# The Economics of Value Investing

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The investment CAPM provides an economic foundation for Graham and Dodd's (1934) *Security Analysis*, without mispricing

# Summary

Cross-sectionally varying expected returns, depending on firms' investment, profitability, and expected investment growth

Empirically, many anomaly variables predict future investment growth, in the same direction in which these variables predict future returns, but the amount of growth predictability is small

The investment CAPM has different properties from the residual income model and the Penman-Reggiani-Richardson-Tuna model

# Outline

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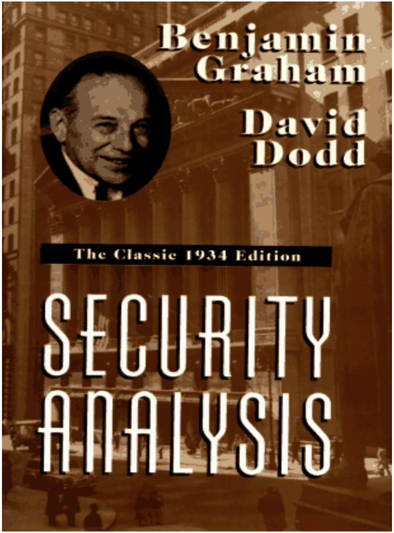
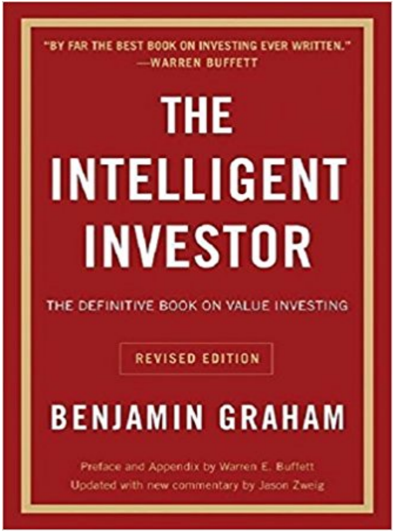

Benjamin Graham
David Dodd
The Classic 1934 Edition
SECURITY ANALYSIS


"BY FAR THE BEST BOOK ON INVESTING EVER WRITTEN."
—WARREN BUFFETT
THE INTELLIGENT INVESTOR
THE DEFINITIVE BOOK ON VALUE INVESTING
REVISED EDITION
BENJAMIN GRAHAM
Preface and Appendix by Warren E. Buffett
Updated with new commentary by Jason Zweig

Invest in undervalued securities selling well below the intrinsic value

The intrinsic value is the value that can be justified by the firm's earnings, assets, and other accounting information

The intrinsic value is distinct from the market value that is subject to artificial manipulation and psychological distortion

Maintain margin of safety, the intrinsic-market value distance

Security analysis is "concerned with the intrinsic value of the security and more particularly with the discovery of discrepancies between the intrinsic value and the market price (p. 17)"

The intelligent investor "would be well advised to devote his attention to the field of undervalued securities—issues, whether bonds or stocks, which are selling well below the levels apparently justified by a careful analysis of the relevant facts (p. 13)"

# Security Analysis

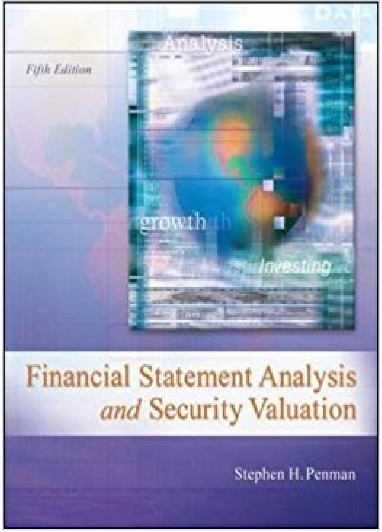
## Warren Buffett (1984), honoring the 50th anniversary of Graham and Dodd (1934)

**The Superinvestors of Graham-and-Doddsville**

By Warren E. Buffett

*"Superinvestor" Warren E. Buffett, who got an A+ from Ben Graham at Columbia in 1951, never stopped making the grade. He made his fortune using the principles of Graham & Dodd's* Security Analysis. *Here, in celebration of the fiftieth anniversary of that classic text, he tracks the records of investors who stick to the "value approach" and have gotten rich going by the book.*

"Our Graham & Dodd investors, needless to say, do not discuss beta, the capital asset pricing model or covariance in returns among securities. These are not subjects of any interest to them. In fact, most of them would have difficulty defining those terms (p. 7)"




"Passive investors accept market prices as fair value. Fundamental investors, in contrast, are active investors. They see that price is what you pay, value is what you get. They understand that the primary risk in investing is the risk of paying too much (or selling for too little). The fundamentalist actively challenges the market price: Is it indeed a fair price (p. 210, original emphasis)?"

# Outline

The first principle of consumption:

$$E_t[M_{t+1} r^S_{it+1}] = 1,$$

in which $r^S_{it+1} = (P_{it+1} + D_{it+1})/P_{it}$ is firm $i$'s stock return, and $M_{t+1} = \rho U'(C_{t+1})/U'(C_t)$ is the stochastic discount factor

Equivalently,

$$E_t[r^S_{it+1}] - r_{ft} = \beta^M_{it} \lambda_{Mt},$$

in which $r_{ft} = 1/E_t[M_{t+1}]$ is the real interest rate,
$\beta^M_{it} = -\text{Cov}(r^S_{it+1}, M_{t+1})/\text{Var}(M_{t+1})$ is the consumption beta, and
$\lambda_{Mt} = \text{Var}(M_{t+1})/E_t[M_{t+1}]$ is the price of the consumption risk

