and February of t. However, firms with fiscal year ending in March of t to May of t can announce their latest earnings before the IBES report in June of t. To match the updated analyst forecasts, we roll forward their book equity using clean surplus accounting as: $B_{t-1}+Y_t-D_t$, in which B_{t-1} is the lagged book equity (relative to the announced earnings), Y_t is the earnings announced after February of t but before the IBES report in June of t, and D_t is dividends.

At the end of June in each year t, we construct the expected Roe for year $t+1$ to $t+5$ as $E_t[\text{Roe}_{t+\tau}] = \text{FEPS}_{t+\tau}/Be_{t+\tau-1}$, in which $\text{FEPS}_{t+\tau}$ is the consensus mean forecast of earnings per share from IBES (unadjusted file, item MEANEST) for year $t+\tau$ (fiscal period indicator $=\tau$) reported in June of t. We require the availability of earnings forecast for years $t+1$ and $t+2$. When the forecast after year $t+2$ is not available, we use the long-term growth rate (item LTG) to construct it as $\text{FEPS}_{t+\tau} = \text{FEPS}_{t+\tau-1} \times (1+\text{LTG})$. If the long-term growth rate is missing, we replace it with the growth rate implied by the forecasts for the previous two years: $\text{FEPS}_{t+\tau} = \text{FEPS}_{t+\tau-1} \times (\text{FEPS}_{t+\tau-1}/\text{FEPS}_{t+\tau-2})$, when $\text{FEPS}_{t+\tau-2}$ and $\text{FEPS}_{t+\tau-1}$ are both positive.

Instead of analysts' earnings forecasts, we also use pooled cross-sectional regressions in equation (B2) to forecast future earnings for up to five years ahead. In June of each year t, we estimate the regression using the pooled panel data from the previous ten years. With a four-month information lag, the accounting data are from fiscal years ending between March of $t-10$ and February of t. We forecast the expected earnings as the estimated regression coefficients times the latest values of the (unwinsorized) predictors from the fiscal year ending between March of $t-1$ and February of t. We implement this modified CT procedure at the firm level. Finally, we also use annual cross-sectional regressions in equation (B3) to forecast future Roe for up to five years ahead. In June of each year t, we estimate the regression using the previous ten years of data. With a four-month information lag, the accounting data are from fiscal years ending between March of $t-10$ and February of t. We directly forecast the expected Roe, $E_t[\text{Roe}_{t+\tau}]$, as the average cross-sectional regression coefficients times the latest values of the predictors from fiscal years ending between March of $t-1$ and February of t. We implement the modified CT procedure at the firm level.

The Ohlson and Juettner-Nauroth (2005, OJ) Procedure. At the end of June in each year t, we construct the IRR as:

$$\text{IRR} = A + \sqrt{A^2 + \frac{E_t[Y_{t+1}]}{P_t} \times (g - (\gamma - 1))}, \tag{B6}$$

in which

$$A \equiv \frac{1}{2}\left((\gamma - 1) + \frac{E_t[D_{t+1}]}{P_t}\right), \tag{B7}$$

$$g \equiv \frac{1}{2}\left(\frac{E_t[Y_{t+3}] - E_t[Y_{t+2}]}{E_t[Y_{t+2}]} + \frac{E_t[Y_{t+5}] - E_t[Y_{t+4}]}{E_t[Y_{t+4}]}\right). \tag{B8}$$

P_t is the market equity in year t, $E_t[Y_{t+\tau}]$ is the expected earnings for year $t+\tau$ based on information available in t, and $E_t[D_{t+1}]$ is the expected dividends for year $t+1$.

