Unemployment Crises

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Abstract

A search and matching model, when calibrated to the mean and volatility of unemployment in the postwar sample, can potentially explain the unemployment crisis in the Great Depression. The limited responses of wages from credible bargaining to labor market conditions, along with the congestion externality from matching frictions, cause the unemployment rate to rise sharply in recessions but decline gradually in booms. The frequency, severity, and persistence of unemployment crises in the model are quantitatively consistent with U.S. historical time series. The welfare gain from eliminating business cycle fluctuations is large.

JEL Classification: E24, E32, J63, J64.

Keywords: Search and matching frictions, unemployment crises, the Great Depression, the unemployment volatility puzzle, nonlinear impulse response functions

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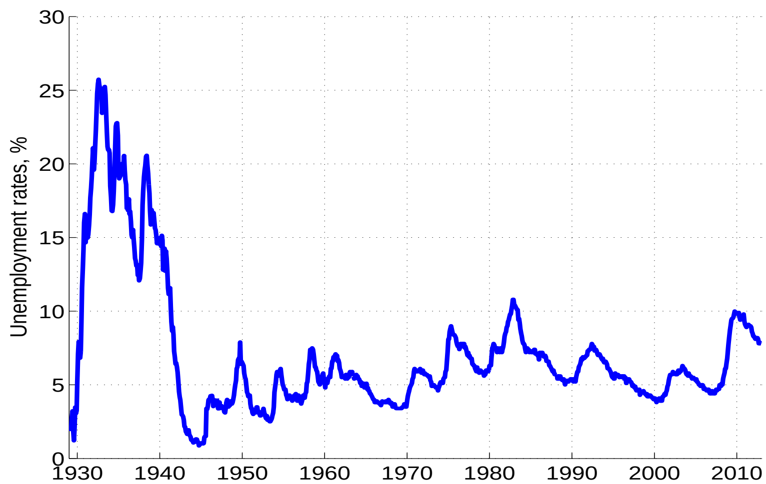
Figure 1 : U.S. Monthly Unemployment Rates, April 1929 to December 2012



1 Introduction

Figure 1 plots the monthly unemployment rate in the United States from April 1929 to December 2012. The mean unemployment rate is 7.1%, and the median rate is 5.7%. The most striking feature of the series is the extraordinarily high levels of unemployment in the 1930s, known as the Great Depression. From January 1931 to December 1939, the average unemployment rate is 18.5%, and the highest unemployment rate reaches 25.7% in August 1932. In contrast, such large dynamics are absent in the postwar sample. We fit a three-state Markov chain on the series via maximum likelihood. Identifying months in which the unemployment rate is above 20% as a crisis state, we estimate the unconditional probability of an unemployment crisis to be 3.47% and its persistence (the probability of a crisis next period conditional on a crisis in the current period) to be 82.35%.

We ask whether a Diamond-Mortensen-Pissarides model of equilibrium unemployment, when calibrated to the mean and volatility of unemployment in the postwar sample, can explain the large unemployment dynamics in the Great Depression. Perhaps surprisingly, the answer is affirmative. In the model, unemployed workers search for vacancies posted by a representative firm. A matching function takes vacancies and unemployed workers as inputs to produce the number of new hires in the labor market. Because of the congestion effect arising from matching frictions, the vacancy filling rate decreases with the tightness of the labor market (the ratio of the number of vacancies over the number of unemployed workers). Deviating from the standard wage determination via a generalized Nash bargaining game, we follow Hall and Milgrom (2008) to derive the equilibrium wage as the outcome of a credible bargaining game. In this setup, both parties make alternating offers that can be accepted, rejected to make a counteroffer, or rejected to take outside options. Relative to the Nash wage, the credible bargaining wage is more insulated from conditions in the labor market.

Our key insight is that the search model endogenizes unemployment crises as in the Great Depression, even though the exogenous driving force is the standard first-order autoregressive process with homoscedastic lognormal shocks. When calibrated to the mean of 5.84% and the volatility of 13.1% for the unemployment rate in the postwar sample, the model implies the persistence of the crisis state to be 84.18% and its unconditional probability 3.21%. These moments are close to 82.35% and 3.47%, respectively, in the long historical series. The unemployment dynamics in the model are large and highly nonlinear. In recessions the unemployment rate rises drastically, whereas in booms it declines only gradually. Its empirical stationary distribution is highly skewed with a long right tail. The skewness in the model is 3.09 (with a cross-simulation standard deviation of 0.90). For comparison, the skewness in the long historical series is 1.98.

The welfare cost of business cycle fluctuations is large in the model. We calculate the welfare cost to be on average 1.2% of consumption in perpetuity, which is 150 times the Lucas (1987) estimate. Intuitively, due to its strong nonlinearity, the mean consumption in simulations from the (stochastic) model is lower than the deterministic steady state of consumption. The difference amounts to 0.95% of the steady state consumption. In addition, recessions are deep and occasionally catastrophic, precisely when a risk averse agent's marginal utility is high. Finally, the welfare cost is countercyclical, and its stationary distribution has a long right tail. In particular, its 5 percentile is −0.38% relative to the median of 1.01%. In contrast, the 95 percentile is further away at 3.42%.

We also shed new light on the second moments of the labor market. Drawing from a variety of data sources, we construct a long vacancy series from April 1929 to December 2012. Together with the unemployment rate series, we calculate the volatility of the labor market tightness to be 0.37 in the long sample. This volatility is even higher than that of 0.27 in the postwar sample after January 1951. Moreover, the U.S. Beveridge curve is flatter in the long sample than in the postwar sample (the unemployment-vacancy correlations are -0.83 and -0.93 across the two samples, respectively). Notably, the model comes close to matching the volatility of the market tightness, 0.33 in crises and 0.27 in normal periods, as well as a flatter Beveridge curve in crises. However, the unemployment-vacancy correlations are somewhat lower in magnitude than those in the data.

Credible bargaining plays a key role in driving our results. As in Hall and Milgrom (2008), by allowing bargaining parties to make alternating offers instead of taking outside options, credible bargaining gives rise to equilibrium wages that are relatively insulated to conditions in the labor market. The congestion externality also plays a role. In recessions many unemployed workers compete for a small pool of vacancies. An extra vacancy is quickly filled, and the vacancy filling rate hardly increases. Consequently, the marginal cost of hiring declines very slowly (downward rigidity). Consider a large negative productivity shock. Output falls, but profits plummet more because the credible bargaining wage is insensitive to aggregate conditions. To make matters worse, the marginal cost of hiring runs into the downward rigidity, failing to decline to offset the impact of falling profits on the firm's incentives of hiring. As a result, unemployment rises drastically, giving rise to crises.

Comparative statics show that the probability of breakdown in bargaining and the delaying cost incurred by the firm during each round of alternating offers (two key parameters in the credible bargaining game) are quantitatively important for the crisis dynamics. A higher probability of breakdown, in which both parties take outside options, brings credible bargaining closer to Nash bargaining and makes the equilibrium wage more responsive to labor market conditions. As such, unemployment crises are dampened. In contrast, a higher delaying cost makes the equilibrium wage more insulated from labor market conditions. As such, crisis dynamics are strengthened.

The theoretical foundations for the search and matching model are provided by Diamond (1982), Mortensen (1982), and Pissarides (1985). Shimer (2005) contributes a simple yet profound insight that the unemployment volatility in the baseline search model is too low relative to that in the data.[1] As noted, our model is built on Hall and Milgrom (2008), who show that replacing the Nash wage with the credible bargaining wage helps explain the unemployment volatility puzzle. Going beyond the second moments of the labor market, we push the search model of unemployment to explain the unemployment crisis in the Great Depression.[2] Our work is also related to Cole and Ohanian (2004) and Ohanian (2009), who quantify important deviations from the neoclassical equilibrium conditions in the labor market in the Great Depression. Their results emphasize the impact of real wage rigidity as well as frictions in the labor market. As such, we view our work based on the search-theoretical approach as complementary to their findings.

The rest of the paper is organized as follows. Section 2 documents the crisis dynamics in the historical U.S. unemployment rates and the Beveridge curve. Section 3 describes the search and matching model with credible bargaining. Section 4 presents the quantitative results. Section 5 calculates the welfare cost of business cycles in the model with log utility. Finally, Section 6 concludes. Supplementary results including detailed data description are provided in the appendices.

2 Evidence

We construct our monthly U.S. unemployment rate series from April 1929 to December 2012 by drawing from NBER macrohistory files and Federal Reserve Economic Data at Federal Reserve Bank of St. Louis. We adjust and concatenate four different series: (i) seasonally adjusted unem-

[1]A large subsequent literature has developed to address the volatility puzzle. Hall (2005) shows how wage stickiness, which satisfies the condition that no worker-firm pair has any unexploited opportunity for mutual gain, increases labor market volatilities. Mortensen and Nagypál (2007) and Pissarides (2009) show that the fixed recruiting cost can explain the volatility puzzle. Hagedorn and Manovskii (2008) show that a calibration with small profits and a low bargaining power for the workers can produce realistic volatilities. Petrosky-Nadeau and Wasmer (2013) show how incorporating financial frictions can generate higher labor market volatilities.

[2]Petrosky-Nadeau, Zhang, and Kuehn (2013) embed the search and matching model into an equilibrium asset pricing framework. Instead of asset prices, we focus on unemployment crises as well as the second moment of the labor market. We also move beyond the Nash bargaining to the Hall and Milgrom (2008) credible bargaining.

ployment rates from April 1929 to February 1940; (ii) seasonally adjusted unemployment rates from March 1940 to December 1946; (iii) unemployment rates (not seasonally adjusted) from January 1947 to December 1947; and (iv) seasonally adjusted civilian unemployment rates from January 1948 to December 2012 from Bureau of Labor Statistics at U.S. Department of Labor. We detail the data sources and our procedures for adjusting the raw series in Appendix A.1.

We also construct a long vacancy rate series stretching back to April 1929 by drawing from four different sources for U.S. job openings: (i) the Metropolitan Life Insurance company help-wanted advertising index in newspapers from January 1919 to August 1960 from the NBER macrohistory files; (ii) the Conference Board help-wanted advertising index from January 1951 to July 2006; (iii) the Barnichon (2010) composite print and online help-wanted index from January 1995 to December 2012; and (iv) the seasonally adjusted job openings series from December 2000 to December 2012 obtained from the Job Openings and Labor Turnover Survey released by U.S. Bureau of Labor Statistics. To convert the help-wanted index to a vacancy rate series, we utilize the series of the civilian labor force over 16 years of age from Current Employment Statistics released by U.S. Bureau of Labor Statistics from January 1948 to December 2012 as well as annual observations of total population from 1929 to 1947 from the U.S. Census Bureau. We detail the data sources and our procedures for adjusting the raw data in Appendix A.2.

2.1 Unemployment Crises

To model the tail behavior in the U.S. unemployment rate series in Figure 1, we follow Chatterjee and Corbae (2007) to fit a three-state Markov chain model via maximum likelihood. The aggregate state of the economy, $\eta \in \{g, b, c\}$, evolves through good (g), bad (b), and crisis (c) states with different employment prospects. Let the transition matrix of the Markov chain be given by:

$$\Lambda = \begin{bmatrix} \lambda_{gg} & \lambda_{bg} & \lambda_{cg} \\ \lambda_{gb} & \lambda_{bb} & \lambda_{cb} \\ \lambda_{gc} & \lambda_{bc} & \lambda_{cc} \end{bmatrix}, \tag{1}$$

in which, for example, $\lambda_{gb} \equiv \text{Prob}\{\eta_{t+1} = g|\eta_t = b\}$ is the probability of the economy being in state g next period conditional on the economy being in state b in the current period.

As discussed in Chatterjee and Corbae (2007), the maximum likelihood estimate of λ_{kj}, which is the (j,k)th element of the aggregate state transition matrix, is the ratio of the number of times the economy switches from state j to state k to the number of times the economy is in state j. Let, for example, $\mathbf{1}_{\{\eta_t=j\}}$ denote the indicator function that takes the value of one if the economy in period t is in state j and zero otherwise. The maximum likelihood estimate of λ_{kj} is given by:

$$\hat{\lambda}_{kj} = \frac{\sum_{t=1}^{T-1} \mathbf{1}_{\{\eta_{t+1}=k\}} \mathbf{1}_{\{\eta_t=j\}}}{\sum_{t=1}^{T-1} \mathbf{1}_{\{\eta_t=j\}}}. \tag{2}$$

In addition, the asymptotic standard error for $\hat{\lambda}_{kj}$ is given by:

$$\text{Ste}(\hat{\lambda}_{kj}) = \sqrt{\frac{\hat{\lambda}_{kj}(1-\hat{\lambda}_{kj})}{\sum_{t=1}^{T} \mathbf{1}_{\{\eta_t=j\}}}}. \tag{3}$$

In practice, we identify the good state, g, as months in which the unemployment rates are below the median unemployment rate of 5.7%. We define the crisis state, c, as months in which the unemployment rates are above or equal to 20%. The bad state, b, is then identified as months in which the unemployment rates are below 20% but above or equal to the median of 5.7%. We choose the crisis cutoff rate of 20% judiciously such that the cutoff rate is relatively high, but there are still a sufficient number of months in which the economy hits the crisis state so that the transition probability estimates can be (relatively) precise.

Table 1 reports the estimated aggregate state transition matrix. The crisis state is persistent in that the probability of the economy remaining in the crisis state conditional on it being in the crisis state is 82.35%. This estimate is also precise, with a small standard error of 0.065. With a probability of 17.65%, the economy switches from the crisis state to the bad state. Unconditionally, the tail probability of the economy being in the crisis state is estimated to be 3.47%.

Table 1 : Estimated Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States, April 1929–December 2012

This table reports the estimated state transition matrix in equation (1). The transition probabilities are defined as in equation (2), and the standard errors (in parentheses) are in equation (3). The last row reports the unconditional probabilities of the states, calculated by raising the transition matrix to the power 1,000.

