

$q^5$

## Stress-testing Factor Models

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Augmenting the Hou-Xue-Zhang (2015)  $q$ -factor model with an expected growth factor to form the  $q^5$  model:

$$E[R_i - R_f] = \beta_{\text{MKT}}^i E[\text{MKT}] + \beta_{\text{Me}}^i E[R_{\text{Me}}] \\ + \beta_{\text{I/A}}^i E[R_{\text{I/A}}] + \beta_{\text{Roe}}^i E[R_{\text{Roe}}] + \beta_{\text{Eg}}^i E[R_{\text{Eg}}]$$

Stress-testing factor models with a large set of 158 anomalies:

- The  $q^5$  model improves on the  $q$ -factor model substantially
- The  $q$ -factor model already compares well with the Fama-French (2018) 6-factor model

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$$E[R_i - R_f] = \beta_{\text{MKT}}^i E[\text{MKT}] + \beta_{\text{Me}}^i E[R_{\text{Me}}] + \beta_{\text{I/A}}^i E[R_{\text{I/A}}] + \beta_{\text{Roe}}^i E[R_{\text{Roe}}]$$

- MKT,  $R_{\text{Me}}$ ,  $R_{\text{I/A}}$ , and  $R_{\text{Roe}}$  are the market, size, **investment**, and **profitability (return on equity, Roe)** factors, respectively
- $\beta_{\text{MKT}}^i$ ,  $\beta_{\text{Me}}^i$ ,  $\beta_{\text{I/A}}^i$ , and  $\beta_{\text{Roe}}^i$  are factor loadings

The  $q$ -factor model largely summarizes the cross section of average stock returns, capturing most (but not all) anomalies that plague the Fama-French 3-factor model and Carhart 4-factor model

The Fama-French (2015) 5-factor model:

$$E[R_i - R_f] = b_i E[\text{MKT}] + s_i E[\text{SMB}] + h_i E[\text{HML}] \\ + r_i E[\text{RMW}] + c_i E[\text{CMA}]$$

- MKT, SMB, HML, RMW, and CMA are the market, size, value, **profitability**, and **investment** factors, respectively
- $b_i, s_i, h_i, r_i,$  and  $c_i$  are factor loadings

Fama and French (2018) add **UMD** to form the 6-factor model

# Background

The  $q$ -factor model predates the Fama-French 5-factor model by 3–6 years

Neoclassical factors

July 2007

An equilibrium three-factor model

January 2009

Production-based factors

April 2009

A better three-factor model

June 2009

that explains more anomalies

An alternative three-factor model

April 2010, April 2011

Digesting anomalies: An investment approach

October 2012, August 2014

Fama and French (2013): A four-factor model for the size, value, and profitability patterns in stock returns

June 2013

Fama and French (2014):

November 2013, September 2014

A five-factor asset pricing model

# Background

Spanning tests in Hou, Mo, Xue, and Zhang (2018, "Which factors?"), 1/1967–12/2016

	$\bar{R}$	$\alpha$	$\beta_{\text{MKT}}$	$\beta_{\text{SMB}}$	$\beta_{\text{HML}}$	$\beta_{\text{RMW}}$	$\beta_{\text{CMA}}$	$\beta_{\text{UMD}}$
$R_{\text{Me}}$	0.31 (2.43)	0.05 (1.58)	0.01 (0.72)	0.97 (64.99)	0.03 (1.63)	-0.03 (-0.98)	0.02 (0.72)	
		0.03 (0.90)	0.01 (1.21)	0.97 (68.50)	0.05 (2.81)	-0.04 (-1.34)	0.01 (0.34)	0.03 (2.57)
$R_{\text{I/A}}$	0.41 (4.92)	0.12 (3.48)	0.01 (0.80)	-0.04 (-3.08)	0.03 (1.32)	0.06 (2.46)	0.82 (31.26)	
		0.11 (3.15)	0.01 (0.97)	-0.05 (-3.06)	0.04 (1.79)	0.06 (2.21)	0.81 (33.12)	0.01 (0.77)
$R_{\text{Roe}}$	0.55 (5.25)	0.47 (5.91)	-0.03 (-1.18)	-0.12 (-2.98)	-0.24 (-3.72)	0.70 12.80	0.10 1.03	
		0.30 (4.50)	0.00 0.03	-0.12 (-3.74)	-0.10 (-2.02)	0.65 (14.77)	-0.01 (-0.21)	0.24 (9.94)