Expected earnings are based on analyst forecasts from IBES or forecasts from regression models. Expected dividends are expected earnings times the current dividend payout ratio, which is computed as dividends (Compustat annual item DVC) divided by earnings (item IB) for profitable firms, or dividends divided by 6% of total assets (item AT) for firms with zero or negative earnings. We follow Gode and Mohanram (2003) and use the average of forecasted near-term growth rate and five-year growth rate as an estimate of g. We require $E_t[Y_{t+2}]$ and $E_t[Y_{t+4}]$ to be positive so that g is well defined. Following Gode and Mohanram (2003), we implement the OJ model on the per share basis with analysts' earnings forecasts. We measure P_t as the June-end share price from CRSP. At the end of June in year t, the expected earnings per share for year $t+\tau$ is the consensus mean forecast from IBES (unadjusted file, item MEANEST) for year $t+\tau$ (fiscal period indicator $=\tau$) reported in June of t. We require the availability of earnings forecast for years $t+1$ and $t+2$. When the forecast after year $t+2$ is not available, we use the long-term growth rate (item LTG) to construct it as: $\text{FEPS}_{t+\tau} = \text{FEPS}_{t+\tau-1} \times (1+\text{LTG})$. If the long-term growth rate is missing, we replace it with the growth rate implied by the forecasts for the previous two years: $\text{FEPS}_{t+\tau} = \text{FEPS}_{t+\tau-1} \times (\text{FEPS}_{t+\tau-1}/\text{FEPS}_{t+\tau-2})$, when $\text{FEPS}_{t+\tau-2}$ and $\text{FEPS}_{t+\tau-1}$ are both positive.

Instead of analysts' earnings forecasts, we also use pooled cross-sectional regressions in equation (B2) to forecast future earnings for up to five years ahead. In June of each year t, we estimate the regression using the pooled panel data from the previous ten years. With a four-month information lag, the accounting data are from fiscal years ending between March of $t-10$ and February of t. We construct the expected earnings as the estimated regression coefficients times the latest values of the (unwinsorized) predictors from the fiscal year ending between March of $t-1$ and February of t. We implement the modified OJ procedure at the firm level.

We also use annual cross-sectional regressions in equation (B3) to forecast future Roe for up to five years ahead. In June of each year t, we estimate the regression using the previous ten years of data. With a four-month information lag, the accounting data are from fiscal years ending between March of $t-10$ and February of t. We forecast the expected Roe, $E_t[\text{Roe}_{t+\tau}]$, as the average cross-sectional regression coefficients times the latest values of the predictors from fiscal years ending between March of $t-1$ and February of t. Expected earnings are then constructed as: $E_t[Y_{t+\tau}] = E_t[\text{Roe}_{t+\tau}] \times Be_{t+\tau-1}$, in which $Be_{t+\tau-1}$ is the book equity in year $t+\tau-1$. We measure current book equity Be_t based on the latest accounting data from the fiscal year ending in March of $t-1$ to February of t, and impute future book equity by applying clean surplus accounting recursively. We implement the modified OJ procedure at the firm level.

Table 1 : Factor Spanning Tests: The q-factor and Q5 Models versus the Fama-French Five- and Six-factor Models, January 1967 to December 2016

m is a factor's average return, α the intercept from a spanning regression, and R^2 its goodness-of-fit coefficient. $R_{\mathrm{Mkt}}, R_{\mathrm{Me}}, R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and Roe factors in the q-factor and Q5 models, respectively, and R_{Eg} the expected growth factor in the Q5 model. MKT, SMB, HML, RMW, and CMA are the market, size, value, profitability, and investment factors in the Fama-French five- and six-factor models, and UMD the momentum factor in the six-factor model. Finally, RMWc is the cash-based profitability factor in the alternative specification of the six-factor model, in which RMW is replaced by RMWc. The t-statistics (reported in the rows denoted [t] beneath the corresponding estimates) are adjusted for heteroscedasticity and autocorrelations.

