

q^5

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Introduction

Theme

Augmenting the q -factor model with an expected growth factor:

$$E[R_i - R_f] = \beta_{\text{MKT}}^i E[\text{MKT}] + \beta_{\text{Me}}^i E[R_{\text{Me}}] \\ + \beta_{\text{I/A}}^i E[R_{\text{I/A}}] + \beta_{\text{Roe}}^i E[R_{\text{Roe}}] + \beta_{\text{Eg}}^i E[R_{\text{Eg}}]$$

The q^5 model improves on the q -factor model substantially

The q -factor model already compares well with the Fama-French 6-factor model in explaining a large set of 158 anomalies

Outline

- 1 Motivation
 - Background
 - Theory
- 2 Modeling the Expected Growth
 - Cross-sectional Forecasts
 - The Expected Growth Factor
- 3 Stress-testing Factor Models
 - Factor Models
 - Testing Deciles
 - Relative Performance
- 4 Individual Factor Regressions

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Motivation

The q -factor model in Hou, Xue, and Zhang (2015)

$$E[R_i - R_f] = \beta_{\text{MKT}}^i E[\text{MKT}] + \beta_{\text{Me}}^i E[R_{\text{Me}}] + \beta_{\text{I/A}}^i E[R_{\text{I/A}}] + \beta_{\text{Roe}}^i E[R_{\text{Roe}}]$$

- MKT , R_{Me} , $R_{\text{I/A}}$, and R_{Roe} are the market, size, **investment**, and **profitability (return on equity, Roe)** factors, respectively
- β_{MKT}^i , β_{Me}^i , $\beta_{\text{I/A}}^i$, and β_{Roe}^i are factor loadings

The q -factor model largely summarizes the cross section of average stock returns, capturing most (but not all) anomalies that plague the Fama-French 3-factor model and Carhart 4-factor model

Motivation

“Endorsement” from Fama and French (2015, 2018)

The Fama-French 5-factor model:

$$E[r_{it} - r_{ft}] = b_i E[\text{MKT}_t] + s_i E[\text{SMB}_t] + h_i E[\text{HML}_t] \\ + r_i E[\text{RMW}_t] + c_i E[\text{CMA}_t]$$

- MKT_t , SMB_t , HML_t , RMW_t , and CMA_t are the market, size, value, **profitability**, and **investment** factors, respectively
- b_i , s_i , h_i , r_i , and c_i are factor loadings

Fama and French (2018) add **UMD** to form the 6-factor model

Motivation

The q -factor model predates the Fama-French 5-factor model by 3–6 years

Neoclassical factors

July 2007

An equilibrium three-factor model

January 2009

Production-based factors

April 2009

A better three-factor model

June 2009

that explains more anomalies

An alternative three-factor model

April 2010, April 2011

Digesting anomalies: An investment approach

October 2012, August 2014

Fama and French (2013): A four-factor model for the size, value, and profitability patterns in stock returns

June 2013

Fama and French (2014):

November 2013, September 2014

A five-factor asset pricing model

Motivation

Spanning tests in Hou, Mo, Xue, and Zhang (2018, "Which factors?"), 1/1967–12/2016

	\bar{R}	α	β_{MKT}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	β_{UMD}
R_{Me}	0.31 (2.43)	0.05 (1.58)	0.01 (0.72)	0.97 (64.99)	0.03 (1.63)	0.03 (0.98)	0.02 (0.72)	
		0.03 (0.90)	0.01 (1.21)	0.97 (68.50)	0.05 (2.81)	0.04 (1.34)	0.01 (0.34)	0.03 (2.57)
$R_{\text{I/A}}$	0.41 (4.92)	0.12 (3.48)	0.01 (0.80)	0.04 (3.08)	0.03 (1.32)	0.06 (2.46)	0.82 (31.26)	
		0.11 (3.15)	0.01 (0.97)	0.05 (3.06)	0.04 (1.79)	0.06 (2.21)	0.81 (33.12)	0.01 (0.77)
R_{Roe}	0.55 (5.25)	0.47 (5.91)	0.03 (1.18)	0.12 (2.98)	0.24 (3.72)	0.70 12.80	0.10 1.03	
		0.30 (4.50)	0.00 0.03	0.12 (3.74)	0.10 (2.02)	0.65 (14.77)	0.01 (0.21)	0.24 (9.94)

Motivation

Spanning tests in Hou, Mo, Xue, and Zhang (2018, "Which factors?"), 1/1967–12/2016

	\bar{R}	α_q	β_{MKT}	β_{ME}	$\beta_{\text{I/A}}$	β_{ROE}
SMB	0.25 (1.92)	0.04 (1.32)	0.01 (0.66)	0.94 (54.18)	0.08 (4.21)	0.09 (5.84)
HML	0.37 (2.71)	0.07 (0.63)	0.04 (1.01)	0.02 (0.31)	1.01 (12.18)	0.19 (2.65)
RMW	0.26 (2.53)	0.01 (0.11)	0.03 (1.21)	0.12 (1.70)	0.03 (0.35)	0.54 (8.53)
CMA	0.33 (3.51)	-0.00 (0.13)	0.04 (3.74)	0.04 (1.90)	0.96 (34.93)	0.10 (3.48)
UMD	0.64 (3.60)	0.11 (0.49)	0.08 (1.24)	0.24 (1.73)	0.00 (0.02)	0.91 (5.88)

The q -factors subsume RMW, CMA, and UMD in the 6-factor model, which in turn cannot subsume the q -factors

Motivation

Spanning tests confirmed, but not reported by Barillas and Shanken (2017, 2018)

HXZ: A Comparison of New Factor Models Discussion

ASU Sonoran Winter Finance Conference

**Jay Shanken
Emory University**

February 20, 2015

Motivation

Spanning tests confirmed, but not reported by Barillas and Shanken (2017, 2018)

Empirical Results: Barillas-Shanken (2015b)

We develop a Bayesian test for comparing models

q-model prob = 97%, FF5 3%

Asness and Frazzini (2013) argue for a **better value factor** than HML

FF (1993) update portfolios once a year using prices lagged 6 months;
ignores recent return

Updating monthly with the most recent stock price gives a factor HML^m
with higher mean and more negatively correlated with momentum

Question: does q-model explain HML^m ?

Answer: HML^m alpha on q-factors is 5.3% ($t = 3.3$)

Also, UMD alpha on q-factors + HML^m is 6.5% ($t = 4.0$)

We explore a 6-factor model $\mathbf{M} = \{\text{Mkt}, \text{SMB}, \text{HML}^m, \text{ROE}, \text{I/A}, \text{UMD}\}$

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Motivation

Cochrane (1991)

In the multiperiod investment framework:

$$r_{it+1}^S = \frac{\Pi_{it+1} + (a/2) (I_{it+1}/A_{it+1})^2 + (1 - \delta) [1 + a (I_{it+1}/A_{it+1})]}{1 + a (I_{it}/A_{it})}$$

The “capital gain” roughly proportional to investment-to-assets growth, $(I_{it+1}/A_{it+1}) / (I_{it}/A_{it})$

Intuition analogous with the profitability-expected return relation

Upgrade the q -factor model with an expected growth factor

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Modeling the Expected Growth

Forecasting framework

Forecast $d^T I/A$, τ -year ahead investment-to-assets changes, via monthly cross-sectional regressions

Motivating predictors based on a priori conceptual arguments:

- Tobin's q : Erickson and Whited (2000)
- Cash flows: Fazzari, Hubbard, and Petersen (1988)
- Change in return on equity: Liu, Whited, and Zhang (2009)

Modeling the Expected Growth

Measuring cash flows per Ball, Gerakos, Linnainmaa, and Nikolaev (2016)

Total revenue minus cost of goods sold, minus selling, general, and administrative expenses, plus research and development expenditures, minus change in accounts receivable, minus change in inventory, minus change in prepaid expenses, plus change in deferred revenue, plus change in trade accounts payable, and plus change in accrued expenses, all scaled by book assets

Due to accounting conservatism, cash flows are better than earnings in capturing expected growth