Spanning tests in Hou, Mo, Xue, and Zhang (2018, "Which factors?"), 1/1967–12/2016

	$\bar{R}$	$\alpha_q$	$\beta_{\text{MKT}}$	$\beta_{\text{ME}}$	$\beta_{\text{I/A}}$	$\beta_{\text{ROE}}$
SMB	0.25 (1.92)	0.04 (1.32)	-0.01 (-0.66)	0.94 (54.18)	-0.08 (-4.21)	-0.09 (-5.84)
HML	0.37 (2.71)	0.07 (0.63)	-0.04 (-1.01)	0.02 (0.31)	1.01 (12.18)	-0.19 (-2.65)
RMW	<b>0.26</b> (2.53)	<b>0.01</b> (0.11)	-0.03 (-1.21)	-0.12 (-1.70)	0.03 (0.35)	0.54 (8.53)
CMA	<b>0.33</b> (3.51)	<b>-0.00</b> (-0.13)	-0.04 (-3.74)	0.04 (1.90)	0.96 (34.93)	-0.10 (-3.48)
UMD	<b>0.64</b> (3.60)	<b>0.11</b> (0.49)	-0.08 (-1.24)	0.24 (1.73)	-0.00 (-0.02)	0.91 (5.88)

The  $q$ -factors subsume RMW, CMA, and UMD in the Fama-French 6-factor model, which in turn cannot subsume the  $q$ -factors

Spanning tests replicated but not reported by Barillas and Shanken (2017, 2018): Slide 6 in Shanken's discussion on "A comparison of new factor models" in February 2015

## Empirical Results: Barillas-Shanken (2015b)

We develop a Bayesian test for comparing models

**q-model prob = 97%**, FF5 3%

Asness and Frazzini (2013) argue for a **better value factor** than HML

FF (1993) update portfolios once a year using prices lagged 6 months;  
ignores recent return

Updating monthly with the most recent stock price gives a factor  $HML^m$   
with higher mean and more negatively correlated with momentum

**Question:** does q-model explain  $HML^m$  ?

**Answer:**  $HML^m$  alpha on q-factors is 5.3% ( $t = 3.3$ )

Also, UMD alpha on q-factors +  $HML^m$  is 6.5% ( $t = 4.0$ )

We explore a 6-factor model  $\mathbf{M} = \{\text{Mkt}, \text{SMB}, \text{HML}^m, \text{ROE}, \text{I/A}, \text{UMD}\}$

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# The Expected Growth Factor

Theoretical motivation

In the multiperiod investment framework (Cochrane 1991):

$$r_{it+1} = \frac{X_{it+1} + (a/2) (I_{it+1}/A_{it+1})^2 + (1 - \delta) [1 + a (I_{it+1}/A_{it+1})]}{1 + a (I_{it}/A_{it})}$$

The “capital gain” roughly proportional to investment-to-assets growth,  $(I_{it+1}/A_{it+1}) / (I_{it}/A_{it})$

Intuition analogous with the **profitability**-expected return relation

Upgrade the  $q$ -factor model with an expected growth factor

# The Expected Growth Factor

Forecasting framework

Forecast  $d^T I/A$ ,  $\tau$ -year ahead investment-to-assets changes, with monthly cross-sectional regressions

Motivating predictors based on *a priori* conceptual arguments:

- Tobin's  $q$ : Erickson and Whited (2000)
- Cash flows: Fazzari, Hubbard, and Petersen (1988)
- Change in return on equity: Liu, Whited, and Zhang (2009)

# The Expected Growth Factor

A priori conceptual arguments

Cash flows: Internal funds available for investments

**Accounting conservatism:** Cash flows better than earnings in capturing expected growth due to intangibles (Ball, Gerakos, Linnainmaa, and Nikolaev 2016)

