The Value Premium

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Outline

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A quantitative neoclassical benchmark framework for the cross section of expected returns

A neoclassical explanation of the value premium:

- Asymmetry causes countercyclical value-minus-growth risk
- Time-varying price of risk propagates risk dynamics

Higher adjustment cost leads to higher risk with production:

- Capital adjustment helps firms smooth out dividend stream
- Adjustment cost is the offsetting force of changing capital

Both in the data and in the model, high (low) book-to-market signals sustained low (high) profitability

The link between book-to-market and risk across business cycles:

- In bad times:

Value Firms $\Rightarrow^{II}$ Burdened With More Unproductive Capital
$\Rightarrow$ Want to Cut More Capital $\Rightarrow$ More Adjustment Cost $\Rightarrow^{I}$ Higher Risk

- In good times:

Growth Firms $\Rightarrow^{II}$ More Productive Capital
$\Rightarrow$ Want to Expand More $\Rightarrow$ More Adjustment Cost $\Rightarrow^{I}$ Higher Risk

- Time-varying price of risk $\Rightarrow$ a positive value premium

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The profit function:

$$\pi_{jt} = e^{(x_t + z_{jt} + p_t)} k_{jt}^{\alpha} - f$$

where

$$\begin{aligned} x_{t+1} &= \bar{x}(1-\rho_x) + \rho_x x_t + \sigma_x \epsilon_{t+1}^{x} \\ z_{jt+1} &= \rho_z z_{jt} + \sigma_z \epsilon_{jt+1}^{z} \end{aligned}$$

The pricing kernel:

$$\begin{aligned} \log M_{t,t+1} &= \log \beta + \gamma_t (x_t - x_{t+1}) \\ \gamma_t &= \gamma_0 + \gamma_1 (x_t - \bar{x}); \qquad \gamma_1 < 0 \end{aligned}$$

Industry demand function: $P_t = Y_t^{-\eta}$; $\eta \in (0,1)$: the inverse price elasticity of demand

The firms' optimal investment problem is:

$$v(k_t, z_t; x_t, p_t) = \max_{i_t} \left\{ \overbrace{e^{x_t + z_t + p_t} k_t^{\alpha} - f - i_t - h(i_t, k_t)}^{\textbf{Current Period Dividend}} + \right.$$

$$\left. \overbrace{\iint M_{t,t+1} v(k_{t+1}, z_{t+1}; x_{t+1}, p_{t+1})\, Q_z\left(dz_{t+1} | z_t\right) Q_x\left(dx_{t+1} | x_t\right)}^{\textbf{Expected Continuation Value}} \right\}$$

subject to the capital accumulation rule: $k_{t+1} = i_t + (1 - \delta) k_t$

Capital adjustment cost is asymmetric and quadratic:

$$h(i_t, k_t) = \frac{\theta_t}{2} \left(\frac{i_t}{k_t} \right)^2 k_t$$

where $\theta^- > \theta^+$ and $\theta_t = \theta^+ \chi_{\{i_t \geq 0\}} + \theta^- \chi_{\{i_t < 0\}}$

Model

Costly reversibility, illustration

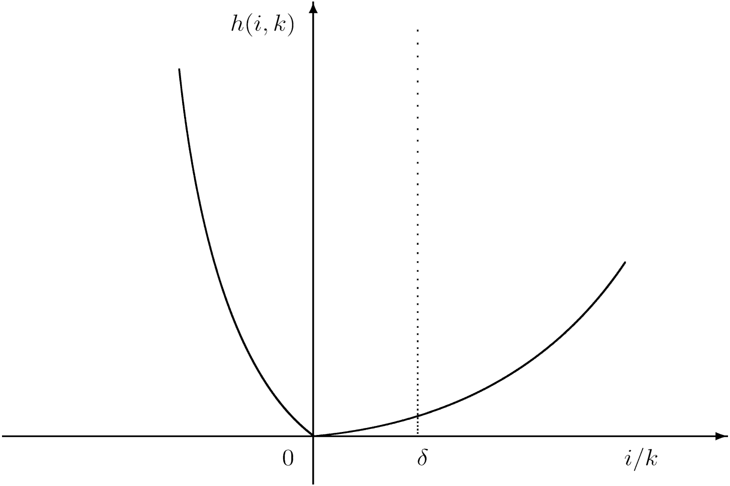

Figure 1. Asymmetric adjustment cost. This figure illustrates the specification of capital ad-

