Aggregation, Capital Heterogeneity, and the Investment CAPM

Andrei S. Gonçalves*
University of North Carolina

Chen Xue†
University of Cincinnati

Lu Zhang‡
Ohio State and NBER

October 2018§

Abstract

A detailed treatment of aggregation and capital heterogeneity substantially improves the performance of the investment CAPM. Firm-level predicted returns are constructed from firm-level accounting variables and aggregated to the portfolio level to match with portfolio-level stock returns. Working capital forms a separate productive input besides physical capital. The model fits well the value, momentum, investment, and profitability premiums simultaneously and partially explains the positive stock-fundamental return correlations, the procyclical and short-term dynamics of the momentum and profitability premiums, as well as the countercyclical and long-term dynamics of the value and investment premiums. However, the model falls short in explaining momentum crashes.

*Kenan-Flagler Business School, University of North Carolina, 4101 McColl Building, 300 Kenan Center Drive, Chapel Hill, NC 27599. Tel: (919) 962-3179. E-mail: Andrei_Goncalves@kenan-flagler.unc.edu.

†Lindner College of Business, University of Cincinnati, 405 Lindner Hall, Cincinnati OH 45221. Tel: (513) 556-7078. E-mail: xuecx@ucmail.uc.edu.

‡Fisher College of Business, The Ohio State University, 760A Fisher Hall, 2100 Neil Avenue, Columbus OH 43210; and NBER. Tel: (614) 292-8644. E-mail: zhanglu@fisher.osu.edu.

§We have benefited from helpful comments from Rüdiger Fahlenbrach, Amit Goyal, Hui Guo, Erwan Morellec, Mehmet Sağlam, Steve Slezak, René Stulz, and Tong Yu, as well as other seminar participants at Shanghai University of Finance and Economics, The Ohio State University, University of Cincinnati, and University of Lausanne. We thank Frederico Belo for helpful conversations on aggregation. This paper supersedes our prior work circulated under the title "Does the investment model explain value and momentum simultaneously?"

1 Introduction

Aggregation and heterogeneity have long been recognized as thorny problems for empirical studies of the investment behavior. Nickell (1978) identifies three major problems on aggregation and heterogeneity. First, "the question arises as to whether one can construct aggregates for the outputs, the capital good inputs and the labour inputs so that it is possible to define a production function which gives aggregate output as a function of the aggregate capital and aggregate labour inputs. The answer, in any realistic case, is that it is not (p. 229–230)." Second, even if the empirical relations at the firm level are good approximations of reality, "it is difficult to develop structural restrictions on the aggregate relationships corresponding to those which theory imposes on the micro-level equations (p. 230)." This difficulty is especially acute, if the micro-level equations are nonlinear. Third, there are serious problems associated with measuring investment and capital stock. Investment data can be "based on orders, deliveries or payments or some mixture of all three (p. 231)" that are not additions to a firm's capital stock. The key problem of measuring the capital stock is "the evaluation of old capital goods for which there exist no active markets (p. 231)."[1]

This paper provides a careful treatment of aggregation, which is the second major problem identified by Nickell (1978). We also address, albeit to a lesser extent, the problem of capital heterogeneity and the measurement of investment and capital, which are the first and third problems in Nickell. We do so in the context of the Investment Capital Asset Pricing Model (the investment CAPM).

Prior studies implement the investment CAPM via structural estimation at the portfolio level (Liu, Whited, and Zhang 2009). Firm-level accounting variables are aggregated to portfolio-level variables, from which portfolio-level investment returns are constructed to match with portfolio-

[1] Aggregation and heterogeneity pose even more challenging problems for empirical studies of the consumption behavior. For example, Blundell and Stoker (2007) write: "[I]t is senseless to ascribe behavioral interpretations to estimated relationships among aggregate data without a detailed treatment of the links between individual and aggregate levels (p. 4614)." "Aggregation problems are among the most difficult problems faced in either the theoretical or empirical study of economics. Heterogeneity across individuals is extremely extensive and its impact is not obviously simplified or lessened by the existence of economic interaction via markets or other institutions. The conditions under which one can ignore a great deal of the evidence of individual heterogeneity are so severe as to make them patently unrealistic. There is no quick, easy or obvious fix to dealing with aggregation problems in general (p. 4658)."

level stock returns. While a useful first stab, this approach has a couple of important drawbacks. First, on economic grounds, it assumes that firms in a given portfolio all follow the same investment decision rule. This assumption is clearly counterfactual. Second, on econometric grounds, this approach misses a substantial amount of heterogeneity in firm-level variables that can help identify structural parameters. We instead use firm-level variables to construct firm-level investment returns, which are then aggregated to the portfolio level to match with portfolio-level stock returns.

In addition, most studies ignore capital heterogeneity, with physical capital (net property, plant, and equipment) as the single productive input. However, net property, plant, and equipment is only a small fraction of total assets on firms' balance sheet. While many possibilities exist to introduce an additional input, we settle on working capital, with no adjustment costs (an assumption that we verify empirically). Consequently, the resulting two-capital model is as parsimonious as the baseline, physical capital model with only two parameters, facilitating comparison with prior work.

Our benchmark model with two capital goods estimated at the firm level goes a long way in resolving the empirical difficulties in prior work. Estimating the physical capital model at the portfolio level, Liu, Whited, and Zhang (2009) show that the marginal product and adjustment costs parameters vary greatly across the value and momentum deciles. If the model is well specified, or "structural," the parameter estimates should be mostly invariant across different testing portfolios. As a result, the baseline model fails to explain value and momentum simultaneously. This weakness applies to the investment CAPM literature more broadly. For example, in a prominent asset pricing textbook, Campbell (2018, p. 213) writes: "This problem, that different parameters are needed to fit each anomaly, is a pervasive one in the q-theoretic asset pricing literature."

The parameter estimates in our benchmark model are relatively stable across the testing deciles on value, momentum, asset growth, and return on equity, separately or jointly. When fitting value and momentum deciles together, with and without also adding the asset growth and return on equity deciles, the scatter plots of average predicted stock returns versus average realized stock returns are mostly aligned with the 45-degree line. For example, when fitting value-weighted value and momentum deciles jointly, the model implies a value premium of 5.2% per annum, with a small alpha of 1.18% ($t = 0.51$), as well as a momentum premium of 14.62%, with an even smaller alpha of 0.35% ($t = 0.12$). However, the model is still rejected by the test of overidentification.

Aggregation is important for the benchmark model's performance. When implemented at the portfolio level, the model yields larger pricing errors. In the joint estimation of value and momentum, the value premium is only 2.88% per annum, with an alpha of 3.51% ($t = 1.23$), although the momentum premium is 13.97%, with a small alpha of 1% ($t = 0.63$). Working capital is also important. In the data the fraction of physical capital in the sum of physical capital and working capital averages only 38%. Accordingly, the average product in the physical capital model is severely misspecified, giving rise to large pricing errors even when estimated at the firm level. Again in the joint estimation of value and momentum, the value premium is 1.64%, with an alpha of 4.75% ($t = 1.8$), and the momentum premium 20.17%, with a large alpha of −9.29% ($t = -2.79$).

We also use the predicted stock return from the benchmark model (dubbed "the fundamental return") to study the dynamics of factor premiums. The model yield significantly positive stock-fundamental return correlations, the short-term dynamics of the momentum and return on equity premiums, as well as the long-term dynamics of the value and investment premiums. The model also partially explains the procyclical variation of the momentum and return on equity premiums as well as the countercyclical variation of the value and investment premiums. However, the model underestimates the volatility, skewness, and kurtosis of factor premiums as well as momentum crashes.

Finally, prior work only examines in-sample fits. We also conduct out-of-sample tests by constructing firm-level 1-period-ahead expected returns from recursively estimating the benchmark two-capital model. The expected return estimates forecast subsequent returns reliably. In contrast, the out-of-sample performance of the physical capital model estimated at the portfolio level and the q-factor model (which is a reduced form implementation of our structural model) is poor.

Building on Cochrane (1991), Liu, Whited, and Zhang (2009) estimate the physical capital model at the portfolio level with data on cross-sectional asset prices. Cooper and Priestley (2016) use the investment framework to study the cost of capital for private firms. Several studies feature additional productive inputs, such real estate (Tuzel 2010), working capital (Wu, Zhang, and Zhang 2010), and inventory (Belo and Lin 2012; Jones and Tuzel 2013). Li (2017) tries to explain value and momentum jointly in a theoretical model. We differ by doing structural estimation on the real data. Aggregation has been largely overlooked in the investment literature.[2] We fill this gap.[3]

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 discusses our econometric design. Section 4 presents our empirical results. Finally, Section 5 concludes.

2 The Model of the Firms

We formulate the two-capital model in Section 2.1 and explain why we include working capital as a productive input in addition to physical capital in Section 2.2.

2.1 Setup

Firms use both short-term working capital and long-term physical capital to produce a homogeneous output. Let $\Pi_{it} \equiv \Pi(K_{it}, W_{it}, X_{it})$ denote the operating profits of firm i at time t, in which K_{it} is physical capital, W_{it} working capital, and X_{it} a vector of exogenous aggregate and firm-specific shocks. We assume that Π_{it} exhibits constant returns to scale, i.e., $\Pi_{it} = K_{it}\partial\Pi_{it}/\partial K_{it} + W_{it}\partial\Pi_{it}/\partial W_{it}$, and that firms have a Cobb-Douglas production function. The marginal product of physical capital can then be parameterized as $\partial\Pi_{it}/\partial K_{it} = \gamma_K Y_{it}/K_{it}$, in which $\gamma_K > 0$ is a technological parameter and Y_{it} sales (Gilchrist and Himmelberg 1998).

[2] In subsequent but independent work, Belo, Gala, and Salomao (2018) study the aggregation issue in the context of equity valuation. However, while it is natural to motivate the portfolio approach for expected return assets (Black, Jensen, and Scholes 1972), it might be difficult, or even unnecessary, to do so for equity valuation tests.

[3] Outside asset pricing, Wildasin (1984) examines optimal investment with many capital goods. Schaller (1990) shows that aggregation is partially responsible for large adjustment costs from aggregate time series. Hayashi and Inoue (1991) derive a one-to-one relation between the growth rate of the capital aggregate and Tobin's q in an investment model with multiple capital goods, and estimate this relation on Japanese firms. Chirinko (1993) estimates the investment model with multiple capital inputs that differ in adjustment technologies. Doyle and Whited (1998) show that smooth industry-level investment results from aggregating asynchronous and lumpy micro-level investment.

