The CAPM Strikes Back?
An Equilibrium Model with Disasters

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Theme

Embedding disasters into a general equilibrium model with heterogeneous firms induces strong nonlinearity in the pricing kernel, helping explain the failure of the (consumption) CAPM

Results

Our single-factor model, in which a nonlinear consumption CAPM holds exactly by construction, reproduces:

- The CAPM fails to explain the value premium in finite samples without disasters, but performs better in samples with disasters
- The beta "anomaly": A flat relation between the pre-ranking market beta and the average return
- The standard consumption CAPM fails in simulations

In a sample without disasters, estimated betas only reflect risk in normal times, but the value premium is driven by disaster risk

Literature

Early investment theories rely on single-factor models, in which the CAPM roughly holds: Gomes, Kogan, and Zhang 2003, Carlson, Fisher, and Giammarino 2004, Zhang 2005, Cooper 2006

Recent models introduce multiple shocks, but inconsistent with the long-sample evidence: Ai and Kiku 2013, Kogan and Papanikolaou 2013, Belo, Lin, and Bazdresch 2014, Koh 2014, Li 2017

Equilibrium with heterogeneous firms: Favilukis and Lin 2016

Disaster models: Rietz 1988, Barro 2006, Gourio 2012, Nakamura, Steinsson, Barro, and Ursua 2013, Wachter 2013

Outline

Outline

Stylized Facts

The CAPM regressions for the b/m deciles, July 1963–June 2017

	L	2	3	4	5	6	7	8	9	H	H−L
m	0.44	0.54	0.59	0.54	0.55	0.66	0.62	0.70	0.86	0.91	0.47
t_m	2.22	3.00	3.26	2.98	3.14	3.88	3.49	3.88	4.41	3.80	2.53
α	−0.11	0.02	0.07	0.03	0.07	0.20	0.15	0.23	0.35	0.32	0.43
t_α	−1.23	0.44	1.17	0.39	0.80	2.21	1.23	2.00	3.03	2.04	1.89
β	1.06	1.00	0.99	0.98	0.91	0.88	0.92	0.91	0.98	1.13	0.07
t_β	41.66	42.06	40.88	32.43	28.19	23.30	19.35	18.26	22.65	17.47	0.86
R^2	0.86	0.91	0.91	0.87	0.83	0.80	0.78	0.76	0.77	0.68	0.00

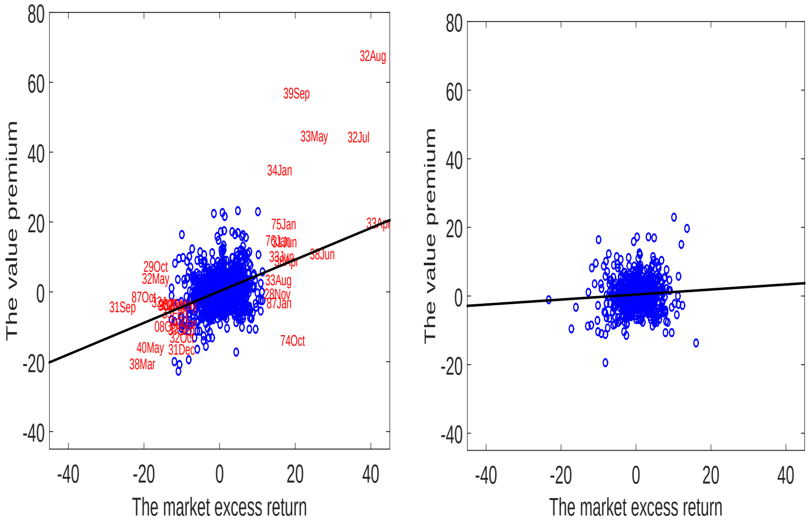
Stylized Facts

The CAPM regressions for the b/m deciles, July 1926–June 2017

	L	2	3	4	5	6	7	8	9	H	H−L
m	0.59	0.69	0.69	0.66	0.72	0.79	0.72	0.91	1.06	1.07	0.48
t_m	3.40	4.28	4.23	3.71	4.19	4.35	3.73	4.49	4.55	3.84	2.50
α	−0.08	0.07	0.05	−0.02	0.07	0.11	0.00	0.16	0.22	0.11	0.19
t_α	−1.21	1.46	1.02	−0.38	0.92	1.32	0.02	1.82	1.94	0.74	0.99
β	1.01	0.95	0.97	1.05	1.00	1.03	1.10	1.14	1.28	1.46	0.45
t_β	52.73	27.62	59.98	22.11	27.29	14.85	17.73	16.11	14.32	14.49	3.87
R^2	0.90	0.91	0.93	0.90	0.89	0.85	0.84	0.83	0.80	0.72	0.14

Stylized Facts

Scatter plots, 1926 versus 1963 onward



Stylized Facts

Large swings in the stock market and the value premium

	MKT	H−L		MKT	H−L
November 1928	11.81	−0.29	August 1933	12.05	3.76
October 1929	−20.12	7.60	January 1934	12.60	35.20
June 1930	−16.27	−3.60	September 1937	−13.61	−10.56
May 1931	−13.24	−3.37	March 1938	−23.82	−20.35
June 1931	13.90	14.57	April 1938	14.51	9.16
September 1931	−29.13	−4.03	June 1938	23.87	11.15
December 1931	−13.53	−16.22	September 1939	16.88	57.22
April 1932	−17.96	−2.65	May 1940	−21.95	−15.59
May 1932	−20.51	4.09	October 1974	16.10	−13.57
July 1932	33.84	44.54	January 1975	13.66	19.72
August 1932	37.06	67.95	January 1976	12.16	15.03
October 1932	−13.17	−12.80	March 1980	−12.90	−8.78
February 1933	−15.24	−5.70	January 1987	12.47	−2.83
April 1933	38.85	20.04	October 1987	−23.24	−1.20
May 1933	21.43	44.85	August 1998	−16.08	−3.27
June 1933	13.11	10.40	October 2008	−17.23	−9.64

Stylized Facts

The CAPM's general problem, the beta "anomaly," July 1963–June 2017, Fama and French (2006)