An individual firm $i$ maximizes:

$$P_{it} + D_{it} \equiv \max_{\{I_{it+s}, A_{it+s+1}, B_{it+s+1}\}_{s=0}^{\infty}} E_t\left[\sum_{s=0}^{\infty} M_{t+s} D_{it+s}\right],$$

in which

$$D_{it} = (1-\tau)[X_{it}A_{it} - \Phi(I_{it}, A_{it})] - I_{it} + B_{it+1} - r_{it}^{B} B_{it} + \tau\delta A_{it} + \tau(r_{it}^{B} - 1)B_{it}$$

The first principle of investment, $E_t[M_{t+1} r^I_{it+1}] = 1$, in which:

$$r^I_{it+1} = \frac{(1-\tau)\left[X_{it+1} + \frac{a}{2}\left(\frac{I_{it+1}}{A_{it+1}}\right)^2\right] + \tau\delta + (1-\delta)\left[1 + (1-\tau)a\left(\frac{I_{it+1}}{A_{it+1}}\right)\right]}{1 + (1-\tau)a\left(\frac{I_{it}}{A_{it}}\right)}$$

The weighted average cost of capital equation (Modigliani and Miller [1958] Proposition III):

$$r^I_{it+1} = w_{it}\, r^{Ba}_{it+1} + (1 - w_{it})\, r^S_{it+1},$$

in which $w_{it}$ is the market leverage, and $r^{Ba}_{it+1}$ is the after-tax corporate bond return

$$r^S_{it+1} = \frac{(1-\tau)\left[X_{it+1} + \frac{a}{2}\left(\frac{I_{it+1}}{A_{it+1}}\right)^2\right] + \tau\delta + (1-\delta)\left[1 + (1-\tau)a\left(\frac{I_{it+1}}{A_{it+1}}\right)\right]}{(1-w_{it})\left[1 + (1-\tau)a\left(\frac{I_{it}}{A_{it}}\right)\right]} - \frac{w_{it}}{1-w_{it}} r^{Ba}_{it+1}$$

- "Dividend yield" and "capital gain"
- Cross-sectionally varying expected returns:
  Investment-to-assets, expected profitability, and expected investment-to-assets growth
- Security analysis in corporate bonds

Both the consumption and investment CAPMs hold in equilibrium, delivering identical expected returns

Complementarity: The consumption CAPM derived from the demand schedule of risky assets, the investment CAPM from supply

Characteristics are sufficient statistics of expected returns: Risk as their sole determinant is a relic and illusion from the CAPM

Relatively immune to the aggregation problem, the investment CAPM is better equipped to explain cross-sectional predictability

## Testing the expected investment growth effect, time series averages of cross-sectional statistics, the full sample, 1/1967–12/2015

| | mean | std | 5% | 25% | 50% | 75% | 95% |
|---|---|---|---|---|---|---|---|
| Sue | 22.11 | 175.55 | −246.34 | −47.79 | 18.75 | 97.18 | 301.99 |
| $R^6$ | 6.78 | 32.37 | −38.46 | −13.23 | 3.10 | 21.35 | 65.60 |
| Ivff | 2.88 | 1.98 | 0.84 | 1.53 | 2.35 | 3.60 | 6.84 |
| Roe | 0.56 | 8.99 | −13.69 | −0.17 | 2.28 | 4.14 | 8.64 |
| log(Bm) | −0.46 | 0.77 | −1.87 | −0.94 | −0.40 | 0.07 | 0.72 |
| I/A | 15.84 | 34.67 | −17.47 | −0.34 | 8.07 | 20.41 | 76.81 |
| Oa | −3.51 | 11.02 | −20.13 | −8.51 | −3.78 | 1.00 | 14.19 |
| Rdm | 7.55 | 9.77 | 0.42 | 1.74 | 4.11 | 9.17 | 27.23 |
| Log(Me) | 4.84 | 1.87 | 1.98 | 3.47 | 4.70 | 6.10 | 8.16 |
| Ep | 1.03 | 21.22 | −32.68 | 0.43 | 6.22 | 9.77 | 16.85 |
| Cop | 12.71 | 12.89 | −9.26 | 6.22 | 13.07 | 19.95 | 33.08 |
| Abr | 0.25 | 8.00 | −12.78 | −3.84 | 0.03 | 4.09 | 13.92 |
| Ilr | 0.96 | 3.67 | −5.06 | −1.40 | 0.96 | 3.29 | 7.06 |
| Nop | 0.61 | 8.67 | −14.53 | −0.49 | 1.12 | 3.89 | 10.22 |
| Dac | −0.24 | 9.30 | −15.50 | −4.33 | −0.16 | 3.91 | 14.75 |
| Nsi | 3.60 | 10.23 | −4.38 | −0.10 | 0.54 | 3.02 | 23.88 |