Modeling the Expected Growth

Monthly cross-sectional regressions of future investment-to-assets changes

τ	$\log(q)$	Cop	dRoe	R^2	Pearson	Rank
1	-0.03 (-5.86)	0.53 (12.82)	0.80 (7.75)	6.64	0.14 [0.00]	0.21 [0.00]
2	-0.08 (-10.09)	0.72 (12.58)	0.93 (10.25)	8.88	0.16 [0.00]	0.23 [0.00]
3	-0.09 (-12.14)	0.76 (12.20)	0.74 (8.62)	9.18	0.16 [0.00]	0.22 [0.00]

Modeling the Expected Growth

An expected growth factor, R_{Eg} , monthly 2×3 sorts on size and $E_t[d^1/A]$

\bar{R}_{Eg}	α	β_{Mkt}	β_{Me}	$\beta_{I/A}$	β_{Roe}	R^2	
0.82 (9.81)	0.63 (9.11)	-0.10 (-6.17)	-0.09 (-3.47)	0.25 (6.26)	0.30 (9.43)	0.48	
	α	β_{Mkt}	β_{Me}	$\beta_{I/A}$	β_{Roe}	$\beta_{\log(q)}$	R^2
	0.63 (9.15)	-0.11 (-6.20)	-0.09 (-3.54)	0.27 (6.00)	0.30 (9.05)	-0.02 (-0.50)	0.48
	α	β_{Mkt}	β_{Me}	$\beta_{I/A}$	β_{Roe}	β_{Cop}	R^2
	0.36 (6.09)	-0.03 (-1.84)	-0.02 (-0.70)	0.32 (10.36)	0.15 (5.07)	0.57 (10.41)	0.66
	α	β_{Mkt}	β_{Me}	$\beta_{I/A}$	β_{Roe}	β_{dRoe}	R^2
	0.59 (8.06)	-0.11 (-6.44)	-0.09 (-3.86)	0.22 (4.81)	0.23 (5.20)	0.15 (2.43)	0.49

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Stress Tests

The playing field: The right-hand side

8 competing factor models:

- The q -factor model, the q^5 model
- The Fama-French 5-factor model, the 6-factor model, the alternative 6-factor model with RMWc
- The Stambaugh-Yuan 4-factor model
- The Barillas-Shanken 6-factor model, including MKT, SMB, $R_{I/A}$, R_{Roe} , the Asness-Frazzini monthly formed HML, UMD
- The Daniel-Hirshleifer-Sun 3-factor model

Stress Tests

The playing field: The left-hand side

158 significant anomalies from Hou, Xue, and Zhang (2018)

- Momentum: 36
- Value-versus-growth: 29
- Investment: 28
- Profitability: 35
- Intangibles: 26
- Trading frictions: 4

Stress Tests

Testing deciles, momentum (36)

Sue1	Earnings surprise (1-month holding period), Foster, Olsen, and Shevlin (1984)	Abr1	Cumulative abnormal returns around earnings announcements (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
Abr6	Cumulative abnormal returns around earnings announcements (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)	Abr12	Cumulative abnormal returns around earnings announcements (12-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
Re1	Revisions in analysts' forecasts (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)	Re6	Revisions in analysts' forecasts (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
R^6 1	Price momentum (6-month prior returns, 1-month holding period), Jegadeesh and Titman (1993)	R^6 6	Price momentum (6-month prior returns, 6-month holding period), Jegadeesh and Titman (1993)
R^6 12	Price momentum (6-month prior returns, 12-month holding period), Jegadeesh and Titman (1993)	R^{11} 1	Price momentum (11-month prior returns, 1-month holding period), Fama and French (1996)

Stress Tests

Testing deciles, momentum (36)

R^{116}	Price momentum, (11-month prior returns, 6-month holding period), Fama and French (1996)	Im1	Industry momentum, (1-month holding period), Moskowitz and Grinblatt (1999)
Im6	Industry momentum (6-month holding period), Moskowitz and Grinblatt (1999)	Im12	Industry momentum (12-month holding period), Moskowitz and Grinblatt (1999)
Rs1	Revenue surprise (1-month holding period), Jegadeesh and Livnat (2006)	dEf1	Analysts' forecast change (1-month hold period), Hawkins, Chamberlin, and Daniel (1984)
dEf6	Analysts' forecast change (6-month hold period), Hawkins, Chamberlin, and Daniel (1984)	dEf12	Analysts' forecast change (12-month hold period), Hawkins, Chamberlin, and Daniel (1984)
Nei1	# of consecutive quarters with earnings increases (1-month holding period), Barth, Elliott, and Finn (1999)	52w6	52-week high (6-month holding period), George and Hwang (2004)
ϵ^6_6	Six-month residual momentum (6-month holding period), Blitz, Huij, and Martens (2011)	ϵ^6_{12}	Six-month residual momentum (12-month holding period), Blitz, Huij, and Martens (2011)

Stress Tests

Testing deciles, momentum (36)

ϵ^{111}	11-month residual momentum, 1-month, Blitz, Huij, and Martens (2011)	ϵ^{116}	11-month residual momentum, 6-month, Blitz, Huij, and Martens (2011)
ϵ^{112}	11-month residual momentum, 12-month, Blitz, Huij, and Martens (2011)	Sm1	Segment momentum 1-month, Cohen and Lou (2012)
llr1	Industry lead-lag effect in prior returns, 1-month, Hou (2007)	llr6	Industry lead-lag effect in prior returns, 6-month, Hou (2007)
llr12	Industry lead-lag effect in prior returns, 12-month, Hou (2007)	lle1	Industry lead-lag effect in earnings news, 1-month, Hou (2007)
Cm1	Customer momentum, 1-month Cohen and Frazzini (2008)	Cm12	Customer momentum, 12-month Cohen and Frazzini (2008)
Sim1	Supplier industries momentum, 1-month, Menzly and Ozbas (2010)	Cim1	Customer industries momentum, 1-month, Menzly and Ozbas (2010)
Cim6	Customer industries momentum, 6-month, Menzly and Ozbas (2010)	Cim12	Customer industries momentum, 12-month, Menzly and Ozbas (2010)

Stress Tests

Testing deciles, value-versus-growth (29)

Bm	Book-to-market equity, Rosenberg, Reid, and Lanstein (1985)	Bmj	Book-to-June-end market equity, Asness and Frazzini (2013)
Bm ^{q12}	Quarterly Book-to-market equity (12-month holding period)	Rev6	Reversal (6-month holding period), De Bondt and Thaler (1985)
Rev12	Reversal (12-month holding period) De Bondt and Thaler (1985)	Ep	Earnings-to-price, Basu (1983)
Ep ^{q1}	Quarterly earnings-to-price (1-month holding period)	Ep ^{q6}	Quarterly earnings-to-price (6-month holding period)
Ep ^{q12}	Quarterly earnings-to-price (12-month holding period)	Cp	Cash flow-to-price, Lakonishok, Shleifer, and Vishny (1994)
Cp ^{q1}	Quarterly Cash flow-to-price (1-month holding period)	Cp ^{q6}	Quarterly Cash flow-to-price (6-month holding period)
Cp ^{q12}	Quarterly Cash flow-to-price (12-month holding period)	Nop	Net payout yield, Boudoukh, Michaely, Richardson, and Roberts (2007)
Em	Enterprise multiple, Loughran and Wellman (2011)	Em ^{q1}	Quarterly enterprise multiple (1-month holding period)

Stress Tests

Testing deciles, value-versus-growth (29)

Em ^{q6}	Quarterly enterprise multiple (6-month holding period)	Em ^{q12}	Quarterly enterprise multiple (12-month holding period)
Sp	Sales-to-price, Barbee, Mukherji, and Raines (1996)	Sp ^{q1}	Quarterly sales-to-price (1-month holding period)
Sp ^{q6}	Quarterly sales-to-price (6-month holding period)	Sp ^{q12}	Quarterly sales-to-price (12-month holding period)
Ocp	Operating cash flow-to-price, Desai, Rajgopal, and Venkatachalam (2004)	Ocp ^{q1}	Operating cash flow-to-price (1-month holding period)
Ir	Intangible return, Daniel and Titman (2006)	Vhp	Intrinsic value-to-market, Frankel and Lee (1998)
Vfp	Analysts-based intrinsic value-to-market, Frankel and Lee (1998)	Ebp	Enterprise book-to-price, Penman, Richardson, and Tuna (2007)
Dur	Equity duration, Dechow, Sloan, and Soliman (2004)		