Similarly, the marginal product of working capital is $\partial \Pi_{it}/\partial W_{it} = \gamma_W Y_{it}/W_{it}$, in which $\gamma_W > 0$.

Taking operating profits as given, firms choose investments in working and physical capital stocks to maximize the market equity. Physical capital evolves as $K_{it+1} = I_{it} + (1-\delta_{it})K_{it}$, in which I_{it} is investment in physical capital, and δ_{it} the rate of depreciation, which firm i takes as given. We allow δ_{it} to be firm-specific and time-varying. Working capital evolves as $W_{it+1} = \triangle W_{it} + W_{it}$, in which $\triangle W_{it}$ is investment in working capital. We assume that working capital does not depreciate.

Firms incur adjustment costs when investing in physical capital, but not in working capital. The adjustment costs function, denoted $\Phi(I_{it}, K_{it})$, is increasing and convex in I_{it}, decreasing in K_{it}, and of constant returns to scale in I_{it} and K_{it}, i.e., $\Phi(I_{it}, K_{it}) = I_{it}\, \partial\Phi(I_{it}, K_{it})/\partial I_{it} + K_{it}\, \partial\Phi(I_{it}, K_{it})/\partial K_{it}$. We adopt the standard quadratic functional form:

$$\Phi_{it} \equiv \Phi(I_{it}, K_{it}) = \frac{a}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}, \tag{1}$$

in which $a > 0$ is the adjustment costs parameter of physical capital.

At the beginning of time t, firm i issues debt, B_{it+1}, which must be repaid at the beginning of $t+1$. When borrowing, firms take as given the gross cost of debt on B_{it}, denoted r^B_{it}, which varies across firms and over time. Taxable corporate profits equal operating profits less physical capital depreciation, adjustment costs, and interest expenses, $\Pi_{it} - \delta_{it}K_{it} - \Phi_{it} - (r^B_{it} - 1)B_{it}$. Let τ_t be the corporate tax rate, $\tau_t\delta_{it}K_{it}$ be depreciation tax shield, and $\tau_t(r^B_i - 1)B_{it}$ be interest tax shield. Firm i's net payout is given by $D_{it} \equiv (1-\tau_t)(\Pi_{it} - \Phi_{it}) - I_{it} - \triangle W_{it} + B_{it+1} - r^B_{it}B_{it} + \tau_t\delta_{it}K_{it} + \tau_t(r^B_{it} - 1)B_{it}$.

Let M_{t+1} be the stochastic discount factor. Taking M_{t+1} as given, firm i chooses the streams of $I_{it}, K_{it+1}, \triangle W_{it}, W_{it+1}$, and B_{it+1} to maximize its cum-dividend market value of equity, $V_{it} \equiv E_t\left[\sum_{s=0}^{\infty} M_{t+s}D_{it+s}\right]$, subject to $\lim_{T\to\infty} E_t\left[M_{t+T}\, B_{it+T+1}\right] = 0$ (the transversality condition), which prevents the firm from borrowing an infinite amount of debt.

The first-order condition for physical investment implies that $E_t[M_{t+1}r^K_{it+1}] = 1$, in which r^K_{it+1}

is the physical capital investment return:

$$r^K_{it+1} \equiv \frac{(1-\tau_{t+1})\left[\gamma_K \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1})\left[1+(1-\tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]}{1+(1-\tau_t)a\left(\frac{I_{it}}{K_{it}}\right)}. \tag{2}$$

Intuitively, the physical investment return is the marginal benefit of physical investment at $t+1$ divided by its marginal cost at t. In the numerator of equation (2), $(1-\tau_{t+1})\gamma_K(Y_{it+1}/K_{it+1})$ is the after-tax marginal product of physical capital, $(1-\tau_{t+1})(a/2)(I_{it+1}/K_{it+1})^2$ is the after-tax marginal reduction in physical adjustment costs, and $\tau_{t+1}\delta_{it+1}$ is the marginal depreciation tax shield. The last term in the numerator is the marginal continuation value of an extra unit of physical capital net of depreciation, in which the marginal continuation value equals the marginal cost of physical investment in the next period, $1+(1-\tau_{t+1})a(I_{it+1}/K_{it+1})$. Finally, $E_t[M_{t+1}r^K_{it+1}]=1$ says that the marginal cost of investment equals the next period marginal benefit discounted to time t.

Similarly, the firm's first-order condition for investment in working capital is $E_t[M_{t+1}r^W_{it+1}]=1$, in which r^W_{it+1} is the working capital investment return:

$$r^W_{it+1} \equiv 1+(1-\tau_{t+1})\gamma_W\frac{Y_{it+1}}{W_{it+1}}. \tag{3}$$

The working capital investment return is again the marginal benefit of working capital investment at $t+1$ divided by its marginal cost at time t. The marginal cost equals one because of no adjustment costs on working capital. For the marginal benefit, $(1-\tau_{t+1})\gamma_W(Y_{it+1}/W_{it+1})$ is the after-tax marginal product of working capital, and without adjustment costs or depreciation, the marginal continuation value of an extra unit of working capital net of depreciation equals one.

Define the after-tax cost of debt as $r^{Ba}_{it+1} \equiv r^B_{it+1}-(r^B_{it+1}-1)\tau_{t+1}$. The firm's first-order condition for new debt implies that $E_t[M_{t+1}r^{Ba}_{it+1}]=1$. Define $P_{it}\equiv V_{it}-D_{it}$ as the ex-dividend market value of equity, $r^S_{it+1}\equiv(P_{it+1}+D_{it+1})/P_{it}$ as the stock return, and $w^B_{it}\equiv B_{it+1}/(P_{it}+B_{it+1})$ as the market leverage. Also, the shadow price of physical capital is marginal q, $q_{it}=1+(1-\tau_t)a(I_{it}/K_{it})$, which in the optimum equals the marginal cost of physical investment. The shadow price of working capital equals one. Finally, define $w^K_{it} \equiv q_{it}K_{it+1}/(q_{it}K_{it+1} + W_{it+1})$ as the weight of the firm's market value attributed to physical capital. Then the weighted average of the two investment returns equals the weighted average of the cost of equity and the after-tax cost of debt (Appendix A):

$$w^K_{it} r^K_{it+1} + (1 - w^K_{it}) r^W_{it+1} = w^B_{it} r^{Ba}_{it+1} + (1 - w^B_{it}) r^S_{it+1}. \quad (4)$$

Solving for the stock return from equation (4) yields the investment CAPM:

$$r^S_{it+1} = r^F_{it+1} \equiv \frac{w^K_{it} r^K_{it+1} + (1 - w^K_{it}) r^W_{it+1} - w^B_{it} r^{Ba}_{it+1}}{1 - w^B_{it}}, \quad (5)$$

in which r^F_{it+1} is the "fundamental" return as a nonlinear function of firm characteristics. If $w^K_{it} = 1$, equation (4) collapses to the equivalence between the physical investment return and the weighted average cost of capital (Liu, Whited, and Zhang 2009). If $w^K_{it} = 1$ and $w^B_{it} = 0$, equation (5) reduces to the equivalence between the stock and physical investment returns (Cochrane 1991).

Equation (5) clearly shows that even without adjustment costs, working capital helps characterize the cross section of expected stock returns more accurately. In this aspect, working capital differs from labor, which does not appear on firms' balance sheet as assets. Firms hire, but do not own, workers. As a result, without adjustment costs on labor hiring, the labor input can be absorbed into the operating profits function and does not affect the cost of equity distribution.

2.2 Why Working Capital?

Short-term working capital is essential for firms' operations. The main components of working capital are cash, account receivables, and inventory (Berk and DeMarzo 2017). Firms hold cash to save transaction costs of raising funds and to avoid liquidation of asses to make payments. Also, firms use cash to finance its day-to-day operations and long-term investments if other financing sources are either unavailable or excessively costly (Opler, Pinkowitz, Stulz, and Williamson 1999).

Trade credit, in the form of accounts receivable and payable, is an important source of short-term external finance among firms (Petersen and Rajan 1997). Suppliers extend trade credit to their customers in the form of accounts receivable to increase sales against their competitors. Relative to financial institutions, suppliers are more inclined to lend to financially constrained firms because of their comparative advantage in obtaining information on the buyers, their ability to liquidate buyers' assets more efficiently, and their implicit equity stake in the buyers.

Inventory is necessary in the production process for a couple of reasons. First, inventory helps avoid stock-outs, in which a firm runs out of its store of commodities and loses sales, or a firm exhausts its store of materials and delays production. Second, inventory helps ensure a more efficient production cycle to meet seasonal demand. Sales can be highly seasonal with upward spikes in the fourth quarter. In contrast, a smooth production process is more desirable to avoid excessive wear and tear on equipment and overtime worker salaries (Berk and DeMarzo 2017).

Several prior asset pricing studies have examined working capital as a separate productive input in addition to physical capital. Wu, Zhang, and Zhang (2010) treat accruals as working capital investment and use the investment theory to interpret the accruals anomaly. Belo and Lin (2012) embed an inventory holding motive into the investment model to explain the negative relation between inventory growth and expected returns. Jones and Tuzel (2013) document the inventory-return relation in both time series and cross section and show that the evidence is consistent with a two-capital investment model with inventory and physical capital as two separate inputs.

While the prior studies model inventory as costly to adjust, we do not include adjustment costs of working capital. We show that the adjustment costs on working capital in an extended model are mostly small and insignificant, especially in the joint estimation with value and momentum (the Internet Appendix). The extended model's performance is also quantitatively close to the simplified model without the extra adjustment costs. As such, we opt for the simpler model for parsimony.

While working capital as a separate input is straightforward to motivate, we do not include other inputs such as labor and intangibles. The crux is measurement errors. Working capital can be accurately measured on firms' balance sheet with relatively few errors. In contrast, in our sample (described in detail in Section 3.3), about 80.1% of wages data (Compustat annual item XLR, total staff expense) are missing at the firm level. In addition, measurement errors are likely even more severe for intangibles. For instance, Peters and Taylor (2017) assume a fixed depreciation rate of 20% for organizational capital and a fixed proportion of 30% of selling, general, and administrative expenses as intangible capital investments. Both rates are assumed to be constant over time and across firms. While these ad hoc assumptions are perhaps unavoidable when measuring intangibles, we hesitate to introduce such measurement errors into our structural estimation.

3 Econometric Design

We describe our structural estimation in Section 3.1 and aggregation in Section 3.2.