## Time series averages of cross-sectional statistics, the all-but-micro sample, 1/1967–12/2015

| | mean | std | 5% | 25% | 50% | 75% | 95% |
|---|---|---|---|---|---|---|---|
| Sue | 44.69 | 185.09 | −228.62 | −35.27 | 33.29 | 126.66 | 358.08 |
| $R^6$ | 10.58 | 27.43 | −26.09 | −6.21 | 6.95 | 22.53 | 59.84 |
| Ivff | 2.00 | 1.10 | 0.79 | 1.25 | 1.75 | 2.46 | 3.99 |
| Roe | 2.76 | 6.07 | −4.16 | 1.55 | 3.10 | 4.78 | 9.28 |
| log(Bm) | −0.68 | 0.72 | −2.01 | −1.12 | −0.60 | −0.17 | 0.36 |
| I/A | 17.97 | 31.63 | −8.15 | 2.87 | 9.92 | 21.36 | 73.52 |
| Oa | −3.65 | 8.99 | −16.36 | −7.68 | −3.91 | −0.10 | 10.04 |
| Rdm | 4.63 | 5.61 | 0.34 | 1.30 | 2.87 | 5.77 | 14.80 |
| Log(Me) | 6.48 | 1.22 | 4.91 | 5.52 | 6.25 | 7.26 | 8.86 |
| Ep | 5.44 | 10.65 | −6.49 | 3.81 | 6.73 | 9.42 | 14.61 |
| Cop | 16.35 | 10.63 | 0.25 | 10.15 | 15.84 | 22.16 | 34.78 |
| Abr | 0.48 | 6.50 | −9.80 | −2.91 | 0.31 | 3.78 | 11.24 |
| Ilr | 0.97 | 3.69 | −5.02 | −1.45 | 0.95 | 3.35 | 7.07 |
| Nop | 1.63 | 6.57 | −8.55 | −0.10 | 1.77 | 4.33 | 9.58 |
| Dac | −0.42 | 7.47 | −12.03 | −3.57 | −0.29 | 2.84 | 10.73 |
| Nsi | 3.09 | 9.05 | −4.23 | −0.36 | 0.58 | 2.98 | 20.10 |

## Univariate cross-sectional regressions of $\tau$-quarter-ahead investment-to-assets changes, the full sample, weighted least squares, 1/1973–12/2015

| $\tau$ | | Sue | $R^6$ | Ivff | Roe | log(Bm) | I/A | Oa | Rdm |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Slope | 0.14 | 0.73 | 0.34 | 0.26 | −0.10 | 0.11 | −0.11 | 0.24 |
| | $t$ | 2.33 | 3.70 | 0.58 | 2.07 | −0.68 | 0.17 | −0.90 | 1.56 |
| 2 | Slope | 0.17 | 0.91 | 0.03 | −0.29 | −0.05 | 0.27 | −0.15 | 0.48 |
| | $t$ | 2.84 | 6.35 | 0.05 | −1.74 | −0.23 | 0.34 | −1.19 | 2.41 |
| 3 | Slope | 0.12 | 0.55 | −0.25 | −0.09 | 0.08 | 0.13 | −0.02 | 0.57 |
| | $t$ | 2.11 | 2.89 | −0.46 | −0.51 | 0.49 | 0.15 | −0.11 | 2.78 |
| 4 | Slope | 0.20 | 0.65 | −0.21 | 0.32 | 0.18 | −0.43 | −0.12 | 0.65 |
| | $t$ | 3.46 | 5.05 | −0.28 | 2.10 | 0.98 | −0.56 | −1.09 | 3.08 |
| 8 | Slope | 0.13 | 0.18 | −0.82 | −0.07 | 0.44 | −0.30 | −0.26 | 0.79 |
| | $t$ | 1.85 | 1.06 | −1.36 | −0.43 | 2.57 | −0.34 | −1.58 | 3.56 |
| 12 | Slope | −0.02 | −0.11 | −0.77 | −0.39 | 0.55 | −0.64 | −0.11 | 0.88 |
| | $t$ | −0.42 | −0.56 | −1.16 | −2.18 | 3.29 | −0.73 | −0.93 | 3.39 |

## Univariate cross-sectional regressions of $\tau$-quarter-ahead investment-to-assets changes, the full sample, weighted least squares, 1/1973–12/2015

| $\tau$ | | log(Me) | Ep | Cop | Abr | Ilr | Nop | Dac | Nsi |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Slope | 0.06 | −0.50 | 0.15 | 0.17 | 0.02 | −0.07 | −0.20 | −0.16 |
| | $t$ | 0.56 | −0.69 | 1.38 | 2.48 | 0.11 | −0.12 | −1.14 | −1.20 |
| 2 | Slope | 0.17 | −0.66 | 0.13 | 0.26 | 0.40 | −0.14 | −0.24 | −0.21 |
| | $t$ | 1.56 | −0.66 | 1.18 | 3.28 | 2.36 | −0.24 | −1.46 | −1.63 |
| 3 | Slope | 0.23 | −0.38 | 0.24 | 0.36 | 0.24 | 1.16 | 0.04 | −0.30 |
| | $t$ | 2.41 | −0.45 | 2.68 | 5.20 | 2.81 | 1.82 | 0.22 | −2.24 |
| 4 | Slope | 0.20 | −0.42 | 0.17 | 0.31 | 0.21 | 0.33 | −0.16 | −0.39 |
| | $t$ | 2.60 | −0.45 | 1.57 | 4.99 | 2.54 | 0.84 | −1.32 | −2.90 |
| 8 | Slope | 0.34 | −0.17 | 0.12 | 0.14 | 0.10 | 0.02 | −0.30 | −0.43 |
| | $t$ | 2.95 | −0.19 | 1.12 | 1.99 | 1.17 | 0.02 | −1.28 | −2.93 |
| 12 | Slope | 0.35 | −0.11 | −0.01 | 0.07 | 0.07 | 0.91 | −0.08 | −0.52 |
| | $t$ | 3.07 | −0.12 | −0.08 | 0.97 | 0.88 | 2.41 | −0.60 | −3.43 |