	Good	Bad	Crisis
Good	0.9586	0.0414	0
	(0.0091)	(0.0091)	(0)
Bad	0.0390	0.9487	0.0123
	(0.0088)	(0.0100)	(0.0050)
Crisis	0	0.1765	0.8235
	(0)	(0.0654)	(0.0654)
Unconditional probability	0.4683	0.4970	0.0347

2.2 The Beveridge Curve

Figure 2 reports the vacancy rates from April 1929 to December 2012 (Panel A) as well as the vacancy-unemployment ratio (labor market tightness) over the sample period (Panel B). We see that World War II is an important outlier during which the vacancy rate, especially the labor market tightness, reaches abnormally high levels that are otherwise absent in other periods.

Figure 3 reports the U.S. Beveridge curve by plotting the vacancy rates against the unemployment rates from April 1929 to December 2012. Several patterns emerge. First, the scatter points display a clear convex shape, a pattern consistent with the congestion externality due to matching frictions in the labor market. Second, the early period prior to January 1951 shows dramatic movements in the unemployment and vacancy rates. In particular, when the unemployment rates exceed 20% in the Great Depression, the vacancy rates are below 0.75%. When the unemployment rates are below 2% during the World War II, the vacancy rates are close to the maximum value of 5%. In contrast, such large movements are absent from the later period. Finally, the Great Depression (with high unemployment and low vacancy rates) makes the Beveridge curve substantially flatter than it otherwise would have been. The observations from the Great Recession (the period after January 2008) seem well aligned with the flattened Beveridge curve.

Figure 2 : U.S. Monthly Vacancy Rates and Labor Market Tightness (the Vacancy-unemployment Ratio), April 1929 to December 2012

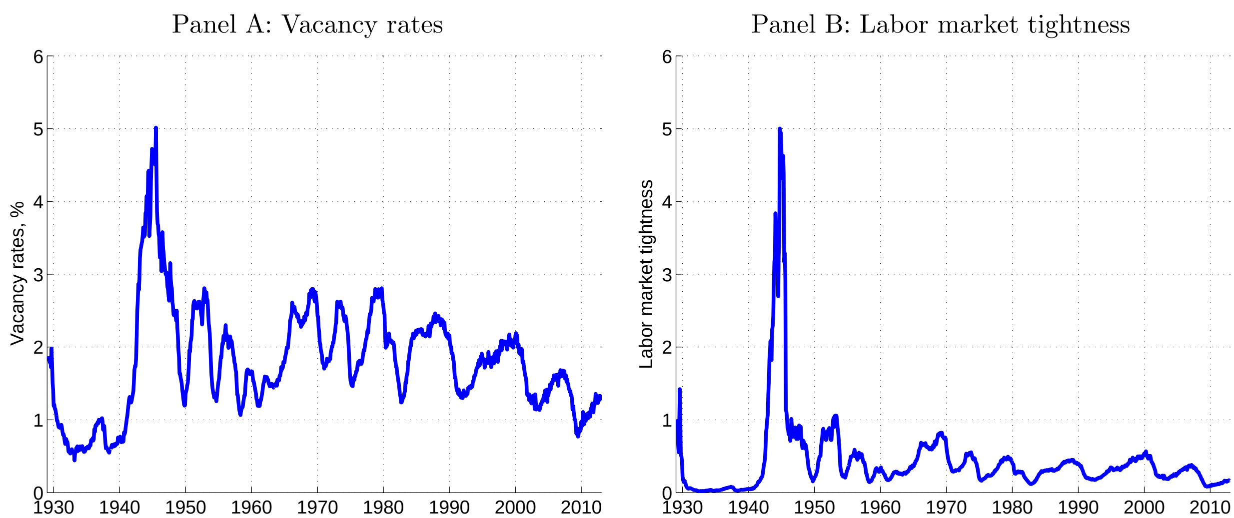
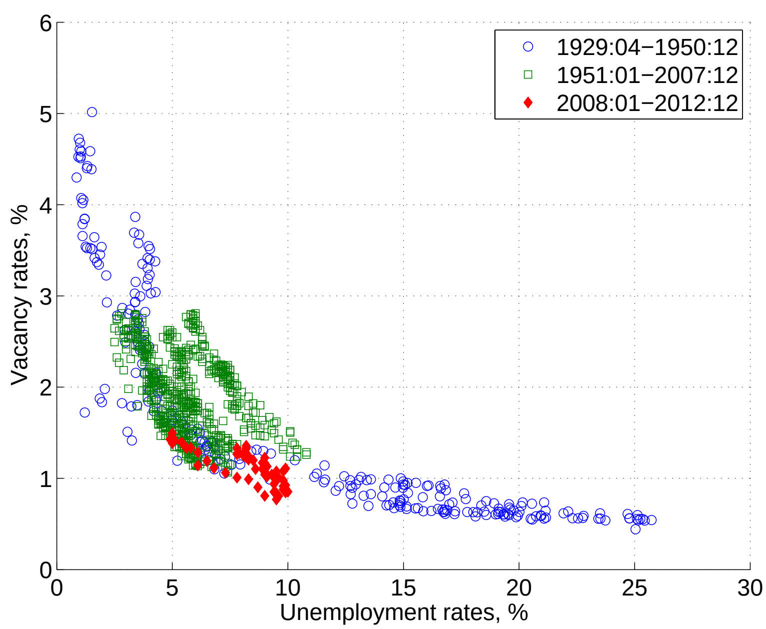


Figure 3 : The U.S. Beveridge Curve, April 1929 to December 2012



2.3 Labor Market Volatilities

We report a standard set of second moments for the labor market for the long sample from April 1929 to December 2012 as well as the postwar sample since January 1951 to compare with existing studies (e.g., Shimer (2005)). We measure the labor productivity as the seasonally adjusted real average output per job in the nonfarm business sector (Series id: PRS85006163) from the Bureau of Labor Statistics. Unlike the unemployment and vacancy rates, the labor productivity is available only for the postwar sample. We take quarterly averages of the monthly series to convert to quarterly series. We detrend the quarterly series in log deviations from the Hodrick-Prescott (HP, 1997) trend with a smoothing parameter of 1,600.

Table 2 reports the data moments. In the long sample that includes the Great Depression, the unemployment volatility is 0.218, which is more than 65% higher than the volatility of 0.131 in the postwar sample. The unemployment-vacancy correlation is -0.827 in the long sample but -0.931 in the postwar sample. As such, the Great Depression flattens somewhat the slope of the Beveridge curve (see also Figure 3). The vacancy volatility is 0.168 in the long sample, which is slightly higher than 0.142 in the postwar sample. The standard deviation for the labor market tightness is 0.368 in the long sample and is higher than 0.269 in the postwar sample. It is clear that the World War II has an important impact on this volatility estimate (see Figure 2).

3 The Model

We construct a search model of unemployment embedded with credible bargaining in determining the equilibrium wage as in Hall and Milgrom (2008).

3.1 The Environment

The model is populated by a representative household and a representative firm that uses labor as the single productive input. Following Merz (1995) and Andolfatto (1996), we use the representative family construct, which implies perfect consumption insurance. The household has a continuum

Table 2 : Labor Market Volatilities in the Data

Both the unemployment rates, U, and the vacancy rates, V, are converted to quarterly averages of monthly series. The labor market tightness is then defined as $\theta = V/U$. The labor productivity, X, is seasonally adjusted real average output per person in the nonfarm business sector from the Bureau of Labor Statistics. The X series is not available in the early sample. All the variables are in log deviations from the HP-trend with a smoothing parameter of 1,600.

		U	V	θ	U	V	θ	X
		Panel A: Crisis sample, April 1929–December 2012			Panel B: Non-crisis sample January 1951–December 2012			
Standard deviation		0.218	0.168	0.368	0.131	0.142	0.269	0.013
Autocorrelation		0.887	0.912	0.907	0.888	0.910	0.905	0.768
Correlation matrix	U		−0.827	−0.967		−0.931	−0.981	−0.232
	V			0.943			0.984	0.391
	θ							0.320

with a unit mass of members who are, at any point in time, either employed or unemployed. The fractions of employed and unemployed workers are representative of the population at large. The household pools the income of all the members together before choosing per capita consumption and asset holdings. Finally, the household is risk neutral with a time discount factor of β.

The representative firm posts a number of job vacancies, V_t, to attract unemployed workers, U_t. Vacancies are filled via a constant returns to scale matching function, $G(U_t, V_t)$, specified as:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{1/\iota}}, \tag{4}$$

in which $\iota > 0$ is a constant parameter. This matching function, specified as in Den Haan, Ramey, and Watson (2000), has the desirable property that matching probabilities fall between zero and one.

Define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate), denoted $f(\theta_t)$, is:

$$f(\theta_t) \equiv \frac{G(U_t, V_t)}{U_t} = \frac{1}{\left(1 + \theta_t^{-\iota}\right)^{1/\iota}}. \tag{5}$$

The probability for a vacancy to be filled per unit of time (the vacancy filling rate), denoted $q(\theta_t)$, is:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1+\theta_t^{\iota})^{1/\iota}}. \tag{6}$$

It follows that $q'(\theta_t) < 0$: An increase in the scarcity of unemployed workers relative to vacancies makes it harder to fill a vacancy. As such, θ_t is labor market tightness from the firm's perspective.

The representative firm incurs costs in posting vacancies. Following Mortensen and Nagypàl (2007) and Pissarides (2009), we incorporate a fixed component in the unit cost per vacancy:

$$\kappa_t \equiv \kappa_0 + \kappa_1 q(\theta_t), \tag{7}$$

in which κ_0 is the proportional cost, κ_1 is the fixed cost, and both are nonnegative. The proportional cost is standard in the literature. The fixed cost captures training, interviewing, and administrative setup costs of adding a worker to the payroll, costs that are paid after a hired worker arrives but before wage bargaining takes place. The marginal cost of hiring arising from the proportional cost, $\kappa_0/q(\theta_t)$, is time-varying, but the marginal cost arising from the fixed cost is constant, κ_1.

Jobs are destroyed at a constant rate of $s > 0$ per period. Employment, N_t, evolves as:

$$N_{t+1} = (1-s)N_t + q(\theta_t)V_t, \tag{8}$$

in which $q(\theta_t)V_t$ is the number of new hires. The size of the population is normalized to be unity, $U_t = 1 - N_t$. As such, N_t and U_t are also the rates of employment and unemployment, respectively.

The firm takes aggregate labor productivity, X_t, as given. The law of motion for $x_t \equiv \log(X_t)$ is:

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \tag{9}$$

in which $\rho \in (0,1)$ is the persistence, $\sigma > 0$ is the conditional volatility, and ϵ_{t+1} is an independently and identically distributed standard normal shock. The firm uses labor to produce output, Y_t, with a constant returns to scale production technology,

$$Y_t = X_t N_t. \tag{10}$$

The dividends to the firm's shareholders are given by:

$$D_t = X_t N_t - W_t N_t - \kappa_t V_t, \tag{11}$$

in which W_t is the equilibrium wage rate. Taking $q(\theta_t)$ and W_t as given, the firm posts an optimal number of job vacancies to maximize the cum-dividend market value of equity, denoted S_t:

$$S_t \equiv \max_{\{V_{t+\triangle t}, N_{t+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t \left[\sum_{\triangle t=0}^{\infty} \beta^{\triangle t} \left[X_{t+\triangle t} N_{t+\triangle t} - W_{t+\triangle t} N_{t+\triangle t} - \kappa_{t+\triangle t} V_{t+\triangle t} \right] \right], \tag{12}$$

subject to the employment accumulation equation (8) and a nonnegativity constraint on vacancies:

$$V_t \geq 0. \tag{13}$$

Because $q(\theta_t) > 0$, this constraint is equivalent to $q(\theta_t)V_t \geq 0$. As such, the only source of job destruction in the model is the exogenous separation of employed workers from the firm.[3]

Let λ_t denote the multiplier on the nonnegativity constraint $q(\theta_t)V_t \geq 0$. From the first-order conditions with respect to V_t and N_{t+1}, we obtain the intertemporal job creation condition:

$$\frac{\kappa_t}{q(\theta_t)} - \lambda_t = E_t \left[\beta \left[X_{t+1} - W_{t+1} + (1-s) \left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda_{t+1} \right] \right] \right]. \tag{14}$$

Intuitively, the marginal cost of hiring at time t equals the marginal value of employment to the firm, which in turn equals the marginal benefit of hiring at period $t+1$, discounted to t. The marginal benefit at $t+1$ includes the marginal product of labor, X_{t+1}, net of the wage rate, W_{t+1}, plus the marginal value of employment, which equals the marginal cost of hiring at $t+1$, net of

[3] This constraint does not bind in the model's simulations under the benchmark calibration. As such, the constraint does not affect our quantitative results. However, the constraint can be binding under at least some alternative parameterizations. As such, we opt to impose the constraint in the solution algorithm for computational accuracy.

separation. The optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0. \tag{15}$$

3.2 Credible Bargaining

To close the model, we need to specify how the wage rate, W_t, is determined in equilibrium. In the standard Diamond-Mortensen-Pissarides model, the wage rate is derived from the sharing rule per the outcome of a generalized Nash bargaining process between the employed workers and the firm (e.g., Pissarides (2000, Section 1.4)). Let $0 < \eta < 1$ be the workers' relative bargaining weight and b the workers' value of unemployment activities. The Nash-bargained wage rate is:

$$W_t = \eta\left(X_t + \kappa_t\theta_t\right) + (1-\eta)b. \tag{16}$$

Although analytically simple, the baseline search model with the Nash wage requires a relatively high replacement ratio (the value of unemployment activities over the average marginal product of labor) to reproduce realistic labor market volatilities (e.g., Hagedorn and Manovskii (2008)).

The Setup

We adopt the credible bargaining wage proposed by Hall and Milgrom (2008). Built on Binmore, Rubinstein, and Wolinsky (1986), Hall and Milgrom place a crucial distinction between a threat point and an outside option in the wage bargaining game. Bargaining takes time. Both parties make alternating offers which can be accepted, rejected to make a counteroffer, or rejected to abandon the bargaining altogether. In the standard Nash bargaining, disagreement leads immediately to the abandonment of the bargaining game, meaning that the relevant threat point is the outside options for both parties. In contrast, in the more realistic alternating bargaining, disagreement only leads to another round of alternating offers. The threat point is the payoff from another round of alternating offers, and outside options are taken only when abandoning the bargaining altogether.