	L	2	3	4	5	6	7	8	9	H	H−L
m	0.52	0.52	0.56	0.58	0.69	0.55	0.67	0.55	0.57	0.55	0.03
t_m	3.85	3.64	3.45	3.38	3.75	2.86	3.14	2.42	2.23	1.72	0.11
α	0.22	0.17	0.13	0.12	0.18	0.01	0.07	−0.08	−0.13	−0.29	−0.52
t_α	2.11	1.76	1.69	1.42	2.17	0.18	0.85	−0.82	−1.10	−1.49	−1.94
β	0.57	0.68	0.82	0.87	0.98	1.03	1.15	1.22	1.34	1.62	1.06
t_β	12.39	17.21	20.57	20.68	28.13	31.21	50.25	41.76	35.41	30.92	11.81
R^2	0.53	0.68	0.77	0.79	0.86	0.86	0.88	0.86	0.84	0.77	0.43

Stylized Facts

The CAPM's general problem, the beta "anomaly," July 1928–June 2017, Fama and French (2006)

	L	2	3	4	5	6	7	8	9	H	H−L
m	0.58	0.63	0.65	0.74	0.83	0.72	0.79	0.73	0.77	0.75	0.16
t_m	5.03	4.66	4.41	4.46	4.54	3.71	3.74	3.11	2.94	2.44	0.66
α	0.22	0.16	0.13	0.14	0.17	0.01	0.02	−0.13	−0.17	−0.33	−0.55
t_α	2.87	2.22	2.21	2.31	2.49	0.20	0.27	−1.51	−1.68	−2.29	−2.81
β	0.57	0.73	0.83	0.94	1.05	1.11	1.22	1.36	1.48	1.70	1.13
t_β	22.86	30.50	36.61	40.31	41.41	39.61	48.26	36.17	26.65	40.93	18.82
R^2	0.66	0.81	0.85	0.88	0.90	0.90	0.91	0.90	0.88	0.84	0.57

Stylized Facts

The consumption CAPM, 25 size and b/m portfolios, annual sample, 1930–2016

	Low	2	3	4	High	Low	2	3	4	High
			m					β^C		
Small	6.04	10.65	13.73	16.82	18.56	2.80	0.66	1.63	1.86	1.58
2	9.02	12.32	13.33	14.90	16.03	1.25	1.72	0.88	1.25	1.68
3	9.27	11.83	11.88	13.73	14.72	0.29	1.11	1.77	2.12	2.15
4	8.82	9.68	11.49	12.83	13.16	0.38	0.37	1.32	1.36	0.47
Big	7.46	7.38	8.90	8.36	11.58	1.05	0.59	1.79	2.26	−0.88

Stylized Facts

The consumption CAPM, 25 size and b/m portfolios, quarterly sample, 1947:Q2–2017:Q2

	Low	2	3	4	High	Low	2	3	4	High
	m					β^C				
Small	1.25	2.58	2.57	3.23	3.65	4.22	4.73	3.43	3.63	3.94
2	1.74	2.58	2.86	3.01	3.38	3.01	2.89	2.91	3.07	3.60
3	1.96	2.61	2.54	2.99	3.26	2.85	2.59	2.57	2.63	2.99
4	2.18	2.18	2.60	2.74	2.93	2.47	2.16	2.54	2.39	3.77
Big	1.90	1.90	2.18	1.98	2.47	2.62	1.94	1.97	2.60	2.80

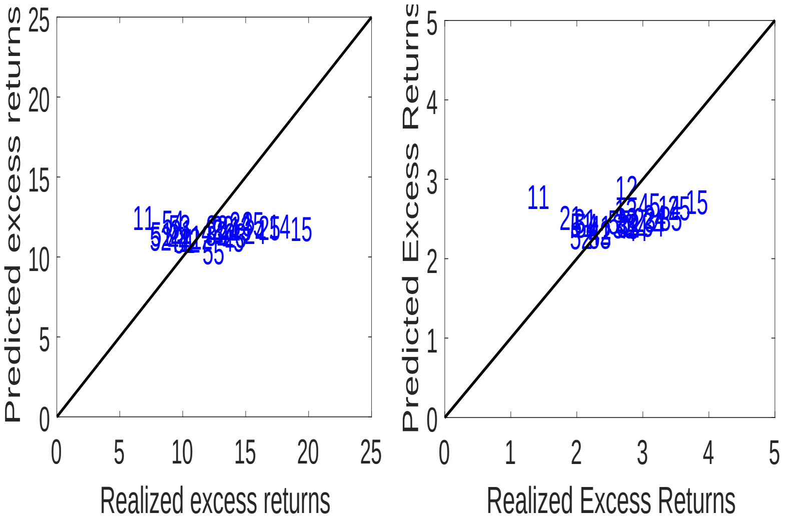
Stylized Facts

The consumption CAPM, 25 size and b/m portfolios, second-stage cross-sectional regressions

	Panel A: Annual, 1930–2016		Panel B: Quarterly, 1947:Q2–2017:Q2	
	ϕ_0	ϕ_1	ϕ_0	ϕ_1
Estimates	10.97	0.58	1.88	0.22
t_{FM}	4.14	1.16	3.73	1.12
t_S	3.99	1.13	3.42	1.03
R^2		0.02		0.07

Stylized Facts

Scatter plots, the consumption CAPM, 25 size and b/m portfolios, annual versus quarterly



Outline

Embedding disasters into a general equilibrium production economy:

- Rare disasters in productivity growth (Rietz 1988, Barro 2006)
- Endogenous cross-sectional distribution with asymmetric adjustment costs (Zhang 2005)
- Approximate aggregation (Krusell and Smith 1998)

Value firms are more exposed to disaster risk than growth firms

The representative household has recursive utility:

$$U_t = \left[(1-\iota) C_t^{1-\frac{1}{\psi}} + \iota \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}},$$

The pricing kernel:

$$M_{t+1} = \iota \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{1-\gamma}}{E_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