3.1 Generalized Method of Moments (GMM)

We use GMM to test the ex ante restriction implied by equation (5):

$$E[r^S_{pt+1} - r^F_{pt+1}] = 0, \tag{6}$$

in which r^S_{pt+1} is the stock return of testing portfolio p, and r^F_{pt+1} is portfolio p's fundamental return given by the right hand side of equation (5). In particular, the pricing error (alpha) from the investment CAPM is defined as $\alpha_p \equiv E_T[r^S_{pt+1} - r^F_{pt+1}]$, in which $E_T[\cdot]$ is the sample mean.

3.1.1 Why Focusing on the First Moment?

Interpreted literally, equation (5) predicts that the stock return equals the fundamental return period by period and state by state. When taking the model to the data, we choose to estimate the structural parameters from the first moment restriction in equation (6), which says that the expected stock return equals the expected fundamental return. We focus on the first moment because the anomalies literature is primarily about the expected return. Why do stocks with high book-to-market, high short-term prior returns, low investment, and high return on equity earn higher average returns than stocks with low book-to-market, low short-term prior returns, high investment, and low return on equity, respectively? These important questions are all about the first moment.

The first moment restriction is also likely to be more reliable in the data. Although equation (5) predicts ex post equivalence between the stock and fundamental returns, it is straightforward to introduce some residuals to break the ex post equivalence. For instance, the marginal product of physical capital, specified as $\gamma_K(Y_{it+1}/K_{it+1})$, might not be exactly proportional to sales-to-physical capital, but come with an additive, zero-mean measurement error, as in $\gamma_K(Y_{it+1}/K_{it+1}) + \epsilon^K_{it+1}$. With such an error, the stock return, which accounts for the error, and the (measured) fundamental return, which does not account for the error, will be equivalent only ex ante, but not ex post. Finally, although we estimate the structural parameters only from the first moment restriction, we push the econometric model as far as possible to explain the second, third, and fourth moments, as well as cross correlations and tail risk, as separate diagnostics of the model (Section 4.4).

3.1.2 Identification, Estimation, and Tests

Although the model has three parameters (γ_K, γ_W, and a), γ_K and γ_W enter the moment condition (6) only in the form of $\gamma \equiv \gamma_K + \gamma_W$. To see this point, we use equations (2) and (3) to rewrite:

$$w^K_{it} r^K_{it+1} + (1 - w^K_{it}) r^W_{it+1} = \frac{(1-\tau_{t+1})(\gamma_K + \gamma_W) Y_{it+1}/(K_{it+1} + W_{it+1})}{q_{it} K_{it+1}/(K_{it+1} + W_{it+1}) + W_{it+1}/(K_{it+1} + W_{it+1})} + w^K_{it} \frac{(1-\tau_{t+1})(a/2)\left(I_{it+1}/K_{it+1}\right)^2 + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1}) q_{it+1}}{q_{it}} + (1 - w^K_{it}). \quad (7)$$

As such, γ_K and γ_W are not separately identifiable, and only their sum, γ, can be estimated. With only two parameters, γ and a, the two-capital model with physical capital and working capital is as parsimonious as the baseline model with only physical capital.

In addition, the numerator of the first term in the right hand side of equation (7) shows that the marginal product in the two-capital model should be measured as proportional to sales over the sum of physical capital and working capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, as opposed to sales-to-physical capital, Y_{it+1}/K_{it+1}, in the physical capital model. Finally, the denominator of the first term can be interpreted as the weighted average of the marginal q of physical capital and that of working capital (one), with the weight given by $K_{it+1}/(K_{it+1}+W_{it+1})$ and $W_{it+1}/(K_{it+1}+W_{it+1})$, respectively.

Formally, let $\mathbf{c} \equiv (\gamma, a)$ denote the model's parameter, and $\mathbf{g}_T$ the sample moments. The GMM objective function is a weighted sum of squares of the alphas across a set of testing portfolios, $\mathbf{g}_T'\mathbf{W}\mathbf{g}_T$, in which we set $\mathbf{W}=\mathbf{I}$, the identity matrix (Cochrane 1996). Let $\mathbf{D}=\partial\mathbf{g}_T/\partial\mathbf{c}$ and $\mathbf{S}$ be a consistent estimate of the variance-covariance matrix of the sample alphas, $\mathbf{g}_T$. The $\mathbf{S}$ estimate accounts for autocorrelations of up to 12 lags. The estimate of $\mathbf{c}$, denoted $\hat{\mathbf{c}}$, is asymptotically normal with the variance-covariance matrix given by $\text{var}(\hat{\mathbf{c}}) = (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}/T$. To construct the standard errors for the pricing errors of individual portfolios, we use the variance-covariance matrix for $\mathbf{g}_T$, $\text{var}(\mathbf{g}_T) = \left[\mathbf{I}-\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\right]\mathbf{S}\left[\mathbf{I}-\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\right]'/T$. Finally, we form a χ^2 test on the null hypothesis that all the alphas are jointly zero, $\mathbf{g}_T'\left[\text{var}(\mathbf{g}_T)\right]^{+}\mathbf{g}_T \sim \chi^2(\#\,\text{moments} - \#\,\text{parameters})$, in which χ^2 is the chi-square distribution with the degrees of freedom given by the number of moments minus the number of parameters, and the superscript $^{+}$ denotes pseudo-inversion (Hansen 1982).

3.2 Aggregation

Prior studies estimate the physical capital model with accounting data aggregated to the portfolio level. Portfolio-level fundamental returns are constructed from portfolio-level characteristics to match with portfolio-level stock returns. Formally, the prior studies estimate:

$$E\left[\sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^{S} - r_{pt+1}^{F}\left(\gamma_K, a; Y_{pt+1}, K_{pt+1}, I_{pt+1}, \delta_{pt+1}, I_{pt}, K_{pt}, r_{pt+1}^{Ba}, w_{pt}^{B}\right)\right] = 0, \tag{8}$$

in which N_{pt} is the number of firms in portfolio p at the beginning of period t, w_{ipt} is the weight of stock i in portfolio p at the beginning of period t, r_{ipt+1}^{S} is the return of stock i in portfolio p over period t, and r_{pt+1}^{F} is the fundamental return for portfolio p. For equal-weighted portfolios, $w_{ipt} = 1/N_{pt}$, and for value-weighted portfolios, w_{ipt} is the market value-weights at the beginning of period t. r_{pt+1}^{F} is constructed from portfolio-level characteristics aggregated from firm-level characteristics, and its functional form does not change with w_{ipt}. To aggregate accounting variables from the firm level to the portfolio level, $I_{pt+1} = \sum_{i=1}^{N_{pt}} I_{ipt+1}$, in which I_{ipt+1} is investment of firm i in portfolio p over period $t+1$, $w_{pt}^{B} = \sum_{i=1}^{N_{pt}} B_{ipt+1} / \sum_{i=1}^{N_{pt}} (P_{ipt} + B_{ipt+1})$, and $r_{pt+1}^{Ba} = (1/N_{pt}) \sum_{i=1}^{N_{pt}} r_{ipt+1}^{Ba}$. Other portfolio-level variables are constructed analogously.

Working with this aggregation procedure, Liu, Whited, and Zhang (2009) show that the physical capital model explains value and momentum separately, but the parameter estimates vary greatly across the two sets of deciles. In addition, Liu and Zhang (2014) document that when forced to use the same parameter values in the joint estimation, the physical capital model manages to capture the momentum premium but fails to explain the value premium altogether.

We explore a new, exact aggregation procedure. We first construct firm-level fundamental returns from firm-level accounting variables and then aggregate to portfolio-level fundamental returns to match with portfolio-level stock returns. Formally, we estimate:

$$E\left[\sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^{S} - \sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^{F}\left(\gamma, a; Y_{ipt+1}, K_{ipt+1}, W_{ipt+1}, I_{ipt+1}, \delta_{ipt+1}, I_{ipt}, K_{ipt}, r_{ipt+1}^{Ba}, w_{ipt}^{B}\right)\right] = 0, \tag{9}$$

in which r_{ipt+1}^{F} is the fundamental return for firm i. As such, aggregating r_{ipt+1}^{S} and r_{ipt+1}^{F} is symmetric, and the portfolio-level fundamental return, $r_{pt+1}^{F} \equiv \sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^{F}$, varies with w_{ipt}.

3.3 Data

We obtain firm-level data from Center for Research in Security Prices (CRSP) monthly stock files and annual Standard and Poor's Compustat industrial files. We exclude firms with primary standard industrial classifications between 6000 and 6999 (financial firms) and firms with total assets, net property, plant, and equipment, or sales either zero or negative at each portfolio formation. These data items are necessary to calculate the firm-level fundamental returns.

3.3.1 Measurement

While largely following Liu, Whited, and Zhang (2009) and Liu and Zhang (2014) in measuring the firm-level variables in the construction of the fundamental returns, we offer a few refinements.

In the model, time-t stock variables are at the beginning of period t, and time-t flow variables are over the course of period t. In Compustat both stock and flow variables are recorded at the end of period t. As such, for the year 2010, for example, we take time-t stock variables from the 2009 balance sheet, and time-t flow variables from the 2010 income or cash flow statement.

We measure output, Y_{it}, as sales (Compustat annual item SALE) and short-term working capital as current assets (item ACT). Total debt, B_{it+1}, is long-term debt (item DLTT, zero if missing) plus short-term debt (item DLC, zero if missing). The market leverage, w_{it}^B, is the ratio of total debt to the sum of total debt and market equity (from CRSP). The tax rate, τ_t, is the statutory corporate income tax rate from the Commerce Clearing House's annual publications. The physical capital, K_{it}, is net property, plant, and equipment (item PPENT).

Departing from the prior studies, we offer several important refinements in measurement. First, the prior studies measure the depreciate rate of physical capital, δ_{it}, as the amount of depreciation and amortization (Compustat annual item DP) divided by physical capital (item PPENT). We subtract the amortization of intangibles (item AM, zero if missing) from item DP, before scaling the difference by item PPENT. This measure is more accurate. In the data, the AM/DP ratio is on average 6.6%, with a standard deviation of 14.3%. The AM/DP distribution has a long right tail. Its median is 0%, but the 75, 90, and 95 percentiles are 4.7%, 25.7%, and 41.3%, respectively.

Second, the prior studies measure investment, I_{it}, as capital expenditures (item CAPX) minus sales of property, plant, and equipment (item SPPE, zero if missing). Despite its simplicity, this I_{it} measure can violate the capital accumulation equation, $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$, in the data. As documented in detail in the Internet Appendix (Table A.3), at the firm level, the differences between CAPX−SPPE and $K_{it+1} - (1 - \delta_{it})K_{it}$ are more than 10.28%, 31.5%, and 57.45% of physical capital, K_{it}, in magnitude, for 25%, 10%, and 5% of the sample observations, respectively.