The risk and expected return of firm j satisfy the linear relationship

$$\mathrm{E}_t[R_{jt+1}] = R_{ft} + \beta_{jt}\lambda_{mt},$$

where R_{ft} is the real interest rate and the stock return is defined as

$$R_{jt+1} \equiv v_{jt+1}/(v_{jt} - d_{jt})$$

and d_{jt} is the dividend at time t, $d_{jt} \equiv \pi_{jt} - i_{jt} - h(i_{jt}, k_{jt})$; the quantity of risk is given by $\beta_{jt} \equiv -\mathrm{Cov}_t[R_{jt+1}, M_{t+1}]/\mathrm{Var}_t[M_{t+1}]$ and the price of risk is given by $\lambda_{mt} \equiv \mathrm{Var}_t[M_{t+1}]/\mathrm{E}_t[M_{t+1}]$

The law of motion of the cross-sectional distribution of firms, μ_t, is:

$$\mu_{t+1}(\Theta; x_{t+1}) = T(\Theta, (k_t, z_t); x_t)\mu_t(k_t, z_t; x_t)$$

where

$$T(\Theta, (k_t, z_t); x_t) \equiv \iint \chi_{\{(k_{t+1}, z_{t+1}) \in \Theta\}} \, Q_z(dz_{t+1}|z_t)Q_x(dx_{t+1}|x_t)$$

Industry output:

$$Y_t \equiv \iint y(k_t, z_t; x_t) \, \mu_t \, (dk, dz; x_t)$$

A competitive equilibrium consists of (T^*, v^*, i^*, p_t^*) such that the following conditions hold: optimality; consistency; and market-clearing

Computation: in standard value functions, the laws of motion of state variables are usually explicit and straightforward

But p_t, depends upon the firm distribution:

Approximate aggregation: Krusell and Smith (1998): approximate the distribution using a finite number of moments, e.g., $\overline{k}, \sigma(k)$.

1. Guess an explicit law of motion:
   $p_{t+1} = a_1 + a_2 p_t + a_3(x_t - \overline{x}) + a_4 \sigma_k$
2. Solve the firms' problem by the value function iteration method
3. Use the optimal investment and exit rules to simulate the industry with 5,000 firms for 12,000 monthly periods
4. Use the data in the stationary region to update the coefficients a_1, a_2, and a_3
5. Check convergence; if yes, go to next step; otherwise go back to step 2
6. Check goodness-of-fit; if yes, done; otherwise try a different specification