# The Investment CAPM

Univariate cross-sectional regressions of $\tau$-quarter-ahead investment-to-assets changes, the all-but-micro sample, ordinary least squares, 1/1973–12/2015

| $\tau$ | | Sue | $R^6$ | Ivff | Roe | log(Bm) | I/A | Oa | Rdm |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Slope | 0.17 | 0.67 | 0.00 | −0.01 | 0.00 | −0.19 | −0.15 | 0.28 |
| | $t$ | 4.98 | 3.56 | 0.02 | −0.12 | 0.00 | −0.79 | −1.28 | 4.55 |
| 2 | Slope | 0.10 | 0.97 | 0.07 | −0.63 | 0.11 | −0.12 | −0.15 | 0.41 |
| | $t$ | 2.59 | 5.89 | 0.29 | −6.78 | 0.50 | −0.30 | −2.62 | 4.98 |
| 3 | Slope | 0.09 | 0.63 | −0.26 | −0.35 | 0.34 | −0.21 | −0.09 | 0.43 |
| | $t$ | 2.32 | 3.64 | −1.36 | −4.11 | 1.68 | −0.48 | −1.20 | 4.48 |
| 4 | Slope | 0.09 | 0.56 | −0.09 | −0.02 | 0.32 | −0.77 | −0.15 | 0.42 |
| | $t$ | 2.80 | 4.61 | −0.21 | −0.32 | 0.94 | −1.87 | −3.17 | 4.66 |
| 8 | Slope | −0.05 | 0.12 | −0.74 | −0.20 | 0.88 | −0.74 | −0.22 | 0.43 |
| | $t$ | −1.45 | 0.87 | −3.31 | −2.67 | 3.99 | −1.76 | −3.95 | 5.18 |
| 12 | Slope | −0.24 | −0.17 | −0.78 | −0.43 | 1.02 | −0.86 | −0.33 | 0.47 |
| | $t$ | −7.34 | −1.03 | −3.27 | −5.05 | 4.44 | −2.05 | −3.54 | 5.45 |

# The Investment CAPM

## Univariate cross-sectional regressions of $\tau$-quarter-ahead investment-to-assets changes, the all-but-micro sample, ordinary least squares, 1/1973–12/2015

| $\tau$ | | log(Me) | Ep | Cop | Abr | Ilr | Nop | Dac | Nsi |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Slope | −0.22 | −0.15 | 0.12 | 0.16 | −0.05 | 0.24 | −0.06 | −0.10 |
| | $t$ | −1.91 | −0.52 | 1.04 | 5.28 | −0.43 | 1.52 | −1.01 | −0.73 |
| 2 | Slope | −0.21 | −0.37 | 0.15 | 0.31 | 0.21 | 0.06 | −0.15 | −0.23 |
| | $t$ | −2.20 | −1.01 | 1.34 | 7.89 | 3.21 | 0.44 | −2.42 | −2.01 |
| 3 | Slope | −0.05 | −0.31 | 0.17 | 0.35 | 0.24 | 0.59 | −0.05 | −0.39 |
| | $t$ | −0.63 | −0.87 | 2.49 | 9.71 | 3.12 | 2.28 | −0.57 | −3.78 |
| 4 | Slope | 0.04 | −0.53 | 0.17 | 0.26 | 0.23 | 0.41 | −0.14 | −0.53 |
| | $t$ | 0.46 | −0.91 | 1.78 | 8.01 | 4.06 | 6.29 | −2.90 | −4.15 |
| 8 | Slope | 0.24 | −0.13 | 0.29 | 0.18 | 0.09 | 0.62 | −0.22 | −0.72 |
| | $t$ | 2.53 | −0.34 | 3.09 | 5.68 | 1.19 | 4.84 | −4.11 | −6.58 |
| 12 | Slope | 0.30 | −0.13 | 0.15 | 0.07 | 0.09 | 0.77 | −0.25 | −0.77 |
| | $t$ | 2.99 | −0.31 | 1.72 | 2.23 | 1.17 | 8.59 | −4.77 | −6.76 |

To what extent can the predictability of future investment growth be exploited in the form of trading strategies?

Form cross-sectional forecasts of future changes in quarterly investment-to-assets:

- Multiple regressions with all 16 anomaly variables
- Multiple regressions with only anomaly variables significant in univariate regressions for each horizon

Cross-sectional forecasts are noisy: Even with all 16 variables, $R^2$ varies from 13.2% to 15.6% in the full sample with WLS, and from 6.7% to 8.8% in the all-but-micro sample with OLS

## Deciles on expected quarterly I/A changes estimated with all 16 variables, NYSE breakpoints with value-weighted returns, July 1976–December 2015

| $\tau$ | | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | H−L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $m$ | 0.49 | 0.70 | 0.68 | 0.59 | 0.62 | 0.65 | 0.63 | 0.53 | 0.59 | 1.02 | 0.53 |
| | $t_m$ | 1.48 | 2.67 | 3.17 | 2.86 | 3.03 | 3.27 | 3.18 | 2.50 | 2.54 | 3.45 | 1.81 |
| | $\alpha_q$ | 0.25 | 0.33 | 0.25 | 0.03 | 0.07 | −0.02 | −0.12 | −0.23 | −0.23 | 0.20 | −0.06 |
| | $t_q$ | 1.12 | 2.08 | 1.90 | 0.30 | 0.67 | −0.19 | −1.52 | −2.62 | −2.14 | 1.01 | −0.14 |
| 2 | $m$ | 0.56 | 0.78 | 0.70 | 0.76 | 0.83 | 0.65 | 0.52 | 0.65 | 0.63 | 1.11 | 0.54 |
| | $t_m$ | 1.62 | 2.59 | 2.63 | 3.09 | 3.35 | 2.77 | 2.19 | 2.68 | 2.47 | 3.40 | 1.84 |
| | $\alpha_q$ | 0.43 | 0.41 | 0.22 | 0.22 | 0.35 | −0.02 | −0.34 | −0.15 | −0.10 | 0.54 | 0.11 |
| | $t_q$ | 1.58 | 2.14 | 1.30 | 1.71 | 2.53 | −0.16 | −3.06 | −1.15 | −0.70 | 2.09 | 0.25 |
| 3 | $m$ | 0.28 | 0.90 | 0.78 | 0.70 | 0.70 | 0.64 | 0.83 | 0.68 | 0.54 | 0.98 | 0.70 |
| | $t_m$ | 0.86 | 3.14 | 2.88 | 2.59 | 2.76 | 2.52 | 3.39 | 2.83 | 2.06 | 3.53 | 2.93 |
| | $\alpha_q$ | 0.04 | 0.51 | 0.46 | 0.27 | 0.18 | 0.02 | 0.26 | −0.01 | −0.20 | 0.27 | 0.23 |
| | $t_q$ | 0.21 | 3.03 | 3.06 | 1.79 | 1.35 | 0.18 | 1.74 | −0.04 | −1.27 | 1.35 | 0.76 |