Panel A: Regressing the q and Q5 factors on the Fama-French factors

	m	α	MKT	SMB	HML	RMW	CMA	UMD	RMWc	R^2
R_{Me}	0.31	0.05	0.01	0.97	0.03	−0.03	0.02			0.95
[t]	2.43	1.58	0.72	64.99	1.63	−0.98	0.72			
R_{Me}		0.03	0.01	0.97	0.05	−0.04	0.01	0.03		0.95
[t]		0.90	1.21	68.50	2.81	−1.34	0.34	2.57		
R_{Me}		0.05	0.00	0.96	0.05		0.02	0.03	−0.08	0.95
[t]		1.43	0.45	73.50	2.69		0.60	2.79	−2.28	
$R_{\mathrm{I/A}}$	0.41	0.12	0.01	−0.04	0.03	0.06	0.82			0.85
[t]	4.92	3.48	0.80	−3.08	1.32	2.46	31.26			
$R_{\mathrm{I/A}}$		0.11	0.01	−0.05	0.04	0.06	0.81	0.01		0.85
[t]		3.15	0.97	−3.06	1.79	2.21	33.12	0.77		
$R_{\mathrm{I/A}}$		0.11	0.01	−0.05	0.04		0.79	0.01	0.05	0.85
[t]		2.84	0.99	−3.04	1.95		31.10	0.77	1.43	
R_{Roe}	0.55	0.47	−0.03	−0.12	−0.24	0.70	0.10			0.52
[t]	5.25	5.91	−1.18	−2.98	−3.72	12.80	1.03			
R_{Roe}		0.30	0.00	−0.12	−0.10	0.65	−0.01	0.24		0.66
[t]		4.50	0.03	−3.74	−2.02	14.77	−0.21	9.94		
R_{Roe}		0.23	0.03	−0.10	−0.04		−0.18	0.24	0.72	0.66
[t]		2.79	1.45	−2.50	−0.51		−2.00	7.07	8.44	
R_{Eg}	0.56	0.56	−0.15	−0.10	−0.05	0.18	0.21			0.38
[t]	6.66	7.55	−7.22	−2.53	−0.92	2.70	3.72			
R_{Eg}		0.50	−0.14	−0.10	0.00	0.16	0.17	0.09		0.42
[t]		7.63	−7.54	−2.84	0.06	2.67	3.23	4.29		
R_{Eg}		0.38	−0.10	−0.03	0.01		0.14	0.08	0.41	0.49
[t]		5.35	−4.39	−1.02	0.10		2.35	3.51	4.72	

Panel B: Regressing the Fama-French factors on the q-factor and Q5 models

	m	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
SMB	0.25	0.04	−0.01	0.94	−0.08	−0.09		0.95
[t]	1.92	1.32	−0.66	54.18	−4.21	−5.84		
SMB		0.07	−0.01	0.94	−0.07	−0.08	−0.04	0.95
[t]		2.13	−1.56	55.15	−3.84	−5.17	−1.81	
HML	0.37	0.07	−0.04	0.02	1.01	−0.19		0.48
[t]	2.71	0.63	−1.01	0.31	12.18	−2.65		
HML		0.02	−0.02	0.03	1.00	−0.21	0.09	0.48
[t]		0.20	−0.62	0.38	11.30	−2.58	0.91	
UMD	0.64	0.11	−0.08	0.24	−0.00	0.91		0.28
[t]	3.60	0.49	−1.24	1.73	−0.02	5.88		
UMD		−0.12	0.00	0.27	−0.06	0.85	0.44	0.30
[t]		−0.54	−0.09	2.02	−0.34	5.34	2.25	
CMA	0.33	−0.00	−0.04	0.04	0.96	−0.10		0.86
[t]	3.51	−0.13	−3.74	1.90	34.93	−3.48		
CMA		−0.06	−0.03	0.05	0.95	−0.11	0.10	0.86
[t]		−1.45	−2.52	2.45	36.63	−4.20	3.30	
RMW	0.26	0.01	−0.03	−0.12	0.03	0.54		0.48
[t]	2.53	0.11	−1.21	−1.70	0.35	8.53		
RMW		−0.05	−0.01	−0.11	0.01	0.52	0.12	0.49
[t]		−0.62	−0.64	−1.38	0.14	7.74	1.08	
RMWc	0.33	0.25	−0.10	−0.18	0.09	0.29		0.56
[t]	4.16	3.83	−6.00	−5.25	2.02	9.88		
RMWc		0.10	−0.06	−0.16	0.05	0.25	0.28	0.61
[t]		1.64	−3.56	−3.54	1.00	7.11	4.56	