Model

Quality of aggregation

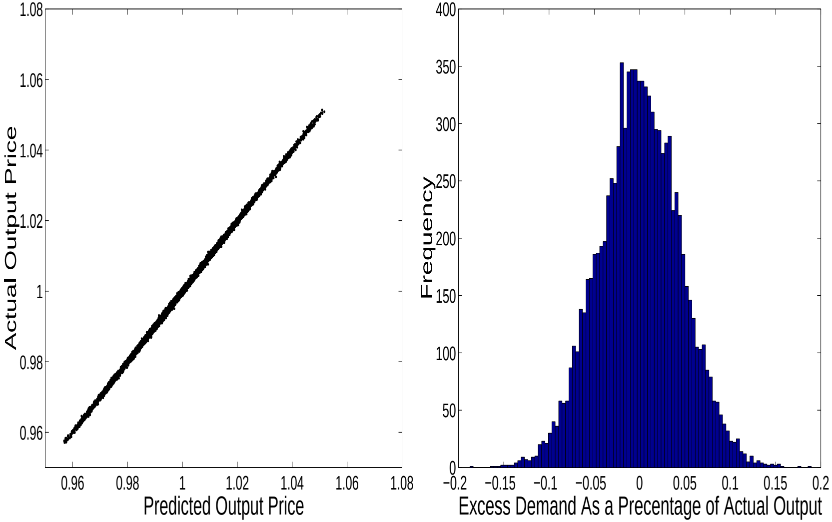

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Table I

Benchmark Parameter Values

This table lists the benchmark parameter values used to solve and simulate the model. I break all the parameters into three groups. Group I includes parameters whose values are restricted by prior empirical or quantitative studies: capital share, α; depreciation, δ; persistence of aggregate productivity, ρ_x; conditional volatility of aggregate productivity, σ_x; and inverse price elasticity of demand, η. Group II includes parameters in the pricing kernel, β, γ_0, and γ_1, which are tied down by matching the average Sharpe ratio and the mean and volatility of real interest rate. The final group of parameters is calibrated with only limited guidance from prior empirical studies. I start with a reasonable set of parameter values and conduct extensive sensitivity analysis in Tables III and IV.

Group I					Group II			Group III				
α	δ	ρ_x	σ_x	η	β	γ_0	γ_1	θ^-/θ^+	θ^+	ρ_z	σ_z	f
0.30	0.01	$0.95^{1/3}$	0.007/3	0.50	0.994	50	−1000	10	15	0.97	0.10	0.0365

Table II

Key Moments under the Benchmark Parametrization

This table reports a set of key moments generated under the benchmark parameters reported in Table I. The data source for the average Sharpe ratio is the postwar sample of Campbell and Cochrane (1999). The moments for the real interest rate are from Campbell et al. (1997). The data moments for the industry returns are computed using the 5-, 10-, 30-, and 48-industry portfolios in Fama and French (1997), available from Kenneth French's web site. The numbers of the average volatility of individual stock return in the data are from Campbell et al. (2001) and Vuolteenaho (2001). The data source for the moments of book-to-market is Pontiff and Schall (1999), and the annual average rates of investment and disinvestment are from Abel and Eberly (2001).

Moments	Model	Data
Average annual Sharpe ratio	0.41	0.43
Average annual real interest rate	0.022	0.018
Annual volatility of real interest rate	0.029	0.030
Average annual value-weighted industry return	0.13	0.12–0.14
Annual volatility of value-weighted industry return	0.27	0.23–0.28
Average volatility of individual stock return	0.286	0.25–0.32
Average industry book-to-market ratio	0.54	0.67
Volatility of industry book-to-market ratio	0.24	0.23
Annual average rate of investment	0.135	0.15
Annual average rate of disinvestment	0.014	0.02

Table III

Properties of Portfolios Sorted on Book-to-Market

This table reports summary statistics for HML and 10 book-to-market portfolios, including mean, m, volatility, σ, and market beta, β. Both the mean and the volatility are annualized. The average HML return (the value premium) is in annualized percent. Panel A reports results from historical data and benchmark model with asymmetry and countercyclical price of risk ($\theta^-/\theta^+ = 10$ and $\gamma_1 = -1000$). Panel B reports results from two comparative static experiments. Model 1 has symmetric adjustment cost and constant price of risk ($\theta^-/\theta^+ = 1$ and $\gamma_1 = 0$), and Model 2 has asymmetry and constant price of risk ($\theta^-/\theta^+ = 10$ and $\gamma_1 = 0$). All the model moments are averaged across 100 artificial samples. All returns are simple returns.