Stress Tests

Testing deciles, investment (28)

Aci	Abnormal corporate investment, Titman, Wei, and Xie (2004)	I/A	Investment-to-assets, Cooper, Gulen, and Schill (2008)
Ia ^{q6}	Quarterly investment-to-assets (6-month holding period)	Ia ^{q12}	Quarterly investment-to-assets (12-month holding period)
dPia	(Changes in PPE and inventory)/assets, Lyandres, Sun, and Zhang (2008)	Noa	Net operating assets, Hirshleifer, Hou, Teoh, and Zhang (2004)
dNoa	Changes in net operating assets, Hirshleifer, Hou, Teoh, and Zhang (2004)	dLno	Change in long-term net operating assets, Fairfield, Whisenant, and Yohn (2003)
Ig	Investment growth, Xing (2008)	2Ig	Two-year investment growth, Anderson and Garcia-Feijoo (2006)
Nsi	Net stock issues, Pontiff and Woodgate (2008)	dli	% change in investment – % change in industry investment, Abarbanell and Bushee (1998)
Cei	Composite equity issuance, Daniel and Titman (2006)	Ivg	Inventory growth, Belo and Lin (2011)

Stress Tests

Testing deciles, investment (28)

Ivc	Inventory changes, Thomas and Zhang (2002)	Oa	Operating accruals, Sloan (1996)
dWc	Change in net non-cash working capital, Richardson, Sloan, Soliman, and Tuna (2005)	dCoa	Change in current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)
dNco	Change in net non-current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)	dNca	Change in non-current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)
dFin	Change in net financial assets, Richardson, Sloan, Soliman, and Tuna (2005)	dFnl	Change in financial liabilities, Richardson, Sloan, Soliman, and Tuna (2005)
dBe	Change in common equity, Richardson, Sloan, Soliman, and Tuna (2005)	Dac	Discretionary accruals, Xie (2001)
Poa	Percent operating accruals, Hafzalla, Lundholm, and Van Winkle (2011)	Pta	Percent total accruals, Hafzalla, Lundholm, and Van Winkle (2011)
Pda	Percent discretionary accruals	Ndf	Net debt finance, Bradshaw, Richardson, and Sloan (2006)

Stress Tests

Testing deciles, profitability (35)

Roe1	Return on equity, 1-month, Hou, Xue, and Zhang (2015)	Roe6	Return on equity, 6-month, Hou, Xue, and Zhang (2015)
dRoe1	Change in Roe, 1-month horizon	dRoe6	Change in Roe, 6-month horizon
dRoe12	Change in Roe, 12-month horizon	Roa1	Return on assets, 1-month horizon, Balakrishnan, Bartov, and Faurel (2010)
dRoa1	Change in Roa, 1-month horizon	dRoa6	Change in Roa, 6-month horizon
Rna ^q 1	Return on net operating assets, 1-month horizon	Rna ^q 6	Return on net operating assets, 6-month horizon
Ato ^q 1	Quarterly asset turnover, 1-month horizon	Ato ^q 6	Quarterly asset turnover, 6-month horizon
Ato ^q 12	Quarterly asset turnover, 12-month horizon	Cto ^q 1	Quarterly capital turnover, 1-month horizon
Cto ^q 6	Quarterly capital turnover, 6-month horizon	Cto ^q 12	Quarterly capital turnover, 12-month horizon
Gpa	Gross profits-to-assets, Novy-Marx (2013)	Gla ^q 1	Gross profits-to-lagged assets, 1-month horizon

Stress Tests

Testing deciles, profitability (35)

Gla ^{q6}	Gross profits-to-lagged assets, 6-month horizon	Gla ^{q12}	Gross profits-to-lagged assets, 12-month horizon
Ole ^{q1}	Operating profits-to-lagged equity, 1-month horizon	Ole ^{q6}	Operating profits-to-lagged equity, 6-month horizon
Opa	Operating profits-to-assets, Ball, Gerakos, Linnainmaa, and Nikolaev (2015)	Ola ^{q1}	Operating profits-to-lagged assets, 1-month horizon
Ola ^{q6}	Operating profits-to-lagged assets, 6-month horizon	Ola ^{q12}	Operating profits-to-lagged assets, 12-month horizon
Cop	Cash-based operating profitability, Ball, Gerakos, Linnainmaa, and Nikolaev (2016)	Cla	Cash-based operating profits-to-lagged assets
Cl ^{q1}	Cash-based operating profits-to-lagged assets, 1-month horizon	Cl ^{q6}	Cash-based operating profits-to-lagged assets, 6-month horizon
Cl ^{q12}	Cash-based operating profits-to-lagged assets, 12-month horizon	F ^{q1}	Quarterly F-score, 1-month horizon
F ^{q6}	Quarterly F-score, 6-month horizon	F ^{q12}	Quarterly F-score, 12-month horizon
Fp ^{q6}	Failure probability, 6-month horizon, Campbell, Hilscher, and Szilagyi (2008)		

Stress Tests

Testing deciles, intangibles (26)

Oca	Organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)	loca	Industry-adjusted organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)
Adm	Advertising expense-to-market, Chan, Lakonishok, and Sougiannis (2001)	Rdm	R&D-to-market, Chan, Lakonishok, and Sougiannis (2001)
Rdm ^{q1}	Quarterly R&D-to-market, 1-month horizon	Rdm ^{q6}	Quarterly R&D-to-market, 6-month horizon
Rdm ^{q12}	Quarterly R&D-to-market, 12-month horizon	OI	Operating leverage, Novy-Marx (2011)
OI ^{q1}	Quarterly operating leverage, 1-month horizon	OI ^{q6}	Quarterly operating leverage, 6-month horizon
OI ^{q12}	Quarterly operating leverage, 12-month horizon	Hs	Industry concentration (sales), Hou and Robinson (2006)
Etr	Effective tax rate, Abarbanell and Bushee (1998)	Rer	Real estate ratio, Tuzel (2010)
Eprd	Earnings predictability, Francis, Lafond, Olsson, and Schipper (2004)	Etl	Earnings timeliness, Francis, Lafond, Olsson, and Schipper (2004)

Stress Tests

Testing deciles, intangibles and trading frictions (4)

Alm^q1	Asset liquidity (market assets), 1-month horizon	Alm^q6	Asset liquidity (market assets), 6-month horizon
Alm^q12	Asset liquidity (market assets), 12-month horizon	R_a^1	12-month-lagged return, Heston and Sadka (2008)
$R_a^{[2,5]}$	Years 2–5 lagged returns, annual Heston and Sadka (2008)	$R_n^{[2,5]}$	Years 2–5 lagged returns, nonannual Heston and Sadka (2008)
$R_a^{[6,10]}$	Years 6–10 lagged returns, annual Heston and Sadka (2008)	$R_n^{[6,10]}$	Years 6–10 lagged returns, nonannual Heston and Sadka (2008)
$R_a^{[11,15]}$	Years 11–15 lagged returns, annual Heston and Sadka (2008)	$R_a^{[16,20]}$	Years 16–20 lagged returns, annual Heston and Sadka (2008)

Trading frictions (4)

$Sv1$	Systematic volatility risk, 1-month horizon, Ang, Hodrick, Xing, and Zhang (2006)	$Dtv12$	Dollar trading volume, 12-month horizon, Brennan, Chordia, and Subrahmanyam (1998)
$Isff1$	Idiosyncratic skewness per the 3-factor model, 1-month horizon	$Isq1$	Idiosyncratic skewness per the q -factor model, 1-month horizon