Table 2 : Factor Spanning Tests: The q-factor and Q5 Models versus the Stambaugh-Yuan "Mispricing" Factor Model, January 1967 to December 2016

m is the average return, α the regression intercept, and R^2 its goodness-of-fit coefficient. $R_{\mathrm{Mkt}}, R_{\mathrm{Me}}, R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and Roe factors in the q-factor and Q5 models, respectively, and R_{Eg} the expected growth factor in the Q5 model. MKT, SMBm, MGMT, and PERF are the market, size, management, and performance factors in the "mispricing" factor model. The t-statistics (reported in the rows denoted [t] beneath the corresponding estimates) are adjusted for heteroscedasticity and autocorrelations.

Panel A: Regressing the q and Q5 factors on the Stambaugh-Yuan factors								Panel B: Regressing the Stambaugh-Yuan factors on the q-factor and Q5 models								
	m	α	MKT	SMBm	MGMT	PERF	R^2		m	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
Original Stambaugh-Yuan "mispricing" factors from Yu Yuan's Web site																
R_{Me}	0.31	−0.04	−0.01	0.97	−0.06	−0.06	0.87	SMBm	0.44	0.16	0.01	0.86	−0.01	0.01		0.86
[t]	2.43	−0.65	−0.67	25.97	−1.71	−2.98		[t]	3.60	3.37	0.57	31.16	−0.23	0.45		
$R_{\mathrm{I/A}}$	0.41	0.08	0.01	0.05	0.53	−0.02	0.61	SMBm		0.16	0.01	0.86	−0.01	0.01	−0.00	0.86
[t]	4.92	1.26	0.52	2.35	15.99	−1.06		[t]		3.01	0.56	30.29	−0.23	0.42	−0.01	
R_{Roe}	0.55	0.33	0.02	−0.20	0.02	0.42	0.46	MGMT	0.61	0.36	−0.17	−0.15	1.00	−0.06		0.69
[t]	5.25	3.55	0.73	−3.44	0.42	11.65		[t]	4.72	4.73	−7.95	−5.02	18.59	−1.33		
R_{Eg}	0.56	0.38	−0.09	−0.07	0.24	0.16	0.48	MGMT		0.18	−0.11	−0.12	0.95	−0.11	0.34	0.72
[t]	6.66	5.84	−4.71	−2.16	6.52	7.70		[t]		2.39	−5.50	−4.00	18.79	−2.92	5.69	
								PERF	0.68	0.34	−0.18	0.11	−0.30	0.95		0.45
								[t]	4.20	2.00	−4.22	1.35	−2.02	10.42		
								PERF		0.05	−0.09	0.15	−0.38	0.88	0.54	0.50
								[t]		0.36	−2.40	2.13	−2.65	10.16	5.27	
Replicated Stambaugh-Yuan "mispricing" factors via the standard construction in the Hou-Xue-Zhang sample																
R_{Me}	0.31	0.01	−0.04	0.95	−0.03	0.10	0.92	SMBm	0.31	0.06	0.06	0.94	0.04	−0.16		0.93
[t]	2.43	0.18	−2.51	29.43	−1.00	4.23		[t]	2.13	1.13	3.37	18.96	0.86	−4.94		
$R_{\mathrm{I/A}}$	0.41	0.07	0.00	0.05	0.70	−0.02	0.71	SMBm		0.14	0.04	0.92	0.06	−0.14	−0.15	0.93
[t]	4.92	1.41	−0.08	2.77	26.78	−0.85		[t]		2.58	3.05	19.13	1.32	−4.16	−2.85	
R_{Roe}	0.55	0.32	0.01	−0.16	−0.04	0.59	0.67	MGMT	0.47	0.20	−0.09	−0.10	0.92	−0.06		0.75
[t]	5.25	4.71	0.49	−4.54	−0.82	20.03		[t]	4.68	3.59	−5.82	−4.10	22.65	−1.68		
R_{Eg}	0.56	0.43	−0.10	−0.09	0.27	0.16	0.48	MGMT		0.05	−0.04	−0.08	0.88	−0.10	0.29	0.79
[t]	6.66	6.88	−5.98	−2.87	7.02	7.85		[t]		0.91	−3.01	−3.62	23.97	−3.49	8.45	
								PERF	0.49	0.03	−0.08	0.08	−0.15	1.00		0.65
								[t]	3.67	0.28	−2.87	1.85	−1.72	13.97		
								PERF		−0.15	−0.03	0.10	−0.20	0.95	0.34	0.68
								[t]		−1.48	−1.05	2.90	−2.41	14.77	5.93	