The outside option for a worker is the value of unemployment. The outside option for the firm is to resume searching in the labor market, and its value is always driven to zero in equilibrium. During a period in which both parties engage in another round of alternating offers, the worker receives the flow value of unemployment activities, b, and the firm incurs the cost of delaying, $\chi > 0$, which can be interpreted as the cost of idle capital (e.g., Hall and Milgrom (2008)). During this period, the negotiation can also break down with a probability of δ.

With this setup of the alternating bargaining game, the indifference condition for a worker when considering a wage offer, W_t, from the firm is:

$$J_{Nt}^{W} = \delta J_{Ut} + (1-\delta)\left(b + E_t[\beta J_{Nt+1}^{W'}]\right), \tag{17}$$

in which $J_t \equiv J(N_t, X_t)$ is the indirect utility function of the representative household, J_{Nt}^{W} is the marginal value of an employed worker to the household when accepting the wage offer from the employer, J_{Ut} is the marginal value of an unemployed worker to the household, and $J_{Nt+1}^{W'}$ is the marginal value of an employed worker to the household when rejecting the firm's wage offer to make a counteroffer of W'_{t+1} in the next period. The indifference condition in equation (17) says that the payoff to the worker when accepting the wage offer from the firm, J_{Nt}^{W}, is just equal to the payoff from rejecting the offer. After rejecting the offer, with a probability of δ, the negotiation breaks down, and the worker returns to the labor market, leaving the household with the marginal value of an unemployed worker. With the probability of $1-\delta$, the worker receives the flow value of unemployment, b, for the current period, and makes a counteroffer of W'_{t+1} to the firm in the next period.

The indifference condition for the firm when considering the worker's counteroffer, W'_t, is:

$$S_{Nt}^{W'} = \delta \times 0 + (1-\delta)\left(-\chi + E_t[\beta S_{Nt+1}^{W}]\right), \tag{18}$$

in which $S_{Nt}^{W'}$ is the marginal value of an employed worker to the firm when accepting the worker's counteroffer, and S_{Nt+1}^{W} is the marginal value of an employed worker to the firm when rejecting the worker's offer to make a counteroffer of W_{t+1} in the next period. Intuitively, equation (18) says that the firm is just indifferent between the payoff from accepting the worker's offer W_t' and the payoff from rejecting the offer to have an opportunity to make a counteroffer of W_{t+1} in the next period. When rejecting the offer, the firm pays the delaying cost of χ if the bargaining does not break down. When the negotiation does break down, the firm's payoff is zero.

The two indifference conditions collapse to the indifference conditions for the standard Nash bargaining when the probability of breakdown, δ, equals one. During the alternating bargaining, it is optimal for each party to make a just acceptable offer. As in Hall and Milgrom (2008), we assume that the firm makes the first offer, which the worker accepts. As such, W_t is the equilibrium wage, and the delaying cost, χ, is never paid in equilibrium.

Equilibrium Wages

The equilibrium wage, W_t, and the worker's counteroffer wage, W_{t+1}', can be characterized further. First, we note that the marginal value of an unemployed worker to the household is:

$$J_{Ut} = b + E_t\left[\beta\left(f_t J_{Nt+1}^W + (1-f_t)J_{Ut+1}\right)\right], \tag{19}$$

in which $f_t \equiv f(\theta_t)$ is the job finding rate. The equation says that the value of unemployment equals the flow value of unemployment activities, b, plus the discounted expected value in the next period. With a probability of f_t, the unemployed worker lands a job, which delivers the value of J_{Nt+1}^W. Otherwise, the worker remains unemployed with a value of J_{Ut+1}.

In addition, the marginal value of an employed worker to the household is:

$$J_{Nt}^W = W_t + E_t\left[\beta\left((1-s)J_{Nt+1}^W + sJ_{Ut+1}\right)\right]. \tag{20}$$

The equation says that the value of employment equals the flow value from the wage, W_t, plus the discounted expected value in the next period. With a probability of s, the employed worker separates from the firm, and returns to the labor market as an unemployed worker with a value of J_{Ut+1}. Otherwise, the worker remains on the job, which delivers the value of J_{Nt+1}^W.

The wage offer by the firm to the worker, W_t, can be expressed as (see Appendix B for details):

$$W_t = b + (1-\delta)\beta E_t\left[J^{W'}_{Nt+1} - J_{Ut+1}\right] - (1 - s - \delta f_t)\,\beta E_t\left[J^{W}_{Nt+1} - J_{Ut+1}\right]. \tag{21}$$

Intuitively, the wage offer from the firm increases in the flow value of unemployment activities b. The second term in equation (21) says that if the bargaining does not breakdown, the wage offer also increases in the surplus that the worker would enjoy after making a counteroffer, W'_{t+1}, to the firm. From the last term in equation (21), the equilibrium wage, W_t, also increases in the separation rate, s. As s goes up, the expected duration of the job shortens. As such, the worker requires a higher wage to remain indifferent between accepting and rejecting the wage offer. Finally, W_t increases in the job finding rate, f_t. As f_t rises, the worker's outside job market prospects improve, and the firm must offer a higher wage to make the worker indifferent. However, this impact of labor market conditions on W_t becomes negligible as the probability of breakdown in the bargaining, δ, goes to zero.

The wage offer of a worker to the firm, W'_t, can be expressed as:

$$W'_t = X_t + (1-\delta)\chi + \beta E_t\left[(1-s)S^{W'}_{Nt+1} - (1-\delta)S^{W}_{Nt+1}\right]. \tag{22}$$

Intuitively, W'_t increases in labor productivity, X_t, and the cost of delay to the firm, χ. A higher χ makes the firm more likely to accept a higher wage offer from the worker to avoid any delay. As W'_t contains a higher constant proportion because of a higher χ, W'_t becomes more insulated from labor market conditions. Further, because W'_t is the flow value of $J^{W'}_{Nt}$ shown in equation (20), $J^{W'}_{Nt}$ also becomes more insulated. More important, as $J^{W'}_{Nt}$ enters the second term in equation (21), the equilibrium wage, W_t, becomes less sensitive to aggregate conditions as a result of a higher χ.

From the last term in equation (22), an increase in the separation rate reduces the wage offer from the worker to the firm, W'_t. As s rises, the present value of profits produced by the worker drops. To make the firm indifferent, the worker must reduce the wage offer. Also, the worker's offer, W'_t, increases (naturally) in the firm's surplus from accepting the offer, $S^{W'}_{Nt+1}$. In contrast, the worker's offer would be lower if the firm's surplus, S^W_{Nt+1}, from rejecting the offer to make a counteroffer, W_t, is higher. However, the quantitative importance of this channel would be negligible if the breakdown probability, δ, goes to one. As such, W'_t increases with δ.

Finally, the two parties of the credible bargaining game would agree to accept the equilibrium wage if the joint surplus of the match is greater than the joint value of the outside options, J_{Ut}, as well as the joint present value of continuous delaying:

$$S^W_{Nt} + J^W_{Nt} > \max\left(J_{Ut}, E_t\left[\sum_{\triangle t=0}^{\infty} \beta^{\triangle t}(b-\chi)\right]\right) = J_{Ut}. \tag{23}$$

The last equality holds because the flow value of unemployment, b, is higher than $b-\chi$ (the delaying cost is positive). We verify that this condition holds in simulations.

3.3 Competitive Equilibrium

In equilibrium, the household receives the firm's dividends, and the goods market clears:

$$C_t + \kappa_t V_t = X_t N_t. \tag{24}$$

The competitive equilibrium consists of vacancy posting, $V_t \geq 0$, multiplier, $\lambda_t \geq 0$, consumption, C_t, and wages, W_t and W'_t, such that: (i) V_t and λ_t satisfy the intertemporal job creation condition (14) and the Kuhn-Tucker conditions (15); (ii) wages, W_t and W'_t, satisfy the indifference conditions (17) and (18); and (iii) the goods market clears as in equation (24).

4 Quantitative Results

We calibrate the model and discuss computational issues in Section 4.1. We examine the model's stationary distribution in Section 4.2 and quantify its performance in explaining higher moments of unemployment in Section 4.3. We then study the second moments of the labor market in Section 4.4, nonlinear impulse response functions in Section 4.5, and comparative statics in Section 4.5.

Table 3 : Parameter Values in the Monthly Calibration for the Benchmark Model

Notation	Parameter	Value
β	Time discount factor	$e^{-5.524/1200}$
ρ	Aggregate productivity persistence	$0.95^{1/3}$
σ	Conditional volatility of productivity shocks	0.00635
s	Job separation rate	0.045
ι	Elasticity of the matching function	1.25
b	The value of unemployment activities	0.71
δ	Probability of breakdown in bargaining	0.1
χ	Cost to employer of delaying in bargaining	0.25
κ_0	The proportional cost of vacancy posting	0.125
κ_1	The fixed cost of vacancy posting	0.2

4.1 Calibration and Computation

Our calibration strategy is to match the mean and the volatility of the unemployment rate in the postwar sample, before quantifying the model's performance in explaining the crisis dynamics documented in Section 2. As the common practice in the macro labor literature, calibrating to the unemployment dynamics in normal periods seems sensible. Because of the strong nonlinearity in the model, steady state relations hold very poorly in simulations. As such, we do not use these relations.

Table 3 lists the parameter values for the monthly benchmark calibration of the model. The time discount factor, β, is set to be $e^{-5.524/1200} = 0.9954$. This value implies a discount rate of 5.524% per annum, which is the leverage-adjusted aggregate discount rate in the 1951–2012 sample.[4] To calibrate the log labor productivity, we set its persistence, ρ, to be $0.95^{1/3} = 0.983$ as in Gertler and Trigari (2009). We then calibrate its conditional volatility, σ, to be 0.00635 to match the standard deviation of 0.013 for the labor productivity in the data (see Table 2).

We set the job separation rate, s, to be 4.5%, which is higher than 3.78% from JOLTS. However, Davis, Faberman, Haltiwanger, and Rucker (2010) show that the JOLTS sample overweights es-

[4]We obtain monthly series of the value-weighted stock market returns, one-month Treasury bill rates, and inflation rates from January 1951 to December 2012 from Center for Research in Security Prices. The average real interest rate (one-month Treasury bill rates minus inflation rates) is 0.895% per annum. The equity premium (the value-weighted market returns in excess of one-month Treasury bill rates) is on average 6.807%. Because we do not model financial leverage, we calculate the leverage-adjusted equity premium as $(1 - 0.32) \times 6.807\% = 4.629\%$, in which 0.32 is the aggregate market leverage ratio of U.S. corporations reported in Frank and Goyal (2008). Taken together, the leverage-adjusted aggregate discount rate in the data is $4.629\% + 0.895\% = 5.524\%$.

tablishments with stable hiring and separation and underweights volatile establishments with rapid growth or contraction. Adjusting for this bias, Davis et al. (Table 5.4) estimate the separation rate to be 4.96%. For the elasticity parameter in the matching function, ι, we set it to be 1.25, which is close to that in Den Haan, Ramey, and Watson (2000).

Following Hall and Milgrom (2008), we calibrate the value of unemployment activities to be 0.71. The probability of breakdown in bargaining, δ, is 0.1 in our monthly frequency, and is close to Hall and Milgrom's value of 0.0055 in their daily calibration (with 20 working days per month). The delaying cost parameter, χ, is set to be 0.25, which is close to 0.27 in Hall and Milgrom. To calibrate the recruiting cost parameters, κ_0 and κ_1, we target the first and the second moments of the unemployment rate in the postwar sample. As noted, the mean unemployment rate in this sample is 5.84%, and the unemployment volatility is 0.131 (see Table 2). We end up with the recruiting cost parameters, $\kappa_0 = 0.125$ and $\kappa_1 = 0.2$, which imply a mean of 5.92% and a volatility of 0.102 for the unemployment rate in normal periods.

Because we focus on the higher moments of unemployment, the standard loglinearization cannot be used. We instead adapt the projection algorithm in Petrosky-Nadeau, Zhang, and Kuehn (2013) to our setting. The state space consists of employment and log productivity, (N_t, x_t). The goal is to solve for the optimal vacancy function, $V_t = V(N_t, x_t)$, the multiplier function, $\lambda_t = \lambda(N_t, x_t)$, and the equilibrium wage, $W_t = W(N_t, x_t)$, from the following five functional equations:

$$
\begin{aligned}
\frac{\kappa_t}{q(\theta_t)} - \lambda(N_t, x_t) &= E_t\left[\beta\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda(N_{t+1}, x_{t+1})\right]\right]\right] \qquad (25)\\
W(N_t, x_t) &= b + (1-\delta)\beta E_t\left[J_N^{W'}(N_{t+1}, x_{t+1}) - J_U(N_{t+1}, x_{t+1})\right]\\
&\quad - (1 - s - \delta f_t)\,\beta E_t\left[J_N^W(N_{t+1}, x_{t+1}) - J_U(N_{t+1}, x_{t+1})\right] \qquad (26)\\
J_U(N_t, x_t) &= b + E_t\left[\beta\left(f_t J_N^W(N_{t+1}, x_{t+1}) + (1-f_t) J_U(N_{t+1}, x_{t+1})\right)\right] \qquad (27)\\
J_N^W(N_t, x_t) &= W_t + E_t\left[\beta\left((1-s) J_N^W(N_{t+1}, x_{t+1}) + s J_U(N_{t+1}, x_{t+1})\right)\right] \qquad (28)\\
J_N^{W'}(N_t, x_t) &= W_t' + E_t\left[\beta\left((1-s) J_N^{W'}(N_{t+1}, x_{t+1}) + s J_U(N_{t+1}, x_{t+1})\right)\right]. \qquad (29)
\end{aligned}
$$

In addition, $V(N_t, x_t)$ and $\lambda(N_t, x_t)$ must also satisfy the Kuhn-Tucker condition (15). Appendix C contains further computational details.