Stress Tests

Relative performance of factor models, all 158 anomalies

	$ \overline{\alpha_{H-L}} $	$\#_{ t \geq 1.96}$	$\#_{ t \geq 3}$	$ \overline{\alpha} $	$\#_{p < 5\%}$
q	0.25	46	17	0.11	98
q^5	0.18	19	4	0.10	58
FF5	0.38	89	61	0.12	113
FF6	0.28	67	33	0.11	95
FF6c	0.25	55	21	0.10	68
BS6	0.28	61	34	0.14	147
SY4	0.27	57	25	0.10	87
DHS	0.42	83	45	0.15	108

Stress Tests

Relative performance of factor models

	$ \overline{\alpha_{H-L}} $	$\#_{ t \geq 1.96}$	$\#_{ t \geq 3}$	$ \overline{\alpha} $	$\#_{p < 5\%}$	$ \overline{\alpha_{H-L}} $	$\#_{ t \geq 1.96}$	$\#_{ t \geq 3}$	$ \overline{\alpha} $	$\#_{p < 5\%}$
	Momentum (36)					Value-versus-growth (29)				
q	0.26	8	1	0.10	23	0.20	4	0	0.11	17
q^5	0.19	6	1	0.09	12	0.19	4	0	0.13	15
FF5	0.64	34	27	0.16	34	0.14	1	0	0.08	9
FF6	0.29	18	8	0.10	25	0.16	4	1	0.09	11
FF6c	0.27	16	5	0.10	18	0.15	4	0	0.09	8
BS6	0.25	12	5	0.13	33	0.24	11	5	0.13	26
SY4	0.34	21	7	0.10	22	0.20	6	2	0.11	15
DHS	0.26	12	2	0.15	26	0.81	29	26	0.23	29

Stress Tests

Relative performance of factor models

	$ \overline{\alpha_{H-L}} $	$\#_{ t \geq 1.96}$	$\#_{ t \geq 3}$	$ \overline{\alpha} $	$\#_{p < 5\%}$	$ \overline{\alpha_{H-L}} $	$\#_{ t \geq 1.96}$	$\#_{ t \geq 3}$	$ \overline{\alpha} $	$\#_{p < 5\%}$
	Investment (28)					Profitability (35)				
q	0.20	9	4	0.10	17	0.23	12	4	0.10	19
q^5	0.10	0	0	0.08	7	0.14	2	0	0.09	12
FF5	0.23	11	6	0.09	17	0.45	28	21	0.12	30
FF6	0.21	10	5	0.09	17	0.32	22	11	0.10	21
FF6c	0.18	7	1	0.08	7	0.26	14	6	0.10	17
BS6	0.20	7	4	0.11	26	0.28	16	11	0.13	34
SY4	0.17	5	3	0.08	17	0.29	15	7	0.09	21
DHS	0.33	19	2	0.10	21	0.19	6	1	0.09	12

Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$ $\#_{ t \geq 1.96}$ $\#_{ t \geq 3}$ $ \alpha $ $\#_{p<5\%}$					$\overline{ \alpha_{H-L} }$ $\#_{ t \geq 1.96}$ $\#_{ t \geq 3}$ $ \alpha $ $\#_{p<5\%}$				
	Intangibles (26)					Trading frictions (4)				
q	0.41	11	8	0.17	19	0.23	2	0	0.09	3
q^5	0.31	7	3	0.13	10	0.17	0	0	0.08	2
FF5	0.41	13	6	0.15	20	0.22	2	1	0.08	3
FF6	0.42	11	8	0.16	18	0.20	2	0	0.08	3
FF6c	0.43	12	9	0.16	17	0.19	2	0	0.07	1
BS6	0.42	13	7	0.19	25	0.21	2	2	0.10	3
SY4	0.33	8	6	0.14	10	0.19	2	0	0.08	2
DHS	0.59	14	12	0.19	17	0.43	3	2	0.16	3

Stress Tests

Summary of empirical horse races

The q^5 model is the best performing model

The q -factor model already compares well with the Fama-French 6-factor and Stambaugh-Yuan models with a lower number of high-minus-low alphas, but a higher number of GRS rejections

The Fama-French 5-factor model, the Barillas-Shanken model, and the Daniel-Hirshleifer-Sun model perform poorly

Outline

- 1 Motivation
 - Background
 - Theory
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 - The Expected Growth Factor
- 3 Stress-testing Factor Models
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Individual Factor Regressions

Momentum, 1/1967–12/2016

	Sue1	Abr1	Abr6	Abr12	Re1	Re6	R^6_1	R^6_6	R^6_{12}	R^{11}_1
α_q	0.06	0.62	0.30	0.24	0.09	-0.02	-0.03	0.25	0.16	0.31
α_{q^5}	-0.04	0.56	0.25	0.20	0.08	-0.08	-0.44	-0.16	-0.06	-0.20
α_{FF5}	0.52	0.82	0.46	0.40	0.78	0.59	0.74	1.00	0.80	1.29
α_{FF6}	0.30	0.64	0.30	0.26	0.37	0.21	-0.21	0.18	0.20	0.21
α_{FF6c}	0.25	0.65	0.30	0.25	0.38	0.21	-0.18	0.16	0.13	0.20
α_{BS6}	0.14	0.67	0.30	0.25	0.08	-0.01	-0.16	0.12	0.12	0.13
α_{SY4}	0.29	0.71	0.36	0.31	0.58	0.35	-0.05	0.28	0.33	0.30
α_{DHS}	-0.35	0.28	0.06	0.04	-0.41	-0.50	-0.70	-0.25	-0.32	-0.33
t_q	0.46	4.25	2.61	2.79	0.38	-0.08	-0.08	0.83	0.81	0.81
t_{q^5}	-0.30	4.00	2.26	2.24	0.31	-0.38	-1.31	-0.60	-0.31	-0.59
t_{FF5}	3.92	5.81	4.58	5.37	3.16	2.73	2.20	3.65	4.16	3.73
t_{FF6}	2.54	4.66	3.30	4.10	1.89	1.26	-1.10	1.77	1.83	1.74
t_{FF6c}	2.10	4.50	3.12	3.69	1.96	1.28	-0.90	1.44	1.19	1.63
t_{BS6}	1.25	4.48	2.93	3.29	0.43	-0.04	-0.76	1.00	0.86	1.01
t_{SY4}	2.42	5.11	3.61	4.19	2.59	1.92	-0.17	1.38	2.10	1.22
t_{DHS}	-3.17	2.20	0.76	0.62	-2.14	-2.91	-1.97	-1.02	-2.08	-1.03

Individual Factor Regressions

Momentum, 1/1967–12/2016

	R^{116}	lm1	lm6	lm12	Rs1	dEf1	dEf6	dEf12	Nei1	52w6
α_q	0.14	0.28	0.07	0.32	0.24	0.55	0.18	0.07	0.12	0.01
α_{q^5}	-0.17	-0.10	-0.33	0.03	0.12	0.48	0.16	0.06	0.02	-0.34
α_{FF5}	1.06	0.74	0.66	0.84	0.56	1.08	0.72	0.50	0.41	0.77
α_{FF6}	0.20	0.09	-0.01	0.30	0.44	0.74	0.40	0.27	0.27	0.03
α_{FF6c}	0.13	0.09	-0.05	0.22	0.41	0.64	0.37	0.22	0.23	0.02
α_{BS6}	0.08	0.20	-0.07	0.23	0.40	0.55	0.20	0.11	0.17	-0.14
α_{SY4}	0.33	0.18	0.08	0.37	0.37	0.90	0.49	0.34	0.27	0.07
α_{DHS}	-0.45	-0.19	-0.29	-0.07	-0.26	0.17	-0.22	-0.27	-0.30	-0.75
t_q	0.49	0.93	0.30	1.45	1.71	2.49	1.08	0.60	1.20	0.02
t_{q^5}	-0.63	-0.34	-1.37	0.13	0.86	2.07	0.92	0.49	0.25	-1.47
t_{FF5}	3.88	2.67	2.81	4.30	4.06	4.68	4.07	3.89	4.28	3.09
t_{FF6}	1.57	0.43	-0.10	1.99	3.27	3.75	3.14	2.60	2.95	0.26
t_{FF6c}	1.03	0.46	-0.35	1.44	3.01	3.06	2.77	2.13	2.33	0.14
t_{BS6}	0.52	0.91	-0.44	1.32	3.15	2.80	1.51	1.05	1.82	-1.08
t_{SY4}	1.55	0.72	0.43	2.08	2.81	4.42	3.31	3.14	2.65	0.42
t_{DHS}	-1.85	-0.70	-1.41	-0.39	-1.80	0.92	-1.70	-2.63	-2.17	-3.08

