

Lecture Notes

Petrosky-Nadeau, Zhang, and Kuehn (2018, “Endogenous Disasters,” *American Economic Review*)

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The textbook Diamond-Mortensen-Pissarides model of equilibrium unemployment gives rise endogenously to rare disasters

Outline

- 1 The Model
- 2 Quantitative Results
- 3 Home Production
- 4 Capital
- 5 Recursive Utility

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Model

Search and matching

The representative firm posts job vacancies, V_t , to attract unemployed workers, U_t , via a CRS matching function:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\nu + V_t^\nu)^{1/\nu}}$$

The job filling rate:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^\nu)^{1/\nu}}$$

in which $\theta_t = V_t/U_t$ is labor market tightness: $q'(\theta_t) < 0$

Model

The unit costs of vacancy

The representative firm incurs costs of vacancy posting, $\kappa_t V_t$, with the unit costs, κ_t , given by:

$$\kappa_t \equiv \kappa_0 + \kappa_1 q(\theta_t)$$

in which

- κ_0 : The proportional costs
- κ_1 : The fixed costs

Fixed matching costs (paid after a hired worker arrives): Training, interviewing, negotiation, and administrative setup costs of adding a worker to the payroll, etc., as in Pissarides (2009)

Model

An occasionally binding constraint on vacancy

Once matched, jobs are destroyed at a constant rate s :

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t$$

in which

$$V_t \geq 0$$

The firm uses labor to produce via a CRS production function:

$$Y_t = X_t N_t \quad \text{in which} \quad \log(X_{t+1}) = \rho \log(X_t) + \sigma \epsilon_{t+1}$$

Model

The representative firm

Dividends to shareholders:

$$D_t = X_t N_t - W_t N_t - \kappa_t V_t$$

in which W_t is the wage rate

Taking the stochastic discount factor, $M_{t+1} = \beta(C_t/C_{t+1})$, as given, the firm maximizes the market value of equity, S_t :

$$S_t \equiv \max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} E_t \left[\sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} \right]$$

subject to $N_{t+1} = (1 - s)N_t + q(\theta_t)V_t$ and $V_t \geq 0$

Model

The intertemporal job creation condition and the stock return

Let λ_t be the multiplier on the $q(\theta_t)V_t \geq 0$ constraint:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t \left[M_{t+1} \left[X_{t+1} - W_{t+1} + (1-s) \left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} \right] \right] \right]$$

The Kuhn-Tucker conditions: $V_t \geq 0, \lambda_t \geq 0, \lambda_t V_t = 0$

Model

Wages divide rents between workers and the firm

Wages as the endogenous outcome of a generalized Nash bargaining process between a worker and the firm:

$$W_t = \eta (X_t + \kappa_t \theta_t) + (1 - \eta)b$$

- η : Relative bargaining weight of the worker
- X_t : Marginal product of labor
- $\kappa_t \theta_t = \kappa_t V_t / U_t$: Vacancy costs per unemployed worker
- b : The flow value of unemployment activities

η and b govern wage elasticity to labor productivity

Model

Equilibrium

The goods market clearing condition:

$$C_t + \kappa_t V_t = X_t N_t$$

The recursive competitive equilibrium consists of vacancies, $V_t^* \geq 0$; multiplier, $\lambda_t^* \geq 0$; consumption, C_t^* ; and indirect utility, J_t^* :

- V_t^* and λ_t^* satisfy the intertemporal job creation condition and the Kuhn-Tucker conditions, while taking the wage equation and the representative household's SDF as given;
- C_t^* and J_t^* satisfy the optimality condition $1 = E_t[M_{t+1}R_{t+1}]$;
- the goods market and the financial market clear

Model

Parameterized expectations per Christiano and Fisher (2000)

Solve for $\lambda_t \equiv \lambda(N_t, X_t)$ from

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t \left[M_{t+1} \left[X_{t+1} - W_{t+1} + (1-s) \left(\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} \right) \right] \right]$$

while obeying the Kuhn-Tucker conditions

$\log(X_t)$ discretized with 17 grid points; cubic splines (50 basis functions) in N for each $\log(X)$ -level

Model

Parameterized expectations per Christiano and Fisher (2000)

Parameterizing the conditional expectation, $\mathcal{E}_t \equiv \mathcal{E}(N_t, X_t)$ eliminates the need to parameterizing λ_t separately:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = \mathcal{E}_t$$