4.2 Properties of the Stationary Distribution

We simulate the economy for one million monthly periods from the model's stationary distribution.[5] Panel A of Figure 4 plots the unemployment rate against labor productivity in simulations. The relation is strongly nonlinear. When labor productivity is above its mean of unity, unemployment goes down only slightly. However, when labor productivity is below its mean, unemployment goes up drastically. The correlation between unemployment and productivity is -0.677. Panel B plots labor market tightness, θ_t, against productivity. Although the relation is nonlinear, the nonlinearity is not nearly as dramatic as that of unemployment in Panel A. Apart from the region when the labor productivity is very low, the θ_t-productivity relation is (virtually) linear.

Panels C and D report the empirical cumulative distribution functions of unemployment and labor market tightness. Unemployment is skewed with a long right tail. The 2.5 percentile, 4.70%, is close to the median of 5.40%, but the 97.5 percentile is far away, 15.15%. The 1 percentile is 4.65%, but the 99 percentile is 20.71%. Finally, the minimum rate is 4.52%, but the maximum is 55.13%. In contrast, the empirical distribution of the labor market tightness is largely symmetric.

Figure 5 illustrates a crisis episode in simulations. Panel A plots the unemployment rate, and Panel B the log productivity. A deep crisis occurs around the 300th month when the log labor productivity drops more than three unconditional standard deviations below the unconditional mean of zero. In response, the unemployment rate rises above 30%. Two other episodes with high unemployment rates around 17% occur shortly after the 400th month and the 700th month, when the log productivity dips just below the two-unconditional-standard-deviation bound.

[5] To reach the stationary distribution, we start at the initial condition of zero for log labor productivity and the deterministic steady state employment and simulate the economy for 6,000 months.

Figure 4 : The Unemployment-productivity Relation, the Labor Market Tightness-productivity Relation, and Empirical Cumulative Distribution Functions of Unemployment and Labor Market Tightness

Results are based on the one-million-month simulated data from the model's stationary distribution.

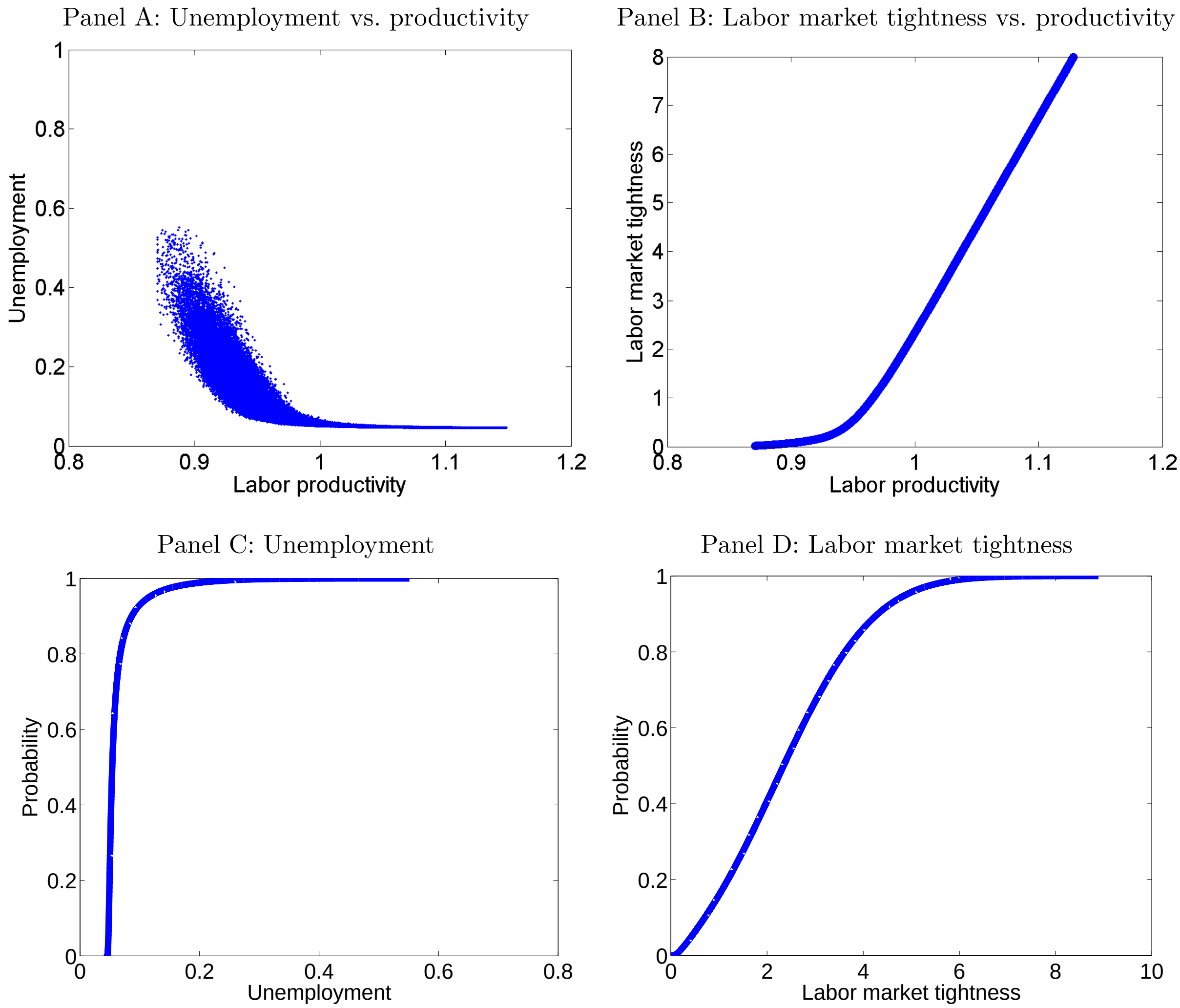
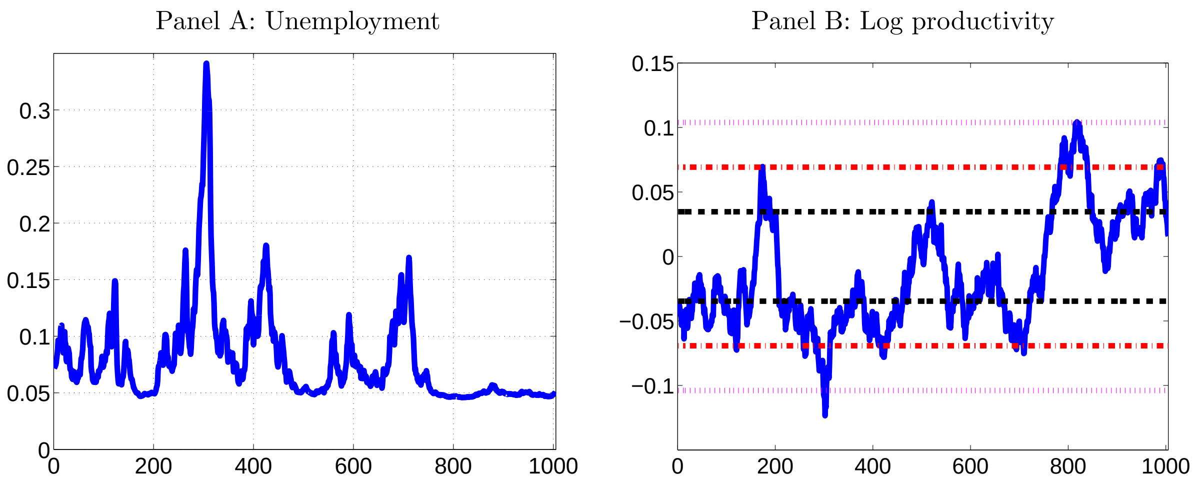


Figure 5 : An Illustrative Crisis Example

Using a crisis episode, we plot the unemployment rate in Panel A and log productivity in Panel B. In Panel B, the black dashed lines indicate one unconditional standard deviation above or below the unconditional mean of zero. The red dashdot lines indicate two unconditional standard deviations above or below zero, and the pink dotted lines indicate three unconditional standard deviations above or low zero.



4.3 Explaining Higher Moments of Unemployment Quantitatively

Can the search and matching model explain quantitatively the unemployment crisis dynamics in the data, including the aggregate state transition matrix and the tail probability of the crisis state in Table 1? To this end, from the model's stationary distribution, we repeatedly simulate 50,000 artificial samples, each of which contains 1,005 monthly periods. The sample length matches the number of months in the data from April 1929 to December 2012.

Because crises, which are rare by definition, do not occur in every simulated sample, we split the 50,000 samples into two groups, non-crisis samples and crisis samples. If the maximum unemployment rate in an artificial sample is greater than or equal to 20%, we categorize it as a crisis sample (otherwise a non-crisis sample). The cutoff threshold of 20% is consistent with our empirical procedure in Section 2. Out of the 50,000 simulations, we have in total 20,057 crisis samples (about 40%). On each crisis sample (i.e., conditional on at least one crisis), we calculate the state transition matrix and unconditional probabilities of the states using the exactly the same procedure as in Table

Table 4 : The Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States, the Benchmark Model with Credible Bargaining

From the model's stationary distribution, we simulate 50,000 artificial samples, each with 1,005 months. We split the samples into two groups, non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). On each crisis sample, we calculate the state transition matrix and unconditional probabilities of the states as in Table 1. We report the cross-simulation averages and standard deviations (in parentheses) across the crisis samples.

	Good	Bad	Crisis
Good	0.9793	0.0207	0
	(0.0067)	(0.0067)	(0)
Bad	0.0217	0.9748	0.0035
	(0.0071)	(0.0075)	(0.0024)
Crisis	0	0.1573	0.8418
	(0)	(0.2214)	(0.2228)
Unconditional probability	0.4942	0.4731	0.0321
	(0.0395)	(0.0448)	(0.0672)

1. We then report the cross-simulation averages and standard deviations across the crisis samples.[6]

Table 4 reports the model output. A comparison with Table 1 shows that the model does a good job in explaining the large unemployment dynamics in the data. In particular, the crisis state is about as persistent in the model as in the data. The probability of the economy remaining in the crisis state next period conditional on the crisis state in the current period is 84.18%, which is close to 82.35% in the data. In addition, the unconditional probability of the crisis state in the model is 3.21%, which is also close to 3.47% in the data. The cross-simulation standard deviation of this estimate is 6.72% in the model. This level of dispersion is perhaps not surprising for a tail probability estimate. In all, the model's estimate seems empirically plausible.

4.4 Labor Market Volatilities

The crisis dynamics in the model have important implications for the second moments such as volatilities and correlations, which are the traditional focus in the macro labor literature. In partic-

[6]We have experimented with simulating only 5,000 artificial samples, and the quantitative results are hardly changed under the benchmark calibration. However, we find that in comparative static experiments (Section 4.5) the results with 5,000 simulations can be sensitive to the increase of the number of simulations. Intuitively, the percentage of crisis samples can be small under alternative parameterizations. As such, the cross-simulation averages can be sensitive because of the small number of crisis samples being averaged over. To ensure accuracy in our quantitative results, we opt to work with 50,000 artificial samples for all the parameterizations in the paper.

Table 5 : Labor Market Volatilities in the Model

We simulate 50,000 artificial samples from the model, with 1,005 months in each sample. We split the samples into two groups, non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). We implement the same empirical procedure as in Table 2 and report cross-simulation averages and standard deviations (in parentheses) conditionally on the non-crisis samples and on the crisis samples.

		U	V	θ	X	U	V	θ	X
		Panel A: Non-crisis samples				Panel B: Crisis samples			
Standard deviation		0.102	0.191	0.274	0.013	0.149	0.216	0.331	0.014
		(0.033)	(0.026)	(0.050)	(0.001)	(0.028)	(0.023)	(0.045)	(0.001)
Autocorrelation		0.789	0.681	0.764	0.773	0.838	0.657	0.781	0.781
		(0.051)	(0.060)	(0.043)	(0.036)	(0.034)	(0.055)	(0.037)	(0.030)
Correlation matrix	U		−0.732	−0.880	−0.742		−0.630	−0.861	−0.710
			(0.069)	(0.026)	(0.062)		(0.055)	(0.025)	(0.057)
	V			0.966	0.950			0.937	0.926
				(0.016)	(0.018)			(0.014)	(0.016)
	θ				0.938				0.925
					(0.022)				(0.020)

ular, it can be misleading to focus only on the second moments in normal periods (non-crisis samples). The second moments in the crisis samples can deviate greatly from those in normal periods.

Panel A of Table 5 reports the results conditional on the non-crisis samples. The unemployment volatility is 0.102, which is lower than but close to 0.131 in the data (see Table 2). Although not a direct target, the standard deviation of labor market tightness is 0.274 in the model, which is close to 0.269 in the data. However, the model predicts a vacancy volatility of 0.191, which overshoots 0.142 in the data. Although negative, the unemployment-vacancy correlation is -0.732 in the model, which is lower in magnitude than -0.931 in the data. Overall, our quantitative results (based on a globally nonlinear algorithm) lend support to Hall and Milgram (2008) in that credible bargaining strengthens the model's ability to explain the unemployment volatility puzzle.

However, focusing only on normal periods suffers from a severe sample selection bias that arises from ignoring the crisis samples. Panel B reports the results conditional on the crisis samples. The unemployment volatility rises to 0.149, which is almost 50% higher than 0.102 in normal periods. However, the estimate of 0.149 falls short of 0.218 in the 1929–2012 sample in the data (see Table

2). The volatilities of vacancy and labor market tightness increase somewhat from 0.191 and 0.274 to 0.216 and 0.331, respectively. In particular, the 0.331 estimate is close to the volatility of 0.368 for the labor market tightness in the long sample. This result seems notable because this volatility in the crisis sample is not an explicit target of our calibration.

The model also predicts that the unemployment-vacancy correlation drops in magnitude from -0.732 in the non-crisis samples to -0.630 in the crisis samples. This result is consistent with a flatter Beveridge curve in the long sample than in the postwar sample, as evident in Figure 3. However, we note that the magnitude of these correlations in the model are lower than those in the data.

These results are intrinsically linked to the nonlinear dynamics in Figures 4. Precisely because unemployment exhibits a long right tail, ignoring crises by focusing only on the second moments in normal periods understates its volatility greatly. Also, because the nonlinearity of the vacancy rate is weaker than that of unemployment, ignoring crises does not materially affect its volatility but does overstate the unemployment-vacancy correlation.