	Panel A: Data and Benchmark						Panel B: Comparative Statics					
	Data			Benchmark			Model 1			Model 2		
	m	β	σ	m	β	σ	m	β	σ	m	β	σ
HML	**4.68**	0.14	0.12	**4.87**	0.43	0.12	**2.19**	0.09	0.04	**2.54**	0.11	0.04
Low	0.11	1.01	0.20	0.09	0.85	0.23	0.08	0.95	0.30	0.08	0.94	0.30
2	0.12	0.98	0.19	0.10	0.92	0.24	0.09	0.97	0.31	0.09	0.97	0.31
3	0.12	0.95	0.19	0.10	0.95	0.25	0.09	0.99	0.31	0.09	0.98	0.31
4	0.11	1.06	0.21	0.11	0.98	0.26	0.09	1.00	0.32	0.10	0.99	0.31
5	0.13	0.98	0.20	0.11	1.01	0.27	0.10	1.00	0.32	0.10	1.00	0.32
6	0.13	1.07	0.22	0.12	1.04	0.28	0.10	1.01	0.32	0.10	1.01	0.32
7	0.14	1.13	0.24	0.12	1.08	0.28	0.10	1.02	0.32	0.10	1.02	0.32
8	0.15	1.14	0.24	0.12	1.12	0.30	0.10	1.03	0.33	0.11	1.04	0.33
9	0.17	1.31	0.29	0.13	1.18	0.31	0.11	1.04	0.33	0.11	1.05	0.33
High	0.17	1.42	0.33	0.15	1.36	0.36	0.11	1.07	0.34	0.12	1.08	0.34

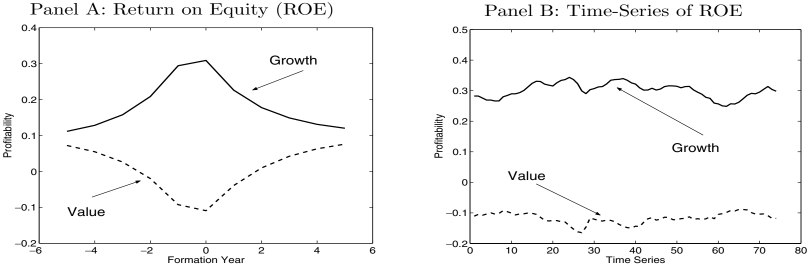

Figure 2. The value factor in profitability (ROE). Following Fama and French (1995), I measure profitability by return on equity, that is $[\triangle k_t + d_t]/k_{t-1}$, where k_t denotes the book value of equity and d_t is the dividend payout. Thus profitability equals the ratio of common equity income for the fiscal year ending in calender year t and the book value of equity for year $t-1$. The profitability of a portfolio is defined as the sum of $[\triangle k_{jt} + d_{jt}]$ for all firms j in the portfolio divided by the sum of k_{jt-1}; thus it is the return on book equity by merging all firms in the portfolio. For each portfolio formation year t, the ratios of $[\triangle k_{t+i} + d_{t+i}]/k_{t+i-1}$ are calculated for year $t+i$, where $i = -5, \dots, 5$. The ratio for year $t+i$ is then averaged across portfolio formation years. Panel A shows the 11-year evolution of profitability for value and growth portfolios. Time 0 on the horizontal axis is the portfolio formation year. Panel B shows the time series of profitability for value and growth portfolios. Value portfolio contains firms in the top 30% of the book-to-market ratios and growth portfolio contains firms in the bottom 30% of the book-to-market ratios. The figure is generated under the benchmark model, and varying θ^-/θ^+ and γ_1 yields similar results.