Mergers and acquisitions (M&As) play an important role in explaining the deviations. We identify M&As by combining the Securities Data Company (SDC) dataset and Compustat (item AQC) and find M&As to be prevalent. The subsample that contains only firms with M&As accounts for 38.63% of the observations in the full sample. More important, the capital accumulation deviations are substantially larger in this subsample than in the full sample. The deviations are more than 19.28%, 53.35%, and 94.59% of physical capital in magnitude for 25%, 10%, and 5% of the observations, respectively, in the subsample with only M&As.

However, M&As do not fully explain the capital accumulation deviations. In the subsample that contains only firms without M&As, the deviations are still substantial, accounting for more than 7.09%, 23.08%, and 43.23% of physical capital for 25%, 10%, and 5% of the observations, respectively. As such, the deviations are more general than M&As. In particular, even without M&As, the measurement errors in I_{it} that amount to more than 23% of K_{it} for as many as 10% of the observations seem excessively large. Because our new aggregation requires the construction of the firm-level fundamental returns, in which investment-to-physical capital is an important component, we opt to measure I_{it} directly as $K_{it+1} - (1 - \delta_{it})K_{it}$.

We emphasize that, as noted, M&As are prevalent in our sample, accounting for 38.63% of the observations. More important, M&As are not random corporate events. Firms with M&As are more likely to be growth firms, momentum winners, high investment firms, and high return on equity firms than firms without M&As. As such, we retain firms with M&As in our sample to facilitate identification. The Internet Appendix shows that our quantitative results are robust if we exclude firms with sizeable M&As, in which the target assets are at least 15% of the acquirer assets (Whited 1992).[4]

Finally, to measure the firm-level pre-tax cost of debt in a broad sample, the prior studies

[4]We should acknowledge that our measure of investment as $K_{it+1} - (1 - \delta_{it})K_{it}$ implicitly assumes that internal growth in physical capital and external growth via M&As face the same adjustment costs technology. This assumption is for parsimony only, as treating M&As separately in the investment framework would take us too far afield and complicate the econometric specification. However, the basic principles of the q-theory of investment also apply to M&As. For example, Jovanovic and Rousseau (2002) show that high-q firms tend to buy low-q firms.

impute credit ratings for firms with no credit ratings data in Compustat and then assign the corporate bond returns for a given credit rating to all the firms with the same credit rating. This imputed measure only captures heterogeneity in the cost of debt across a few categories of credit ratings. The imputation also likely introduces estimation errors into the cost of debt measure. We instead compute the pre-tax cost of debt directly as the ratio of total interest and related expenses (Compustat annual item XINT) scaled by total debt, B_{it+1}. This simpler measure increases the sample coverage by 12.7% and also facilitates our goal of accounting for firm-level heterogeneity.[5]

3.3.2 Timing Alignment

We follow Liu and Zhang (2014) in aligning the timing of stock returns and accounting variables. In particular, the momentum and Roe deciles are rebalanced monthly, but accounting variables in Compustat are annual. Due to the large number of data items required to construct the firm-level fundamental return, we do not work with the Compustat quarterly files because of their limited coverage for many of these data items. We construct monthly fundamental returns from annual accounting variables to match with monthly stock returns. For each month, we take firm-level accounting variables from the fiscal year end that is closest to the month in question to measure (flow) variables dated t in the model and take accounting variables from the subsequent fiscal year end to measure (flow) variables dated $t + 1$ in the model. Because the portfolio composition can change monthly, the portfolio fundamental returns aggregated from the firm level also change monthly.

While portfolio stock returns are in monthly terms and in monthly frequency, portfolio fundamental returns are in monthly frequency but in annual terms, constructed from annual accounting variables. To align the units, Liu and Zhang (2014) annualize monthly portfolio stock returns in a given month to match with portfolio fundamental returns constructed for the month in question. This procedure creates potential timing mismatch. The crux is that the portfolio stock returns are for a given month, but the matching fundamental returns are constructed from annual accounting

[5]The Internet Appendix shows that our quantitative results are robust if we instead use the imputed cost of debt measure. The crux is that the identifying information in the structural estimation comes mostly from the cross section of the cost of equity. Relative to the cost of equity, the dispersion in the cost of debt is economically small.

variables both prior to and after the month. To better align the timing, we instead compound the portfolio stock returns within a 12-month rolling window with the month in question in the middle of the window. In particular, we multiply simple gross portfolio stock returns from month $t-5, t-4, \ldots, t, t+1, \ldots$, and $t+6$ to match with the fundamental returns constructed in month t.

3.3.3 Testing Portfolios

We use 40 testing deciles formed on book-to-market, momentum, asset growth, and return on equity, either separately or jointly, in the moment condition (6). Book-to-market and momentum are classic anomalies. We also include asset growth and return on equity, both of which feature prominently in a new generation of factor models (Hou, Xue, and Zhang 2015). Although we construct the fundamental returns at the firm level, our structural estimation still relies on the cross-sectional variation of average returns to identify the model parameters. To the extent that forming portfolios on value, momentum, asset growth, and return on equity yields economically large and statistically reliable average return spreads across the testing deciles, using these testing portfolios facilitates the identification of the structural parameters (Black, Jensen, and Scholes 1972).

Sorting on the relevant, separate components of the fundamental returns, such as investment and profitability, is exactly the idea behind the q-factor model, from which we include the asset growth and return on equity deciles. Sales-to-total capital, $Y_{it}/(K_{it} + W_{it})$, in the fundamental returns is economically related to return on equity. Both are measures of profitability. While return on equity accounts for operating costs, sales do not. Investment-to-physical capital, I_{it}/K_{it}, is economically related to asset growth, in which investment is measured as the change in total assets (including both short-term and long-term investments). Finally, market leverage, w_{it}^{B}, is closely related to book-to-market, and momentum to return on equity (Hou, Xue, and Zhang 2015).

To control for microcaps (stocks smaller than the 20 percentile of market equity of NYSE stocks), we form testing deciles with NYSE breakpoints and value-weighted returns (Hou, Xue, and Zhang 2018). In the Internet Appendix, we detail the results with testing deciles formed with all-but-micro breakpoints and equal-weighted returns. We first exclude microcaps from our sample, sort the remaining stocks into deciles, and calculate equal-weighted returns. Our quantitative results are robust with equal-weighted returns (and are in fact overall stronger).

To form the book-to-market (Bm) deciles, at the end of June of each year t, we sort stocks on Bm, which is the book equity for the fiscal year ending in calendar year $t-1$ divided by the market equity (from CRSP) at the end of December of $t-1$. For firms with more than one share class, we merge the market equity for all share classes before computing Bm. Monthly decile returns are calculated from July of year t to June of $t+1$, and the deciles are rebalanced in June of $t+1$.[6]

To form the momentum (R^{11}) deciles, we split all stocks at the beginning of each month t based on their prior 11-month returns from month $t-12$ to $t-2$. Skipping month $t-1$, we calculate monthly decile returns for month t, and rebalance the deciles at the beginning of month $t+1$ (Fama and French 1996). Liu and Zhang (2014) follow Jegadeesh and Titman (1993), sort on the prior 6-month return, skip one month, and hold the deciles for the subsequent 6-month period. To simplify the portfolio construction, we avoid the resulting six overlapping sets of momentum deciles with only 1-month holding period. In any event, the momentum profits from the R^{11} deciles are higher than those in Liu and Zhang, raising the hurdle for the structural model to explain.

To form the asset growth (I/A) deciles, at the end of June of each year t, we sort stocks on I/A, defined as total assets (Compustat annual item AT) for the fiscal year ending in calendar year $t-1$ divided by total assets for the fiscal year ending in $t-2$ (Cooper, Gulen, and Schill 2008). Monthly decile returns are from July of year t to June of $t+1$, and the deciles are rebalanced in June of $t+1$.

We measure return on equity (Roe) as income before extraordinary items (Compustat quarterly item IBQ) divided by 1-quarter-lagged book equity (Hou, Xue, and Zhang 2015).[7] At the beginning

[6]Book equity is stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (Compustat annual item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if it is available. If not, we measure stockholders' equity as the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock.

[7]From 1972 onward, quarterly book equity is shareholders' equity, plus balance sheet deferred taxes and invest-

of each month t, we sort all stocks into deciles based on their most recent past Roe. Before 1972, we use the most recent Roe computed with quarterly earnings from fiscal quarters ending at least four months ago. Starting from 1972, we use Roe computed with quarterly earnings from the most recent quarterly earnings announcement dates (Compustat quarterly item RDQ). For a firm to enter the portfolio formation, we require the end of the fiscal quarter that corresponds to its most recent Roe to be within six months prior to the portfolio formation, and its earnings announcement date to be after the corresponding fiscal quarter end. Monthly decile returns are calculated for the current month t, and the deciles are rebalanced at the beginning of month $t + 1$.

Table 1 shows the descriptive statistics of the monthly returns of the 40 testing deciles and the high-minus-low deciles from January 1967 to June 2017. In our structural estimation, we use the 12-month rolling procedure described in Section 3.3.2 to convert these monthly returns to the monthly observations of annual portfolio stock returns from June 1967 to December 2016 to match with the fundamental returns constructed over the same sample period.

From Panel A, the value premium (the average return of the high-minus-low Bm decile) is 0.47% per month ($t = 2.15$). Panel B shows that the momentum premium (the average return of the high-minus-low R^{11} decile) is much larger, 1.12% ($t = 3.88$). The investment premium (the average return of the high-minus-low I/A decile) is −0.36% ($t = -2.2$) (Panel C). Finally, Panel D shows that the Roe premium (the average return of the high-minus-low Roe decile) is 0.68% ($t = 3.01$).

3.3.4 Properties of the Accounting Variables

Table 2 reports descriptive statistics for firm-level accounting variables in the fundamental returns. The sample period for the fundamental returns is from June 1967 to December 2016 to align with the portfolio stock returns from the 12-month rolling procedure. However, it is important to note

ment tax credit (item TXDITCQ) if available, minus the book value of preferred stock (item PSTKQ). Depending on availability, we use stockholders' equity (item SEQQ), or common equity (item CEQQ) plus the book value of preferred stock, or total assets (item ATQ) minus total liabilities (item LTQ) in that order as shareholders' equity. Prior to 1972, we expand the sample coverage by using book equity from Compustat annual files and imputing quarterly book equity with clean surplus accounting (Hou, Xue, and Zhang 2018).

that the accounting variables underlying the fundamental returns for June 1967 can come from the fiscal year ending in calendar year as early as 1966, and the accounting variables underlying the fundamental returns for December 2016 can come from as late as 2018. In all, the guiding principle in our sample construction is to maximize the data coverage both across firms and over time.