4.5 Intuition: What Drives Unemployment Crises?

We turn to the economic mechanisms behind the unemployment crises in the model. We illustrate the intuition in two ways, impulse responses and comparative statics.

Impulse Responses

We calculate the impulse responses from three different starting points: bad, median, and good. The bad economy is the 5 percentile of the model's bivariate stationary distribution of employment and log productivity, the median is the median, and the good economy is the 95 percentile. (Across the bad, median, and good economies, the unemployment rates are 11.54%, 5.40%, and 4.75%, and the log labor productivity levels are $-0.0567, 0$, and 0.0567, respectively.) Both positive and negative one-standard-deviation shocks to the log productivity are examined.

Panels A to C of Figure 6 report responses in the labor market tightness. We observe that the responses are substantially larger in the bad economy than in the good economy. In the bad economy, a positive one-standard-deviation shock to the log labor productivity increases the tightness by 32.45%. In contrast, the response is only 6.12% in the good economy, and is less than 20% of the response in the bad economy. A negative impulse decreases the tightness by 6.08% in the good economy, which is around 20% of the response in the bad economy, 28.94%.

We also see large and asymmetric responses in unemployment. After a negative impulse, the unemployment rate shoots up 1.58% in the bad economy (Panel D). This response is about 23 times as large as that of 0.07% in the good economy (Panel F). From Panel E, the response in the median economy is only 0.20%, which is closer to that in the good economy than to that in the bad economy.

Panels G to I demonstrate the limited wage responses in the model. Starting from the good economy, the negative impulse reduces wages by 0.42%, which is only about 1.3% of the response in the market tightness, 32.45%. Even starting from the bad economy, a one-standard-deviation negative impulse reduces wages by about 0.60%, which is only slightly more than 2% of the response in the market tightness, 28.94%. These results clearly show that, consistent with Hall and Milgrom (2008), wages in the credible bargaining model are relatively insulated from conditions in the labor market. Most important, this insulation holds up quantitatively even in the bad economy, forcing negative shocks to be mostly absorbed into unemployment to give rise to crises.

Comparative Statics

We also conduct an array of comparative statics to gain more intuition. We perform six computational experiments: (i) raising the probability of breakdown in bargaining to $\delta = 0.15$; (ii) reducing the delaying cost to $\chi = 0.20$; (iii) lowering the proportional cost of vacancy posting to $\kappa_0 = 0.05$; (iv) decreasing the fixed cost of vacancy posting with $\kappa_1 = 0.1$; (v) lowering the job separation rate to $s = 0.035$; and (vi) cutting the matching function function parameter to $\iota = 0.9$. In each experiment, all the other parameters remain fixed as in the benchmark calibration. We quantify how the results reported in Tables 4 and 5 change as we vary each of the parameter values.

Figure 6 : Impulse Response Functions

We compute the impulse response functions from three different initial points: the five percentile, the median, and the 95 percentile of the model's bivariate distribution of employment and log productivity, respectively. The responses in wages and labor market tightness are in percentage deviations from the values at a given initial point, and the responses in unemployment are in levels. We average the impulse responses across 5,000 simulations, each of which has 120 months. The blue solid (red broken) lines are the responses to a positive (negative) one-standard-deviation shock to log productivity.

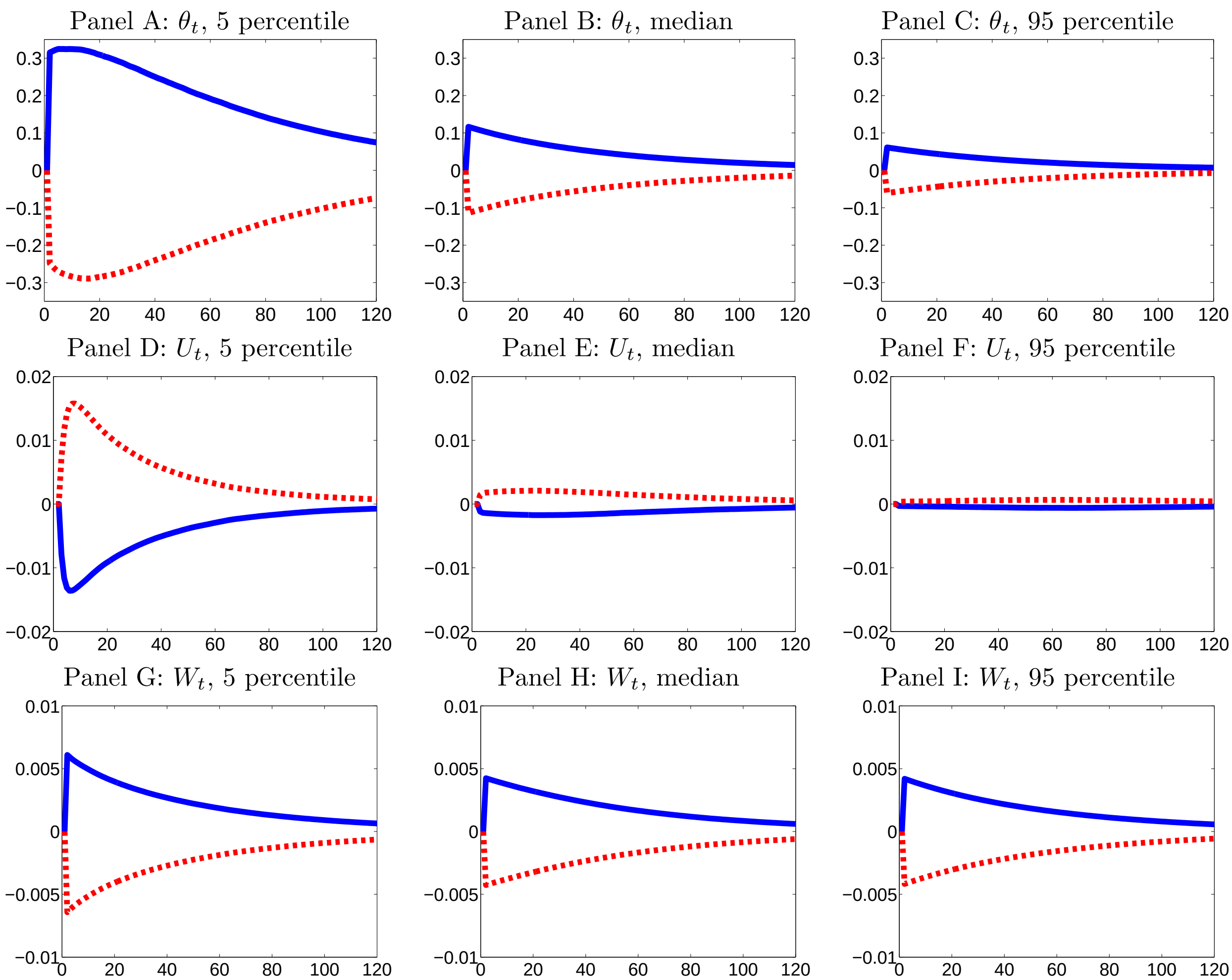


Table 6 reports the results for the crisis moments. From Panel A, increasing the probability of breakdown in negotiation, δ, weakens the crisis dynamics. The percentage of the crisis samples out of 50,000 simulations drops from 40% under the benchmark calibration to only 1.85%. Conditional on the crisis samples, the persistence of crisis weakens somewhat from 0.84 to 0.79, and the unconditional crisis probability falls from 3.21% to 2.01%. Panel A of Table 7 shows further that raising δ decreases somewhat the labor market volatilities in both non-crisis and crisis samples.

Intuitively, a higher probability of breakdown in negotiation brings credible bargaining closer to Nash bargaining. In the extreme case of $\delta = 1$, credible bargaining collapses to Nash bargaining, which implies more flexible wages. As such, a higher δ makes equilibrium wages less insulated to labor market conditions. In bad times, the productivity drops, but wages also fall with the deteriorating labor market, providing the firm with more incentives to creating jobs. As such, the unemployment volatility falls, consistent with Hall and Milgrom (2008). More important, we push their argument further by quantifying the impact of wage rigidity on unemployment crises.

The delaying cost is also quite important for explaining the crisis moments. From Panel B of Table 6, reducing χ from 0.25 under the benchmark calibration to 0.2 lowers the percentage of the crisis samples out of 50,000 simulations from 40% to only 1.54%. Conditional on the crisis samples, the persistence of crisis weakens from 0.84 to 0.73, and the unconditional crisis probability falls from 3.21% to 1.70%. Panel B of Table 7 shows further that reducing χ lowers the mean unemployment rate to 4.90% and the unemployment volatility to 0.032 in normal times and 0.108 in crises. Intuitively, a lower delaying cost makes equilibrium wages more responsive to labor market conditions. As such, the crisis dynamics are weakened, and the labor market volatilities are lowered. Also, the unemployment-vacancy correlation is -0.847 in normal periods but -0.596 in crises. As the crises become more infrequent, the difference across normal periods and crises becomes starker.

The proportional and the fixed costs of vacancy posting impact the results in the same direction as the cost of delaying, but to a lesser extent quantitatively. From Table 6, reducing κ_0 to 0.05 lowers

Table 6 : Comparative Statics, Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States

We consider six comparative static experiments: (i) increasing the probability of breakdown in bargaining to $\delta = 0.15$; (ii) reducing the delaying cost to $\chi = 0.2$; (iii) lowering the proportional cost of vacancy t0 $\kappa_0 = 0.05$; (iv) reducing the fixed cost of vacancy, $\kappa_1 = 0.1$; (v) decreasing the job separation rate to $s = 0.035$; and (vi) cutting the curvature parameter of the matching function to $\iota = 0.9$. In each experiment, all the other parameters remain identical to those in the benchmark calibration. Under each alternative calibration, we simulate 50,000 artificial samples (each with 1,005 months) from the model's stationary distribution. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). On each crisis sample, we calculate the state transition matrix and unconditional probabilities of the states per the procedure in Table 1 and report cross-simulation averages.

	Good	Bad	Crisis	Good	Bad	Crisis
	Panel A: $\delta = 0.15$ (% crisis samples = 1.85)			Panel B: $\chi = 0.2$ (% crisis samples = 1.54)		
Good	0.9807	0.0193	0	0.9801	0.0199	0
Bad	0.0197	0.9780	0.0023	0.0199	0.9779	0.0023
Crisis	0	0.2127	0.7873	0	0.2660	0.7340
Unconditional probability	0.4946	0.4852	0.0201	0.4901	0.4929	0.0170
	Panel C: $\kappa_0 = 0.05$ (% crisis samples = 26.21)			Panel D: $\kappa_1 = 0.1$ (% crisis samples = 12.73)		
Good	0.9784	0.0216	0	0.9798	0.0202	0
Bad	0.0214	0.9758	0.0028	0.0208	0.9766	0.0026
Crisis	0	0.1725	0.8264	0	0.1531	0.8458
Unconditional probability	0.4830	0.4888	0.0276	0.4926	0.4810	0.0257
	Panel E: $s = 0.035$ (% crisis samples = 8.80)			Panel F: $\iota = 0.9$ (% crisis samples = 41.71)		
Good	0.9801	0.0199	0	0.9794	0.0206	0
Bad	0.0204	0.9771	0.0025	0.0217	0.9747	0.0036
Crisis	0	0.2160	0.7833	0	0.1424	0.8570
Unconditional probability	0.4935	0.4839	0.0222	0.4954	0.4701	0.0340

Table 7 : Comparative Statics, Labor Market Volatilities

We consider six comparative static experiments: (i) increasing the probability of breakdown in bargaining to $\delta = 0.15$; (ii) reducing the delaying cost to $\chi = 0.2$; (iii) reducting the proportional cost of vacancy t0 $\kappa_0 = 0.05$; (iv) reducing the fixed cost of vacancy, $\kappa_1 = 0.1$; (v) reducing the job separation rate to $s = 0.035$; and (vi) reducing the curvature parameter of the matching function to $\iota = 0.9$. In each experiment, all the other parameters remain identical to those in the benchmark calibration. Under each alternative calibration, we simulate 50,000 artificial samples (each with 1,005 months) from the model's stationary distribution. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). We implement the same procedure as in Table 2 and report the cross-simulation averages.

	U	V	θ	X		U	V	θ	X
	Non-crisis samples					Crisis samples			
	Panel A: $\delta = 0.15$ (mean U in normal times: 5.72%)								
Standard deviation	0.070	0.150	0.209	0.013		0.106	0.162	0.245	0.014
Autocorrelation	0.792	0.708	0.772	0.775		0.849	0.686	0.791	0.785
Correlation matrix		−0.781	−0.895	−0.792	U		−0.650	−0.863	−0.735
			0.977	0.970	V			0.944	0.950
				0.960	θ				0.949
	Panel B: $\chi = 0.2$ (mean U in normal times: 4.91%)								
Standard deviation	0.032	0.128	0.155	0.013		0.108	0.173	0.253	0.014
Autocorrelation	0.763	0.747	0.769	0.775		0.855	0.709	0.803	0.786
Correlation matrix		−0.847	−0.901	−0.776	U		−0.596	−0.834	−0.502
			0.993	0.968	V			0.939	0.883
				0.951	θ				0.819
	Panel C: $\kappa_0 = 0.05$ (mean U in normal times: 4.91%)								
Standard deviation	0.074	0.241	0.298	0.013		0.146	0.283	0.390	0.014
Autocorrelation	0.741	0.727	0.759	0.774		0.833	0.709	0.785	0.783
Correlation matrix		−0.736	−0.838	−0.613	U		−0.607	−0.814	−0.546
			0.985	0.934	V			0.955	0.913
				0.903	θ				0.869
	Panel D: $\kappa_1 = 0.1$ (mean U in normal times: 5.46%)								
Standard deviation	0.076	0.171	0.233	0.013		0.131	0.200	0.300	0.014
Autocorrelation	0.779	0.707	0.766	0.775		0.846	0.679	0.788	0.784
Correlation matrix		−0.763	−0.882	−0.728	U		−0.618	−0.849	−0.640
			0.976	0.953	V			0.939	0.918
				0.932	θ				0.895
	Panel E: $s = 0.035$ (mean U in normal times: 4.51%)								
Standard deviation	0.093	0.185	0.260	0.013		0.141	0.205	0.312	0.014
Autocorrelation	0.791	0.695	0.768	0.775		0.851	0.674	0.789	0.785
Correlation matrix		−0.732	−0.876	−0.730	U		−0.603	−0.849	−0.682
			0.968	0.953	V			0.933	0.929
				0.937	θ				0.922
	Panel F: $\iota = 0.9$ (mean U in normal times: 7.25%)								
Standard deviation	0.097	0.175	0.257	0.013		0.130	0.200	0.303	0.014
Autocorrelation	0.813	0.663	0.767	0.773		0.848	0.651	0.781	0.781
Correlation matrix		−0.759	−0.896	−0.815	U		−0.677	−0.873	−0.779
			0.969	0.950	V			0.949	0.926
				0.956	θ				0.944

the persistence of the crisis state slightly to 0.83 and the unconditional probability to 2.76%. The percentage of the crisis samples also drops to 26.21% (Panel C). Similarly, reducing κ_1 to 0.1 does not seem to affect the persistence of the crisis state but lowers its unconditional probability to 2.57%. Also, the percentage of crisis samples drops to 12.73%. From Table 7, reducing κ_0 and κ_1 also lowers the unemployment volatility, especially in normal times. Intuitively, lowering vacancy costs stimulates job creation flows to starve off unemployment crises. In particular, the fixed matching cost buttresses the downward rigidity in the marginal cost of hiring. Reducing the fixed cost weakens the rigidity, allowing the marginal cost of hiring to decline and more jobs to be created in recessions.