Operating profits:

$$\Pi_{it} = (X_t Z_{it})^{1-\xi} K_{it}^{\xi} - f K_{it}$$

Aggregate log productivity growth:

$$g_{xt} = \overline{g} + g_t$$

Firm-specific log productivity:

$$z_{it+1} = (1 - \rho_z)\bar{z} + \rho_z z_{it} + \sigma_z e_{it+1}$$

The Model

Normal times, Rouwenhorst 1995

g_t follows a discretized autoregressive process:

- Five states: $\{g_1, g_2, g_3, g_4, g_5\}$
- Transition matrix: $p_{ij} \equiv \text{Prob}(g_{t+1} = g_i | g_t = g_j)$:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{15} \\ p_{21} & p_{22} & \cdots & p_{25} \\ \vdots & \vdots & \ddots & \vdots \\ p_{51} & p_{52} & \cdots & p_{55} \end{bmatrix}$$

Insert the disaster state, $g_0 = \lambda_D < 0$ and the recovery state, $g_6 = \lambda_R > 0$

Modify the transition matrix:

$$P = \begin{bmatrix} \theta & 0 & 0 & \dots & 0 & 1-\theta \\ \eta & p_{11}-\eta & p_{12} & \dots & p_{15} & 0 \\ \eta & p_{21} & p_{22}-\eta & \dots & p_{25} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta & p_{51} & p_{52} & \dots & p_{55}-\eta & 0 \\ 0 & (1-\nu)/5 & (1-\nu)/5 & \dots & (1-\nu)/5 & \nu \end{bmatrix}$$

η: disaster probability, θ: persistence, ν: recovery persistence

Capital accumulation:

$$K_{it+1} = I_{it} + (1-\delta)K_{it}$$

Asymmetric capital adjustment costs:

$$\Phi(I_{it}, K_{it}) = \begin{cases} a^{+}K_{it} + \frac{c^{+}}{2}\left(\frac{I_{it}}{K_{it}}\right)^{2} K_{it} & \text{for} \quad I_{it} > 0 \\ 0 & \text{for} \quad I_{it} = 0 \\ a^{-}K_{it} + \frac{c^{-}}{2}\left(\frac{I_{it}}{K_{it}}\right)^{2} K_{it} & \text{for} \quad I_{it} < 0 \end{cases}$$

in which $c^{-} > c^{+} > 0$ and $a^{-} > a^{+} > 0$

The cross-sectional distribution of firms, μ_t, including K_{it} and z_{it}:

$$\mu_{t+1} = \Upsilon(\mu_t, X_t, X_{t+1}).$$

Value maximization:

$$V_{it} = \max_{\{\chi_{it}\}} \left(\max_{\{I_{it}\}} \; D_{it} + E_t \left[M_{t+1} V(K_{it+1}, Z_{it+1}; X_{t+1}, \mu_{t+1}) \right], \; sK_{it} \right),$$

in which $D_{it} = \Pi_{it} - I_{it} - \Phi(I_{it}, K_{it})$ is net payout

Entry and exit, delisting return, reorganizational costs

$E_t[M_{t+1}R_{it+1}] = 1$ implies:

$$E_t[R_{it+1}] = r_{ft} + \left(-\frac{\text{Cov}_t[R_{it+1}, M_{t+1}]}{\text{Var}_t[M_{t+1}]}\right)\frac{\text{Var}_t[M_{t+1}]}{E_t[M_{t+1}]} \equiv r_{ft} + \beta_{it}^M \phi_{Mt}$$

- $r_{ft} \equiv 1/E_t[M_{t+1}]$: The real interest rate
- β_i^M: The true beta
- ϕ_{Mt}: The price of consumption risk

A competitive equilibrium consists of an optimal investment rule, $I(K_{it}, Z_{it}; X_t, \mu_t)$; an optimal exit rule, $\chi(K_{it}, Z_{it}; X_t, \mu_t)$; a value function, $V(K_{it}, Z_{it}; X_t, \mu_t)$; and an equilibrium law of motion for the firm distribution, $\Upsilon(\mu_t, X_t, X_{t+1})$, such that:

- Optimality: $I(K_{it}, Z_{it}; X_t, \mu_t)$, $\chi(K_{it}, Z_{it}; X_t, \mu_t)$, and $V(K_{it}, Z_{it}; X_t, \mu_t)$ solve the firms' problem

- Consistency: The aggregate behavior of the economy is consistent with the optimal behavior of all firms:

$$
\begin{aligned}
Y_t &= \int Y_{it}\, \mu_t(dK_{it}, dZ_{it}) \\
I_t &= \int I_{it}\, \mu_t(dK_{it}, dZ_{it}) \\
K_t &= \int K_{it}\, \mu_t(dK_{it}, dZ_{it}) \\
\Phi_t &= \int \Phi_{it}\, \mu_t(dK_{it}, dZ_{it})
\end{aligned}
$$

- Consistency: The law of motion for the firm distribution, Υ, is consistent with the optimal decisions of firms. Let Θ be any measurable set in the product space of K_{it} and Z_{it}:

$$\mu_{t+1}(\Theta, X_{t+1}) = T(\Theta, (K_{it}, Z_{it}), X_t)\, \mu_t(K_{it}, Z_{it}, X_t),$$

in which $T(\Theta, (K_{it}, Z_{it}), X_t) \equiv$:

$$\iint \mathbf{1}_{\{(I_{it}+(1-\delta)K_{it}, Z_{it+1})\in\Theta\}}\, Q_Z(dZ_{it+1}|Z_{it}) Q_X(dX_{t+1}|X_t),$$

and $\mathbf{1}_{\{\cdot\}}$ is an indicator function that takes the value of one if the event described in $\{\cdot\}$ is true, and zero otherwise, and Q_Z and Q_X are the transition functions for Z_{it} and X_t, respectively

- Market clearing: Aggregate consumption equals aggregate output minus investment:

$$C_t = Y_t - I_t \qquad \Rightarrow \qquad C_t = D_t + fK_t + \Phi_t$$

The fixed costs of production, fK_t, and capital adjustment costs, Φ_t, as compensation to labor and part of consumption, driving a wedge between C_t and D_t (Abel 1999)