- Total revenue minus cost of goods sold, minus SG&A, plus R&D, minus change in accounts receivable, minus change in inventory, minus change in prepaid expenses, plus change in deferred revenue, plus change in trade accounts payable, and plus change in accrued expenses, all scaled by book assets

dRoe: Capturing short-term dynamics of investment growth

# The Expected Growth Factor

Monthly cross-sectional regressions of  
future investment-to-assets changes, 7/1963–12/2016

$\tau$	$\log(q)$	Cop	dRoe	$R^2$	Pearson	Rank
1	-0.03 (-5.86)	<b>0.53</b> (12.82)	<b>0.80</b> (7.75)	6.64	0.14 [0.00]	<b>0.21</b> [0.00]
2	-0.08 (-10.09)	0.72 (12.58)	0.93 (10.25)	8.88	0.16 [0.00]	0.23 [0.00]
3	-0.09 (-12.14)	0.76 (12.20)	0.74 (8.62)	9.18	0.16 [0.00]	0.22 [0.00]

Relatively reliable out-of-sample correlations with subsequent,  
realized investment-to-assets changes

# The Expected Growth Factor

Properties of the expected growth deciles, 1/1967–12/2016

Low	2	3	4	5	6	7	8	9	High	H-L
Average excess returns, $\bar{R}$										
-0.12	0.26	0.33	0.45	0.46	0.51	0.57	0.65	0.75	0.95	1.06
-0.39	1.05	1.43	2.09	2.31	2.64	3.03	3.41	3.99	4.57	6.25
The expected 1-year-ahead growth, $E_t[d^1 A]$										
-15.21	-7.70	-5.61	-4.18	-2.99	-1.92	-0.80	0.55	2.62	7.79	23.00
-35.58	-30.23	-24.17	-19.68	-15.25	-10.44	-4.63	3.50	17.61	39.62	44.31
Average future 1-year-ahead realized growth, $d^1 A$										
-17.43	-12.37	-3.83	-3.51	-1.22	-0.35	-0.42	0.56	1.64	6.09	23.52
-12.01	-8.33	-6.44	-5.19	-2.36	-0.73	-0.90	1.01	3.72	9.15	15.03

$E_t[d^1|A]$  and  $d^1|A$  aligned at the portfolio level (Corr = 0.66)



# The Expected Growth Factor

$R_{Eg}$ , independent  $2 \times 3$  monthly sorts on size and  $E_t[d^1/A]$ , 1/1967–12/2016

$\bar{R}_{Eg}$	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{I/A}$	$\beta_{Roe}$	$R^2$	
0.82 (9.81)	0.63 (9.11)	-0.10 (-6.17)	-0.09 (-3.47)	0.25 (6.26)	0.30 (9.43)	0.48	
	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{I/A}$	$\beta_{Roe}$	$\beta_{\log(q)}$	$R^2$
	0.63 (9.15)	-0.11 (-6.20)	-0.09 (-3.54)	0.27 (6.00)	0.30 (9.05)	-0.02 (-0.50)	0.48
	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{I/A}$	$\beta_{Roe}$	$\beta_{Cop}$	$R^2$
	0.36 (6.09)	-0.03 (-1.84)	-0.02 (-0.70)	0.32 (10.36)	0.15 (5.07)	0.57 (10.41)	0.66
	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{I/A}$	$\beta_{Roe}$	$\beta_{dRoe}$	$R^2$
	0.59 (8.06)	-0.11 (-6.44)	-0.09 (-3.86)	0.22 (4.81)	0.23 (5.20)	0.15 (2.43)	0.49

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8 competing factor models:

- The  $q$ -factor model, the  $q^5$  model
- The Fama-French 5-factor model, the 6-factor model, the alternative 6-factor model with RMWc
- The Stambaugh-Yuan 4-factor model
- The Barillas-Shanken 6-factor model, including MKT, SMB,  $R_{I/A}$ ,  $R_{Roe}$ , the Asness-Frazzini monthly formed HML, UMD
- The Daniel-Hirshleifer-Sun 3-factor model

Use the replicated Stambaugh-Yuan and Daniel-Hirshleifer-Sun models via the common construction (Hou, Mo, Xue, and Zhang 2018, “Which factors?” Review of Finance)