From Panel E of Table 6, reducing the job separation rate, s, to 3.5% makes unemployment crises less frequent and less persistent. The persistence of the crisis state falls from 0.84 to 0.78, and the unconditional crisis probability from 3.21% to 2.22%. The percentage of the crisis samples also drops from 40% to 8.8%. From Table 7, a lower s also reduces the mean unemployment rate to 4.51% but leaves the labor market volatilities largely unaffected. Intuitively, because jobs are destroyed at a lower rate, all else equal, the economy is more capable of offsetting job destruction flows through job creation. As such, the mean unemployment rate is reduced, and the crisis dynamics dampened.

From Panel F of Table 6, reducing the curvature of the matching function, ι, from 1.25 to 0.9 strengthens the crisis dynamics. The persistence of the crisis state increases from 0.84 slightly to 0.86, and the unconditional crisis probability from 3.21% from 3.40%. Intuitively, a decrease in ι increases the elasticity of new hires with respect to vacancies. As vacancies fall in recessions, new hires drop faster with a lower ι, meaning that the congestion effect for unemployed workers becomes more severe. As such, the crisis dynamics are reinforced. A lower ι also increases the mean unemployment rate to 7.25% in normal times but reduce slightly the unemployment volatilities (Table 7).

5 The Welfare Cost of Business Cycles

Lucas (1987, 2003) argues that the welfare cost of business cycles is negligible. Assuming log utility for the representative consumer and a log-normal distribution for the consumption growth, Lucas calculates that the consumer would only sacrifice a mere 0.008% of their consumption in perpetuity to get rid of all its aggregate fluctuations. However, Lucas's analysis might underestimate the welfare cost by overlooking crisis states, in which the agent's marginal utility is high (e.g., Chatterjee and Corbae (2007)). In addition, eliminating business cycles can also affect the mean consumption level. We show that the welfare cost in our model (with linear utility replaced by log utility) is about two orders of magnitude larger than the Lucas estimate.[7]

Following Lucas (1987), we define the welfare cost of business cycles as the permanent percentage of the consumption flow that the representative household would sacrifice to eliminate aggregate fluctuations. Formally, for a given current state of the economy, (N_t, x_t), at date t, we calculate the welfare cost, denoted $\psi_t \equiv \psi(N_t, x_t)$, implicitly from:

$$E_t \left[\sum_{\triangle t=0}^{\infty} \beta^{\triangle t} \log \left[(1+\psi_t) C_{t+\triangle t} \right] \right] = \sum_{\triangle t=0}^{\infty} \beta^{\triangle t} \log \left(C^{\star} \right), \tag{30}$$

in which $C^{\star}$ is the aggregate consumption at the deterministic steady state. $C^{\star}$ is the constant level of consumption implied by the model after the shocks are eliminated. Solving for ψ_t yields:

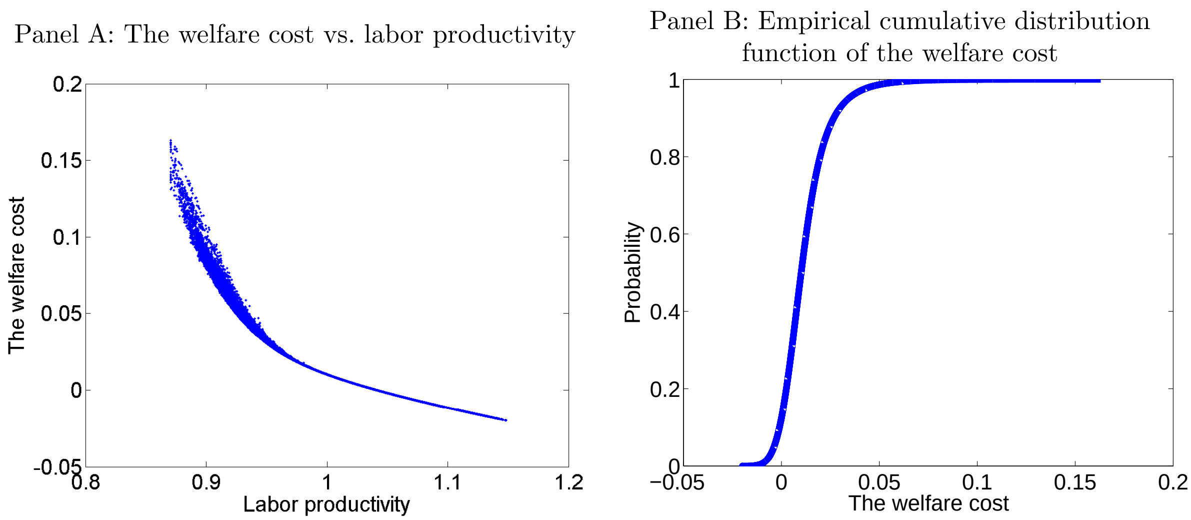
$$\psi_t = \exp \left(\log \left(C^{\star} \right) - (1-\beta) J_t \right) - 1, \tag{31}$$

in which $J_t \equiv E_t \sum_{\triangle t=0}^{\infty} \log \left(C_{t+\triangle t} \right)$ is the indirect utility function. In practice, we solve for $J_t = J(N_t, x_t)$ on the N-x grid from the recursion $J(N_t, x_t) = \log C(N_t, x_t) + \beta E_t[J(N_{t+1}, x_{t+1})]$.

To evaluate the magnitude of the welfare cost in our model, we simulate one million months of ψ_t from the model's stationary distribution. Using the simulated data, we calculate the mean of the welfare cost to be 1.2%, which is 150 times of the Lucas estimate of 0.008%. The mean consumption in the stochastic model is 0.95% lower than the steady state consumption. In addition, the welfare cost is time-varying. Panel A of Figure 7 plots the welfare cost function, ψ_t, against the labor productivity. We see that ψ_t is clearly countercyclical in that it decreases almost

[7] Replacing the linear utility with log utility does not change the quantitative results of our model. In Appendix D, we report that the persistence of the crisis state and its unconditional probability as well as the second moments of the labor market in the model with log utility are quantitatively similar to those from the model with linear utility.

Figure 7 : The Welfare Cost of Business Cycles in the Model with Log Utility



monotonically with productivity. Panel B plots the empirical cumulative distribution function for ψ_t. The welfare cost has a long right tail. Its median is 1.01%, and the 2.5 and 5 percentiles are -0.61% and -0.38%, whereas the 95 and 97.5 percentiles are 3.42% and 4.23%, respectively. The maximum welfare cost in the simulated data can reach as high as 16.3%.

6 Conclusion

A search and matching model, when calibrated to the mean and volatility of unemployment in the postwar sample, can potentially explain the unemployment crisis in the Great Depression. The key ingredient of the model is the Hall and Milgrom (2008) credible bargaining game for determining equilibrium wages. The limited responses of the wages to conditions in the labor market, along with the congestion externality from matching frictions, cause the unemployment rate to rise sharply in recessions but decline gradually in booms. The frequency, severity, and persistence of unemployment crises in the model are quantitatively consistent with U.S. historical time series. The welfare gain from eliminating business cycle fluctuations is large, amounting to 1.2% of consumption in perpetuity, which is two orders of magnitude larger than the Lucas (1987) estimate.

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A Data

We describe our data on unemployment rates in Section A.1 and on vacancy rates in Section A.2.

A.1 Unemployment Rates

We construct our monthly U.S. unemployment rate series from April 1929 to December 2012 by drawing from NBER macrohistory files (chapter 8: Income and employment) and Federal Reserve Economic Data (FRED) at Federal Reserve Bank of St. Louis.

We concatenate four different U.S. unemployment rate series:

- The seasonally adjusted unemployment rates from April 1929 to February 1940 (NBER data series m08292a, FRED series id: M0892AUSM156SNBR. Source: National Industrial Conference Board, published by G. H. Moore Business Cycle Indicators, vol. II, p. 35 and p. 123).

- The seasonally adjusted unemployment rates from March 1940 to December 1946 (NBER data series m08292b, FRED series id: M0892BUSM156SNBR. Source: U.S. Bureau of the Census, Current Population Reports, Labor Force series P-50, no. 2, 13, and 19).

- The seasonally adjusted unemployment rates from January 1947 to December 1947. To construct this series, we first obtain the monthly unemployment rates (not seasonally adjusted) from January 1947 to December 1966 (NBER data series m08292c, FRED series id: M0892CUSM156NNBR. Source: Employment and Earnings and Monthly Report on the Labor Force, vol. 13, no. 9, March 1967). We then pass the entire monthly series from 1947 to 1966 through the X-12-ARIMA seasonal adjustment program from the U.S. Census Bureau and take the seasonally adjusted series from January to December of 1947.

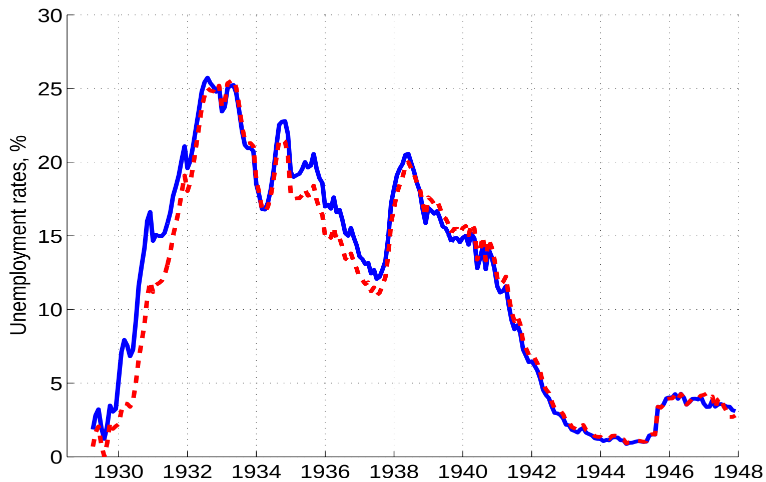
- The seasonally adjusted civilian unemployment rates from January 1948 to December 2012 from Bureau of Labor Statistics at U.S. Department of Labor (FRED series id: UNRATE).

Following Owyang, Ramey, and Zubairy (2013), we adjust the pre-1948 unemployment rate series as follows. We use the monthly unemployment rates from January 1930 to December 1947 to interpolate annual unemployment rates data from Weir (1992) using the Denton (1971) proportional interpolation procedure. In addition, we scale the nine monthly unemployment rates from April to December 1929 so that their average matches the annual unemployment rate for 1929 reported in Weir. (We cannot apply the Denton procedure on the nine monthly observations because the procedure requires the complete data for 12 months in a given year.)

Figure A.1 plots the adjusted monthly U.S. unemployment rate series from April 1929 to December 1947, along with the raw series from the NBER macrohistory files. The two series seem to track each other closely. The mean unemployment rate for the adjusted series is 11.66%, which is slightly higher than 11.03% for the raw series. To compute the unemployment volatilities, we convert both

Figure A.1 : The Adjusted and Raw Monthly U.S. Unemployment Rate Series, April 1929 to December 1947

The blue solid line plots the adjusted unemployment rate series from the Denton proportional interpolation procedure. The red broken line plots the unadjusted raw series from the NBER macrohistory files.



monthly series to quarterly averages and then take the HP-filtered proportional deviations from the mean. The volatility of the adjusted unemployment rates is 22.18%, which is somewhat lower than 25.85% for the raw series. Their first-order autocorrelations are also close, 0.85 versus 0.89.