After obtaining \mathcal{E}_t , calculate $\tilde{q}(\theta_t) = \kappa_0 / (\mathcal{E}_t - \kappa_1)$:

- If $\tilde{q}(\theta_t) \geq 1$ (binding constraint): set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$
- If $\tilde{q}(\theta_t) < 1$ (nonbinding constraint): set $\lambda_t = 0$, $q(\theta_t) = \tilde{q}(\theta_t) \Rightarrow \theta_t = q^{-1}(\tilde{q}(\theta_t))$, $V_t = \theta_t(1 - N_t)$

Model

Monthly calibration

Rate of time preference, β	0.9954	
Aggregate productivity persistence, ρ	0.983	Gertler and Trigari (2009)
Conditional volatility of shocks, σ	0.01	Gertler and Trigari (2009)
Workers' bargaining weight, η	0.04	Hagedorn and Manovskii (2008)
Job destruction rate, s	0.04	Davis et al. (2006)
Elasticity of matching function, ι	1.25	Den Haan et al. (2000)
Value of unemployment activities, b	0.85	Hagedorn and Manovskii (2008)
The proportional costs, κ_0	0.5	
The fixed costs, κ_1	0.5	

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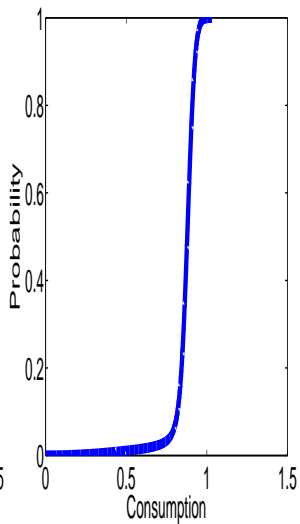
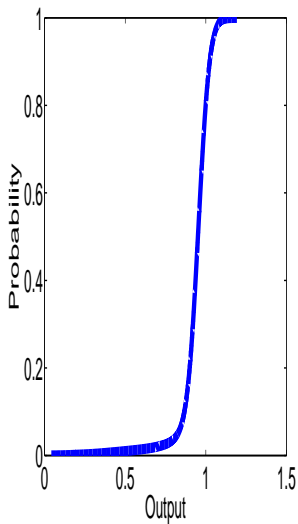
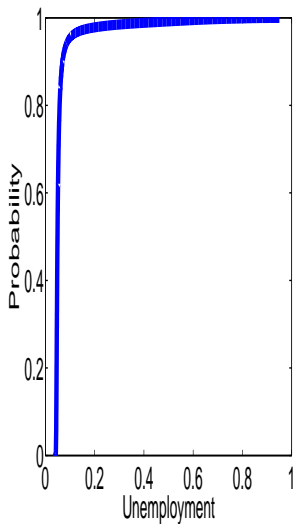
Quantitative Results

Basic moments

Panel A: Log output growth						Panel B: Log consumption growth					
	data	mean	5%	95%	p-value		data	mean	5%	95%	p-value
σ_Y	5.63	5.31	2.90	11.56	0.31	σ_C	6.37	4.65	2.12	11.26	0.21
S_Y	-1.02	0.85	-0.39	3.35	0.99	S_C	-0.55	0.91	-0.50	3.57	0.96
K_Y	11.87	12.8	3.08	34.99	0.41	K_C	9.19	14.4	3.16	38.59	0.54
ρ_1^Y	0.16	0.24	0.01	0.63	0.58	ρ_1^C	0.07	0.23	-0.02	0.65	0.79
ρ_2^Y	0	-0.11	-0.31	0.23	0.16	ρ_2^C	0.03	-0.12	-0.33	0.24	0.14
ρ_3^Y	0.02	-0.12	-0.32	0.08	0.1	ρ_3^C	0	-0.12	-0.34	0.09	0.15
ρ_4^Y	-0.02	-0.11	-0.31	0.08	0.23	ρ_4^C	-0.02	-0.11	-0.32	0.09	0.23
Panel C: Unemployment											
	data	mean	5%	95%	p-value		data	mean	5%	95%	p-value
$E[U]$	6.98	6.28	4.83	10.54	0.19	S_U	2.02	3.52	1.49	5.82	0.85
K_U	7.26	19.18	5.24	41.78	0.87	σ_U	21.76	23.41	5.45	53.49	0.43