158 anomalies with NYSE breakpoints and value-weighted returns significant at the 5% level (Hou, Xue, and Zhang 2018, “Replicating anomalies”)

- Momentum: 36
- Value-versus-growth: 29
- Investment: 28
- Profitability: 35
- Intangibles: 26
- Trading frictions: 4

# Stress Tests

Testing deciles, momentum (36)

- |                   |  |                   |   |
|-------------------|--|-------------------|---|
| Sue1              | Earnings surprise (1-month holding period), Foster, Olsen, and Shevlin (1984)  | Abr1              | Cumulative abnormal returns around earnings announcements (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)  |
| Abr6              | Cumulative abnormal returns around earnings announcements (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996) | Abr12             | Cumulative abnormal returns around earnings announcements (12-month holding period), Chan, Jegadeesh, and Lakonishok (1996) |
| Re1               | Revisions in analysts' forecasts (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)                          | Re6               | Revisions in analysts' forecasts (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)                           |
| R <sup>6</sup> 1  | Price momentum (6-month prior returns, 1-month holding period), Jegadeesh and Titman (1993)                                | R <sup>6</sup> 6  | Price momentum (6-month prior returns, 6-month holding period), Jegadeesh and Titman (1993)                                 |
| R <sup>6</sup> 12 | Price momentum (6-month prior returns, 12-month holding period), Jegadeesh and Titman (1993)                               | R <sup>11</sup> 1 | Price momentum (11-month prior returns, 1-month holding period), Fama and French (1996)                                     |

# Stress Tests

## Testing deciles, momentum (36)

$R^{116}$	Price momentum, (11-month prior returns, 6-month holding period), Fama and French (1996)	Im1	Industry momentum, (1-month holding period), Moskowitz and Grinblatt (1999)
Im6	Industry momentum (6-month holding period), Moskowitz and Grinblatt (1999)	Im12	Industry momentum (12-month holding period), Moskowitz and Grinblatt (1999)
Rs1	Revenue surprise (1-month holding period), Jegadeesh and Livnat (2006)	dEf1	<b>Analysts' forecast change</b> (1-month hold period), Hawkins, Chamberlin, and Daniel (1984)
dEf6	Analysts' forecast change (6-month hold period), Hawkins, Chamberlin, and Daniel (1984)	dEf12	Analysts' forecast change (12-month hold period), Hawkins, Chamberlin, and Daniel (1984)
Nei1	# of consecutive quarters with earnings increases (1-month holding period), Barth, Elliott, and Finn (1999)	52w6	52-week high (6-month holding period), George and Hwang (2004)
$\epsilon^6$	Six-month residual momentum (6-month holding period), Blitz, Huij, and Martens (2011)	$\epsilon^6$ 12	Six-month residual momentum (12-month holding period), Blitz, Huij, and Martens (2011)

# Stress Tests

## Testing deciles, momentum (36)

$\epsilon^{111}$	11-month residual momentum, 1-month, Blitz, Huij, and Martens (2011)	$\epsilon^{116}$	11-month residual momentum, 6-month, Blitz, Huij, and Martens (2011)
$\epsilon^{112}$	11-month residual momentum, 12-month, Blitz, Huij, and Martens (2011)	Sm1	Segment momentum 1-month, Cohen and Lou (2012)
llr1	Industry lead-lag effect in prior returns, 1-month, Hou (2007)	llr6	Industry lead-lag effect in prior returns, 6-month, Hou (2007)
llr12	Industry lead-lag effect in prior returns, 12-month, Hou (2007)	lle1	Industry lead-lag effect in earnings news, 1-month, Hou (2007)
Cm1	Customer momentum, 1-month Cohen and Frazzini (2008)	Cm12	Customer momentum, 12-month Cohen and Frazzini (2008)
Sim1	Supplier industries momentum, 1-month, Menzly and Ozbas (2010)	Cim1	Customer industries momentum, 1-month, Menzly and Ozbas (2010)
Cim6	Customer industries momentum, 6-month, Menzly and Ozbas (2010)	Cim12	Customer industries momentum, 12-month, Menzly and Ozbas (2010)