To control for outliers, we winsorize the accounting variables except for market leverage at the 2.5–97.5% level at the portfolio formation. We do not winsorize the market leverage because it is bounded in $[0, 1]$. The mean physical investment-to-capital, I_{it}/K_{it}, is 0.36, with a large standard deviation of 0.44. For comparison, the mean working capital investment rate, $\triangle W_{it}/W_{it}$, is 0.13, with a standard deviation of 0.32. Disinvestment in working capital is much more frequent than disinvestment in physical capital, as the 5 percentile of $\triangle W_{it}/W_{it}$ is -0.3 but -0.03 for I_{it}/K_{it}.

On average, physical capital accounts for only 38% of the sum of physical capital and working capital, and the 25 and 75 percentiles of this fraction are 18% and 55%, respectively. This evidence indicates the potential importance of accounting for capital heterogeneity in the data. The ratio of sales to the sum of the two capital goods, $Y_{it+1}/(K_{it+1} + W_{it+1})$, is on average 1.62, which is close to the median of 1.5, and its standard deviation is only 0.93. In contrast, sales-to-physical capital, Y_{it+1}/K_{it+1}, has a mean of 9.05, a median of 5.24, and a standard deviation of 11.59. As such, Y_{it+1}/K_{it+1} is much more volatile and skewed than $Y_{it+1}/(K_{it+1}+W_{it+1})$. The evidence again indicates the extreme importance of accounting for capital heterogeneity. The key is that $Y_{it+1}/(K_{it+1}+W_{it+1})$ is a more appropriate measure of the average product of capital than Y_{it+1}/K_{it+1} in the two-capital model and in the data. The rate of physical capital depreciation is on average 19%, with a standard deviation of 12%. The market leverage, w_{it}^{B}, is on average 0.26, with a standard deviation of 0.22. For the pre-tax cost of debt, the mean is 8.74%, and the standard deviation 5.77%.

Table 2 also reports pairwise correlations of the accounting variables. The investment rate in physical capital, I_{it}/K_{it}, and the investment rate in working capital, $\triangle W_{it}/W_{it}$, have a positive correlation of 0.30. I_{it}/K_{it} has an autocorrelation of 0.32. In contrast, $\triangle W_{it}/W_{it}$ has an autocorrelation of only 0.04, which accords well with our assumption of zero adjustment costs on working capital. I_{it+1}/K_{it+1} has positive correlations of 0.36 and 0.2 with sales-to-physical capital, Y_{it+1}/K_{it+1}, and sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, respectively, but a zero correlation with Y_{it+1}/W_{it+1}. Similarly, $\triangle W_{it+1}/W_{it+1}$ have positive correlations of 0.25 and 0.2 with Y_{it+1}/W_{it+1} and $Y_{it+1}/(K_{it+1} + W_{it+1})$, respectively, but a small correlation of 0.09 with Y_{it+1}/K_{it+1}. The fraction of physical capital in total capital, $K_{it+1}/(K_{it+1} + W_{it+1})$, has negative correlations of -0.28, -0.6, and -0.33 with I_{it+1}/K_{it+1}, Y_{it+1}/K_{it+1}, and $Y_{it+1}/(K_{it+1} + W_{it+1})$, respectively, but a positive correlation of 0.46 with Y_{it+1}/W_{it+1}.

Figure 1 reports the histograms of the accounting variables both at the firm level and the portfolio level. Aggregating firm-level variables to the portfolio level eliminates a great deal of heterogeneity. Firm-level investment-to-physical capital, I_{it}/K_{it}, varies from -0.5 to 2.5, but the portfolio-level I_{it}/K_{it} lies between -0.5 and one, while centering about 0.25. Firm-level sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, varies from zero to 4.5, whereas the portfolio-level variable from 0.4 to 2.5. The firm-level Y_{it+1}/K_{it+1} distribution is much more dispersed, ranging from zero to 50, whereas the portfolio-level Y_{it+1}/K_{it+1} ranges from zero to seven. The firm-level pre-tax cost of debt, r^B_{it+1}, varies from zero to 0.4, whereas the portfolio-level r^B_{it+1} mostly from zero to 0.12. The firm-level distribution of r^B_{it+1} has a spike at zero because we treat zero-debt firms as having zero cost of debt.

4 Estimation Results

We first replicate the key findings in the prior studies that estimate the physical capital model at the portfolio level in Section 4.1. In Section 4.2, we report the results from the benchmark two-capital model estimated at the firm level. In Section 4.3, we quantify the impact of aggregation and capital heterogeneity by estimating the two-capital model at the portfolio level and the physical capital model at the firm level, respectively. In Section 4.4, we use the fundamental returns implied from the benchmark two-capital model estimated at the firm level to examine the dynamics of factor premiums. Finally, in Section 4.5, we examine the out-of-sample performance of the benchmark model.

4.1 Replicating the Prior Studies

Panel A of Table 3 reports the GMM estimation and tests for the physical capital model estimated directly at the portfolio level, without first constructing firm-level fundamental returns. Consistent with the prior studies, the physical capital model does a good job in accounting for value and momentum separately but fails to do so jointly. The failure in the joint estimation is reflected in the parameter instability across the testing deciles when estimated separately. The marginal product parameter, γ_K, is 0.166 with the book-to-market deciles, but 0.12 with the momentum deciles. For the adjustment costs parameter, a, the contrast is between 6.27 and 1.28. The average absolute high-minus-low alpha in the joint value and momentum estimation is 7.02% per annum, which is substantially larger than 0.32% and 1.46% in the separate estimation.[8]

Figure 2 reports the alphas of individual deciles by plotting average predicted stock returns against average realized stock returns across the value and momentum deciles as well as across all the 40 testing deciles in the joint estimation. The physical capital model manages to explain the momentum premium but fails entirely for the value premium. Panel A shows that with value and momentum jointly, the model predicts a *negative* value premium of −2.46% per annum, in contrast to 6.39% in the data. The high-minus-low alpha is economically large, 8.85%, and statistically significant ($t = 2.76$). The model also predicts a momentum premium of 20.17% and overshoots the data moment of 14.97%, giving rise to a high-minus-low alpha of −5.2% ($t = -2.63$).[9]

From Panel B, adding the asset growth and Roe deciles exacerbates the model's failure in explaining the value premium in the joint estimation. With all 40 testing deciles together, the model predicts a value premium of −4.72% per annum, giving rise to a large alpha of 11.11% ($t = 3.89$).

[8]The prior studies use equal-weighted decile returns. The Internet Appendix (Table A4) shows that the joint estimation failure is more severe with equal-weighted deciles. The marginal product parameter, γ_K, is estimated to be 0.251 and the adjustment costs parameter, a, 15.03 with the book-to-market deciles, but 0.128 and 1.34, respectively, with the momentum deciles. In the joint estimation, the γ_K estimate is 0.142, and a 3.19. As a result, the average absolute high-minus-low alpha in the joint estimation is 12.49% per annum, which is substantially higher than 3.25% and 0.12% in the separate estimation of value and momentum, respectively.

[9]The failure in fitting the equal-weighted deciles is more severe. The Internet Appendix (Figure A1) shows that the model predicts a large, negative value premium of −7.52% per annum, in contrast to an observed value premium of 8.89%. The high-minus-low alpha is massive, 16.41% ($t = 5.05$). The model implied momentum premium is 24.8%, relative to the data moment of 16.24%, giving rise to a high-minus-low alpha of −8.57% ($t = -4.28$).

The model does well in predicting a momentum premium of 16.17%, with a small alpha of −1.2% ($t = -0.48$), and an investment premium of −6.88%, with an alpha of 1.57% ($t = 0.79$). Finally, the model predicts an Roe premium of 11.02%, with an alpha of −2.59% ($t = -1.05$).

4.2 The Benchmark Specification

From Panel B of Table 3, our benchmark two-capital model estimated at the firm level succeeds in explaining value and momentum simultaneously. A first indication is that the parameter estimates are relatively stable across the testing deciles. In particular, the marginal product parameter, γ, is 0.176 with the book-to-market deciles and 0.134 with the momentum deciles. For the adjustment costs parameter, a, the contrast is between 3.75 and 8.11. With value and momentum jointly, the average absolute high-minus-low alpha is only 0.77% per annum, which is close to an order of magnitude smaller than 7.02% from the physical capital model estimated at the portfolio level. The mean absolute alpha is also smaller in the benchmark model than in the physical capital model, 1.27% versus 2.9%. However, the benchmark model is still rejected by the overidentification test. Finally, adding the asset growth and Roe deciles does not materially change the results.[10]

Figure 3 plots average predicted stock returns from the benchmark estimation against average realized stock returns across the testing deciles. The model performs well, and the scatter points are mostly aligned with the 45-degree line. In particular, Panel A shows that when fitting value and momentum deciles jointly, the model predicts a value premium of 5.2% per annum (6.39% in the data), giving rise to a small alpha of 1.18% ($t = 0.51$). The model also predicts a momentum premium of 14.62% (14.97% in the data), with an even smaller alpha of 0.35% ($t = 0.12$).[11]

Panel B shows that the alphas from the benchmark model increase only slightly in magnitude

[10] The improvement relative to the physical capital model estimated at the portfolio level is more visible in the equal-weighted returns. The Internet Appendix (Table A4) shows that the marginal product parameter, γ, is 0.167 with the book-to-market deciles and 0.165 with the momentum deciles. For the adjustment costs parameter, a, the contrast is between 3.93 and 3.02. As a result, the average absolute high-minus-low alpha in the joint value and momentum estimation is only 1.23% per annum, which is an order of magnitude smaller than 12.49% in the physical capital model. The mean absolute alpha is also much smaller, 0.83% versus 4.06%.

[11] For equal-weighted deciles, the Internet Appendix (Figure A2) shows that the model predicts a large, positive value premium of 7.48% per annum (8.89% in the data), with a relatively small alpha of 1.41% ($t = 0.67$). The model implied momentum premium is 17.28% (16.24% in the data), giving rise to a small alpha of −1.04% ($t = -0.34$).

after adding the asset growth and Roe deciles. The scatter plots continue to align largely along the 45-degree line. The model predicts a value premium of 3.29% per annum, with an alpha of 3.09% ($t = 1.37$), and a momentum premium of 13.42%, with an alpha of 1.55% ($t = 0.5$). The investment premium is -5.05% in the model (-5.11% in the data), giving rise to a tiny alpha of -0.06% ($t = -0.04$). Finally, the Roe premium is 6.2% in the model (8.43% in the data), with an alpha of 2.23% ($t = 0.89$). In all, although the alpha for the value premium, 3.29%, is not small, the improvement of the benchmark model estimated at the firm level over the physical capital model estimated at the portfolio level (which yields an alpha of 11.11%) is substantial.