Replicated Stambaugh-Yuan "mispricing" factors via the standard construction in the Stambaugh-Yuan sample

	m	α	MKT	SMBm	MGMT	PERF	R^2
R_{Me}	0.31	−0.00	0.01	1.03	−0.01	0.01	0.91
[t]	2.43	−0.11	0.40	26.50	−0.46	0.29	
$R_{\mathrm{I/A}}$	0.41	0.09	0.02	0.06	0.69	−0.02	0.67
[t]	4.92	1.59	1.19	2.82	18.38	−0.69	
R_{Roe}	0.55	0.31	0.01	−0.22	0.15	0.54	0.44
[t]	5.25	3.51	0.48	−3.90	1.91	10.01	
R_{Eg}	0.56	0.41	−0.09	−0.10	0.32	0.21	0.47
[t]	6.66	6.56	−5.52	−3.26	7.56	8.19	

	m	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
SMBm	0.31	0.06	0.01	0.87	0.00	−0.05		0.92
[t]	2.51	1.42	0.38	22.52	0.07	−2.16		
SMBm		0.11	−0.01	0.86	0.02	−0.03	−0.10	0.92
[t]		2.86	−1.01	22.66	0.52	−1.33	−2.45	
MGMT	0.44	0.19	−0.12	−0.10	0.86	−0.02		0.73
[t]	4.26	3.26	−7.00	−4.33	23.19	−0.61		
MGMT		0.06	−0.08	−0.08	0.83	−0.05	0.24	0.75
[t]		0.96	−5.14	−3.68	22.84	−2.09	5.66	
PERF	0.44	0.24	−0.09	0.09	−0.37	0.66		0.41
[t]	3.70	1.92	−2.78	1.72	−4.00	10.78		
PERF		0.07	−0.03	0.12	−0.42	0.61	0.34	0.44
[t]		0.58	−1.18	2.44	−4.61	9.82	4.36	

Table 3 : Correlation Matrix: The q and Q5 Factors, The Fama-French Factors, and the Stambaugh-Yuan "Mispricing" Factors, January 1967 to December 2016

$R_{\mathrm{Mkt}}, R_{\mathrm{Me}}, R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and Roe factors in the q-factor and Q5 models, respectively, and R_{Eg} the expected growth factor in the Q5 model. SMB, HML, RMW, and CMA are the size, value, profitability, and investment factors in the Fama-French five- and six-factor models, and UMD the momentum factor in the six-factor model. RMWc is the cash-based profitability factor in the alternative specification of the six-factor model, in which RMW is replaced by RMWc. SMBm, MGMT, and PERF are the size, management, and performance factors in the replicated M4 model via the standard construction in our sample. The p-values testing that a given correlation equals zero are reported in the rows beneath the correlations.