Individual Factor Regressions

Momentum, 1/1967–12/2016

	ϵ^6	ϵ^6_{12}	ϵ^{11}_1	ϵ^{11}_6	ϵ^{11}_{12}	Sm1	llr1	llr6	llr12	lle1
α_q	0.26	0.20	0.26	0.22	0.12	0.59	0.73	0.19	0.19	0.32
α_{q^5}	0.06	0.05	0.02	0.05	0.01	0.40	0.50	0.01	0.02	0.11
α_{FF5}	0.47	0.43	0.57	0.56	0.42	0.66	0.80	0.37	0.39	0.70
α_{FF6}	0.20	0.19	0.21	0.23	0.16	0.58	0.66	0.10	0.12	0.49
α_{FF6c}	0.18	0.17	0.22	0.21	0.14	0.55	0.65	0.09	0.10	0.44
α_{BS6}	0.17	0.18	0.14	0.16	0.12	0.64	0.77	0.15	0.13	0.43
α_{SY4}	0.27	0.25	0.28	0.31	0.22	0.64	0.67	0.17	0.17	0.45
α_{DHS}	0.07	-0.01	0.07	0.02	-0.09	0.56	0.51	0.00	0.00	0.01
t_q	1.64	1.57	1.25	1.31	0.82	2.15	2.94	1.45	1.80	1.84
t_{q^5}	0.38	0.35	0.08	0.29	0.08	1.37	2.03	0.04	0.22	0.59
t_{FF5}	3.45	3.78	3.03	3.62	3.16	2.77	3.41	3.11	3.83	4.21
t_{FF6}	1.76	2.16	1.32	1.90	1.51	2.43	3.03	1.22	2.06	2.92
t_{FF6c}	1.54	1.76	1.37	1.73	1.30	2.10	2.74	1.01	1.57	2.53
t_{BS6}	1.36	1.85	0.84	1.23	1.08	2.55	3.30	1.59	2.04	2.38
t_{SY4}	1.91	2.27	1.53	2.11	1.84	2.41	2.94	1.62	2.04	2.64
t_{DHS}	0.47	-0.07	0.35	0.16	-0.72	2.00	2.15	0.02	0.05	0.04

Individual Factor Regressions

Momentum and value-versus-growth, 1/1967–12/2016

	Cm1	Cm12	Sim1	Cim1	Cim6	Cim12	Bm	Bmj	Bm ^q 12	Rev6
α_q	0.70	0.05	0.57	0.64	0.06	0.08	0.15	0.28	0.37	-0.21
α_{q^5}	0.68	-0.02	0.25	0.36	-0.17	-0.12	0.08	0.30	0.38	-0.07
α_{FF5}	0.75	0.13	0.75	0.74	0.25	0.29	-0.10	-0.13	-0.12	-0.01
α_{FF6}	0.74	0.02	0.60	0.62	-0.01	0.04	-0.08	0.07	0.16	-0.10
α_{FF6c}	0.72	0.02	0.56	0.54	0.02	0.03	-0.08	0.10	0.18	-0.15
α_{BS6}	0.74	0.03	0.57	0.66	0.02	0.03	-0.29	-0.11	-0.04	-0.06
α_{SY4}	0.75	0.03	0.56	0.57	0.01	0.05	0.03	0.08	0.23	0.10
α_{DHS}	0.76	0.02	0.47	0.40	-0.05	-0.04	0.87	1.05	1.20	-0.99
t_q	2.84	0.55	1.87	2.36	0.35	0.65	0.99	1.59	2.18	-1.20
t_{q^5}	2.70	-0.23	0.82	1.25	-1.05	-1.03	0.51	1.77	2.25	-0.37
t_{FF5}	3.38	1.45	2.72	3.02	1.76	2.48	-0.88	-0.95	-0.84	-0.04
t_{FF6}	3.00	0.23	2.36	2.66	-0.07	0.56	-0.70	0.54	1.35	-0.59
t_{FF6c}	2.84	0.19	2.11	2.32	0.20	0.38	-0.63	0.79	1.47	-0.85
t_{BS6}	3.09	0.38	2.11	2.66	0.16	0.32	-2.17	-0.80	-0.37	-0.30
t_{SY4}	3.05	0.33	2.07	2.35	0.05	0.52	0.20	0.48	1.77	0.59
t_{DHS}	2.92	0.24	1.51	1.59	-0.35	-0.42	4.16	5.09	6.11	-3.88

Individual Factor Regressions

Value-versus-growth, 1/1967–12/2016

	Rev12	Ep	Ep ^{q1}	Ep ^{q6}	Ep ^{q12}	Cp	Cp ^{q1}	Cp ^{q6}	Cp ^{q12}	Nop
α_q	-0.13	0.02	0.41	0.09	-0.04	0.04	0.42	0.31	0.16	0.35
α_{q^5}	-0.01	-0.07	0.52	0.10	-0.04	0.02	0.53	0.37	0.21	0.20
α_{FF5}	-0.02	-0.10	0.41	0.08	-0.07	-0.22	0.05	-0.05	-0.15	0.22
α_{FF6}	-0.06	-0.14	0.55	0.17	-0.03	-0.18	0.40	0.23	0.02	0.24
α_{FF6c}	-0.09	-0.21	0.47	0.10	-0.09	-0.25	0.37	0.19	-0.01	0.16
α_{BS6}	-0.00	-0.52	-0.04	-0.32	-0.46	-0.47	0.01	-0.09	-0.26	0.12
α_{SY4}	0.11	0.04	0.70	0.32	0.13	0.07	0.48	0.32	0.18	0.17
α_{DHS}	-0.84	0.56	0.94	0.55	0.41	0.67	1.19	1.00	0.82	0.38
t_q	-0.78	0.12	1.74	0.46	-0.25	0.20	1.96	1.65	0.95	2.42
t_{q^5}	-0.08	-0.37	2.25	0.58	-0.25	0.10	2.59	2.11	1.36	1.33
t_{FF5}	-0.14	-0.81	2.38	0.56	-0.57	-1.74	0.28	-0.31	-1.23	1.83
t_{FF6}	-0.41	-1.04	3.21	1.23	-0.21	-1.48	2.91	1.81	0.18	1.92
t_{FF6c}	-0.56	-1.59	2.85	0.69	-0.68	-2.08	2.69	1.51	-0.07	1.22
t_{BS6}	-0.01	-3.05	-0.23	-2.27	-3.66	-3.02	0.06	-0.69	-2.13	0.83
t_{SY4}	0.66	0.19	3.77	1.99	0.90	0.39	2.97	2.18	1.27	1.35
t_{DHS}	-3.40	3.12	4.34	3.06	2.63	3.75	5.92	5.78	5.13	3.16