The Model

Approximate aggregation, Krusell and Smith 1998

Detrending with X_{t-1}, e.g., $\widehat{K}_{it} = K_{it}/X_{t-1}$

Assume the average detrended capital, $\overline{K}_t$, contains all the information in μ_t relevant for forecasting M_{t+1}

The detrended value function, $\widehat{V}(\widehat{K}_{it}, Z_{it}, g_t, \overline{K}_t) =$

$$\max_{\{\chi_{it}\}} \left[\max_{\{\widehat{K}_{it+1}\}} \widehat{D}_{it} + E_t \left[M_{t+1} \widehat{V}(\widehat{K}_{it+1}, Z_{it+1}, g_{t+1}, \overline{K}_{t+1}) \right] \exp(g_{xt}),\ s\widehat{K}_{it} \right]$$

The Model

Approximate aggregation, Krusell and Smith 1998

The equilibrium laws of motion:

$$\begin{aligned} \log \widehat{C}_t^{(j+1)}(g_t = g_i) &= a_{0i}^{(j+1)} + a_{1i}^{(j+1)} \log \overline{K_t} + a_{2i}^{(j+1)} \left(\log \overline{K_t}\right)^2 \\ \log \overline{K}_{t+1}^{(j+1)}(g_t = g_i) &= b_{0i}^{(j+1)} + b_{1i}^{(j+1)} \log \overline{K_t} + b_{2i}^{(j+1)} \left(\log \overline{K_t}\right)^2 \end{aligned}$$

Check the convergence for the coefficients, for $l = \{0, 1, 2\}$:

$$\max_{i \in [1,7]} |a_{li}^{(j+1)} - a_{li}^{(j)}| < 10^{-2}, \quad \max_{i \in [1,7]} |b_{li}^{(j+1)} - b_{li}^{(j)}| < 10^{-3}$$

Otherwise update the coefficients with the Newton method

$R^2 = 0.9999983$ for $\overline{K}_t$ and 0.99494656 for $\widehat{C}_t$, with $N = 30,000$

Outline

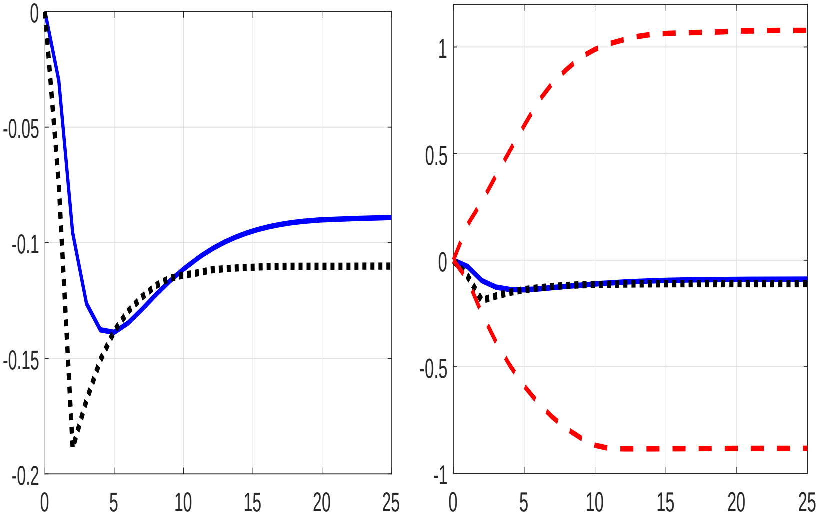
Calibration and Basic Moments

The benchmark monthly calibration

ι	γ	ψ	$\bar{g}$	ρ_g	σ_g
0.9945	5	1.5	0.019/12	0.6	0.003
η	λ_D	θ	λ_R	ν	ξ
0.02/12	-0.0275	$0.914^{1/3}$	0.015	0.964	0.65
δ	f	$\bar{z}$	ρ_z	σ_z	a^+
0.01	0.005	-8.52	0.985	0.5	0.035
a^-	c^+	c^-	s	κ	$\widetilde{R}$
0.05	75	150	0	0.25	-12.33%

Calibration and Basic Moments

The impulse response of consumption to a disaster shock mimics that in Nakamura, Steinsson, Barro, and Ursua (2013)



Calibration and Basic Moments

Aggregate output growth

	Samples with disasters, annual							Samples without disasters, quarterly					
	Data	mean	2.5%	50%	97.5%	p		Data	mean	2.5%	50%	97.5%	p
Vol	4.79	4.41	1.37	4.26	8.50	0.41		0.94	0.50	0.44	0.49	0.65	0.00
Skew	−0.29	−1.89	−4.32	−2.09	2.07	0.15		−0.18	0.02	−0.32	−0.02	1.02	0.88
Kurt	6.14	11.43	2.95	9.54	27.52	0.78		4.51	3.05	2.41	2.90	5.11	0.04
ρ_1	0.54	0.69	0.27	0.73	0.93	0.80	ρ_1	0.37	0.43	0.30	0.42	0.63	0.82
ρ_2	0.19	0.38	−0.15	0.40	0.82	0.74	ρ_4	−0.07	0.11	−0.06	0.09	0.35	0.99
ρ_3	−0.14	0.23	−0.22	0.21	0.72	0.92	ρ_8	−0.02	0.07	−0.09	0.06	0.26	0.82
ρ_4	−0.34	0.14	−0.26	0.12	0.62	0.99	ρ_{12}	−0.12	0.05	−0.10	0.04	0.24	0.99
ρ_5	−0.19	0.09	−0.25	0.07	0.53	0.94	ρ_{20}	0.05	0.02	−0.13	0.02	0.19	0.35