Quantitative Results

Empirical cumulative distribution functions: U_t , Y_t , and C_t



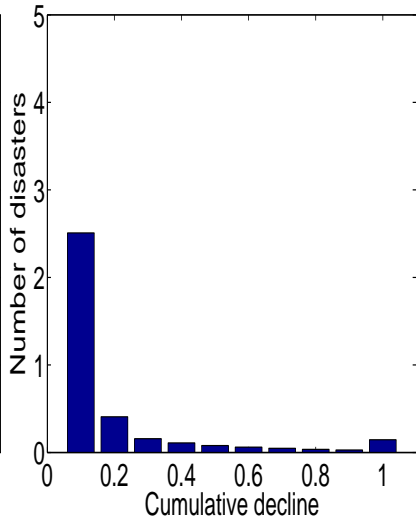
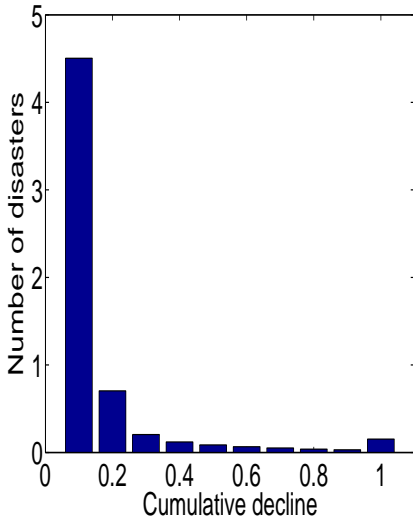
Quantitative Results

Moments of rare disasters

	Data	Model			
		Mean	5%	95%	p-value
Panel A: Output					
Probability	7.83	5.04	2.24	8.57	0.09
Size	21.99	22.22	12.7	46.24	0.33
Duration	3.72	4.44	3.2	6	0.79
Panel B: Consumption					
Probability	8.57	2.86	0.71	5.83	0.00
Size	23.16	25.64	11.26	62.13	0.36
Duration	3.75	4.91	3	7	0.81

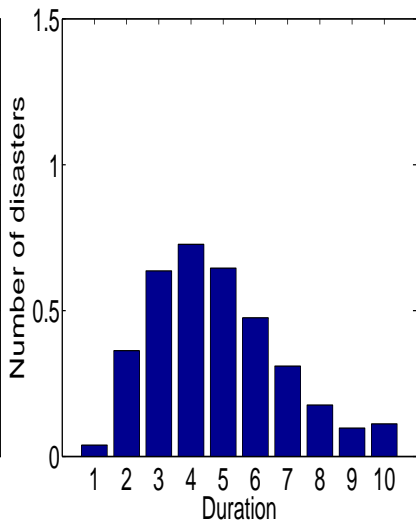
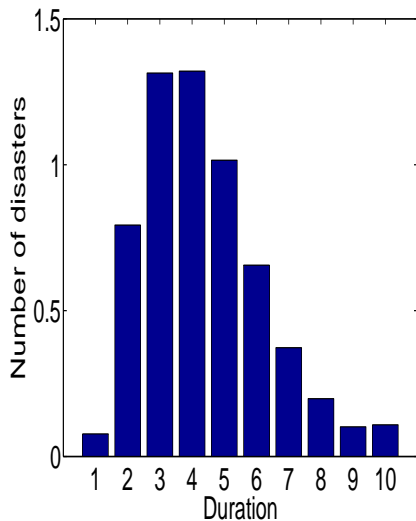
Quantitative Results