Individual Factor Regressions

Value-versus-growth, 1/1967–12/2016

	Em	Em ^{q1}	Em ^{q6}	Em ^{q12}	Sp	Sp ^{q1}	Sp ^{q6}	Sp ^{q12}	Ocp	Ocp ^{q1}
α_q	-0.24	-0.48	-0.21	-0.19	-0.05	0.20	0.14	0.05	0.36	0.48
α_{q^5}	-0.05	-0.45	-0.15	-0.12	0.05	0.36	0.28	0.18	0.24	0.43
α_{FF5}	-0.05	-0.33	-0.04	-0.03	-0.26	-0.21	-0.23	-0.24	-0.02	0.12
α_{FF6}	-0.01	-0.45	-0.14	-0.08	-0.16	0.13	0.05	-0.04	0.06	0.41
α_{FF6c}	0.13	-0.31	-0.01	0.05	-0.18	0.10	0.03	-0.06	0.01	0.40
α_{BS6}	0.17	-0.17	0.07	0.12	-0.47	-0.25	-0.28	-0.36	-0.15	0.31
α_{SY4}	-0.16	-0.55	-0.27	-0.23	-0.09	0.16	0.10	0.02	0.30	0.60
α_{DHS}	-0.70	-0.86	-0.59	-0.54	0.73	1.15	1.07	0.91	1.01	1.13
t_q	-1.40	-2.00	-0.99	-1.03	-0.28	0.70	0.59	0.23	1.98	1.62
t_{q^5}	-0.27	-1.91	-0.72	-0.65	0.30	1.44	1.33	1.00	1.28	1.66
t_{FF5}	-0.35	-1.70	-0.20	-0.21	-1.82	-1.04	-1.38	-1.60	-0.12	0.53
t_{FF6}	-0.06	-2.55	-0.87	-0.51	-1.22	0.70	0.33	-0.33	0.47	2.59
t_{FF6c}	0.94	-1.77	-0.05	0.34	-1.33	0.55	0.22	-0.48	0.07	2.46
t_{BS6}	1.07	-1.01	0.43	0.75	-3.01	-1.27	-1.73	-2.38	-0.92	1.81
t_{SY4}	-0.94	-2.79	-1.47	-1.37	-0.57	0.78	0.60	0.13	1.70	3.04
t_{DHS}	-4.07	-4.16	-3.38	-3.17	3.53	4.05	4.42	4.25	5.01	4.50

Individual Factor Regressions

Value-versus-growth and investment, 1/1967–12/2016

	lr	Vhp	Vfp	Ebp	Dur	Aci	I/A	la ⁹⁶	la ⁹¹²	dPia
α_q	-0.16	0.01	0.12	0.06	-0.03	-0.16	0.07	-0.11	0.00	-0.18
α_{q^5}	-0.02	-0.11	0.11	0.08	0.06	-0.16	0.08	0.00	0.09	-0.11
α_{FF5}	0.13	-0.15	0.09	-0.22	0.11	-0.30	0.02	0.01	0.03	-0.30
α_{FF6}	0.05	-0.15	0.08	-0.13	0.12	-0.21	0.02	-0.06	0.02	-0.26
α_{FF6c}	0.03	-0.22	0.02	-0.11	0.14	-0.20	0.00	-0.12	-0.04	-0.29
α_{BS6}	0.18	-0.48	-0.25	-0.33	0.48	-0.17	0.14	0.00	0.10	-0.18
α_{SY4}	0.02	0.05	0.28	-0.03	-0.02	-0.19	0.16	0.14	0.19	-0.05
α_{DHS}	-1.00	0.55	0.50	0.85	-0.53	-0.20	-0.43	-0.67	-0.53	-0.43
t_q	-1.05	0.06	0.55	0.42	-0.17	-1.02	0.62	-1.09	0.03	-1.47
t_{q^5}	-0.12	-0.61	0.49	0.49	0.30	-1.07	0.63	0.02	0.77	-0.91
t_{FF5}	0.96	-1.06	0.50	-1.70	0.80	-2.05	0.17	0.10	0.36	-2.61
t_{FF6}	0.39	-1.06	0.42	-1.09	0.91	-1.37	0.22	-0.56	0.18	-2.26
t_{FF6c}	0.21	-1.48	0.09	-0.86	0.99	-1.29	-0.02	-1.23	-0.46	-2.29
t_{BS6}	1.20	-2.71	-1.23	-2.65	3.07	-1.00	1.28	0.01	0.92	-1.42
t_{SY4}	0.14	0.29	1.31	-0.18	-0.09	-1.31	1.30	1.30	1.83	-0.43
t_{DHS}	-4.25	3.05	2.21	4.32	-2.79	-1.27	-2.49	-3.77	-2.99	-2.76

Individual Factor Regressions

Investment, 1/1967–12/2016

	Noa	dNoa	dLno	lg	2lg	Nsi	dli	Cei	lvg	lvc
α_q	-0.45	-0.15	0.03	-0.07	0.06	-0.29	0.11	-0.29	0.01	-0.28
α_{q^5}	-0.13	-0.10	0.13	-0.12	0.06	-0.12	0.10	-0.04	0.10	0.01
α_{FF5}	-0.53	-0.26	-0.09	-0.18	-0.07	-0.30	0.00	-0.30	-0.08	-0.36
α_{FF6}	-0.45	-0.23	-0.04	-0.15	0.01	-0.28	0.09	-0.26	-0.03	-0.30
α_{FF6c}	-0.44	-0.22	-0.12	-0.19	-0.03	-0.20	0.09	-0.17	-0.01	-0.24
α_{BS6}	-0.61	-0.07	0.01	-0.02	0.09	-0.22	0.27	-0.08	0.09	-0.25
α_{SY4}	-0.17	-0.09	0.19	-0.04	0.08	-0.15	0.09	-0.22	0.03	-0.19
α_{DHS}	-0.32	-0.42	-0.22	-0.37	-0.31	-0.29	-0.13	-0.30	-0.21	-0.48
t_q	-2.59	-1.04	0.19	-0.59	0.49	-2.32	1.06	-2.25	0.09	-2.08
t_{q^5}	-0.88	-0.66	0.79	-0.90	0.44	-0.89	0.83	-0.31	0.75	0.08
t_{FF5}	-3.37	-1.81	-0.62	-1.65	-0.59	-2.58	-0.01	-2.92	-0.66	-2.97
t_{FF6}	-3.18	-1.64	-0.28	-1.37	0.08	-2.39	0.89	-2.33	-0.26	-2.44
t_{FF6c}	-2.88	-1.64	-0.82	-1.57	-0.23	-1.60	0.86	-1.56	-0.06	-1.89
t_{BS6}	-4.02	-0.53	0.05	-0.13	0.71	-1.65	2.37	-0.55	0.69	-1.78
t_{SY4}	-1.21	-0.66	1.36	-0.37	0.69	-1.36	0.79	-1.91	0.27	-1.45
t_{DHS}	-2.20	-2.94	-1.23	-3.17	-1.73	-2.52	-1.03	-2.72	-1.53	-2.95

Individual Factor Regressions

Investment, 1/1967–12/2016

	Oa	dWc	dCoa	dNco	dNca	dFin	dFnl	dBe	Dac	Poa
α_q	-0.56	-0.51	0.08	-0.06	-0.02	0.43	-0.07	0.12	-0.67	-0.13
α_{q^5}	-0.23	-0.22	0.20	0.05	0.03	0.12	0.01	0.17	-0.28	-0.01
α_{FF5}	-0.52	-0.50	0.05	-0.20	-0.15	0.50	-0.17	0.13	-0.64	-0.13
α_{FF6}	-0.47	-0.45	0.06	-0.17	-0.14	0.48	-0.15	0.13	-0.63	-0.10
α_{FF6c}	-0.31	-0.30	0.09	-0.17	-0.17	0.36	-0.13	0.07	-0.53	0.01
α_{BS6}	-0.54	-0.40	0.18	-0.08	-0.05	0.53	-0.12	0.19	-0.72	-0.04
α_{SY4}	-0.44	-0.43	0.13	0.00	0.01	0.38	-0.06	0.28	-0.50	-0.15
α_{DHS}	-0.33	-0.28	-0.33	-0.33	-0.35	0.26	-0.24	-0.32	-0.43	-0.28
t_q	-4.10	-3.80	0.78	-0.50	-0.21	3.00	-0.62	0.97	-4.73	-1.00
t_{q^5}	-1.51	-1.62	1.66	0.41	0.24	0.81	0.12	1.19	-1.91	-0.05
t_{FF5}	-4.20	-3.90	0.55	-1.62	-1.26	4.17	-1.63	1.17	-4.90	-1.13
t_{FF6}	-3.42	-3.45	0.56	-1.39	-1.18	3.86	-1.39	1.18	-4.55	-0.88
t_{FF6c}	-2.04	-2.14	0.76	-1.38	-1.37	2.65	-1.19	0.67	-3.63	0.05
t_{BS6}	-3.68	-2.74	1.55	-0.71	-0.43	3.71	-1.06	1.44	-4.94	-0.32
t_{SY4}	-3.23	-3.33	1.14	-0.02	0.11	2.90	-0.60	2.19	-3.45	-1.19
t_{DHS}	-2.30	-1.74	-2.08	-2.30	-2.49	2.08	-1.83	-1.69	-2.96	-2.04