Calibration and Basic Moments

Aggregate consumption growth

	Samples with disasters, annual							Samples without disasters, quarterly					
	Data	mean	2.5%	50%	97.5%	p		Data	mean	2.5%	50%	97.5%	p
Vol	2.13	4.28	1.30	4.13	8.28	0.87		0.50	0.46	0.40	0.45	0.60	0.09
Skew	−1.48	−1.93	−4.42	−2.14	2.13	0.32		−0.41	0.02	−0.31	−0.03	1.14	0.99
Kurt	8.09	11.66	2.98	9.63	28.82	0.63		4.17	3.10	2.44	2.93	5.83	0.04
ρ_1	0.48	0.69	0.24	0.74	0.93	0.85	ρ_1	0.31	0.44	0.31	0.44	0.66	0.97
ρ_2	0.18	0.39	−0.15	0.42	0.83	0.75	ρ_4	0.10	0.13	−0.05	0.12	0.39	0.61
ρ_3	−0.05	0.24	−0.22	0.23	0.72	0.86	ρ_8	−0.02	0.08	−0.08	0.08	0.30	0.86
ρ_4	−0.19	0.16	−0.24	0.13	0.63	0.95	ρ_{12}	0.08	0.06	−0.10	0.05	0.28	0.35
ρ_5	0.00	0.10	−0.24	0.08	0.55	0.70	ρ_{20}	−0.04	0.03	−0.13	0.03	0.21	0.83

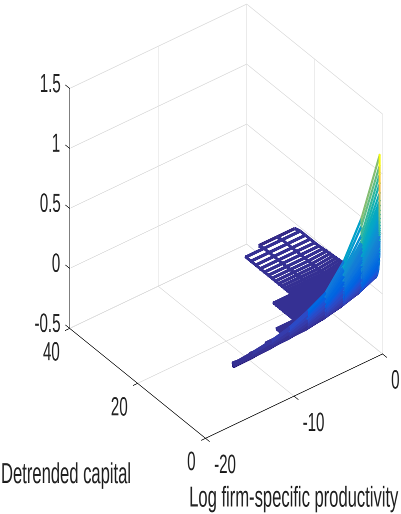
Calibration and Basic Moments

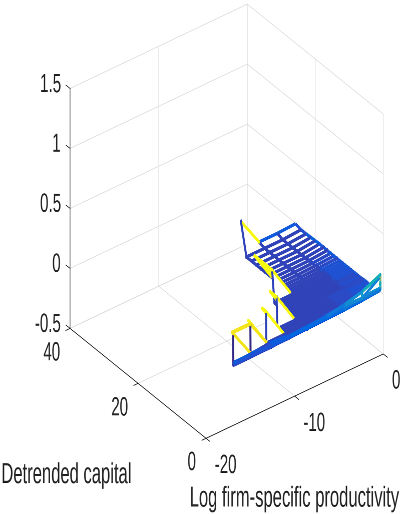
Aggregate investment growth

	Samples with disasters, annual							Samples without disasters, quarterly					
	Data	mean	2.5%	50%	97.5%	p		Data	mean	2.5%	50%	97.5%	p
Vol	13.53	19.56	3.10	12.28	71.84	0.45		2.40	1.09	0.98	1.08	1.33	0.00
Skew	−1.33	−0.17	0.02	−1.56	2.69	0.68		−0.53	−0.20	−0.58	−0.20	0.25	0.96
Kurt	7.07	27.45	6.68	19.50	100.98	0.96		4.73	3.70	2.85	3.41	5.26	0.03
ρ_1	0.41	0.18	0.00	0.23	0.59	0.17	ρ_1	0.46	0.24	0.11	0.24	0.38	0.01
ρ_2	−0.15	−0.06	0.00	0.00	−0.44	0.71	ρ_4	−0.03	−0.00	−0.12	−0.01	0.14	0.63
ρ_3	−0.33	−0.07	0.00	0.00	0.38	0.96	ρ_8	−0.18	−0.01	−0.12	−0.01	0.11	1.00
ρ_4	−0.17	−0.06	−0.00	0.00	−0.07	0.84	ρ_{12}	−0.09	−0.01	−0.13	−0.01	0.11	0.90
ρ_5	−0.05	−0.05	−0.00	−0.05	−0.06	0.57	ρ_{20}	0.03	−0.00	−0.12	0.00	0.11	0.29