Distributions of disasters by size



Quantitative Results

Distributions of disasters by duration



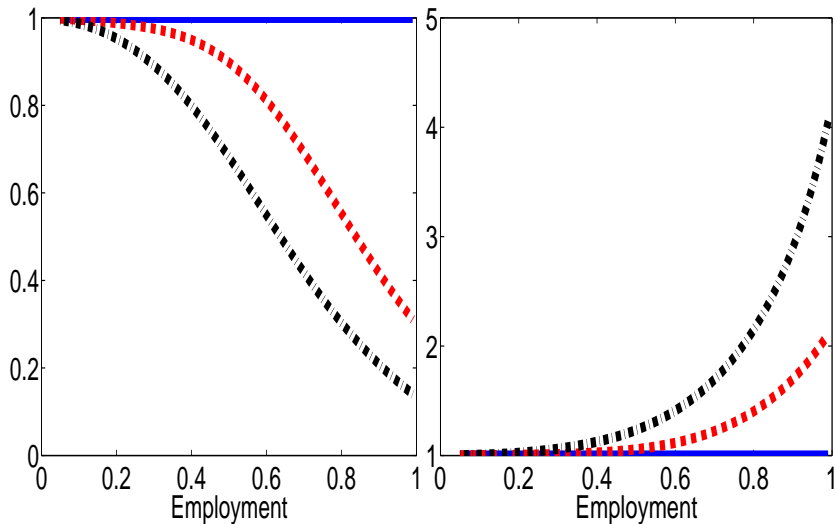
Quantitative Results

Comparative statics

	Baseline	$b = 0.825$	$b = 0.4$	$s = 0.035$	$\kappa_t = 0.7$	$\iota = 1.1$	$\eta = 0.05$
Panel A: Output							
Probability	5.04	3.61	2.53	4.42	4.05	5.29	5.57
Size	22.22	16.07	13.41	19.87	18.2	21.97	22.69
Duration	4.44	4.57	4.7	4.5	4.51	4.41	4.4
Panel B: Consumption							
Probability	2.86	1.62	1.32	2.43	1.85	3.04	3.59
Size	25.64	16.31	12.35	22.25	20.19	25.05	25.21
Duration	4.91	5.19	5.2	4.97	5.1	4.88	4.78

Quantitative Results

Downward rigidity in marginal hiring costs



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Home Production

Model

Let C_{mt} be market consumption, C_{ht} home consumption, and the composite consumption bundle:

$$C_t \equiv [aC_{mt}^e + (1-a)C_{ht}^e]^{1/e},$$

in which $e \in (0, 1]$ and $a \in [0, 1]$

The stochastic discount factor:

$$M_{t+1} = \beta \left(\frac{C_{mt+1}}{C_{mt}} \right)^{e-1} \left(\frac{C_t}{C_{t+1}} \right)^e$$

Home Production

Model

Home production technology:

$$C_{ht} = X_h U_t, \quad \text{with} \quad X_h > 0$$

The equilibrium Nash-wage becomes:

$$W_t = \eta(X_t + \kappa_t \theta_t) + (1 - \eta)z_t,$$

with

$$z_t \equiv X_h \left(\frac{1 - a}{a} \right) \left(\frac{C_{mt}}{C_{ht}} \right)^{1-e} + b$$

The market clearing condition: $C_{mt} + \kappa_t V_t = X_t N_t$

Home Production

Quantitative Results

	Data					Model					
σ		0.01	0.014	0.014	0.014	σ		0.01	0.014	0.014	0.014
a		0.8	0.8	0.8	0.85	a		0.8	0.8	0.8	0.85
e		0.85	0.85	0.9	0.85	e		0.85	0.85	0.9	0.85
σ_Y	5.63	3.41	5.29	4.62	3.9	σ_C	6.37	2.91	4.67	3.74	2.9
S_Y	-1.02	0.06	0.15	0.13	0.01	S_C	-0.55	0.09	0.2	0.2	0.03
K_Y	11.87	3.83	4.92	4.95	3.42	K_C	9.19	4.22	5.73	5.97	3.48
ρ_1^Y	0.16	0.15	0.16	0.15	0.14	ρ_1^C	0.07	0.15	0.16	0.15	0.14
ρ_2^Y	0	-0.13	-0.13	-0.13	-0.12	ρ_2^C	0.03	-0.13	-0.14	-0.14	-0.12
ρ_3^Y	0.02	-0.1	-0.11	-0.1	-0.1	ρ_3^C	0	-0.1	-0.11	-0.11	-0.1
ρ_4^Y	-0.02	-0.08	-0.08	-0.08	-0.08	ρ_4^C	-0.02	-0.08	-0.09	-0.08	-0.08
Prob_Y	7.83	5	9.95	8.2	6.88	Prob_C	8.57	3.35	7.52	4.95	3.43
Size_Y	21.99	15	18.58	17.21	15.43	Size_C	23.16	14.42	18.06	16.34	13.81
Dur_Y	3.74	4.32	3.74	3.88	3.98	Dur_C	3.75	4.65	3.99	4.31	4.59
$E[U]$	6.98	5.97	6.58	5.33	4.5	S_U	2.02	1.86	2.44	3.06	2.1
K_U	7.26	7.48	10.42	15.73	9.87	σ_U	21.76	10.75	19.53	15.3	4.07