Individual Factor Regressions

Investment and profitability, 1/1967–12/2016

	Pta	Pda	Ndf	Roe1	Roe6	dRoe1	dRoe6	dRoe12	Roal	dRoal
α_q	-0.19	-0.39	0.01	-0.03	-0.16	0.34	-0.03	-0.10	0.04	0.06
α_{q^5}	-0.04	-0.12	0.10	-0.17	-0.29	0.10	-0.21	-0.18	-0.20	-0.13
α_{FF5}	-0.16	-0.42	-0.07	0.53	0.32	0.79	0.40	0.26	0.53	0.53
α_{FF6}	-0.16	-0.37	-0.06	0.35	0.16	0.56	0.21	0.11	0.30	0.31
α_{FF6c}	-0.13	-0.34	-0.03	0.23	0.04	0.56	0.19	0.09	0.16	0.28
α_{BS6}	-0.08	-0.40	0.01	-0.07	-0.20	0.35	-0.05	-0.11	-0.02	0.11
α_{SY4}	-0.07	-0.26	-0.03	0.35	0.16	0.55	0.18	0.10	0.31	0.35
α_{DHS}	-0.29	-0.41	-0.16	-0.46	-0.63	0.12	-0.18	-0.21	-0.48	-0.05
t_q	-1.42	-2.60	0.11	-0.28	-1.32	2.37	-0.31	-1.11	0.34	0.37
t_{q^5}	-0.34	-0.78	0.84	-1.40	-2.53	0.68	-1.76	-1.93	-1.75	-0.72
t_{FF5}	-1.32	-3.01	-0.67	3.98	2.49	5.54	3.31	2.58	3.80	3.41
t_{FF6}	-1.32	-2.57	-0.58	2.86	1.33	4.36	1.98	1.28	2.51	2.01
t_{FF6c}	-1.04	-2.28	-0.23	1.45	0.24	4.24	1.77	0.93	1.10	1.84
t_{BS6}	-0.54	-2.54	0.11	-0.56	-1.55	2.61	-0.46	-1.25	-0.19	0.62
t_{SY4}	-0.61	-1.92	-0.27	2.20	0.98	3.93	1.62	1.11	2.01	2.18
t_{DHS}	-2.25	-2.73	-1.21	-2.47	-3.37	0.93	-1.77	-2.24	-2.61	-0.33

Individual Factor Regressions

Profitability, 1/1967–12/2016

	dRoa6	Rna ^{q1}	Rna ^{q6}	Ato ^{q1}	Ato ^{q6}	Ato ^{q12}	Cto ^{q1}	Cto ^{q6}	Cto ^{q12}	Gpa
α_q	-0.19	0.19	0.10	0.35	0.34	0.32	-0.10	-0.08	-0.06	0.17
α_{q^5}	-0.27	-0.04	-0.15	0.11	0.11	0.11	-0.16	-0.14	-0.11	0.04
α_{FF5}	0.25	0.57	0.38	0.52	0.50	0.45	0.07	0.07	0.08	0.26
α_{FF6}	0.05	0.42	0.28	0.42	0.40	0.36	0.03	0.02	0.04	0.24
α_{FF6c}	0.05	0.30	0.14	0.37	0.34	0.29	-0.09	-0.10	-0.09	0.17
α_{BS6}	-0.19	0.19	0.11	0.52	0.53	0.52	-0.04	-0.02	0.03	0.31
α_{SY4}	0.08	0.44	0.30	0.25	0.24	0.20	-0.11	-0.10	-0.08	0.05
α_{DHS}	-0.24	-0.25	-0.30	0.34	0.25	0.18	-0.10	-0.10	-0.08	0.08
t_q	-1.38	1.41	0.79	2.06	2.09	2.03	-0.60	-0.50	-0.35	1.24
t_{q^5}	-1.77	-0.29	-1.24	0.62	0.69	0.67	-0.95	-0.81	-0.66	0.29
t_{FF5}	1.83	4.08	3.02	3.17	3.41	3.19	0.47	0.50	0.58	2.06
t_{FF6}	0.42	3.22	2.37	2.74	2.85	2.61	0.21	0.15	0.29	1.86
t_{FF6c}	0.35	1.96	1.04	2.28	2.23	1.97	-0.50	-0.62	-0.58	1.24
t_{BS6}	-1.42	1.39	0.83	3.24	3.67	3.61	-0.25	-0.10	0.16	2.14
t_{SY4}	0.58	2.58	1.90	1.65	1.67	1.42	-0.69	-0.69	-0.51	0.35
t_{DHS}	-1.71	-1.34	-1.68	1.67	1.33	0.99	-0.53	-0.52	-0.45	0.49

Individual Factor Regressions

Profitability, 1/1967–12/2016

	Gla ^{q1}	Gla ^{q6}	Gla ^{q12}	Ole ^{q1}	Ole ^{q6}	Opa	Ola ^{q1}	Ola ^{q6}	Ola ^{q12}	Cop
α_q	0.21	0.11	0.14	0.03	-0.11	0.46	0.40	0.26	0.32	0.69
α_{q^5}	0.04	-0.04	0.00	-0.17	-0.31	-0.04	-0.08	-0.20	-0.10	0.10
α_{FF5}	0.41	0.28	0.26	0.32	0.12	0.57	0.74	0.54	0.54	0.82
α_{FF6}	0.33	0.22	0.22	0.18	0.02	0.52	0.58	0.41	0.43	0.73
α_{FF6c}	0.25	0.13	0.13	0.04	-0.14	0.41	0.50	0.32	0.33	0.51
α_{BS6}	0.31	0.20	0.22	-0.20	-0.30	0.58	0.48	0.34	0.38	0.82
α_{SY4}	0.23	0.14	0.16	0.21	0.06	0.39	0.55	0.41	0.45	0.58
α_{DHS}	0.08	-0.04	-0.01	-0.28	-0.37	-0.01	0.00	-0.13	-0.08	0.19
t_q	1.59	0.93	1.17	0.18	-0.79	2.96	2.64	1.89	2.49	5.04
t_{q^5}	0.31	-0.28	0.01	-1.17	-2.23	-0.28	-0.59	-1.79	-0.92	0.89
t_{FF5}	3.01	2.34	2.27	2.35	0.99	3.60	4.47	3.85	4.24	6.53
t_{FF6}	2.51	1.90	1.97	1.35	0.20	3.67	3.89	3.25	3.75	6.15
t_{FF6c}	1.80	1.08	1.06	0.23	-0.89	2.55	2.87	2.10	2.44	4.28
t_{BS6}	2.20	1.63	1.78	-1.33	-2.08	3.57	3.23	2.52	3.01	5.93
t_{SY4}	1.66	1.12	1.30	1.23	0.40	2.44	3.60	2.91	3.44	4.51
t_{DHS}	0.54	-0.28	-0.10	-1.62	-2.24	-0.04	0.01	-0.71	-0.47	1.16