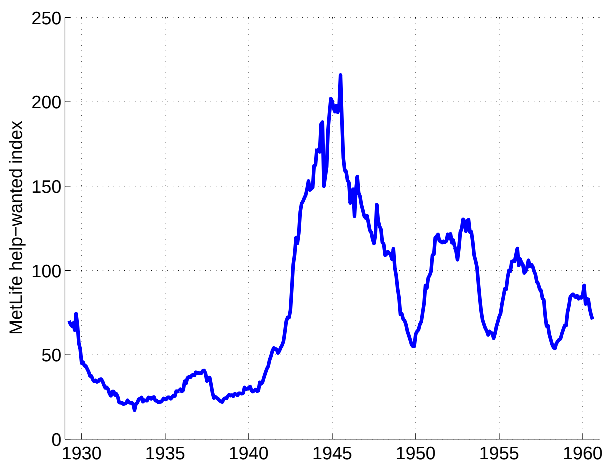
A.2 Vacancy Rates

To construct a long vacancy rate series from April 1929 to December 2012, we draw from four different series of U.S. job openings:

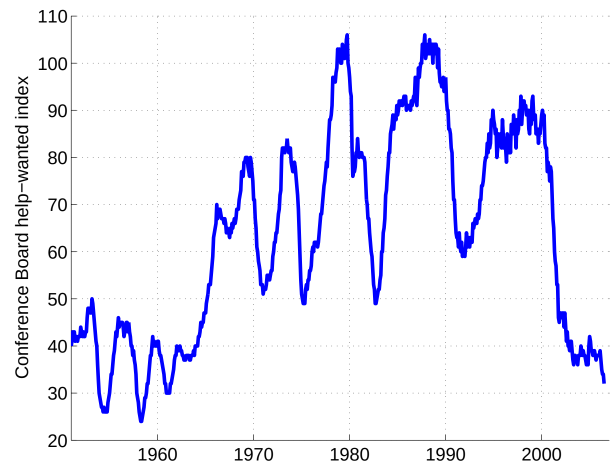
- The Metropolitan Life Insurance company (MetLife) help-wanted advertising index in newspaper from April 1929 to August 1960. The series (not seasonally adjusted) is obtained from the NBER macrohistory files (series id: m08082a, FRED series id: M0882AUSM349NNBR). The NBER scales the series to average 100 over the 1947–1949 period. To seasonally adjust the series, we pass the raw series through the X-12-ARIMA program from the U.S. Census Bureau.
- The help-wanted advertising index from the Conference Board, seasonally adjusted, from January 1951 to July 2006. The Conference Board scales the series to average 100 in 1987.
- The composite print and online help-wanted index from Barnichon (2010). The series, ranging from January 1995 to December 2012, is obtained from Regis Barnichon's Web site.
- The seasonally adjusted job openings series (total nonfarm, level in thousands) from the Job Openings and Labor Turnover Survey (JOLTS) released by U.S. Bureau of Labor Statistics. We obtain the series from December 2000 to December 2012 from FRED (id: JTSJOL).

Figure A.2 : Four Series of U.S. Job Openings, All Seasonally Adjusted

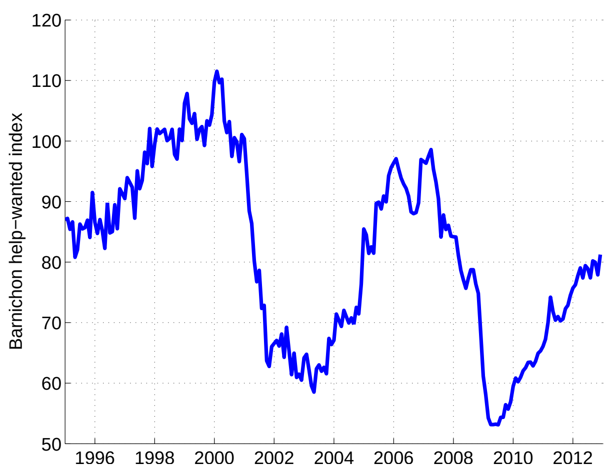
Panel A: MetLife help-wanted index, April 1929 to August 1960



Panel B: Conference Board help-wanted index, January 1951 to July 2006



Panel C: Barnichon's help-wanted index, January 1995 to December 2012



Panel D: JOLTS job openings, December 2000 to December 2012

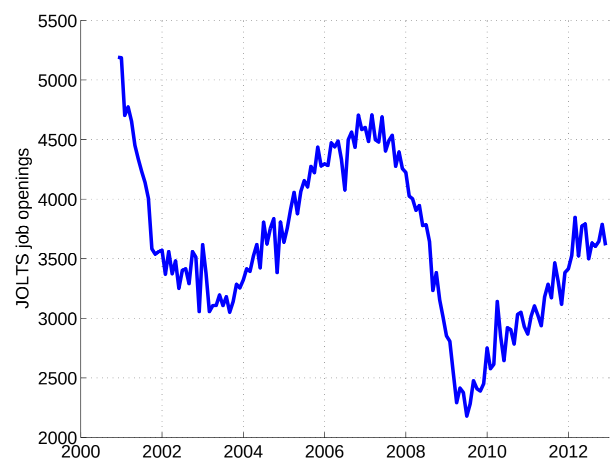


Figure A.2 plots the four original series of U.S. job openings, all seasonally adjusted.

The MetLife help-wanted index, developed and overseen by William A. Berridge, was initiated in 1927 to construct national measures of the labor demand (e.g., Berridge (1929, 1961)) and to provide MetLife with a statistical tool to evaluate the performance of their sales force (e.g., Zagorsky (1998)). Past issues of print newspapers were collected to construct a historical series back to January 1919. However, not all newspapers were able to provide MetLife with back issues, particularly prior to 1923. We use the data after April 1929 to be consistent with the unemployment rate series.

According to Zagorsky (1998), Berridge worked with the Conference Board as he neared retirement for it to take over the help-wanted index program. The Conference Board index is close, statistically and methodologically, to the MetLife series. Zagorsky also details the construction of the MetLife and Conference Board help-wanted indices. The Conference Board began its series in 1960 and constructed a historical series dating back to 1951 with old print issues submitted by newspapers. However, 14% of the newspapers were not able to provide data back to 1951.

To compare the MetLife series with the Conference Board series, Figure A.3 plots both for the overlapping period from January 1951 to August 1960. We scale the Conference Board index (multiply the index by 2.08) so that its value for January 1960 equals the MetLife index value for the same month.[8] The figure shows that the two series track each other tightly in the late 1950s and in 1960. However, the Conference Board series has in general lower values in the early sample, likely because fewer back print issues were provided by the newspapers. As such, we use the MetLife index from April 1929 to December 1959 and the Conference Board series thereafter.

Since the mid 1990s, as advertising for jobs over the internet becomes more and more prevalent, the print help-wanted index from the Conference Board has become increasingly unrepresentative. In response, the Conference Board started in 2005 to publish an online help-wanted index. Combining the information on both print and online advertising, Barnichon (2010) constructs a composite index for total (print and online) help-wanted advertising. Figure A.4 plots the Barnichon composite index together with the Conference Board print index. The two series have diverged significantly since 1996. As such, we use the Conference Board index until December 1994 and the Barnichon index thereafter. In addition, because the two series have the same unit, we scale the Barnichon index in the same way as we scale the Conference Board index to concatenate with the MetLife series.

As noted, the JOLTS job openings series becomes available after December 2000. Figure A.5 plots the JOLTS series together with the Barnichon series for the overlapping period after December 2000. To be comparable, we scale the JOLTS series (multiply by 0.0195) so that its value in December 2000 equals the Barnichon index value for the same month. We see that the two series track each other tightly up to 2005. Since then the JOLTS series has taken somewhat lower values

[8]It is important to emphasize that scaling different vacancy indices (to make their units comparable) does not affect the standard deviations of the vacancy rate and the vacancy-unemployment ratio. The reason is that we calculate the standard deviations of their HP-filtered cyclical components, as in Shimer (2005). The different scales are absorbed into the trend components that do not enter the calculations.

Figure A.3 : A Comparison of the MetLife and the Conference Board Help-wanted Indices, January 1951 to August 1960

The blue solid line plots the MetLife help-wanted index. The red broken line plots the Conference Board index, scaled to equate its index value for January 1960 with the MetLife index value for the same month.

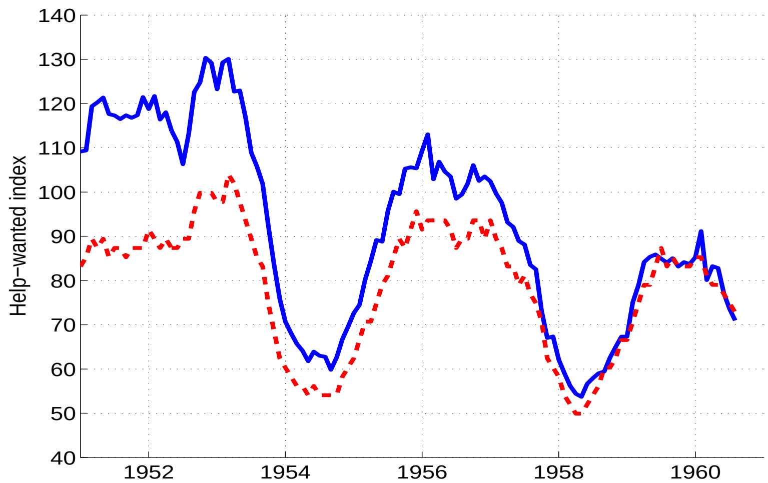


Figure A.4 : A Comparison of the Barnichon Composite Help-wanted Index and the Conference Board Print Help-wanted Index, January 1995 to June 2006

The blue solid line plots the Barnichon (2010) composite (print and online) help-wanted index. The red broken line plots the Conference Board print help-wanted index.

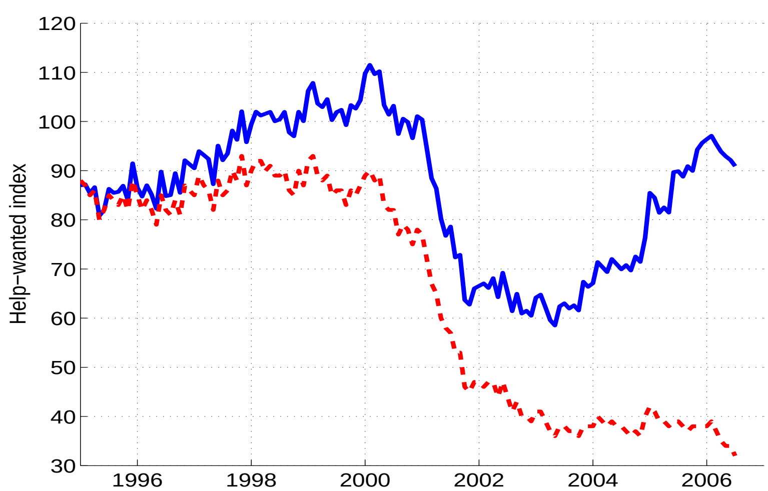
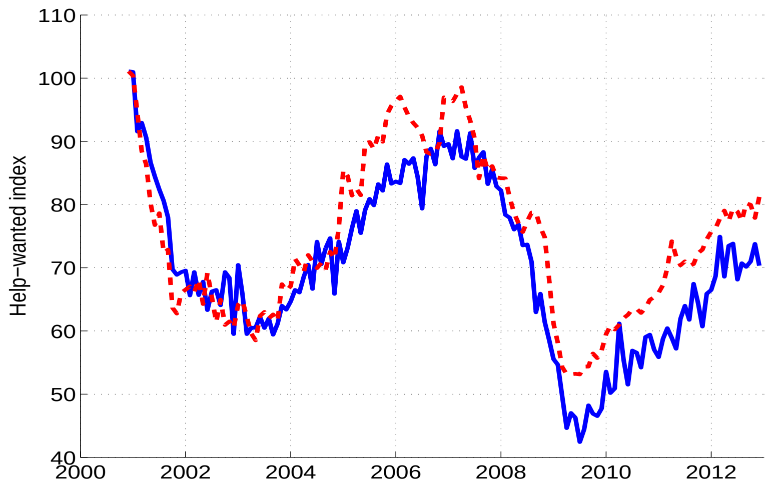


Figure A.5 : A Comparison of the (Scaled) JOLTS Job Openings Series and the Barnichon Composite Help-wanted Index, December 2000 to December 2012

The blue solid line plots the (scaled) JOLTS job openings series. The red broken line plots the Barnichon (2010) composite help-wanted index.



than the Barnichon series for most months. Because of the deviation, we use the JOLTS series (scaled by $2.08 \times 0.0195 = 0.04$) after December 2000 in our overall help-wanted index.

Figure A.6 reports our overall help-wanted index from April 1929 to December 2012.

To convert the help-wanted index into vacancy rates, we need to construct a labor force series. We first obtain the civilian labor force over 16 years of age in thousands of persons from FRED (series id: CLF16OV). The series (seasonally adjusted) is based on Current Employment Statistics released by U.S. Bureau of Labor Statistics. The sample is from January 1948 to December 2012. To construct the labor force series for the period from April 1929 to December 1947, we obtain the annual observations of total population from the U.S. Census.[9] We then construct a monthly series by linearly interpolating two adjacent annual observations across the 12 months in question. Finally, we multiply the total population estimates by the fraction of the population over 16 years of age in 1948 and the average labor force participation rate in 1948. The implicit assumption is that both rates are largely constant from 1929 to 1947. Figure A.7 reports the overall labor force series.

The last step in constructing the vacancy rate series is to divide the overall help-wanted index by the labor force series, while rescaling the resulting series to a known estimate of the job vacancy rate. In particular, we multiply the resulting series by 13.47 so that the series averages to 2.05% in 1965, which is the vacancy rate documented by Zagorsky (1998, Table A1). The scaling again does not affect our key results (see footnote 8).

[9]See http://www.census.gov/popest/data/national/totals/pre-1980/tables/popclockest.txt.