# Stress Tests

## Testing deciles, value-versus-growth (29)

Bm	Book-to-market equity, Rosenberg, Reid, and Lanstein (1985)	Bmj	Book-to-June-end market equity, Asness and Frazzini (2013)
Bm <sup>q</sup> 12	Quarterly book-to-market equity (12-month holding period)	Rev6	Reversal (6-month holding period), De Bondt and Thaler (1985)
Rev12	Reversal (12-month holding period) De Bondt and Thaler (1985)	Ep	Earnings-to-price, Basu (1983)
Ep <sup>q</sup> 1	Quarterly earnings-to-price (1-month holding period)	Ep <sup>q</sup> 6	Quarterly earnings-to-price (6-month holding period)
Ep <sup>q</sup> 12	Quarterly earnings-to-price (12-month holding period)	Cp	Cash flow-to-price, Lakonishok, Shleifer, and Vishny (1994)
Cp <sup>q</sup> 1	Quarterly cash flow-to-price (1-month holding period)	Cp <sup>q</sup> 6	Quarterly cash flow-to-price (6-month holding period)
Cp <sup>q</sup> 12	Quarterly cash flow-to-price (12-month holding period)	Nop	Net payout yield, Boudoukh, Michaely, Richardson, and Roberts (2007)
Em	Enterprise multiple, Loughran and Wellman (2011)	Em <sup>q</sup> 1	Quarterly enterprise multiple (1-month holding period)



# Stress Tests

## Testing deciles, value-versus-growth (29)

Em<sup>q6</sup> Quarterly enterprise multiple  
(6-month holding period)

Sp Sales-to-price,  
Barbee, Mukherji, and Raines (1996)

Sp<sup>q6</sup> Quarterly sales-to-price  
(6-month holding period)

Ocp Operating cash flow-to-price, Desai,  
Rajgopal, and Venkatachalam (2004)

Ir Intangible return,  
Daniel and Titman (2006)

Vfp Analysts-based intrinsic value-to-market,  
Frankel and Lee (1998)

Dur Equity duration,  
Dechow, Sloan, and Soliman (2004)

Em<sup>q12</sup> Quarterly enterprise multiple  
(12-month holding period)

Sp<sup>q1</sup> **Quarterly sales-to-price**  
(1-month holding period)

Sp<sup>q12</sup> Quarterly sales-to-price  
(12-month holding period)

Ocp<sup>q1</sup> Operating cash flow-to-price  
(1-month holding period)

Vhp Intrinsic value-to-market,  
Frankel and Lee (1998)

Ebp Enterprise book-to-price, Penman,  
Richardson, and Tuna (2007)

# Stress Tests

## Testing deciles, investment (28)

Aci	Abnormal corporate investment, Titman, Wei, and Xie (2004)	I/A	Investment-to-assets, Cooper, Gulen, and Schill (2008)
la <sup>q6</sup>	Quarterly investment-to-assets (6-month holding period)	la <sup>q12</sup>	Quarterly investment-to-assets (12-month holding period)
dPia	(Changes in PPE and inventory)/assets, Lyandres, Sun, and Zhang (2008)	Noa	<b>Net operating assets</b> , Hirshleifer, Hou, Teoh, and Zhang (2004)
dNoa	Changes in net operating assets, Hirshleifer, Hou, Teoh, and Zhang (2004)	dLno	Change in long-term net operating assets, Fairfield, Whisenant, and Yohn (2003)
Ig	Investment growth, Xing (2008)	2Ig	Two-year investment growth, Anderson and Garcia-Feijoo (2006)
Nsi	<b>Net stock issues</b> , Pontiff and Woodgate (2008)	dli	% change in investment – % change in industry investment, Abarbanell and Bushee (1998)
Cei	Composite equity issuance, Daniel and Titman (2006)	Ivg	Inventory growth, Belo and Lin (2011)