4.2.1 Intuition: Current Investment, Expected Investment, and Expected Returns

What are the economic mechanisms behind the value, momentum, investment, and Roe premiums in the benchmark model? To shed light on the underlying intuition, we conduct comparative statics on key variables, including the current investment-to-physical capital, I_{it}/K_{it}, the next period investment-to-physical capital, I_{it+1}/K_{it+1}, and the next period average product of total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$. Other variables also matter, but their impact is not nearly as important.

In the experiment on I_{it}/K_{it}, we set I_{it}/K_{it} to be its cross-sectional median at period t across all firms. We use the parameter estimates from the benchmark estimation with all the 40 deciles jointly to reconstruct the fundamental returns and recalculate the model's alphas as the average portfolio stock-minus-fundamental returns. If the resulting alphas are large relative to those from the benchmark estimation, we can infer that the I_{it}/K_{it} spread is quantitatively important to explain the average return spreads. The other comparative statics are designed analogously.

Table 4 shows that I_{it}/K_{it} and I_{it+1}/K_{it+1} are the two most important drivers of the expected stock return. I_{it}/K_{it} is more important than I_{it+1}/K_{it+1} for the value and investment premiums, but I_{it+1}/K_{it+1} is more important than I_{it}/K_{it} for the momentum and Roe premiums. Intuitively, current investment and expected investment are locked in a tug of war within the model. When current investment overpower expected investment, the model predicts the value and investment premiums. Otherwise, it predicts the momentum and Roe premiums. A somewhat surprising insight is that the four seemingly different factor premiums are all driven by closely related, if not identical, mechanisms. $Y_{it+1}/(K_{it+1}+W_{it+1})$ also plays a role, especially for the Roe premium.

From Panel A, I_{it}/K_{it} is essential for the value premium. Removing its cross-sectional variation gives rise to a large, positive alpha of 36.28% per annum for the value premium. Intuitively, firms that invest more are growth firms with high marginal q, which equals the marginal cost of physical investment, $q_{it} = 1 + (1-\tau)a(I_{it}/K_{it})$. Firms that invest less are value firms with low marginal q. In the data, the average cross-sectional correlation between I_{it}/K_{it} and book-to-market is -0.23. Because the marginal cost of investment is in the denominator of equation (2), growth firms with high I_{it}/K_{it} have lower fundamental returns than value firms with low I_{it}/K_{it}. Fixing I_{it}/K_{it}, we hold the denominator of equation (2) constant, shutting down this mechanism to yield a large alpha.

The next period investment-to-physical capital, I_{it+1}/K_{it+1}, is the countervailing force of current investment, I_{it}/K_{it}. Fixing I_{it+1}/K_{it+1} across firms yields a large, negative alpha of -27.79% per annum for the value premium. Intuitively, growth firms also invest more and have higher marginal q next period than value firms. In the data, the average cross-sectional correlation between I_{it+1}/K_{it+1} and book-to-market is -0.19. Because I_{it+1}/K_{it+1} appears in the numerator of equation (2), the I_{it+1}/K_{it+1} spread implies that growth firms should have higher expected returns than value firms, countervailing I_{it}/K_{it} in the denominator. On net, current investment dominates expected investment across the book-to-market deciles, allowing the model to yield a positive value premium.

Panel B shows that the next period investment-to-physical capital, I_{it+1}/K_{it+1}, is the most important driver of momentum, and the current I_{it}/K_{it} is the countervailing force. Fixing I_{it+1}/K_{it+1} across firms yields a large, positive alpha of 20.71% per annum for the momentum premium. Intuitively, winners are expected to invest more than losers. In the data the average cross-sectional correlation between I_{it+1}/K_{it+1} and prior 11-month returns, R^{11}, is 0.19. This expected investment mechanism implies that winners should have higher expected returns than losers. The current

I_{it}/K_{it} is the offsetting force, but weaker. Fixing its cross-sectional variation yields a negative alpha of -7.65%, but its magnitude is substantially smaller than 20.71% from fixing I_{it+1}/K_{it+1}. In the data the average cross-sectional correlation between I_{it}/K_{it} and R^{11} is lower, 0.09. On net, expected investment dominates current investment, allowing the model to explain momentum.

Not surprisingly, Panel C shows that current investment, I_{it}/K_{it}, is the most important driver for the investment premium. Fixing I_{it}/K_{it} across firms yields an alpha of -21.75% per annum for the investment premium. In the data the average cross-sectional correlation between I_{it}/K_{it} and asset growth is 0.18. The next period I_{it+1}/K_{it+1} is the countervailing force. Fixing its cross-sectional variation yield an alpha of 13.36%, but its magnitude is smaller than -21.75% from fixing I_{it}/K_{it}. The average cross-sectional correlation between asset growth and I_{it+1}/K_{it+1} is 0.09. The economic mechanism for the investment premium is similar to that for the value premium.

Finally, from Panel D, the next period investment-to-physical capital, I_{it+1}/K_{it+1}, is the most important driver of the Roe premium. Fixing I_{it+1}/K_{it+1} across firms yields an alpha of 14.83% per annum for the Roe premium. Sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, reinforces I_{it+1}/K_{it+1} in the numerator of equation (2). Removing its dispersion yields an alpha of 8.72%. The current I_{it}/K_{it} is the countervailing force. Fixing its cross-sectional variation yields an alpha of -6.34%. On net, the numerator effect from I_{it+1}/K_{it+1} and $Y_{it+1}/(K_{it+1} + W_{it+1})$ dominates the denominator effect from I_{it}/K_{it}, allowing the model to yield a positive Roe premium.

4.3 Alternative Econometric Specifications

To understand the sources of the improvement of the benchmark model relative to prior studies, we quantify the impact of aggregation and capital heterogeneity in this subsection.

4.3.1 Aggregation

Panel A of Table 5 estimates the two-capital model at the portfolio level. In particular, instead of constructing firm-level fundamental returns, we aggregate firm-level accounting variables to the portfolio level and then construct fundamental returns directly at the portfolio level. The portfolio-level estimation yields larger alphas. For example, with value and momentum jointly, the mean absolute alpha is 1.52% per annum, and the average absolute high-minus-low alpha 2.26%. Both are larger than 1.27% and 0.77%, respectively, from the firm-level estimation (Panel B of Table 3).

Figure 4 shows the scatter plots of average predicted stock returns from the portfolio-level estimation of the two-capital model versus average realized stock returns. The model struggles to fit the value premium in the joint estimation. With value and momentum jointly (Panel A), the value premium is only 2.88% in the model, with an alpha of 3.51%, albeit insignificant ($t = 1.23$). With asset growth and Roe added to the joint estimation (Panel B), the value premium drops further to 1.45% in the model, with an alpha of 4.94% ($t = 1.93$). Intuitively, the amount of heterogeneity in the accounting variables is substantial at the firm level (Figure 1). This heterogeneity is dampened greatly once the variables are aggregated to the portfolio level. As such, estimating the two-capital model at the firm level is more "structural" (and more accurate) than at the portfolio level.

4.3.2 Capital Heterogeneity

To quantify the impact of introducing working capital as a separate input in the benchmark two-capital model, we estimate the physical capital model at the firm level. Panel B of Table 5 shows that without working capital, the physical capital model with the new aggregation yields a mean absolute alpha of 2.43% per annum and an average absolute high-minus-low alpha of 7.02%. These alphas are much larger than 1.27% and 0.77%, respectively, from the benchmark two-capital model.

The γ_K estimates in Panel B of Table 5 are lower than those from the portfolio-level estimation (Panel A of Table 3). The crux is that the firm-level distribution of sales-to-capital, Y_{it+1}/K_{it+1}, is highly skewed, but the portfolio-level Y_{it+1}/K_{it+1} distribution is substantially less dispersed (Figure 1). The lower γ_K estimates reflect the different Y_{it+1}/K_{it+1} distribution at the firm level. The γ_K estimates are also lower than the γ estimates in the two-capital model at the firm level. The crux is that sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, is much less dispersed and skewed than sales-to-physical capital, Y_{it+1}/K_{it+1}. As noted, physical-to-total capital, $K_{it+1}/(K_{it+1} + W_{it+1})$, is on average only 0.38 (Table 2). As such, incorporating working capital better characterizes the firm-level distribution of the average product of capital and that of the fundamental return.

Figure 5 shows the scatter plots of average predicted stock returns from the firm-level estimation of the physical capital model versus average realized stock returns. The model struggles to explain the average returns across the testing deciles. With value and momentum jointly (Panel A), the value premium is 1.64% per annum in the model, with an alpha of 4.75% ($t = 1.8$). The model also exaggerates the momentum premium to 20.17%, yielding a large alpha of −9.29% ($t = -2.79$). With asset growth and Roe added to the joint estimation (Panel B), the value premium deteriorates further to −2.14% per annum in the model, giving rise to a large alpha of 8.52% ($t = 3.41$). The momentum premium becomes 21.36%, with an alpha of −6.39% ($t = -1.89$).

4.4 Diagnostics: The Dynamics of Factor Premiums

In this subsection we use the fundamental returns implied from the benchmark two-capital model estimated at the firm level to study the dynamics of factor premiums. Because the parameters are estimated from only matching the average returns across the testing portfolios, the dynamics are economically important in serving as separate diagnostics on the model's performance. We examine calendar- and event-time dynamics. Finally, to construct the fundamental returns, we always use the parameter estimates from the joint estimation of all the 40 value-weighted testing deciles.

4.4.1 Correlations between Stock and Fundamental Returns

Taken literally, equation (5) implies that the stock and fundamental returns are equal ex post. However, Liu, Whited, and Zhang (2009) document a correlation puzzle that the contemporaneous correlations between the stock and fundamental returns are weakly *negative*, but the correlations between the one-year-lagged stock returns and the fundamental returns are significantly positive.

Liu, Whited, and Zhang (2009) align the timing of annual stock returns from July of year t to

June of $t + 1$ with the fundamental returns constructed from the accounting variables at fiscal year end of t and $t+1$. We instead follow Liu and Zhang (2014) in constructing monthly fundamental returns from annual accounting data. As noted, for each month, we take accounting variables from the fiscal year end that is closest to the month to measure period-t variables in the model and take accounting variables from the subsequent fiscal year end to measure period-t+1 variables in the model. However, differing from Liu and Zhang, we match the fundamental returns for the month in equation (value-weighted to the portfolio level) with portfolio stock returns compounded across the 12-month rolling window with the month in question in the middle of the window. This rolling procedure better aligns the timing of stock and fundamental returns, helping resolve the correlation puzzle.