Outline

Intuition

The value factor in corporate investment

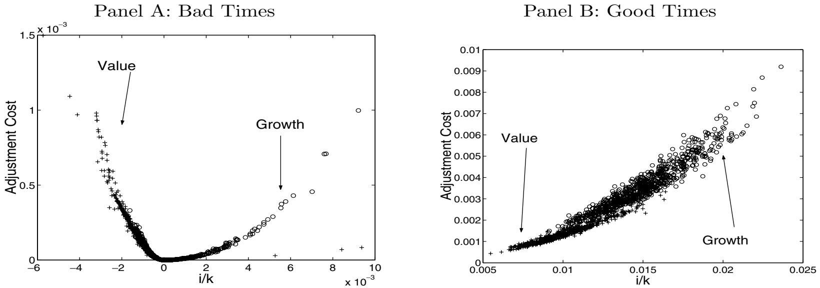

Figure 3. The value factor in corporate investment. This figure illustrates the value factor in corporate investment under the benchmark model. Panel A plots the adjustment cost, $h(i_t, k_t) = \frac{\theta_t}{2}(\frac{i_t}{k_t})^2 k_t$, as a function of the investment rate, i_t/k_t, in bad times for value firms (the "+"s) and growth firms (the "o"s). Panel B presents the same plot in good times. Good times are defined as times when the aggregate productivity, x_t, is more than one unconditional standard deviation, $\sigma_x/\sqrt{1-\rho_x^2}$, above its unconditional mean, $\overline{x}$. Bad times are defined as times when x_t is more than one standard deviation below its long-run level. Within each simulated sample, the investment rates and adjustment costs are averaged across all the good or the bad times for value and growth firms. I then repeat the simulation 100 times and plot the cross-simulation average adjustment costs against the cross-simulation average investment rates. The figure is generated within the benchmark model, and varying θ^-/θ^+ and γ_1 yields similar results.

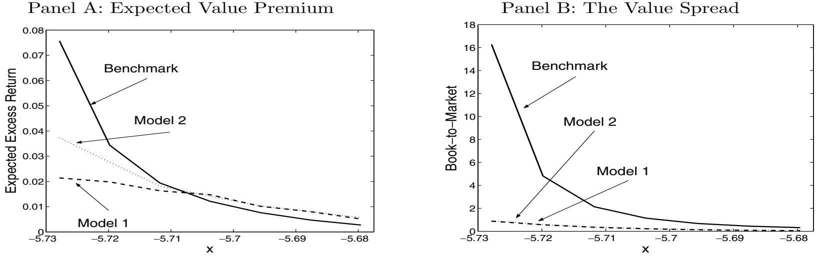

Figure 4. Time-varying spreads in expected excess return and in book-to-market between low-productivity (value) and high-productivity (growth) firms. This figure plots the spread in expected excess returns (Panel A) and the spread in book-to-market (Panel B) between firms with low idiosyncratic productivity and firms with high idiosyncratic productivity as functions of aggregate productivity, x. As is evident from Figure 2, sorting on firm-level productivity, z_t, in the model is equivalent to sorting on book-to-market. In effect, Panel A plots the time-varying expected value premium, and Panel B plots the time-varying spread in book-to-market (which Cohen et al. (2003) call the value spread) across business cycles. Three versions of the model are considered. The solid lines are for the benchmark model with asymmetry and countercyclical price of risk ($\theta^-/\theta^+ = 10$ and $\gamma_1 = -1000$). The broken lines are for Model 1 with symmetric adjustment cost and a constant price of risk ($\theta^-/\theta^+ = 1$ and $\gamma_1 = 0$). Finally, the dotted lines are for Model 2 with asymmetry and constant price of risk ($\theta^-/\theta^+ = 10$ and $\gamma_1 = 0$). The figure is generated with firm-level capital k and log output price p at their long-run average levels. Other values of k and p yield similar results.

Outline

Impact

Overcoming initial resistance

A quantitative neoclassical benchmark framework for the cross section of expected returns:

- A careful quantitative evaluation based on nonlinear solution methods for asset pricing dynamics, deviating from continuous time analytics and macro loglinearization

Empirically, the key theoretical prediction (value is riskier than growth in bad times) appeared contradicting Lakonishok, Shleifer, and Vishny (1994), resolved in Petkova and Zhang (2005)

Over 700 google scholar cites as of January 2014

Smith-Breeden for 2005

A top-five most cited article in the anomalies literature since 2000

A top-24 most cited article in JF since 2004

Featured in depth in Bodie, Kane, and Marcus's Investments

Model extended to include debt dynamics, labor, inventory, real estate, financial frictions for richer risk dynamics

Model inconsistent with the failure of the CAPM: models with second shocks including investment shocks, shocks to adjustment costs, long-run risks, and uncertainty shocks

- Bai, Hou, Kung, and Zhang (2014, "Exhuming" the CAPM) retain the single shock structure but incorporate disaster risk

Empirics: structural estimation per Liu, Whited, and Zhang (2009) and factor regressions per Hou, Xue, and Zhang (2014)