# Stress Tests

## Testing deciles, investment (28)

Ivc	Inventory changes, Thomas and Zhang (2002)	Oa	Operating accruals, Sloan (1996)
dWc	Change in net non-cash working capital, Richardson, Sloan, Soliman, and Tuna (2005)	dCoa	Change in current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)
dNco	Change in net non-current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)	dNca	Change in non-current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)
dFin	Change in net financial assets, Richardson, Sloan, Soliman, and Tuna (2005)	dFnl	Change in financial liabilities, Richardson, Sloan, Soliman, and Tuna (2005)
dBe	Change in common equity, Richardson, Sloan, Soliman, and Tuna (2005)	Dac	Discretionary accruals, Xie (2001)
Poa	Percent operating accruals, Hafzalla, Lundholm, and Van Winkle (2011)	Pta	Percent total accruals, Hafzalla, Lundholm, and Van Winkle (2011)
Pda	Percent discretionary accruals	Ndf	Net debt finance, Bradshaw, Richardson, and Sloan (2006)

# Stress Tests

## Testing deciles, profitability (35)

Roe1	Return on equity, 1-month, Hou, Xue, and Zhang (2015)	Roe6	Return on equity, 6-month, Hou, Xue, and Zhang (2015)
dRoe1	Change in Roe, 1-month horizon	dRoe6	Change in Roe, 6-month horizon
dRoe12	Change in Roe, 12-month horizon	Roa1	Return on assets, 1-month horizon, Balakrishnan, Bartov, and Faurel (2010)
dRoa1	Change in Roa, 1-month horizon	dRoa6	Change in Roa, 6-month horizon
Rna <sup>q1</sup>	Return on net operating assets, 1-month horizon	Rna <sup>q6</sup>	Return on net operating assets, 6-month horizon
Ato <sup>q1</sup>	Quarterly asset turnover, 1-month horizon	Ato <sup>q6</sup>	Quarterly asset turnover, 6-month horizon
Ato <sup>q12</sup>	Quarterly asset turnover, 12-month horizon	Cto <sup>q1</sup>	Quarterly capital turnover, 1-month horizon
Cto <sup>q6</sup>	Quarterly capital turnover, 6-month horizon	Cto <sup>q12</sup>	Quarterly capital turnover, 12-month horizon
Gpa	Gross profits-to-assets, Novy-Marx (2013)	Gla <sup>q1</sup>	Gross profits-to-lagged assets, 1-month horizon

# Stress Tests

## Testing deciles, profitability (35)

- |                    |  |                    |  |
|--------------------|--|--------------------|--|
| Gla <sup>q6</sup>  | Gross profits-to-lagged assets, 6-month horizon                                    | Gla <sup>q12</sup> | Gross profits-to-lagged assets, 12-month horizon               |
| Ole <sup>q1</sup>  | Operating profits-to-lagged equity, 1-month horizon                                | Ole <sup>q6</sup>  | Operating profits-to-lagged equity, 6-month horizon            |
| Opa                | Operating profits-to-assets, Ball, Gerakos, Linnainmaa, and Nikolaev (2015)        | Ola <sup>q1</sup>  | Operating profits-to-lagged assets, 1-month horizon            |
| Ola <sup>q6</sup>  | Operating profits-to-lagged assets, 6-month horizon                                | Ola <sup>q12</sup> | Operating profits-to-lagged assets, 12-month horizon           |
| Cop                | Cash-based operating profitability, Ball, Gerakos, Linnainmaa, and Nikolaev (2016) | Cla                | Cash-based operating profits-to-lagged assets                  |
| Cla <sup>q1</sup>  | Cash-based operating profits-to-lagged assets, 1-month horizon                     | Cla <sup>q6</sup>  | Cash-based operating profits-to-lagged assets, 6-month horizon |
| Cla <sup>q12</sup> | Cash-based operating profits-to-lagged assets, 12-month horizon                    | F <sup>q1</sup>    | Quarterly F-score, 1-month horizon                             |
| F <sup>q6</sup>    | Quarterly F-score, 6-month horizon   | F <sup>q12</sup>   | Quarterly F-score, 12-month horizon                            |
| Fp <sup>q6</sup>   | Failure probability, 6-month horizon, Campbell, Hilscher, and Szilagyi (2008)      |                    |  |