Individual Factor Regressions

Profitability and intangibles, 1/1967–12/2016

	Cla	Cla ^{q1}	Cla ^{q6}	Cla ^{q12}	F ^{q1}	F ^{q6}	F ^{q12}	Fp ^{q6}	Oca	loca
α_q	0.75	0.46	0.41	0.45	0.13	0.14	0.05	-0.18	0.13	0.07
α_{q^5}	0.17	-0.02	-0.03	0.04	0.24	0.28	0.18	0.33	-0.13	-0.02
α_{FF5}	0.85	0.63	0.57	0.60	0.37	0.37	0.26	-0.86	0.36	0.30
α_{FF6}	0.78	0.54	0.47	0.51	0.23	0.26	0.18	-0.35	0.34	0.17
α_{FF6c}	0.56	0.45	0.37	0.40	0.25	0.24	0.12	-0.32	0.43	0.16
α_{BS6}	0.89	0.53	0.46	0.51	0.05	0.09	0.00	-0.24	0.27	0.03
α_{SY4}	0.66	0.41	0.40	0.43	0.33	0.38	0.28	-0.28	0.00	0.09
α_{DHS}	0.19	0.10	0.11	0.15	0.05	0.04	-0.04	0.56	0.19	0.20
t_q	5.23	3.02	2.97	3.63	0.58	0.85	0.36	-0.68	0.69	0.57
t_{q^5}	1.40	-0.13	-0.28	0.41	1.21	1.67	1.28	1.39	-0.63	-0.16
t_{FF5}	6.82	4.28	4.35	5.07	1.78	2.23	1.94	-3.18	1.80	2.40
t_{FF6}	6.36	3.93	4.10	4.84	1.15	1.53	1.30	-2.17	1.71	1.41
t_{FF6c}	4.68	3.16	3.06	3.71	1.20	1.33	0.82	-1.85	1.90	1.21
t_{BS6}	6.22	3.69	3.70	4.63	0.27	0.53	0.02	-1.40	1.39	0.25
t_{SY4}	4.87	2.97	3.44	4.23	1.59	2.24	1.85	-1.93	-0.01	0.72
t_{DHS}	1.12	0.67	0.85	1.26	0.23	0.22	-0.24	2.29	0.88	1.40

Individual Factor Regressions

Intangibles, 1/1967–12/2016

	Adm	Rdm	Rdm ^{q1}	Rdm ^{q6}	Rdm ^{q12}	OI	OI ^{q1}	OI ^{q6}	OI ^{q12}	Hs
α_q	0.09	0.72	1.39	0.95	0.81	0.01	0.08	0.10	0.12	-0.30
α_{q^5}	0.06	0.25	1.07	0.54	0.37	0.06	0.11	0.04	0.04	-0.12
α_{FF5}	-0.09	0.57	0.89	0.63	0.60	0.14	0.26	0.25	0.29	-0.41
α_{FF6}	0.04	0.60	1.33	0.92	0.77	0.13	0.26	0.25	0.27	-0.34
α_{FF6c}	0.03	0.76	1.36	1.01	0.88	0.13	0.24	0.24	0.26	-0.32
α_{BS6}	-0.26	0.73	1.40	0.96	0.80	-0.02	0.10	0.09	0.11	-0.46
α_{SY4}	0.08	0.30	1.14	0.63	0.47	0.02	0.15	0.14	0.14	-0.26
α_{DHS}	0.93	1.13	1.82	1.48	1.36	0.13	0.15	0.18	0.18	-0.17
t_q	0.35	3.11	3.06	2.87	3.01	0.06	0.48	0.61	0.77	-1.56
t_{q^5}	0.27	1.13	2.26	1.57	1.31	0.33	0.61	0.21	0.25	-0.55
t_{FF5}	-0.50	2.55	2.26	1.98	2.22	0.94	1.51	1.54	1.81	-2.50
t_{FF6}	0.21	2.77	3.58	3.05	3.00	0.88	1.56	1.57	1.71	-1.96
t_{FF6c}	0.13	3.34	3.65	3.36	3.51	0.81	1.29	1.35	1.52	-1.85
t_{BS6}	-1.14	3.09	3.44	2.89	2.84	-0.10	0.59	0.53	0.64	-2.46
t_{SY4}	0.35	1.34	2.87	2.13	1.84	0.17	0.93	0.86	0.96	-1.44
t_{DHS}	3.07	4.31	3.91	3.34	3.34	0.76	0.83	0.94	0.98	-1.01

Individual Factor Regressions

Intangibles, 1/1967–12/2016

	Etr	Rer	Eprd	Etl	Alm ^{q1}	Alm ^{q6}	Alm ^{q12}	R_a^1	$R_a^{[2,5]}$	$R_n^{[2,5]}$
α_q	0.09	0.34	-0.55	0.27	0.25	0.24	0.14	0.58	0.81	-0.20
α_{q^5}	0.11	0.18	-0.43	0.18	0.24	0.22	0.17	0.50	0.85	-0.09
α_{FF5}	0.20	0.29	-0.91	0.36	0.02	0.09	0.05	0.67	0.73	0.05
α_{FF6}	0.17	0.27	-0.81	0.26	0.09	0.10	0.03	0.48	0.74	-0.05
α_{FF6c}	0.23	0.25	-0.85	0.33	0.09	0.10	0.02	0.41	0.66	-0.05
α_{BS6}	0.13	0.35	-0.81	0.35	-0.03	-0.03	-0.12	0.47	0.78	0.12
α_{SY4}	0.13	0.20	-0.58	0.18	0.15	0.17	0.12	0.59	0.83	0.05
α_{DHS}	0.13	0.15	0.00	0.37	0.94	0.89	0.76	0.32	0.58	-0.77
t_q	0.75	2.05	-3.02	1.56	1.68	1.78	1.01	2.75	4.06	-1.06
t_{q^5}	0.82	1.14	-2.49	1.10	1.61	1.62	1.22	2.25	4.02	-0.42
t_{FF5}	1.83	1.86	-5.56	2.48	0.18	0.76	0.44	3.58	4.03	0.27
t_{FF6}	1.51	1.73	-4.97	1.90	0.71	0.93	0.28	2.67	3.80	-0.31
t_{FF6c}	2.03	1.59	-5.08	2.41	0.72	0.90	0.21	2.12	3.24	-0.28
t_{BS6}	0.97	2.08	-4.64	2.23	-0.20	-0.28	-0.93	2.23	3.73	0.69
t_{SY4}	1.10	1.23	-3.71	1.30	1.06	1.28	0.90	3.16	4.21	0.29
t_{DHS}	1.13	0.82	0.01	2.38	4.77	4.49	3.93	1.31	2.56	-3.24

Individual Factor Regressions

Intangibles and trading frictions, 1/1967–12/2016

	$R_a^{[6,10]}$	$R_n^{[6,10]}$	$R_a^{[11,15]}$	$R_a^{[16,20]}$	Sv1	Dtv12	lsff1	lsq1
α_q	1.11	0.03	0.60	0.62	-0.22	-0.13	0.27	0.29
α_{q^5}	0.95	0.05	0.55	0.61	-0.16	-0.15	0.20	0.19
α_{FF5}	1.05	-0.08	0.68	0.60	-0.26	-0.06	0.30	0.28
α_{FF6}	1.11	0.00	0.65	0.60	-0.25	-0.06	0.26	0.24
α_{FF6c}	1.11	-0.03	0.66	0.63	-0.18	-0.09	0.27	0.24
α_{BS6}	1.11	0.33	0.58	0.59	-0.21	-0.01	0.31	0.33
α_{SY4}	1.01	-0.09	0.59	0.56	-0.24	-0.03	0.24	0.25
α_{DHS}	1.13	-0.36	0.52	0.59	-0.11	-0.95	0.27	0.38
t_q	5.05	0.15	3.48	3.22	-0.90	-1.72	2.56	2.84
t_{q^5}	4.74	0.24	3.16	2.83	-0.59	-1.94	1.73	1.76
t_{FF5}	5.37	-0.47	3.91	3.72	-1.15	-0.77	3.05	2.89
t_{FF6}	5.69	-0.02	4.13	3.43	-1.08	-0.79	2.76	2.54
t_{FF6c}	5.25	-0.19	3.76	3.30	-0.75	-1.13	2.63	2.35
t_{BS6}	4.73	1.70	3.16	3.32	-0.84	-0.11	3.12	3.14
t_{SY4}	4.97	-0.50	3.85	3.01	-0.98	-0.35	2.36	2.39
t_{DHS}	5.43	-1.72	3.07	3.08	-0.52	-4.41	2.68	3.19

Conclusion

 q^5

The q^5 model substantially improves the q -factor model, which in turn compares well with the Fama-French 6-factor model