Theoretical Properties

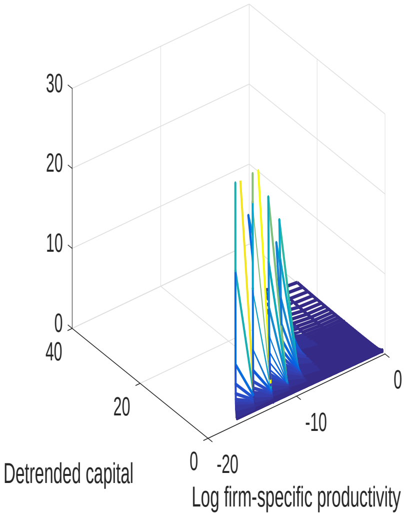
Optimal policy functions

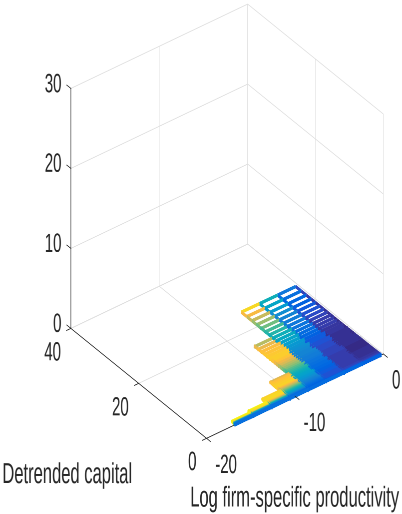




Theoretical Properties

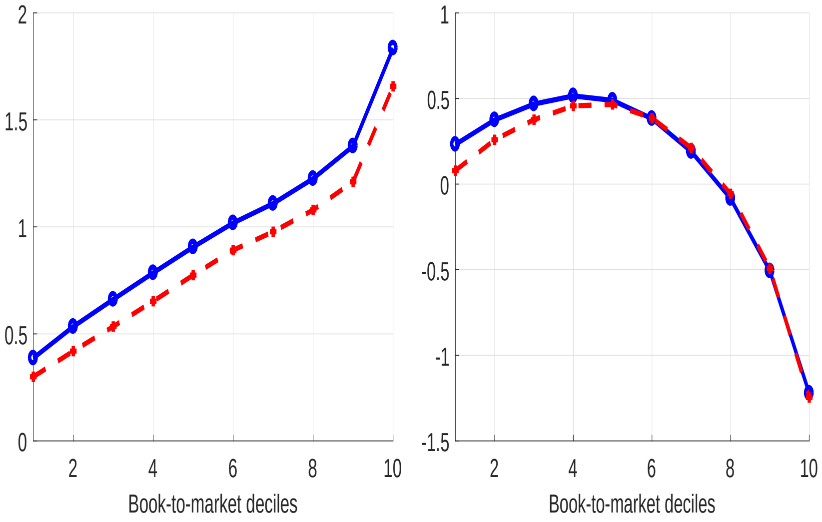
The true beta





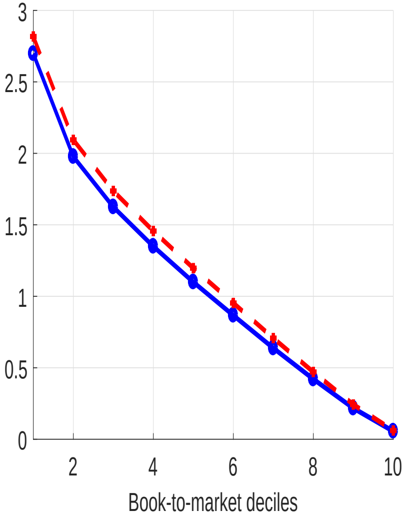
Theoretical Properties

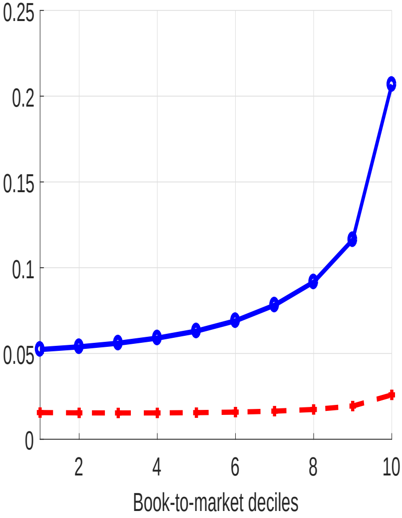
Value versus growth: $\widehat{K}_{it}$ and z_{it}



Theoretical Properties

Value versus growth: $\widehat{I}_{it}/\widehat{K}_{it}$ and β_{it}^M





The CAPM regressions for the b/m deciles, no-disaster samples

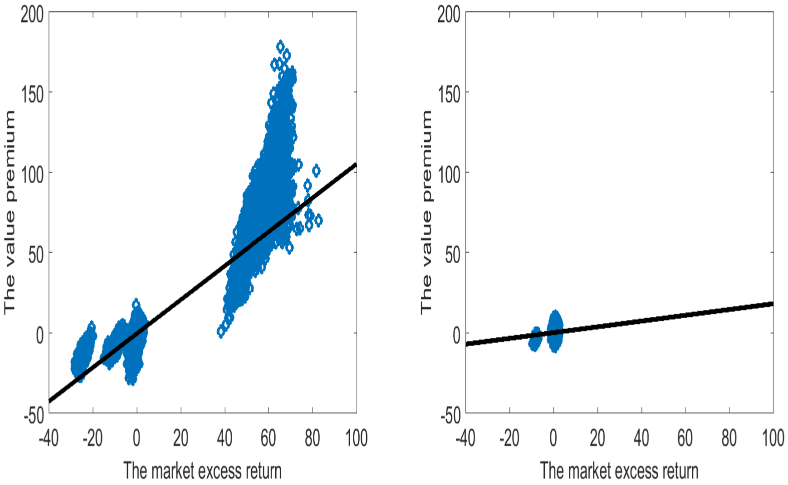
	L	2	3	4	5	6	7	8	9	H	H−L
m	0.77	0.76	0.75	0.74	0.75	0.76	0.78	0.82	0.91	1.16	0.40
t_m	23.37	23.02	22.48	22.05	22.08	21.79	22.75	23.93	25.51	28.69	7.72
α	0.10	0.04	−0.02	−0.07	−0.10	−0.13	−0.07	0.02	0.13	0.35	0.25
t_α	1.46	0.57	−0.22	−0.99	−1.37	−1.80	−0.93	0.32	1.83	4.25	2.26
α, 2.5%	−0.04	−0.09	−0.16	−0.20	−0.24	−0.26	−0.20	−0.12	−0.00	0.17	0.02
α, 97.5%	0.25	0.18	0.12	0.08	0.05	0.00	0.06	0.16	0.27	0.51	0.49
t_α, 2.5%	−0.55	−1.21	−2.21	−2.82	−3.24	−3.62	−2.78	−1.63	−0.01	1.77	0.18
t_α, 97.5%	3.61	2.68	1.68	1.16	0.88	0.02	0.88	2.46	3.87	6.61	4.37
β	0.83	0.90	0.96	1.02	1.06	1.10	1.06	1.00	0.97	1.01	0.18
t_β	11.04	11.91	12.60	13.23	13.69	14.06	13.58	12.94	11.89	10.64	1.44
β, 2.5%	0.67	0.74	0.81	0.86	0.89	0.97	0.90	0.84	0.80	0.80	−0.09
β, 97.5%	0.98	1.05	1.11	1.18	1.23	1.24	1.22	1.15	1.13	1.20	0.47
t_β, 2.5%	8.56	9.05	10.40	10.63	11.07	11.51	10.92	10.29	9.55	7.66	−0.70
t_β, 97.5%	14.75	16.68	15.84	16.53	16.60	17.57	17.53	17.10	15.10	13.49	3.59
R^2	0.10	0.12	0.13	0.14	0.15	0.16	0.15	0.13	0.12	0.10	0.00