# Stress Tests

## Testing deciles, intangibles (26)

Oca	Organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)	loca	Industry-adjusted organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)
Adm	Advertising expense-to-market, Chan, Lakonishok, and Sougiannis (2001)	Rdm	<b>R&amp;D-to-market</b> , Chan, Lakonishok, and Sougiannis (2001)
Rdm <sup>q1</sup>	<b>Quarterly R&amp;D-to-market</b> , 1-month horizon	Rdm <sup>q6</sup>	Quarterly R&D-to-market, 6-month horizon
Rdm <sup>q12</sup>	Quarterly R&D-to-market, 12-month horizon	OI	Operating leverage, Novy-Marx (2011)
OI <sup>q1</sup>	Quarterly operating leverage, 1-month horizon	OI <sup>q6</sup>	Quarterly operating leverage, 6-month horizon
OI <sup>q12</sup>	Quarterly operating leverage, 12-month horizon	Hs	Industry concentration (sales), Hou and Robinson (2006)
Etr	Effective tax rate, Abarbanell and Bushee (1998)	Rer	Real estate ratio, Tuzel (2010)
Eprd	Earnings predictability, Francis, Lafond, Olsson, and Schipper (2004)	Etl	Earnings timeliness, Francis, Lafond, Olsson, and Schipper (2004)

# Stress Tests

## Testing deciles, intangibles and trading frictions (4)

Alm<sup>a</sup>1 Asset liquidity (market assets),  
1-month horizon

Alm<sup>a</sup>12 Asset liquidity (market assets),  
12-month horizon

$R_a^{[2,5]}$  Years 2–5 lagged returns, annual  
Heston and Sadka (2008)

$R_a^{[6,10]}$  Years 6–10 lagged returns, annual  
Heston and Sadka (2008)

$R_a^{[11,15]}$  Years 11–15 lagged returns, annual  
Heston and Sadka (2008)

Alm<sup>a</sup>6 Asset liquidity (market assets),  
6-month horizon

$R_a^1$  12-month-lagged return,  
Heston and Sadka (2008)

$R_n^{[2,5]}$  Years 2–5 lagged returns, nonannual  
Heston and Sadka (2008)

$R_n^{[6,10]}$  Years 6–10 lagged returns, nonannual  
Heston and Sadka (2008)

$R_a^{[16,20]}$  Years 16–20 lagged returns, annual  
Heston and Sadka (2008)

### Trading frictions (4)

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Sv1 Systematic volatility risk,  
1-month horizon, Ang, Hodrick,  
Xing, and Zhang (2006)

Isff1 Idiosyncratic skewness  
per the 3-factor model,  
1-month horizon

Dtv12 Dollar trading volume, 12-month  
horizon, Brennan,  
Chordia, and Subrahmanyam (1998)

Isq1 Idiosyncratic skewness  
per the  $q$ -factor model,  
1-month horizon

# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\# t  \geq 1.96$	$\# t  \geq 3$	$\overline{ \alpha }$	$\#_{p < 5\%}^{GRS}$
	All (158)				
$q$	0.25	46	17	0.11	98
$q^5$	0.18	19	4	0.10	58
FF5	0.38	89	61	0.12	113
FF6	0.28	67	33	0.11	95
FF6c	0.25	55	21	0.10	68
BS6	0.28	61	34	0.14	147
SY4	0.27	57	25	0.10	87
DHS	0.42	83	45	0.15	108



# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{GRS}$
Momentum (36)					
$q$	0.26	8	1	0.10	23
$q^5$	0.19	6	1	0.09	12
FF5	0.64	34	27	0.16	34
FF6	0.29	18	8	0.10	25
FF6c	0.27	16	5	0.10	18
BS6	0.25	12	5	0.13	33
SY4	0.34	21	7	0.10	22
DHS	0.26	12	2	0.15	26

# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\# t  \geq 1.96$	$\# t  \geq 3$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
Value-versus-growth (29)					
$q$	0.20	4	0	0.11	17
$q^5$	0.19	4	0	0.13	15
FF5	0.14	1	0	0.08	9
FF6	0.16	4	1	0.09	11
FF6c	0.15	4	0	0.09	8
BS6	0.24	11	5	0.13	26
SY4	0.20	6	2	0.11	15
DHS	0.81	29	26	0.23	29

# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{GRS}$
Investment (28)					
$q$	0.20	9	4	0.10	17
$q^5$	0.10	0	0	0.08	7
FF5	0.23	11	6	0.09	17
FF6	0.21	10	5	0.09	17
FF6c	0.18	7	1	0.08	7
BS6	0.20	7	4	0.11	26
SY4	0.17	5	3	0.08	17
DHS	0.33	19	2	0.10	21

# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\# t  \geq 1.96$	$\# t  \geq 3$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
Profitability (35)					
$q$	0.23	12	4	0.10	19
$q^5$	0.14	2	0	0.09	12
FF5	0.45	28	21	0.12	30
FF6	0.32	22	11	0.10	21
FF6c	0.26	14	6	0.10	17
BS6	0.28	16	11	0.13	34
SY4	0.29	15	7	0.09	21
DHS	0.19	6	1	0.09	12

# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{GRS}$
Intangibles (26)					
$q$	0.41	11	8	0.17	19
$q^5$	0.31	7	3	0.13	10
FF5	0.41	13	6	0.15	20
FF6	0.42	11	8	0.16	18
FF6 <sub>c</sub>	0.43	12	9	0.16	17
BS6	0.42	13	7	0.19	25
SY4	0.33	8	6	0.14	10
DHS	0.59	14	12	0.19	17

# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{GRS}$
	Trading frictions (4)				
$q$	0.23	2	0	0.09	3
$q^5$	0.17	0	0	0.08	2
FF5	0.22	2	1	0.08	3
FF6	0.20	2	0	0.08	3
FF6 <sub>c</sub>	0.19	2	0	0.07	1
BS6	0.21	2	2	0.10	3
SY4	0.19	2	0	0.08	2
DHS	0.43	3	2	0.16	3

The  $q^5$  model is the best performing model

The  $q$ -factor model already compares well with the Fama-French 6-factor model, with a lower number of significant high-minus-low alphas but a higher number of GRS rejections

The Fama-French 5-factor model, the Barillas-Shanken model, and the Daniel-Hirshleifer-Sun model all perform poorly

- 1 Background
- 2 The Expected Growth Factor
- 3 Stress-testing Factor Models
- 4 Examples of Individual Factor Regressions**



# Individual Factor Regressions

Examples, 1/1967–12/2016

	Sue1	$R^6$	Bm	Nop	Nsi	Oa	dFin	Dac	Cop	Rdm
$\bar{R}$	0.46	0.82	0.54	0.63	-0.64	-0.27	0.28	-0.39	0.63	0.70
$t_{\bar{R}}$	3.48	3.50	2.61	3.40	-4.46	-2.19	2.39	-2.95	3.57	2.75
$\alpha_q$	0.06	0.25	0.15	0.35	-0.29	-0.56	0.43	-0.67	0.69	0.72
$\alpha_{q^5}$	-0.04	-0.16	0.08	0.20	-0.12	-0.23	0.12	-0.28	0.10	0.25
$t_q$	0.46	0.83	0.99	2.42	-2.32	-4.10	3.00	-4.73	5.04	3.11
$t_{q^5}$	-0.30	-0.60	0.51	1.33	-0.89	-1.51	0.81	-1.91	0.89	1.13
$\alpha_{FF5}$	0.52	1.00	-0.10	0.22	-0.30	-0.52	0.50	-0.64	0.82	0.57
$\alpha_{FF6}$	0.30	0.18	-0.08	0.24	-0.28	-0.47	0.48	-0.63	0.73	0.60
$\alpha_{FF6c}$	0.25	0.16	-0.08	0.16	-0.20	-0.31	0.36	-0.53	0.51	0.76
$t_{FF5}$	3.92	3.65	-0.88	1.83	-2.58	-4.20	4.17	-4.90	6.53	2.55
$t_{FF6}$	2.54	1.77	-0.70	1.92	-2.39	-3.42	3.86	-4.55	6.15	2.77
$t_{FF6c}$	2.10	1.44	-0.63	1.22	-1.60	-2.04	2.65	-3.63	4.28	3.34

The  $q^5$  model with an expected growth factor substantially improves the  $q$ -factor model, which already compares well with the Fama-French 6-factor model