Table 6 shows that the contemporaneous correlations between stock and fundamental returns from the benchmark model are significantly positive. From Panel A, the time series average of cross-sectional correlations of the two types of returns is 0.11 across all firms and 0.19 across the 40 testing deciles. Both correlations are significant at the 1% level. At the firm level, the lead-lag correlations are all positive within the 12-month horizon but turn negative at longer horizons. At the portfolio level, the lead-lag correlations are all positive across the horizons within 60 months.

Panel B shows the time series correlation between the stock and fundamental returns for each testing decile. The correlations are positive and mostly significant for the extreme deciles and high-minus-low deciles. In particular, the correlations are 0.26 for the value premium and 0.42 for the investment premium. Both are significant at the 1% level. The correlations are 0.14 for the momentum premium and 0.16 for the Roe premium, but are only marginally significant. Finally, we emphasize that equation (5) predicts perfect stock-fundamental returns correlations across firms (and portfolios). The correlations in Table 6, while mostly positive, are far from perfect. Measurement and specification errors in the firm-level fundamental returns are likely to blame.

4.4.2 Market States and Factor Premiums

Cooper, Gutierrez, and Hameed (2004) show that momentum is large and positive following nonnegative prior 36-month market returns (Up markets) but negative following negative prior 36-month market returns (Down markets). Liu and Zhang (2014) show that the physical capital model estimated at the portfolio level fails to explain this evidence in that it predicts weakly *countercyclical* momentum profits. We reexamine this issue on momentum with our benchmark model and also extend the evidence to the value, investment, and Roe factor premiums.

Panel A of Table 7 shows that the value premium is stronger following Down than Up markets identified with prior 36-month market returns, 17.19% versus 4.47% per annum. The model succeeds in explaining the countercyclical variation, 17.43% versus 0.7%. From Panel B, the momentum premium is stronger following Up than Down markets. With the market states again identified with prior 36-month market returns, the momentum premium is 19.36% following Up markets but −9.49% following Down markets. The contrast is 14.08% versus 9.78% in the model.

Panel C shows that the investment premium is stronger following Down than Up markets. With prior 12-month market returns defining the market states, the investment premium is −10.94% per annum following Down markets but −3.35% following Up markets. In the model the contrast is only −5.54% versus −4.86%, albeit going in the right direction. Finally, from Panel D, the Roe premium is stronger following Up than Down markets. With prior 36-month market returns identifying the market states, the Roe premium is 11.13% following Up markets but −5.88% following Down markets. In the model the contrast is between 7.02% and 1.44%.

In all, improving on Liu and Zhang (2014), our benchmark model succeeds in explaining the procyclical dynamics of the momentum and Roe premiums as well as the countercyclical dynamics of the value and investment premiums. (However, the magnitude of the dynamics in the model is still weaker than that in the data.) Intuitively, the rolling procedure described in Section 3.3.2 allows us to better align the timing between stock and fundamental returns. In contrast, Liu and

Zhang aligns the timing of portfolio stock returns for a given month with the month's fundamental returns that are constructed with accounting variables from both before and after the month. This crude procedure creates a timing mismatch between stock and fundamental returns, messing up the cross correlations between the model's factor premiums and observed stock market returns.

4.4.3 Persistence of Factor Premiums

Fama and French (1995) show that the value premium subsists for three to five years after the portfolio formation, whereas Chan, Jegadeesh, and Lokonishok (1996) show that momentum profits are more short-lived, positive within the 12-month horizon but negative afterward. Liu and Zhang (2014) show that the physical capital model estimated at the portfolio level explains the short-lived dynamics of momentum. We show that the two-capital model estimated at the firm level retains this performance and also extend the evidence to the value, investment and Roe premiums.

Figure 6 reports the event-time dynamics of stock and fundamental returns of the high and low deciles during 36 months after the portfolio formation. From Panels A–D, in the data the value premium persists even after three years, whereas the momentum premium converges to zero after about ten months. The investment premium lasts about two years, and the Roe premium converges to zero within ten months. Panels E–H show that the benchmark model succeeds in explaining the short-lived nature of the momentum and Roe premiums as well as the long-lived nature of the value and investment premiums, as the fundamental returns mimic the stock returns in event-time dynamics.[12]

4.4.4 Higher Moments

Table 8 compares higher moments including volatility, skewness, and kurtosis of stock returns with those of fundamental returns. Several patterns emerge. First, the fundamental returns are less volatile, echoing Cochrane (1991) at the aggregate level. The stock return volatilities of the value,

[12]To shed light on the model's performance, the Internet Appendix (Figure A6) shows that the growth rate of marginal q_{it} for physical capital exhibits the same short- and long-term dynamics as the fundamental returns. The marginal q growth reflects the tug of war between current investment, I_{it}/K_{it}, and future investment, I_{it+1}/K_{it+1}. The evidence shows that when current investment dominates future investment, the impact is more long-lasting. However, when future investment overpowers current investment, the impact is more short-lived.

momentum, investment, and Roe premiums are 20%, 28%, 14%, and 20% per annum, in contrast to the fundamental return volatilities of 18%, 13%, 11%, and 14%, respectively. For individual deciles, the fundamental return volatilities are often less than one half of their stock return volatilities.

Second, the benchmark model largely fails to explain the negative skewness of momentum. Daniel and Moskowitz (2016) show that momentum tends to experience infrequent and persistent negative returns. Such crashes yield a negative skewness for the momentum premium. Panel B replicates their evidence. The momentum premium has a skewness of −1.78, albeit significant only at the 10% level. In contrast, the fundamental momentum premium shows a positive but small skewness of 0.3. Panel D extends the Daniel-Moskowitz evidence to the Roe premium. Its skewness is −0.84, which is significant at the 10% level, and the model predicts a skewness of −0.38. Finally, the model does better in explaining kurtosis. For the value premium, the kurtosis is 3.28 for stock returns and 4.03 for fundamental returns. The fundamental returns also match the kurtosis of the stock returns for the investment premium, 3.44 versus 3.18. However, the fundamental returns fall far short for momentum, 11.59 versus 5.29, but come close for the Roe premium, 5.75 versus 4.45.

Figure 7 plots the time series of stock and fundamental factor premiums. The fundamental returns track the stock returns well, reflecting the economically large and statistically significant correlations in Table 6. However, the fundamental returns clearly fall short in explaining the extreme movements in the momentum and Roe premiums. In particular, the momentum premium experiences a crash of −168% in August 2009, but its fundamental return falls no more than 50%. The Roe premium experiences a crash of −110% in August 1999, but its fundamental return is positive, 8.7%.

In all, unlike the first moment, the benchmark model's performance in explaining higher moments of returns leaves much to be desired. Intuitively, as noted in Section 3.1.1, zero-mean measurement and specification errors in the fundamental returns can be averaged out when matching the first moment. However, the errors do impact on higher moments.

4.5 Out-of-sample Performance

We examine the model's out-of-sample performance in two ways. First, we recursively estimate the model's parameters and evaluate the fit with 1-period-ahead alphas (Section 4.5.1). Second, we construct the cross-sectional forecasts of 1-period-ahead sales growth and investment-to-physical capital, combine the forecasts with the recursively estimated parameters to form expected returns estimates, and evaluate the forecasting power of the estimates for subsequent realized returns (Section 4.5.2). For comparison with the two-capital model estimated at the firm level, we also report the out-of-sample tests for the physical capital model estimated at the portfolio level and for the q-factor model, which is a reduced-form implementation of the fundamental return (5).

4.5.1 Recursive Estimation

The first estimation window in the recursive estimation is from June 1967 to July 1985. The starting point of the window, June 1967, is identical to that of the in-sample estimation in that the accounting variables underlying the fundamental returns for June 1967 can come from as early as 1966. However, crucially, differing from the in-sample estimation, the endpoint of the first recursive window, July 1985, implies that the latest accounting variables in the estimation must come from the fiscal year ending at least four months prior to the *beginning* of July 1985. We impose this timing restriction to ensure no look-ahead bias in the expected returns estimates. As such, the first recursive window contains roughly 20 years of data to ensure that the point estimates are relatively precise. Recursive estimation means that we expand the recursive window one month at a time until December 2017 to yield the time series of the structural parameters.[13]

In the joint estimation of the two-capital model with all 40 testing deciles, the marginal product parameter, γ, is stable around 0.185. The adjustment costs parameter, a, is also stable but with a higher volatility relative to its mean. Although the marginal product parameter, γ_K, is stable in the

[13] Because of the 4-month-lagged timing restriction, we can end the last recursive window at the beginning of December 2017. The sample end differs from December 2016 for the in-sample estimation, in which the accounting variables underlying the last period fundamental returns can come from as late as 2018. Again, when constructing our sample, we follow the guiding principle of maximizing the data coverage both across firms and over time.

physical capital model, the adjustment costs parameter shows large low-frequency movements. It is relatively low at the beginning and the end, but high during the recursive period from 1985 to 2017. Such movements are largely absent from the two-capital model (the Internet Appendix, Figure A11).

Armed with the recursive parameters, we calculate the 1-month-ahead fundamental returns with the next month's out-of-sample accounting variables and compare the fundamental returns to the 1-month-ahead stock returns. The differences between the 1-month-ahead stock and fundamentals are defined as the 1-month-ahead alphas. This procedure, which combines the recursive parameters with the next month's realized accounting variables (instead of their forecasts), follows Fama and French (1997). We tackle the forecasting problem in Section 4.5.2.

In particular, for the q-factor model, we use the 60-month rolling window (36 months minimum) to estimate the factor loadings, which are in turn combined with the next month's realized q-factor premiums to generate the 1-month-ahead predicted returns for the testing deciles. The predicted returns are in monthly terms for a given month. To ease comparison with the structural models, we use the 12-month rolling procedure described in Section 3.3.2 to convert the monthly to annual predicted returns, which are compared with the annual stock returns from the same rolling procedure.