The CAPM regressions for the b/m deciles, disaster samples

	L	2	3	4	5	6	7	8	9	H	H−L
m	0.75	0.74	0.74	0.74	0.75	0.77	0.81	0.86	0.96	1.20	0.46
t_m	11.17	10.95	10.73	10.50	10.29	10.02	9.83	9.59	9.29	8.94	4.92
α	0.08	0.06	0.04	0.03	0.01	−0.02	−0.05	−0.09	−0.15	−0.27	−0.35
t_α	1.75	1.55	1.22	0.74	0.18	−0.54	−1.10	−1.60	−2.05	−2.32	−2.44
α, 2.5%	−0.03	−0.03	−0.04	−0.05	−0.08	−0.12	−0.19	−0.29	−0.42	−0.70	−0.86
α, 97.5%	0.21	0.16	0.13	0.10	0.08	0.06	0.05	0.04	0.02	0.00	0.00
t_α, 2.5%	−0.84	−0.91	−1.03	−1.53	−2.11	−3.01	−3.63	−4.19	−4.16	−4.33	−4.53
t_α, 97.5%	4.43	3.99	3.56	2.92	2.29	1.90	1.39	1.07	0.57	0.05	0.05
β	0.83	0.85	0.87	0.89	0.93	0.99	1.07	1.19	1.40	1.84	1.01
t_β	35.57	42.36	51.84	69.25	74.28	65.01	53.50	38.76	25.28	18.49	7.85
β, 2.5%	0.66	0.73	0.79	0.84	0.87	0.90	0.94	1.00	1.13	1.47	0.52
β, 97.5%	0.98	0.96	0.94	0.96	1.04	1.18	1.33	1.57	1.85	2.32	1.61
t_β, 2.5%	8.64	12.52	17.67	22.78	18.11	12.68	10.11	8.22	7.10	7.45	3.47
t_β, 97.5%	132.89	133.16	145.36	174.58	184.14	169.89	166.37	139.47	77.09	42.45	17.28
R^2	0.77	0.78	0.79	0.79	0.80	0.81	0.83	0.85	0.86	0.87	0.57

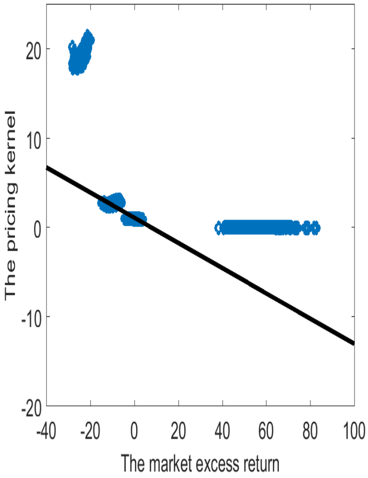
The CAPM

Nonlinearity in the CAPM regressions, disasters vs. no disasters



The CAPM

Nonlinearity in the pricing kernel, disasters vs. no disasters



The Beta "Anomaly"

Deciles formed on rolling market betas, disaster samples

	L	2	3	4	5	6	7	8	9	H	H−L
m	0.77	0.79	0.81	0.83	0.82	0.85	0.85	0.85	0.85	0.83	0.06
t_m	10.48	10.68	10.54	10.26	9.78	9.83	9.57	9.27	8.69	8.31	0.85
α	0.03	0.05	0.04	0.02	−0.02	−0.03	−0.05	−0.09	−0.16	−0.21	−0.24
t_α	0.70	1.36	1.17	0.46	−0.49	−0.53	−0.91	−1.23	−1.64	−2.15	−1.74
α, 2.5%	−0.12	−0.04	−0.04	−0.06	−0.13	−0.15	−0.22	−0.29	−0.47	−0.55	−0.67
α, 97.5%	0.16	0.15	0.12	0.11	0.09	0.09	0.09	0.08	0.07	0.04	0.11
t_α, 2.5%	−2.92	−1.17	−1.09	−1.75	−3.10	−3.33	−3.80	−4.06	−4.39	−4.78	−4.52
t_α, 97.5%	3.66	3.77	3.26	2.84	2.29	2.33	2.33	2.17	1.84	1.05	1.86
β	0.92	0.92	0.96	1.01	1.05	1.09	1.12	1.16	1.25	1.28	0.37
t_β	35.79	48.38	62.91	74.19	61.90	48.67	41.79	36.71	28.14	20.98	2.57
β, 2.5%	0.78	0.84	0.90	0.93	0.94	0.96	0.95	0.95	0.94	0.92	−0.09
β, 97.5%	1.12	1.03	1.04	1.08	1.17	1.24	1.31	1.41	1.64	1.72	0.93
t_β, 2.5%	9.71	15.76	21.14	21.10	17.95	13.39	9.96	7.94	5.54	5.57	−2.85
t_β, 97.5%	134.73	167.31	168.44	192.12	192.23	156.44	157.07	154.49	140.85	79.15	7.00
R^2	0.81	0.81	0.82	0.82	0.82	0.84	0.84	0.84	0.85	0.85	0.21

The Beta "Anomaly"

Deciles formed on rolling market betas, no-disaster samples

	L	2	3	4	5	6	7	8	9	H	H−L
m	0.78	0.81	0.82	0.83	0.81	0.84	0.83	0.81	0.79	0.76	−0.02
t_m	23.48	23.66	23.62	23.53	21.72	23.38	23.13	22.88	22.50	21.87	−0.48
α	−0.05	0.07	0.11	0.13	0.01	0.13	0.10	0.04	−0.05	−0.25	−0.21
t_α	−0.69	0.99	1.58	1.82	0.16	1.77	1.27	0.53	−0.67	−3.67	−1.96
α, 2.5%	−0.17	−0.06	−0.04	−0.01	−0.14	−0.01	−0.04	−0.14	−0.20	−0.37	−0.39
α, 97.5%	0.07	0.22	0.27	0.29	0.16	0.29	0.25	0.17	0.08	−0.12	−0.02
t_α, 2.5%	−2.43	−0.84	−0.47	−0.13	−1.77	−0.16	−0.55	−1.80	−2.73	−5.62	−3.91
t_α, 97.5%	1.01	3.24	3.75	4.14	2.08	3.96	3.26	2.39	1.36	−1.91	−0.15
β	1.03	0.92	0.88	0.87	0.99	0.88	0.91	0.96	1.05	1.26	0.23
t_β	14.06	12.43	11.32	11.06	11.88	11.17	11.18	12.21	13.84	16.87	1.98
β, 2.5%	0.89	0.75	0.72	0.68	0.83	0.69	0.74	0.81	0.88	1.10	−0.00
β, 97.5%	1.17	1.08	1.04	1.02	1.15	1.04	1.08	1.15	1.20	1.41	0.46
t_β, 2.5%	11.01	9.35	8.88	8.28	9.19	8.06	8.43	9.32	10.32	13.45	−0.01
t_β, 97.5%	17.00	16.61	14.24	15.16	15.76	13.54	14.07	14.84	16.66	21.02	4.19
R^2	0.16	0.12	0.11	0.10	0.12	0.10	0.11	0.13	0.15	0.22	0.00