Figure A.6 : The Overall Help-wanted Index, April 1929 to December 2012

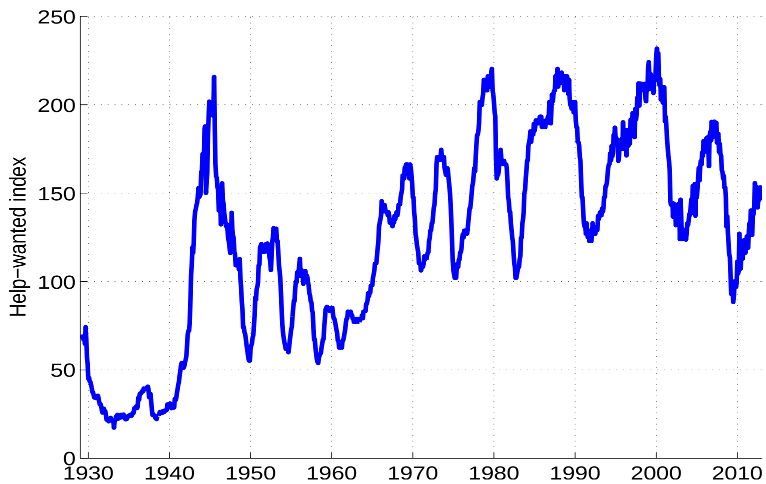
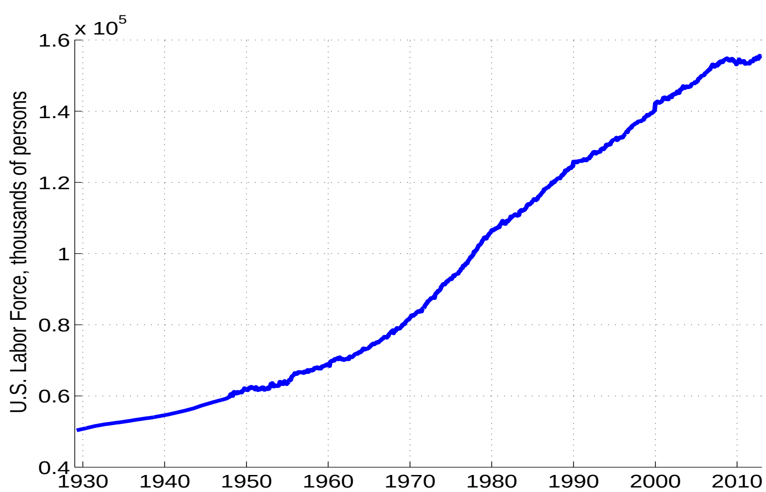


Figure A.7 : The U.S. Labor Force in Thousands of Persons, April 1929 to December 2012



B Derivations

To prove equation (21), we plug equations (19) and (20) into equation (17) to obtain:

$$W_t + E_t\left[\beta\left((1-s)J^W_{Nt+1} + sJ_{Ut+1}\right)\right] = b + \delta E_t\left[\beta\left(f_t J^W_{Nt+1} + (1-f_t)J_{Ut+1}\right)\right] + (1-\delta)E_t\left[\beta J^{W'}_{Nt+1}\right]. \tag{B.1}$$

Solving for W_t yields:

$$W_t = b + [\delta f_t - (1-s)]E_t\left[\beta J^W_{Nt+1}\right] + [\delta(1-f_t) - s]E_t\left[\beta J_{Ut+1}\right] + (1-\delta)E_t\left[\beta J^{W'}_{Nt+1}\right]. \tag{B.2}$$

Rearranging the right-hand side yields equation (21).

To characterize the worker's counteroffer, W_t', as in equation (22), we first rewrite equation (12) recursively (while making explicitly the dependence of S_t on W_t with the notation S_t^W):

$$S_t^W = X_t N_t - W_t N_t - \kappa_t V_t + \lambda_t q(\theta_t)V_t + E_t\left[\beta S^W_{t+1}\right], \tag{B.3}$$

The first-order condition with respect to V_t yields:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t\left[\beta S^W_{Nt+1}\right]. \tag{B.4}$$

Also, replacing W_t with W_t' in equation (B.3) and differentiating with respect to N_t yield:

$$S^{W'}_{Nt} = X_t - W_t' + (1-s)E_t\left[\beta S^{W'}_{Nt+1}\right]. \tag{B.5}$$

Plugging equation (B.4) into the firm's indifference condition (18) yields:

$$S^{W'}_{Nt} = (1-\delta)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi\right]. \tag{B.6}$$

Combining with equation (B.5) yields:

$$X_t - W_t' + (1-s)E_t\left[\beta S^{W'}_{Nt+1}\right] = (1-\delta)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi\right]. \tag{B.7}$$

Isolating W_t' to one side of the equality:

$$\begin{aligned} W_t' &= X_t + (1-\delta)\chi + (1-s)E_t\left[\beta S^{W'}_{Nt+1}\right] - (1-\delta)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t\right] && \text{(B.8)} \\ &= X_t + (1-\delta)\chi + (1-s)E_t\left[\beta S^{W'}_{Nt+1}\right] - (1-\delta)E_t\left[\beta S^W_{Nt+1}\right] && \text{(B.9)} \\ &= X_t + (1-\delta)\chi + \beta E_t\left[(1-s)S^{W'}_{Nt+1} - (1-\delta)S^W_{Nt+1}\right], && \text{(B.10)} \end{aligned}$$

which is identical to equation (22). Leading equation (B.6) by one period, plugging it along with equation (B.4) into equation (B.7), and solving for W_t' yield:

$$W_t' = X_t - (1-\delta)\left[\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi\right] - (1-s)E_t\left[\beta\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} - \chi\right]\right]\right]. \quad \text{(B.11)}$$

We further characterize the agreement condition (23) as follows. Rewriting equation (B.5) with W_t and combining with equation (B.4) yield $S_{Nt}^W = X_t - W_t + (1-s)\left[\kappa_0/q(\theta_t) + \kappa_1 - \lambda_t\right]$. As such, the agreement condition becomes:

$$X_t - W_t + (1-s)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t\right] + J_{Nt}^W > J_{Ut}. \quad \text{(B.12)}$$

Although equations (22) and (23) are easier to interpret, we implement equations (B.11) and (B.12) in our numerical algorithm.

C Computation

We parameterize the conditional expectation in the right-hand side of equation (25) as $\mathcal{E}_t \equiv \mathcal{E}(N_t, x_t)$, as well as four other functions, $W(N_t, x_t)$, $J_U(N_t, x_t)$, $J_N^W(N_t, x_t)$, and $J_N^{W'}(N_t, x_t)$. As in Christiano and Fisher (2000), we then exploit a convenient mapping from $\mathcal{E}_t$ to policy and multiplier functions to eliminate the need to parameterize the multiplier function separately. Specifically, after obtaining the parameterized $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) \equiv \kappa_t/\mathcal{E}_t$. If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$ from equation (6), and $V_t = \theta_t(1-N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the nonnegativity constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_t - \mathcal{E}_t$.

We approximate the log productivity process, x_t, in equation (9) based on the discrete state space method of Rouwenhorst (1995).[10] We use 17 grid points to cover the values of x_t, which are precisely within four unconditional standard deviations from the unconditional mean of zero. For the N_t grid, we set the minimum value to be 0.001 and the maximum 0.99, and N_t never hits one of the bounds in simulations. On each grid point of x_t, we use cubic splines with 20 basis functions on the N_t space to approximate the five functions. We use extensively the approximation toolkit in the Miranda and Fackler (2002) CompEcon Toolbox in MATLAB. To obtain an initial guess, we use the loglinear solution to a simplified model without the fixed matching cost.

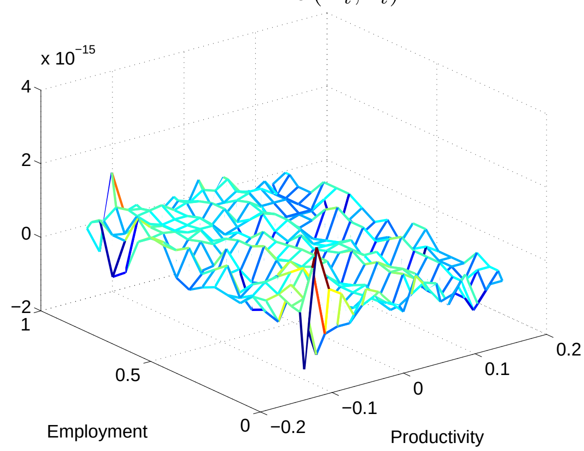
Figure C.1 reports the approximation errors for the five functional equations from (25) to (29). The error for each equation is the left-hand side minus the right-hand side of the equation. The errors, in the magnitude no greater than 10^{-13}, are extremely small. As such, our projection algorithm does an accurate job in characterizing the competitive search equilibrium.

[10] Kopecky and Suen (2010) show that the Rouwenhorst method is more accurate than other methods in approximating highly persistent autoregressive processes.

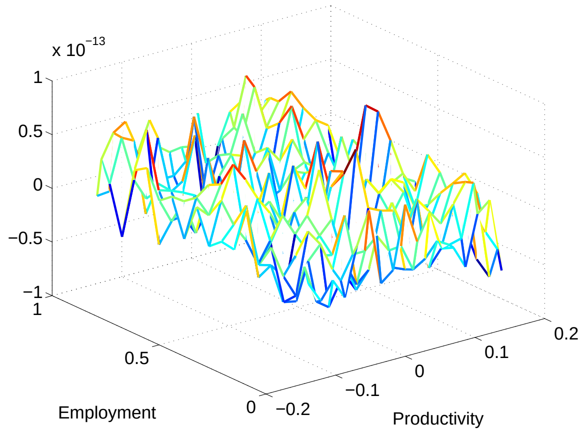
Figure C.1 : Approximation Errors

The approximation errors are for the five functional equations from (25) to (29). The error for each equation is the left-hand side minus the right-hand side of the equation.

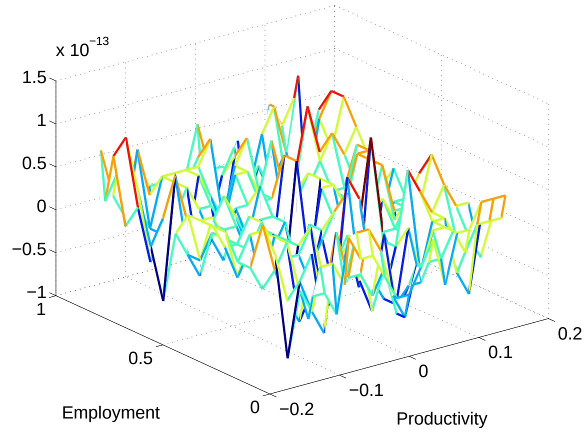
Panel A: The $\mathcal{E}(N_t, x_t)$ error



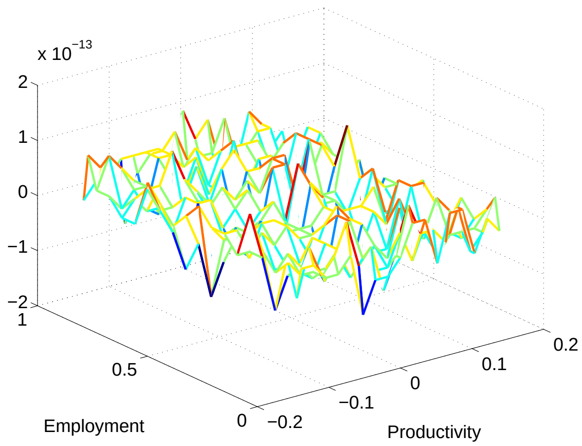
Panel B: The $W(N_t, x_t)$ error



Panel C: The $J_U(N_t, x_t)$ error



Panel D: The $J_N^W(N_t, x_t)$ error



Panel E: The $J_N^{W'}(N_t, x_t)$ error

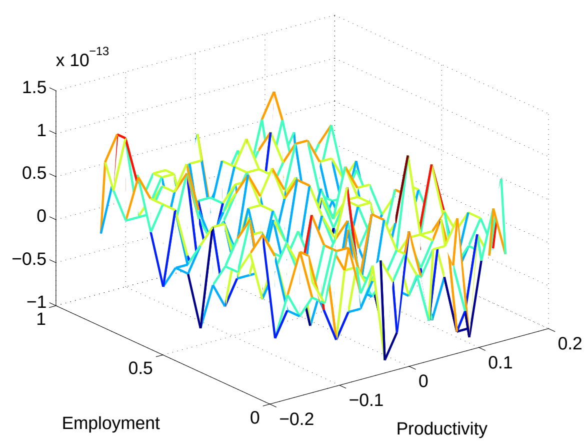


Table D.1 : Aggregate State Transition Matrix and Unconditional Probabilities of the Three Economic States, the Model with Log Utility

From the model's stationary distribution, we simulate 50,000 artificial samples, each with 1,005 months. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). On each crisis sample, we calculate the state transition matrix and unconditional probabilities of the states as in Table 1. We report the cross-simulation averages and standard deviations (in parentheses) across the crisis samples.

	Good	Bad	Crisis
Good	0.9794	0.0206	0
	(0.0068)	(0.0068)	(0)
Bad	0.0218	0.9750	0.0032
	(0.0072)	(0.0074)	(0.0021)
Crisis	0	0.1228	0.8764
	(0)	(0.1854)	(0.1869)
Unconditional probability	0.4939	0.4673	0.0383
	(0.0443)	(0.0492)	(0.0781)

D The Log Utility Model

We show that replacing the linear utility with log utility does not materially affect the quantitative results from the benchmark model with linear utility. Other than the change in utility, all the parameter values remain unchanged from the benchmark model.

Table D.1 repeats the analysis in Table 4 but with log utility. The results are quantitatively similar to those in Table 4 in the benchmark model with linear utility. The probability of the economy remaining in the crisis state next period conditional on being in the crisis state in the current period is 87.64%, which is slightly higher than 84.18% with linear utility. The unconditional probability of the crisis state in the log utility model is 3.83%, which is slightly higher than 3.21% with linear utility.

Table D.2 shows further that the change to log utility does not affect materially the second moments of the labor market either. From Panel A, the unemployment volatility in the non-crisis samples is 0.105, which is very close to 0.102 with linear utility (Panel B of Table 5). In addition, the unemployment-vacancy correlation is -0.742, which is also close to -0.732 with linear utility. In the crisis samples, the unemployment volatility is 0.147, and the unemployment-vacancy correlation is -0.663, which are close to those with linear utility, 0.149 and -0.630, respectively.

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Table D.2 : Labor Market Volatilities, the Model with Log Utility

We simulate 50,000 artificial samples from the model's stationary distribution, with 1,005 months in each sample. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 20%) and crisis samples (in which the maximum rate is greater than or equal to 20%). We take the quarterly averages of monthly U, V, and X to convert to quarterly series. We implement the same procedure as in Table 2 and report cross-simulation averages and standard deviations (in parentheses).

		U	V	θ	X	U	V	θ	X
		Panel A: Non-crisis samples				Panel B: Crisis samples			
Standard deviation		0.105	0.191	0.277	0.013	0.147	0.207	0.323	0.014
Autocorrelation		0.806	0.708	0.781	0.773	0.855	0.706	0.807	0.782
Correlation matrix	U		-0.742	-0.887	-0.733		-0.663	-0.876	-0.696
	V			0.967	0.963			0.941	0.958
	θ				0.939				0.929

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