## Deciles on expected quarterly I/A changes estimated with all 16 variables, NYSE breakpoints with value-weighted returns, July 1976–December 2015

| $\tau$ | | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | H−L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | $m$ | 0.35 | 0.78 | 0.66 | 0.69 | 0.93 | 0.56 | 0.60 | 0.81 | 0.66 | 0.87 | 0.51 |
| | $t_m$ | 1.08 | 2.60 | 2.32 | 2.63 | 3.84 | 2.34 | 2.27 | 3.54 | 2.59 | 3.15 | 2.38 |
| | $\alpha_q$ | 0.12 | 0.37 | 0.34 | 0.19 | 0.35 | −0.07 | 0.05 | 0.14 | −0.04 | 0.39 | 0.27 |
| | $t_q$ | 0.67 | 2.24 | 1.75 | 1.43 | 2.74 | −0.65 | 0.42 | 1.04 | −0.27 | 2.20 | 1.20 |
| 8 | $m$ | 0.38 | 0.83 | 0.84 | 0.62 | 0.73 | 0.90 | 0.86 | 0.62 | 0.64 | 0.86 | 0.49 |
| | $t_m$ | 1.01 | 2.44 | 2.89 | 2.21 | 2.80 | 3.35 | 3.40 | 2.58 | 2.80 | 3.92 | 1.61 |
| | $\alpha_q$ | 0.13 | 0.52 | 0.40 | −0.18 | 0.14 | 0.48 | 0.28 | 0.01 | −0.06 | 0.26 | 0.13 |
| | $t_q$ | 0.63 | 3.02 | 2.73 | −1.07 | 0.80 | 2.67 | 1.71 | 0.07 | −0.51 | 2.06 | 0.51 |
| 12 | $m$ | 0.86 | 0.73 | 0.66 | 0.89 | 0.82 | 1.02 | 0.80 | 0.78 | 0.88 | 0.69 | −0.17 |
| | $t_m$ | 2.49 | 2.16 | 2.21 | 3.18 | 3.18 | 3.77 | 3.14 | 3.12 | 3.71 | 2.96 | −0.67 |
| | $\alpha_q$ | 0.24 | 0.22 | 0.14 | 0.08 | 0.08 | 0.24 | 0.06 | 0.14 | 0.12 | 0.12 | −0.12 |
| | $t_q$ | 1.26 | 1.22 | 0.85 | 0.52 | 0.42 | 1.24 | 0.40 | 0.91 | 0.90 | 0.95 | −0.49 |

## Deciles on expected quarterly I/A changes estimated with all 16 variables, all-but-micro breakpoints with equal-weighted returns, July 1976–December 2015

| $\tau$ | | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | H−L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $m$ | 0.46 | 0.72 | 0.69 | 0.74 | 0.75 | 0.93 | 0.80 | 0.83 | 1.14 | 1.43 | 0.96 |
| | $t_m$ | 1.15 | 2.13 | 2.33 | 2.69 | 2.82 | 3.51 | 2.97 | 2.90 | 3.67 | 3.78 | 3.52 |
| | $\alpha_q$ | 0.24 | 0.34 | 0.17 | 0.15 | 0.01 | 0.22 | 0.08 | 0.14 | 0.48 | 0.87 | 0.63 |
| | $t_q$ | 1.10 | 1.57 | 1.04 | 1.40 | 0.09 | 2.38 | 0.91 | 1.28 | 2.82 | 3.67 | 1.64 |
| 2 | $m$ | 0.66 | 0.95 | 0.75 | 0.80 | 0.96 | 0.86 | 0.83 | 0.85 | 0.97 | 1.00 | 0.35 |
| | $t_m$ | 1.66 | 2.81 | 2.43 | 2.74 | 3.50 | 3.29 | 3.07 | 3.02 | 3.08 | 2.69 | 1.27 |
| | $\alpha_q$ | 0.34 | 0.57 | 0.19 | 0.16 | 0.28 | 0.11 | 0.08 | 0.16 | 0.32 | 0.49 | 0.15 |
| | $t_q$ | 1.29 | 3.02 | 1.27 | 1.12 | 2.30 | 0.97 | 0.88 | 1.22 | 1.85 | 2.10 | 0.34 |
| 3 | $m$ | 0.54 | 0.82 | 0.81 | 0.90 | 0.74 | 0.95 | 0.88 | 0.84 | 0.98 | 1.08 | 0.53 |
| | $t_m$ | 1.36 | 2.38 | 2.57 | 3.13 | 2.67 | 3.44 | 3.26 | 2.97 | 3.23 | 2.94 | 2.05 |
| | $\alpha_q$ | 0.28 | 0.41 | 0.36 | 0.27 | 0.07 | 0.19 | 0.16 | 0.13 | 0.32 | 0.54 | 0.26 |
| | $t_q$ | 1.16 | 2.21 | 2.23 | 2.09 | 0.59 | 1.58 | 1.78 | 1.05 | 1.91 | 2.30 | 0.65 |