The Consumption CAPM

First-stage regressions, 25 size and b/m portfolios, annual samples with disasters

	Low	2	3	4	High	Low	2	3	4	High
	m					β^C				
Small	13.69	14.54	15.95	17.90	23.37	−0.64	−0.77	−0.93	−1.15	−1.28
2	12.33	13.29	14.21	15.45	18.90	−0.49	−0.59	−0.72	−0.89	−1.34
3	12.05	12.17	12.42	12.95	14.62	−0.43	−0.47	−0.53	−0.64	−0.74
4	10.57	10.40	10.42	10.85	13.84	−0.32	−0.33	−0.36	−0.46	−0.69
Big	7.96	7.92	8.18	8.86	10.14	−0.07	−0.08	−0.10	−0.22	−0.23

The Consumption CAPM

First-stage regressions, 25 size and b/m portfolios, quarterly samples without disasters

	Low	2	3	4	High	Low	2	3	4	High
			m					β^C		
Small	3.16	3.31	3.56	3.92	5.17	0.11	0.12	0.12	0.13	0.27
2	2.89	3.08	3.24	3.45	4.09	0.12	0.12	0.13	0.13	0.18
3	2.84	2.85	2.88	2.96	3.33	0.16	0.13	0.14	0.16	0.25
4	2.53	2.48	2.47	2.53	3.19	0.16	0.18	0.22	0.24	0.24
Big	1.93	1.91	1.96	2.07	2.42	0.74	0.93	1.08	0.94	0.85

The Consumption CAPM

Second-stage regressions, 25 size and b/m portfolios

	Annual, disasters		Quarterly, no disasters	
	ϕ_0	ϕ_1	ϕ_0	ϕ_1
Estimates	9.02	−6.59	3.34	−1.19
2.5%	5.24	−13.52	3.16	−1.73
97.5%	13.41	1.09	3.53	−0.76
t_{FM}	15.26	−6.41	73.52	−13.68
2.5%	6.50	−13.36	50.28	−18.35
97.5%	50.48	1.00	83.57	−8.61
t_S	7.97	−3.34	44.02	−9.16
2.5%	3.84	−5.82	27.09	−10.89
97.5%	23.10	0.99	57.68	−6.85
R^2		0.63		0.29
2.5%		0.00		0.12
97.5%		0.94		0.49

The Consumption CAPM

First-stage regressions with M_{t+1}, 25 size and b/m portfolios, annual samples with disasters

	Low	2	3	4	High	Low	2	3	4	High
	$\hat{\beta}^M$					$t_{\hat{\beta}^M}$				
Small	0.04	0.04	0.04	0.05	0.07	8.26	7.87	7.58	7.20	7.08
2	0.03	0.04	0.04	0.04	0.05	8.51	8.25	8.04	7.85	7.71
3	0.03	0.03	0.03	0.04	0.04	8.26	8.53	8.34	8.03	7.49
4	0.03	0.03	0.03	0.03	0.04	8.94	8.79	8.63	8.16	8.47
Big	0.02	0.02	0.02	0.02	0.03	8.79	8.53	8.26	7.76	7.49

The Consumption CAPM

First-stage regressions with M_{t+1}, 25 size and b/m portfolios, quarterly samples without disasters

	Low	2	3	4	High	Low	2	3	4	High
			$\hat{\beta}^M$					$t_{\hat{\beta}^M}$		
Small	0.12	0.13	0.14	0.15	0.25	7.78	5.74	5.20	5.32	6.11
2	0.12	0.12	0.13	0.14	0.17	5.09	5.26	5.36	5.49	5.17
3	0.12	0.12	0.12	0.12	0.15	2.99	5.10	5.32	4.95	3.70
4	0.11	0.11	0.11	0.11	0.15	4.29	4.27	4.08	3.53	3.57
Big	0.09	0.10	0.10	0.10	0.12	2.66	2.81	2.91	2.99	2.90

The Consumption CAPM

Second-stage regressions with M_{t+1}, 25 size and b/m portfolios

	Annual, disasters		Quarterly, no disasters	
	$\hat{\phi}_0$	$\hat{\phi}_M$	$\hat{\phi}_0$	$\hat{\phi}_M$
Estimates	0.01	5.18	0.02	0.11
2.5%	−0.01	0.35	0.01	0.06
97.5%	0.06	7.66	0.02	0.26
t_{FM}	2.44	8.36	19.22	15.47
2.5%	−1.48	3.17	7.33	8.77
97.5%	17.72	18.90	30.19	20.62
t_{S}	0.91	3.56	6.80	5.42
2.5%	−0.61	1.54	1.94	3.78
97.5%	5.74	7.09	14.16	8.11
R^2		0.89		0.43
2.5%		0.52		0.13
97.5%		0.97		0.79

Conclusion

Bai, Hou, Kung, Li, and Zhang (2018)

A general equilibrium heterogeneous firms economy with disasters explains the failure of the (consumption) CAPM with strong nonlinearity in the pricing kernel

The widely documented failures of the (consumption) CAPM might have more to do with deficiencies of empirical tests, rather than deficiencies of standard economic theory