	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	SMB	HML	RMW	CMA	UMD	RMWc	SMBm	MGMT	PERF
R_{Mkt}	0.27	−0.38	−0.21	−0.53	0.28	−0.27	−0.24	−0.40	−0.15	−0.48	0.35	−0.49	−0.23
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R_{Me}		−0.15	−0.31	−0.32	0.97	−0.04	−0.37	−0.06	−0.02	−0.53	0.95	−0.28	−0.20
		0.00	0.00	0.00	0.00	0.36	0.00	0.14	0.60	0.00	0.00	0.00	0.00
$R_{\mathrm{I/A}}$			0.04	0.32	−0.19	0.67	0.10	0.91	0.03	0.26	−0.15	0.84	−0.02
			0.34	0.00	0.00	0.00	0.02	0.00	0.53	0.00	0.00	0.00	0.55
R_{Roe}				0.33	−0.37	−0.14	0.67	−0.08	0.50	0.57	−0.42	0.05	0.80
				0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.25	0.00
R_{Eg}					−0.35	0.22	0.35	0.34	0.28	0.60	−0.42	0.51	0.40
						0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SMB						−0.08	−0.36	−0.10	−0.05	−0.53	0.96	−0.32	−0.22
						0.05	0.00	0.02	0.20	0.00	0.00	0.00	0.00
HML							0.09	0.70	−0.19	0.16	−0.01	0.66	−0.23
							0.03	0.00	0.00	0.00	0.74	0.00	0.00
RMW								−0.01	0.11	0.76	−0.37	0.16	0.55
								0.86	0.01	0.00	0.00	0.00	0.00
CMA									0.00	0.19	−0.07	0.84	−0.08
									1.00	0.00	0.07	0.00	0.04
UMD										0.17	−0.19	0.03	0.71
										0.00	0.00	0.47	0.00
RMWc											−0.56	0.37	0.46
											0.00	0.00	0.00
SMBm												−0.29	−0.31
												0.00	0.00
MGMT													0.04
													0.36

Table 4 : Estimates of the Internal Rates of Returns for the Fama-French (2015) Factors

AR, IRR, and Diff (all in annual percent) are the average return, the internal rate of return, and AR minus IRR, respectively. SMB, HML, RMW, and CMA are the Fama-French (2015) size, value, profitability, and investment factors, respectively. IRR is measured at the June of of each year t, and AR from the July of year t to June of $t+1$. Panel A uses the analysts' earnings forecasts, Panel B the Hou-van Dijk-Zhang (2012) cross-sectional earnings forecasts, and Panel C the Tang-Wu-Zhang (2014) cross-sectional Roe forecasts in estimating the IRRs. GLS denotes the Gebhardt-Lee-Swaminathan model, Easton the Easton model, CT the Claus-Thomas model, OJ the Ohlson-Juettner-Nauroth model, and Average the averages across the models. Appendix B details the estimation methods.