## Deciles on expected quarterly I/A changes estimated with all 16 variables, all-but-micro breakpoints with equal-weighted returns, July 1976–December 2015

| $\tau$ | | Low | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | High | H−L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | $m$ | 0.36 | 0.86 | 0.75 | 0.95 | 0.84 | 0.94 | 0.89 | 0.91 | 0.94 | 1.31 | 0.95 |
| | $t_m$ | 0.88 | 2.46 | 2.39 | 3.34 | 2.91 | 3.46 | 3.35 | 3.24 | 3.25 | 3.70 | 3.92 |
| | $\alpha_q$ | 0.16 | 0.45 | 0.34 | 0.39 | 0.11 | 0.17 | 0.09 | 0.11 | 0.16 | 0.67 | 0.51 |
| | $t_q$ | 0.75 | 2.74 | 2.46 | 2.95 | 1.08 | 1.69 | 0.89 | 1.10 | 1.10 | 3.13 | 1.57 |
| 8 | $m$ | 0.34 | 0.85 | 0.78 | 0.94 | 0.93 | 1.01 | 0.87 | 0.90 | 0.79 | 0.89 | 0.55 |
| | $t_m$ | 0.83 | 2.35 | 2.30 | 3.13 | 3.18 | 3.60 | 3.16 | 3.11 | 2.82 | 2.83 | 2.35 |
| | $\alpha_q$ | 0.16 | 0.49 | 0.40 | 0.47 | 0.31 | 0.36 | 0.10 | 0.14 | 0.03 | 0.17 | 0.01 |
| | $t_q$ | 0.96 | 3.52 | 2.96 | 4.35 | 3.40 | 3.75 | 1.05 | 1.49 | 0.28 | 1.17 | 0.06 |
| 12 | $m$ | 0.62 | 0.96 | 0.84 | 0.99 | 1.16 | 0.95 | 1.02 | 0.97 | 0.97 | 0.87 | 0.25 |
| | $t_m$ | 1.51 | 2.66 | 2.52 | 3.19 | 3.82 | 3.35 | 3.60 | 3.52 | 3.37 | 2.63 | 0.93 |
| | $\alpha_q$ | 0.24 | 0.44 | 0.33 | 0.42 | 0.47 | 0.24 | 0.23 | 0.13 | 0.15 | 0.07 | −0.17 |
| | $t_q$ | 1.36 | 3.36 | 3.05 | 3.60 | 3.90 | 2.22 | 2.07 | 1.25 | 1.26 | 0.43 | −0.70 |

Miller and Modigliani (1961) and Ohlson (1995):

$$P_{it} = \sum_{\tau=1}^{\infty} \frac{E[D_{it+\tau}]}{(1+r_i)^\tau} \quad \Rightarrow \quad \frac{P_{it}}{Be_{it}} = \frac{\sum_{\tau=1}^{\infty} E[Y_{it+\tau} - \triangle Be_{it+\tau}]/(1+r_i)^\tau}{Be_{it}}$$

Fama-French (2006, 2015) derive three predictions, all else equal:

- A lower $P_{it}/Be_{it}$ means a higher $r_i$
- A higher $E[Y_{it+\tau}]$ means a higher $r_i$
- A higher $E[\triangle Be_{it+\tau}]/Be_{it}$ means a lower $r_i$

# Comparison with the Residual Income Model

## The internal rate of return is not equal to the one-period-ahead expected return

Fama and French (2015, p. 2): "Most asset pricing research focuses on short-horizon returns—we use a one-month horizon in our tests. If each stock's short-horizon expected return is positively related to its internal rate of return—if, for example, the expected return is the same for all horizons—the valuation equation... (our emphasis)"

Assumption clearly contradicting price and earnings momentum

# Comparison with the Residual Income Model

## The internal rate of return is not equal to the one-period-ahead expected return, IRRs estimated with IBES earnings forecasts, 1979–2015

| | AR | IRR | Diff | AR | IRR | Diff |
|---|---|---|---|---|---|---|
| | GLS | | | Easton | | |
| SMB | 1.75 | 0.87 | 0.88 | 1.87 | 2.52 | −0.65 |
| [$t$] | 0.87 | 4.56 | 0.44 | 0.97 | 14.63 | −0.34 |
| HML | 3.26 | 3.50 | −0.23 | 3.16 | 3.30 | −0.14 |
| [$t$] | 1.48 | 18.39 | −0.11 | 1.37 | 7.31 | −0.06 |
| RMW | 3.75 | −1.18 | 4.93 | 4.43 | −3.29 | 7.72 |
| [$t$] | 2.59 | −8.32 | 3.49 | 2.70 | −9.42 | 4.60 |
| CMA | 3.34 | 0.64 | 2.70 | 3.46 | 2.45 | 1.00 |
| [$t$] | 2.69 | 4.69 | 2.26 | 2.95 | 7.80 | 0.94 |
| | CT | | | OJ | | |
| SMB | 1.79 | 1.10 | 0.69 | 1.94 | 0.67 | 1.27 |
| [$t$] | 0.89 | 6.79 | 0.35 | 0.97 | 4.43 | 0.65 |
| HML | 3.38 | 0.06 | 3.32 | 2.23 | 0.70 | 1.53 |
| [$t$] | 1.50 | 0.17 | 1.46 | 1.03 | 3.55 | 0.70 |
| RMW | 3.71 | 0.34 | 3.37 | 4.11 | −0.03 | 4.14 |
| [$t$] | 2.64 | 2.20 | 2.42 | 3.36 | −0.17 | 3.36 |
| CMA | 3.31 | 0.08 | 3.23 | 3.03 | 0.07 | 2.96 |
| [$t$] | 2.71 | 0.58 | 2.65 | 2.59 | 0.42 | 2.53 |
| | Average | | | | | |
| SMB | 1.73 | 1.72 | 0.01 | | | |
| [$t$] | 0.90 | 10.65 | 0.01 | | | |
| HML | 3.15 | 2.03 | 1.13 | | | |
| [$t$] | 1.36 | 8.64 | 0.50 | | | |
| RMW | 4.47 | −1.59 | 6.06 | | | |
| [$t$] | 2.76 | −9.74 | 3.76 | | | |
| CMA | 3.26 | 1.15 | 2.11 | | | |
| [$t$] | 2.72 | 7.01 | 1.86 | | | |