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Capital Model

Production $Y_t = X_t K_t^\alpha N_t^{1-\alpha}$, $\alpha \in (0, 1)$, and $x_t = \log(X_t)$:

$$x_{t+1} = (1 - \rho)\bar{x} + \rho x_t + \sigma \epsilon_{t+1}$$

Capital accumulation:

$$K_{t+1} = (1 - \delta)K_t + \Phi(I_t, K_t),$$

in which δ is the capital depreciation rate, I_t is investment, and

$$\Phi(I_t, K_t) \equiv \left[a_1 + \frac{a_2}{1 - 1/\nu} \left(\frac{I_t}{K_t} \right)^{1-1/\nu} \right] K_t$$

Capital Model

The investment Euler equation:

$$\frac{1}{a_2} \left(\frac{I_t}{K_t} \right)^{1/\nu} = E_t \left[M_{t+1} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1}{a_2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^{1/\nu} (1 - \delta + a_1) + \frac{1}{\nu - 1} \frac{I_{t+1}}{K_{t+1}} \right) \right]$$

The intertemporal job creation condition:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t \left[M_{t+1} \left((1 - \alpha) \frac{Y_{t+1}}{N_{t+1}} - W_{t+1} + (1 - s) \left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} \right] \right) \right]$$

The equilibrium wage:

$$W_t = \eta \left[(1 - \alpha) \frac{Y_t}{N_t} + \kappa_t \theta_t \right] + (1 - \eta)b$$

The goods market clearing condition: $C_t + I_t + \kappa_t V_t = Y_t$

Capital

Quantitative results

	Data					Model					
σ		0.01	0.014	0.014	0.014	σ		0.01	0.014	0.014	0.014
ν		2	2	1.5	0.5	ν		2	2	1.5	0.5
σ_Y	5.63	3.35	5.11	5.1	4.93	σ_C	6.37	2.38	3.74	4	4.75
S_Y	-1.02	0.1	0.12	0.11	0.1	S_C	-0.55	0.08	0.12	0.14	0.17
K_Y	11.87	4.11	4.5	4.49	4.34	K_C	9.19	4.67	5.18	5.1	4.79
ρ_1^Y	0.16	0.18	0.19	0.19	0.17	ρ_1^C	0.07	0.21	0.22	0.2	0.17
ρ_2^Y	0	-0.1	-0.09	-0.1	-0.12	ρ_2^C	0.03	-0.08	-0.07	-0.09	-0.12
Prob_Y	7.83	4.55	9.45	9.4	9.07	Prob_C	8.57	2.08	5.31	5.95	8.18
Size_Y	21.99	15.76	18.97	18.81	18.08	Size_C	23.16	14.9	17.69	17.68	17.98
Dur_Y	3.72	4.58	3.89	3.87	3.8	Dur_C	3.75	5.39	4.51	4.33	3.9
σ_I	23.33	4.52	6.98	6.06	2.88	$E[U]$	6.98	5.98	7.46	7.45	6.92
S_I	-0.79	0.2	0.2	0.17	0	S_U	2.02	2.51	2.55	2.55	2.64
K_I	8.72	4.51	4.94	4.92	4.66	K_U	7.26	11	11.09	11.12	11.65
ρ_1^I	0.22	0.17	0.17	0.18	0.19	σ_U	21.76	14	22.51	22.57	22.27
ρ_2^I	-0.04	-0.12	-0.12	-0.11	-0.1						

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Recursive Utility

Model

Preferences:

$$J_t = \max_{\{C_t\}} \left[(1 - \beta) C_t^{1 - \frac{1}{\psi}} + \beta \left(E_t [J_{t+1}^{1-\gamma}] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}$$

Optimality condition: $1 = E_t[M_{t+1}R_{t+1}]$:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{J_{t+1}}{E_t[J_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma}$$

$$R_{t+1} = \frac{X_{t+1} - W_{t+1} + (1 - s) [\kappa_0/q(\theta_{t+1}) + \kappa_1 - \lambda_{t+1}]}{\kappa_0/q(\theta_t) + \kappa_1 - \lambda_t}$$