	Panel A: IBES earnings forecasts (1979–2016)						Panel B: Cross-sectional earnings forecasts (1967–2016)						Panel C: Cross-sectional Roe forecasts (1967–2016)					
	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff
	GLS			Easton			GLS			Easton			GLS			Easton		
SMB	1.51	0.88	0.64	1.57	2.51	−0.94	2.57	1.52	1.04	2.38	5.11	−2.72	2.91	0.06	2.84	2.71	1.14	1.58
[t]	0.76	4.62	0.33	0.82	14.85	−0.50	1.25	4.02	0.53	1.18	6.96	−1.44	1.49	0.26	1.49	1.38	4.82	0.80
HML	2.98	3.50	−0.52	2.90	3.26	−0.36	3.36	5.57	−2.20	3.27	7.23	−3.96	3.64	5.21	−1.57	3.87	7.60	−3.73
[t]	1.39	18.91	−0.25	1.29	7.45	−0.17	1.83	27.23	−1.24	1.77	15.32	−2.08	1.98	32.87	−0.89	2.04	16.10	−2.08
RMW	3.72	−1.19	4.91	4.48	−3.27	7.75	3.46	−1.43	4.89	4.06	−3.66	7.72	3.13	−1.35	4.47	3.35	−6.41	9.76
[t]	2.64	−8.27	3.58	2.82	−9.44	4.75	2.72	−6.54	4.00	2.77	−10.22	5.00	2.54	−7.81	3.77	2.65	−19.72	7.19
CMA	3.46	0.64	2.82	3.58	2.45	1.13	3.72	1.59	2.13	4.46	4.06	0.40	3.50	1.09	2.41	3.63	4.52	−0.88
[t]	2.87	4.69	2.42	3.14	7.91	1.08	3.17	9.09	1.87	4.34	11.56	0.37	3.19	5.75	2.24	3.10	9.97	−0.78
	CT			OJ			CT			OJ			CT			OJ		
SMB	1.55	1.11	0.43	1.70	0.68	1.02	2.65	2.71	−0.06	3.73	3.21	0.52	3.23	−1.17	4.41	3.21	−1.40	4.62
[t]	0.78	6.84	0.22	0.86	4.57	0.53	1.29	4.23	−0.03	1.91	5.39	0.28	1.65	−3.65	2.29	1.59	−3.86	2.26
HML	3.12	0.14	2.98	2.05	0.73	1.32	3.14	3.68	−0.54	2.85	4.74	−1.89	2.95	2.64	0.31	3.02	4.23	−1.22
[t]	1.42	0.42	1.33	0.98	3.77	0.63	1.71	16.21	−0.29	1.69	17.64	−1.10	1.74	5.90	0.18	1.58	9.36	−0.63
RMW	3.66	0.29	3.38	4.07	−0.05	4.12	3.18	−0.01	3.19	3.16	−1.57	4.72	2.57	−0.04	2.61	2.07	−2.11	4.17
[t]	2.68	1.94	2.48	3.41	−0.27	3.41	3.01	−0.03	3.04	3.14	−5.50	4.45	2.56	−0.20	2.59	2.04	−9.38	3.85
CMA	3.41	0.11	3.30	3.15	0.08	3.07	3.32	1.52	1.81	3.89	2.12	1.77	2.91	0.62	2.29	3.26	0.66	2.60
[t]	2.88	0.79	2.78	2.79	0.48	2.70	2.92	11.29	1.54	3.35	11.11	1.54	2.68	4.32	2.06	2.67	3.06	2.04
	Average						Average						Average					
SMB	1.44	1.72	−0.28				2.53	3.22	−0.69				2.90	−0.25	3.15			
[t]	0.76	10.74	−0.15				1.23	5.60	−0.35				1.49	−0.93	1.64			
HML	2.90	2.04	0.86				3.52	5.31	−1.79				3.60	5.14	−1.54			
[t]	1.28	9.07	0.39				1.88	25.28	−0.97				1.96	17.72	−0.85			
RMW	4.52	−1.58	6.10				3.61	−1.84	5.45				3.14	−2.47	5.61			
[t]	2.88	−9.66	3.90				2.66	−9.41	4.07				2.54	−21.47	4.52			
CMA	3.40	1.16	2.24				3.81	2.64	1.17				3.44	2.02	1.43			
[t]	2.92	7.09	2.02				3.34	19.06	1.04				3.17	13.47	1.34			

Table 5 : Annual Cross-sectional Regressions of Future Book Equity Growth Rates and Operating Profitability, 1963–2016

The sample contains all common stocks on NYSE, Amex, and Nasdaq. We do not exclude financial firms, because these stocks are included in the construction of the Fama-French (2015) five factors. All the regressions are annual cross-sectional regressions. A_{it} is total assets for firm i at year t, $\triangle A_{it} \equiv A_{it} - A_{it-1}$, Be_{it} is book equity for firm i at year t, $\triangle Be_{it} \equiv Be_{it} - Be_{it-1}$, and Op_{it} is operating profitability for firm i at year t. Book equity is measured as in Davis, Fama, and French (2000), and operating profitability is measured as in Fama and French (2015). Variables dated t are measured at the end of the fiscal year ending in calendar year t. To avoid the excess influence of small firms, we follow Fama and French (2006) and exclude those with total assets below \$5 million or book equity below \$2.5 million in year t in Panel A. The cutoffs are \$25 million and \$12.5 million in Panel B. We winsorize all regression variables at the 1st and 99th percentiles of the cross-sectional distribution each year.