HML redundant in Fama and French (2015), inconsistent with their conceptual argument

Consistent with the investment CAPM:

$$1 + (1 - \tau) a \frac{I_{it}}{A_{it}} = \frac{P_{it}}{A_{it+1}}$$

i.e., marginal cost of investment equals marginal $q$, which is in turn average $q$ (Hayashi 1982)

This investment-value linkage allows us to replace HML with the investment factor

Reformulating the residual income model with $E_t[r_{it+1}]$:

$$
\begin{aligned}
P_{it} &= \frac{E_t[Y_{it+1} - \triangle Be_{it+1}] + E_t[P_{it+1}]}{1 + E_t[r_{it+1}]}, \\
\frac{P_{it}}{Be_{it}} &= \frac{E_t\left[\frac{Y_{it+1}}{Be_{it}}\right] - E_t\left[\frac{\triangle Be_{it+1}}{Be_{it}}\right] + E_t\left[\frac{P_{it+1}}{Be_{it+1}}\left(1 + \frac{\triangle Be_{it+1}}{Be_{it}}\right)\right]}{1 + E_t[r_{it+1}]}, \\
\frac{P_{it}}{Be_{it}} &= \frac{E_t\left[\frac{Y_{it+1}}{Be_{it}}\right] + E_t\left[\frac{\triangle Be_{it+1}}{Be_{it}}\left(\frac{P_{it+1}}{Be_{it+1}} - 1\right)\right] + E_t\left[\frac{P_{it+1}}{Be_{it+1}}\right]}{1 + E_t[r_{it+1}]}.
\end{aligned}
$$

Lettau and Ludvigson (2002), Fama and French (2006), our evidence on the expected investment growth effect

# Comparison with the Residual Income Model

Past investment does not forecast future investment, annual cross-sectional regressions of future book equity growth on past asset growth (book equity growth), 1963–2015

| | | $\frac{\triangle Be_{it+\tau}}{Be_{it+\tau-1}} = \gamma_0 + \gamma_1 \frac{\triangle A_{it}}{A_{it-1}} + \epsilon_{t+\tau}$ | | | | | $\frac{\triangle Be_{it+\tau}}{Be_{it+\tau-1}} = \gamma_0 + \gamma_1 \frac{\triangle Be_{it}}{Be_{it-1}} + \epsilon_{t+\tau}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\tau$ | #firms | $\gamma_0$ | $t(\gamma_0)$ | $\gamma_1$ | $t(\gamma_1)$ | $R^2$ | $\gamma_0$ | $t(\gamma_0)$ | $\gamma_1$ | $t(\gamma_1)$ | $R^2$ |
| 1 | 3,106 | 0.09 | 14.57 | 0.22 | 14.38 | 0.05 | 0.09 | 13.14 | 0.21 | 8.64 | 0.06 |
| 2 | 2,844 | 0.10 | 14.57 | 0.10 | 7.58 | 0.01 | 0.10 | 14.65 | 0.10 | 5.23 | 0.02 |
| 3 | 2,624 | 0.10 | 14.98 | 0.06 | 6.25 | 0.01 | 0.10 | 14.91 | 0.06 | 4.09 | 0.01 |
| 4 | 2,429 | 0.10 | 16.19 | 0.05 | 5.47 | 0.00 | 0.10 | 16.20 | 0.05 | 3.73 | 0.00 |
| 5 | 2,256 | 0.10 | 15.03 | 0.04 | 3.41 | 0.00 | 0.10 | 16.04 | 0.03 | 1.91 | 0.00 |
| 6 | 2,099 | 0.10 | 15.26 | 0.05 | 4.57 | 0.00 | 0.10 | 15.01 | 0.03 | 2.24 | 0.00 |
| 7 | 1,955 | 0.10 | 15.45 | 0.04 | 4.39 | 0.00 | 0.10 | 15.58 | 0.03 | 2.69 | 0.00 |
| 8 | 1,821 | 0.09 | 15.36 | 0.03 | 3.98 | 0.00 | 0.10 | 15.65 | 0.01 | 1.60 | 0.00 |
| 9 | 1,697 | 0.09 | 15.28 | 0.03 | 3.45 | 0.00 | 0.10 | 15.42 | 0.01 | 1.19 | 0.00 |
| 10 | 1,584 | 0.09 | 14.57 | 0.04 | 4.50 | 0.00 | 0.09 | 14.77 | 0.02 | 2.17 | 0.00 |

Consistent with the lumpy investment literature in economics (Dixit and Pindyck 1994, Doms and Dunne 1998, Whited 1998)

# Outline

Building on Easton, Harris, and Ohlson (1992), PRRT work with:

$$E_t[r^S_{it+1}] = E_t\left[\frac{P_{it+1} + D_{it+1} - P_{it}}{P_{it}}\right]$$
$$= \frac{E_t[Y_{it+1}]}{P_{it}} + E_t\left[\frac{(P_{it+1} - Be_{it+1}) - (P_{it} - Be_{it})}{P_{it}}\right]$$

The expected change in the market-minus-book equity linked to the expected earnings growth