Recursive Utility

Quantitative results

	Data					Model					
γ		10	7.5	10	1	γ		10	7.5	10	1
ψ		1.5	1.5	1	1	ψ		1.5	1.5	1	1
σ_Y	5.63	5.67	4.97	4.99	4.11	σ_C	6.37	5.05	4.35	4.35	3.44
S_Y	-1.02	0.87	0.81	0.76	0.61	S_C	-0.55	0.88	0.81	0.81	0.67
K_Y	11.87	15.47	14.18	12.36	10.36	K_C	9.19	17.09	15.69	14.2	11.9
ρ_1^Y	0.16	0.21	0.19	0.23	0.20	ρ_1^C	0.07	0.19	0.18	0.22	0.19
ρ_2^Y	0	-0.14	-0.14	-0.12	-0.12	ρ_2^C	0.03	-0.15	-0.15	-0.13	-0.14
ρ_3^Y	0.02	-0.13	-0.12	-0.12	-0.12	ρ_3^C	0	-0.13	-0.12	-0.13	-0.12
ρ_4^Y	-0.02	-0.1	-0.1	-0.11	-0.1	ρ_4^C	-0.02	-0.1	-0.09	-0.11	-0.1
Prob $_Y$	7.83	4.49	4.03	4.53	5.03	Prob $_C$	8.57	2.51	2.12	2.51	2.84
Size $_Y$	21.99	23.92	22.17	21.92	22.25	Size $_C$	23.16	28.86	26.51	25.6	25.7
Dur $_Y$	3.72	4.46	4.56	4.5	4.45	Dur $_C$	3.75	4.84	5.01	4.9	4.93
$E[U]$	6.98	6.26	5.88	6.23	5.7	$E[R - R^f]$	4.69	4.45	1.1	4.97	0.22
S_U	2.02	3.66	3.57	3.46	3.29	$E[R^f]$	1.04	2.58	2.87	2.6	2.93
K_U	7.26	20.71	20.75	18.48	18	σ_R	20	15.79	15.15	15.73	14.5
σ_U	21.76	25.67	21.99	22.93	17.88	σ_{R^f}	12.32	1.64	1.39	1.98	1.54

Recursive Utility

Predicting excess returns and consumption growth with log price-to-consumption

H	1y	2y	3y	4y	5y	1q	4q	8q	12q	16q	20q
	U.S. annual data, 1836–2013					U.S. quarterly data, 1947q2–2013q4					
β_R	-3.68	-7.64	-10.43	-13.66	-16.65	-0.74	-3.39	-6.78	-9.69	-12.18	-15.03
t_R	-4.54	-5.49	-5.2	-5.57	-6.43	-2.31	-2.75	-2.94	-3.24	-3.36	-3.68
R_R^2	8.1	17.09	23.65	31.31	38.97	1.75	8.06	16.64	24.56	30.67	36.69
β_C	0.46	0.21	0.05	-0.10	-0.15	0.06	0.07	-0.05	-0.16	-0.24	-0.34
t_C	1.69	0.43	0.07	-0.10	-0.13	1.66	0.40	-0.13	-0.27	-0.31	-0.34
R_C^2	1.48	0.15	0.01	0.02	0.03	1.23	0.27	0.05	0.31	0.51	0.77
	Recursive utility					Recursive utility					
β_R	-1.91	-3.44	-4.71	-5.75	-6.62	-0.74	-2.72	-4.89	-6.68	-8.20	-9.49
t_R	-1.83	-2.17	-2.24	-2.28	-2.31	-1.65	-2.04	-2.19	-2.38	-2.60	-2.82
R_R^2	2.04	3.49	4.65	5.59	6.35	1.17	4.14	7.35	9.96	12.14	13.97
β_C	-0.67	-1.68	-2.69	-3.62	-4.42	-0.00	-0.56	-1.53	-2.51	-3.41	-4.19
t_C	-2.19	-2.82	-3.37	-3.83	-4.19	-1.11	-2.14	-2.98	-3.65	-4.24	-4.76
R_C^2	7.62	11.95	16.28	20.1	23.32	2.56	8.54	16.09	22.43	27.63	31.82

Conclusion

Endogenous disasters

The textbook Diamond-Mortensen-Pissarides model of equilibrium unemployment gives rise endogenously to rare disasters