		$\frac{\triangle Be_{it+\tau}}{Be_{it+\tau-1}} = \gamma_0 + \gamma_1 \frac{\triangle A_{it}}{A_{it-1}} + \epsilon_{t+\tau}$					$\frac{\triangle Be_{it+\tau}}{Be_{it+\tau-1}} = \gamma_0 + \gamma_1 \frac{\triangle Be_{it}}{Be_{it-1}} + \epsilon_{t+\tau}$					$Op_{it+\tau} = \gamma_0 + \gamma_1 Op_{it} + \epsilon_{t+\tau}$				
τ	#firms	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2
Panel A: Firms with assets ≥ \$5 million and book equity ≥ \$2.5 million																
1	3,105	0.09	14.46	0.22	13.94	0.05	0.09	13.10	0.20	8.47	0.06	0.03	4.73	0.80	43.30	0.54
2	2,843	0.10	14.32	0.10	7.60	0.01	0.10	14.43	0.10	5.21	0.02	0.05	6.24	0.67	27.12	0.36
3	2,624	0.10	14.71	0.06	6.31	0.01	0.10	14.70	0.06	4.05	0.01	0.07	7.84	0.59	24.18	0.27
4	2,431	0.10	15.78	0.05	5.53	0.00	0.10	15.88	0.05	3.69	0.00	0.09	9.32	0.53	22.64	0.22
5	2,259	0.10	14.76	0.04	3.44	0.00	0.10	15.71	0.02	1.92	0.00	0.10	11.18	0.49	22.78	0.19
6	2,103	0.10	14.99	0.05	4.57	0.00	0.10	14.71	0.03	2.27	0.00	0.11	13.03	0.45	23.22	0.16
7	1,961	0.09	15.15	0.04	4.43	0.00	0.10	15.26	0.03	2.68	0.00	0.11	14.62	0.43	21.87	0.15
8	1,828	0.09	15.07	0.03	4.14	0.00	0.10	15.35	0.01	1.71	0.00	0.12	15.86	0.40	19.23	0.13
9	1,706	0.09	15.09	0.03	3.37	0.00	0.10	15.16	0.01	1.19	0.00	0.12	15.08	0.39	17.63	0.12
10	1,593	0.09	14.47	0.04	4.32	0.00	0.09	14.61	0.02	2.13	0.00	0.12	14.14	0.38	16.68	0.11
Panel B: Firms with assets ≥ \$25 million and book equity ≥ \$12.5 million																
1	2,492	0.08	15.77	0.23	16.94	0.05	0.08	14.11	0.24	10.36	0.07	0.03	7.15	0.82	58.34	0.61
2	2,284	0.09	15.41	0.13	10.30	0.02	0.09	15.84	0.12	7.08	0.02	0.06	8.95	0.70	34.86	0.42
3	2,109	0.09	16.25	0.08	7.65	0.01	0.09	16.18	0.08	5.75	0.01	0.08	10.74	0.62	30.44	0.32
4	1,956	0.09	16.71	0.07	6.75	0.01	0.09	16.69	0.07	4.41	0.01	0.09	13.11	0.56	29.61	0.26
5	1,821	0.09	16.35	0.05	3.94	0.01	0.09	16.97	0.04	2.85	0.01	0.10	15.92	0.52	30.60	0.22
6	1,699	0.09	16.24	0.05	5.50	0.00	0.09	16.04	0.04	3.37	0.00	0.11	17.80	0.48	31.61	0.19
7	1,588	0.09	16.48	0.05	4.96	0.00	0.09	16.58	0.04	3.12	0.00	0.12	19.76	0.45	31.23	0.17
8	1,485	0.09	15.84	0.03	3.99	0.00	0.09	15.83	0.02	2.62	0.00	0.13	20.33	0.43	27.16	0.15
9	1,388	0.09	15.68	0.03	3.84	0.00	0.09	15.80	0.02	2.10	0.00	0.13	18.98	0.42	22.62	0.14
10	1,298	0.09	14.50	0.05	5.23	0.00	0.09	14.71	0.03	2.98	0.00	0.13	17.77	0.41	21.77	0.13