Mark-to-market:

$$P_{it} = Be_{it} \quad \Rightarrow \quad E_t[r^S_{it+1}] = \frac{E_t[Y_{it+1}]}{P_{it}}$$

No earnings growth:

$$P_{it+1} - Be_{it+1} = P_{it} - Be_{it} \quad \Rightarrow \quad E_t[r^S_{it+1}] = \frac{E_t[Y_{it+1}]}{P_{it}}$$

Growth unrelated to risk and return

Growth related to risk and return, the expected return is a weighted average of the forward earnings yield and book-to-market (a proxy for the expected earnings growth)

Penman and Reggiani (2013): Deferring earnings recognition raises the expected earnings growth, connected to risk (uncertainty)

Penman and Zhu (2014): Many anomaly variables forecast the forward earnings yield and two-year-ahead earnings growth in the cross section, in the same direction of forecasting returns

Penman and Zhang (2015): Accounting conservatism expenses R&D and advertising, inducing high expected earnings growth

Penman and Zhu (2016): Regress future returns on anomaly variables a priori connected to expected earnings growth to estimate costs of capital

Focus on the one-period-ahead expected return, not IRR

The one-period-ahead expected earnings and expected growth as key drivers of the expected return

The PRRT model uses accounting insights to link the expected market-minus-book equity change to the expected earnings growth

- The investment CAPM uses the investment-value linkage to replace capital gain with investment growth

The PRRT model still has the market equity in its formulation

- The investment CAPM is purely "fundamental," thanks again to the investment-value linkage

As key determinants of the expected return, the PRRT model picks earnings yield to proxy for the forward earnings yield, and book-to-market to proxy for the expected earnings growth

- The investment CAPM picks investment-to-assets and profitability (with the expected I/A growth)

# Comparison with the PRRT Model

## Factor spanning tests indicate a stronger explanatory power of the $q$-factor model, explaining the PRRT factors, 1967–2015

| | $m$ | $\alpha_q$ | $\beta_{MKT}$ | $\beta_{ME}$ | $\beta_{I/A}$ | $\beta_{ROE}$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| $f_{ME}$ | 0.24 | 0.09 | 0.01 | 0.88 | −0.19 | −0.09 | 0.91 |
| | 1.85 | 2.11 | 0.96 | 44.45 | −4.77 | −3.17 | |
| $f_{EP}$ | 0.25 | −0.01 | −0.12 | −0.12 | 0.40 | 0.34 | 0.38 |
| | 1.97 | −0.11 | −3.61 | −1.74 | 4.60 | 4.33 | |
| $f_{BM}$ | 0.16 | 0.06 | −0.07 | 0.01 | 0.74 | −0.31 | 0.47 |
| | 1.43 | 0.72 | −2.59 | 0.16 | 12.38 | −4.81 | |
| | | $\alpha_C$ | $\beta_{MKT}$ | $\beta_{SMB}$ | $\beta_{HML}$ | $\beta_{UMD}$ | $R^2$ |
| $f_{ME}$ | | 0.02 | 0.03 | 0.93 | 0.05 | −0.02 | 0.92 |
| | | 0.50 | 2.47 | 74.18 | 1.93 | −1.14 | |
| $f_{EP}$ | | 0.18 | −0.12 | −0.25 | 0.41 | 0.05 | 0.46 |
| | | 1.96 | −4.83 | −4.34 | 8.17 | 1.12 | |
| $f_{BM}$ | | −0.06 | −0.05 | 0.07 | 0.70 | −0.02 | 0.71 |
| | | −0.93 | −2.93 | 3.47 | 22.00 | −1.21 | |

# Comparison with the PRRT Model

## Factor spanning tests indicate a stronger explanatory power of the $q$-factor model, the PRRT four-factor regressions, 1967–2015

| | $m$ | $\alpha_P$ | $\beta_{MKT}$ | $\beta_{ME}$ | $\beta_{EP}$ | $\beta_{BM}$ | $R^2$ |
|---|---|---|---|---|---|---|---|
| SMB | 0.21 | 0.01 | −0.02 | 0.97 | −0.07 | −0.01 | 0.92 |
| | 1.59 | 0.16 | −1.35 | 41.62 | −2.37 | −0.38 | |
| HML | 0.34 | 0.05 | 0.06 | 0.06 | 0.43 | 0.89 | 0.82 |
| | 2.53 | 0.84 | 3.42 | 2.12 | 9.22 | 37.49 | |
| UMD | 0.69 | 0.86 | −0.20 | −0.02 | 0.00 | −0.41 | 0.07 |
| | 3.82 | 4.82 | −2.47 | −0.14 | 0.01 | −3.23 | |
| $r_{ME}$ | 0.31 | 0.03 | 0.00 | 1.00 | 0.08 | 0.09 | 0.90 |
| | 2.34 | 0.76 | 0.14 | 42.80 | 2.15 | 3.96 | |
| $r_{I/A}$ | 0.41 | 0.36 | −0.06 | −0.06 | 0.10 | 0.39 | 0.45 |
| | 4.88 | 5.72 | −3.76 | −1.96 | 1.86 | 12.93 | |
| $r_{ROE}$ | 0.57 | 0.61 | −0.05 | −0.17 | 0.39 | −0.45 | 0.39 |
| | 5.36 | 7.78 | −1.68 | −3.75 | 5.40 | −9.04 | |

Cross-sectionally varying expected returns, depending on firms' investment, profitability, and expected investment growth

Empirically, many anomaly variables predict future investment growth, in the same direction in which these variables predict future returns, but the amount of growth predictability is small

The investment CAPM has more appealing properties than the Miller-Modigliani (1961) and Ohlson (1995) residual income model and the Penman-Reggiani-Richardson-Tuna (2017) model

Value investing is consistent with efficient markets