Figure 8 reports the 1-period-ahead fits of the 40 testing deciles via recursive estimation. From Panel A, the scatter plots of average predicted against average realized stock returns for the two-capital model estimated at the firm level are largely aligned with the 45-degree line. The alpha of the value premium is 1.87% per annum ($t = 0.28$), and the alpha of the momentum premium -1.28% ($t = -0.19$). The t-values are smaller due to a shorter period for the 1-period-ahead evaluation. Perhaps surprisingly, the 1-period-ahead alphas are similar in magnitude as the in-sample alphas (Figure 3). The average absolute high-minus-low alpha and mean absolute alpha are 1.2% and 1.92%, which are comparable with 1.73% and 1.33% from the in-sample estimation (Table 3), respectively.

Panel B shows a poor 1-period-ahead fit for the physical capital model estimated at the portfolio level. The value premium is 4.31% per annum in the data but -10.77% in the model, yielding a massive alpha of 15.08% ($t = 2.31$). The model also overshoots the momentum premium, which is 11.82% in the data but 18.76% in the model, with an alpha of −6.94% ($t = -1.01$). The average absolute high-minus-low alpha is 7.72%, and the mean absolute alpha 3.41%. Both are larger than 4.12% and 2.96% from the in-sample fit of the physical capital model and 1.2% and 1.92% from the 1-period-ahead fit of the two-capital model estimated at the firm level.

Finally, Panel C shows a good 1-period-ahead fit for the q-factor model. The average absolute high-minus-low alpha is 1.14%, and the mean absolute alpha 1.41%. In particular, the alpha for the value premium is 2.18% ($t = 0.41$), and the alpha for the momentum premium 1.49% ($t = 0.21$).

4.5.2 Expected Returns Estimates

The fundamental return (5) provides a detailed, theoretical description of the 1-period-ahead expected stock return, $E_t[r^F_{it+1}]$. To construct $E_t[r^F_{it+1}]$, we need to form expectations for the stochastic variables in equation (5), including sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, investment-to-physical capital, I_{it+1}/K_{it+1}, as well as the after-tax cost of debt, r^{Ba}_{it+1}, the tax rate, τ_{t+1}, and the depreciation rate, δ_{it+1}. To reduce estimation errors, we set the expected r^{Ba}_{it+1}, τ_{t+1}, and δ_{it+1} values to their current values from the most recent fiscal year ending at least four months ago. Finally, due to the 1-period time-to-build, although dated $t + 1$, the two capital goods, K_{it+1} and W_{it+1} are known at the beginning of time t. As such, the crux is to forecast I_{it+1} and Y_{it+1}.

To form $E_t[I_{it+1}]$, we forecast I_{it+1}/K_{it+1} on lagged Tobin's Q_{it}, sales-to-total capital, $Y_{it}/(K_{it} + W_{it})$, and investment-to-physical capital, I_{it}/K_{it}.[14] To form $E_t[Y_{it+1}]$, we forecast annual sales growth, Y_{it+1}/Y_{it} on the quarter-to-quarter sales growth rates of prior four quarters.[15] We use pooled panel regressions and allow industry-specific intercepts across the Fama-French (1997) 48 industries. Excluding four financial industries (banks, insurance, real estate, and trading) leaves 44

[14] Hou et al. (2018a) use a similar specification to forecast investment-to-assets changes to construct their expected investment growth factor. In addition, to reduce estimation errors, we do not separately forecast $(I_{it+1}/K_{it+1})^2$ in the numerator of equation (2), which in turn appears in equation (5). Instead, we compute $E_t[I_{it+1}/K_{it+1}]^2$ as $(E_t[I_{it+1}/K_{it+1}])^2$. $(I_{it+1}/K_{it+1})^2$ is relatively small, meaning that the ignored Jensen's inequity term is even smaller.

[15] Fairfield, Ramnath, and Yohn (2009) use a similar specification to forecast sales growth.

industries. To accord with value-weighted returns, we use weighted least squares with the ratio of a firm's market equity divided by the market's median equity in the same month as the weights.[16]

At the beginning of each month t from July 1985 to December 2017, we use the prior 120-month rolling window to estimate the I_{it+1}/K_{it+1} and Y_{it+1}/Y_{it} panel forecasting regressions. The I_{it+1} and Y_{it+1} data are from the most recent fiscal year ending at least four months prior to month t, and the instruments in the forecasting regressions are further lagged accordingly. We then combine the regression coefficients with the latest known instruments (which are lagged by at least four months as of month t) to compute $E_t[I_{it+1}/K_{it+1}]$ and $E_t[Y_{it+1}/Y_{it}]$, from which we calculate $E_t[Y_{it+1}/(K_{it+1}+W_{it+1})]$. Finally, we plug all the expectations, data items, and recursive parameters as of month t into equation (5) to construct the 1-period-ahead expected stock return, $E_t[r^F_{it+1}]$.

With the $E_t[r^F_{it+1}]$ estimates in hand at the beginning of month t, we use their NYSE breakpoints to split NYSE, Amex, and NASDAQ stocks into deciles. We calculate the monthly decile returns for three different holding periods (1-, 6-, and 12-month), over the current month t, from month t to $t+5$, and from month t to $t+11$. The 6-month horizon means that for a given decile in each month, there exist six sub-deciles, each initiated in a different month in the prior 6-month period. We take the simple average of the sub-decile returns as the monthly return for the decile.

Panel A of Table 9 shows that $E_t[r^F_{it+1}]$ from the two-capital model estimated at the firm level forecasts subsequent returns reliably. At the 1-month horizon, the high-minus-low decile earns an average return of 0.44% per month ($t = 2.32$). The average return spread declines somewhat to 0.36% ($t = 2.1$) at the 6-month and further to 0.24% ($t = 1.44$) at the 12-month. This evidence is important. A voluminous literature shows that the expected return estimates from accounting-based valuation models often do not forecast the 1-period-ahead realized return (Easton and Monahan 2005). Intuitively, the accounting-based expected returns are estimates of the (constant) internal

[16] The Internet Appendix illustrates the pooled panel forecasting regressions in the full sample from June 1967 to December 2017. Lagged Tobin's Q_{it}, sales-to-total capital, $Y_{it}/(K_{it}+W_{it})$, and investment-to-physical capital, I_{it}/K_{it}, all forecast I_{it+1}/K_{it+1} with significantly positive slopes, with an R^2 of 47.44%. The quarter-to-quarter sales growth rates of prior four quarters all forecast annual sales growth with significantly positive slopes, with an R^2 of 19.88%.

rate of return, which should not forecast returns in the time series (Hou et al. 2018b). In contrast, $E_t[r^F_{it+1}]$ is the 1-period-ahead expected return, which can vary both over time and across firms.

Panel B shows that $E_t[r^F_{it+1}]$ from the physical capital model estimated at the portfolio level does not forecast subsequent returns. To construct $E_t[r^F_{it+1}]$, we start with the same firm-level $E_t[I_{it+1}]$ and $E_t[Y_{it+1}]$ estimates as in the benchmark specification and aggregate them to the portfolio level. We then plug in the expected values, portfolio-level accounting variables, and recursive parameters, all known at the beginning of month t, into equation (5) to construct $E_t[r^F_{it+1}]$ for all the 40 testing deciles. Finally, we assign the expected portfolio returns back to the firm level. Because a given stock can appear simultaneously in four different portfolios, we take the simple average. The high-minus-low $E_t[r^F_{it+1}]$ decile earns on average only 0.17% per month ($t = 0.8$) at the 1-month horizon, which further declines to 0.04% ($t = 0.28$) at the 12-month.

Finally, the expected return estimates from the q-factor model do not forecast returns (Panel C). At the beginning of each month t, we estimate the q-factor loadings for a given stock from the prior 60-month rolling window (36 months minimum) and then combine the loadings with the factor premiums averaged over the expanding window from January 1967 to month $t-1$ to calculate the stock's expected return. The high-minus-low decile earns insignificant average returns of only 0.14%, 0.13%, and 0.22% per month ($t = 0.61, 0.52$, and 0.88) at the 1-, 6-, and 12-month, respectively.

The weak out-of-sample performance is generic to all factor models. Fama and French (1997) show that the industry cost of equity estimates from their 3-factor model are very imprecise, and firm-level estimates are surely even less accurate. As such, the main application of factor models ought to describe the common cross-sectional variation of returns to facilitate risk management and portfolio optimization for investment managers (Bodie, Kane, and Marcus 2014, Chapter 8). In contrast, in the same spirit as the accounting-based implied costs of capital (in terms of inferring discount rates from accounting variables), while allowing time-varying and cross-sectionally varying expected returns, our structural model seems more promising for estimating expected returns.

5 Conclusion

Aggregation and heterogeneity are thorny challenges for empirical investment studies. This paper provides a detailed treatment of aggregation, and to a lesser extent, capital heterogeneity in the context of the investment CAPM. Instead of forming fundamental returns from portfolio-level accounting variables aggregated from the firm level, we construct firm-level fundamental returns from firm-level variables and then aggregate firm-level fundamental returns to the portfolio level to match with portfolio-level stock returns. We also introduce working capital as a separate productive input from physical capital. Both innovations make the empirical specification of the fundamental return more "structural," help stabilize parameter estimates, and better describe the cross-sectional returns distribution. Most important, our benchmark two-capital model estimated at the firm level largely succeeds in explaining value and momentum simultaneously.

References

Belo, Frederico, and Xiaoji Lin, 2012, The inventory growth spread, *Review of Financial Studies* 25, 278–313.

Belo, Frederico, Vito D. Gala, and Juliana Salomao, 2018, Decomposing firm value, working paper, University of Minnesota.

Berk, Jonathan, and Peter DeMarzo, 2017, *Corporate Finance* 4th edition, Pearson.

Black, Fischer, Michael C. Jensen, and Myron Scholes, 1972, The Capital Asset Pricing Model: Some empirical tests, in Michael C. Jensen (ed.), *Studies in the Theory of Capital Markets*, Praeger Publisher Inc, New York.

Blundell, Richard, Thomas M. Stoker, 2007, Models of aggregate economic relationships that account for heterogeneity, in *Handbook of Econometrics*, volume 6A, Chapter 68, 4609–4666, Elsevier B. V.

Bodie, Zvi, Alex Kane, and Alan J. Marcus, 2014, *Investments*, 10th ed., McGraw-Hill, New York.

Campbell, John Y., 2018, *Financial Decisions and Markets: A Course in Asset Pricing*, Princeton University Press.

Chan, Louis K. C., Narasimhan Jegadeesh, and Josef Lakonishok, 1996, Momentum strategies, *Journal of Finance* 51, 1681–1713.

Chirinko, Robert S., 1993, Multiple capital inputs, Q, and investment spending, *Journal of Economic Dynamics and Control* 17, 907–928.

Cochrane, John H., 1991, Production-based asset pricing and the link between stock returns and economic fluctuations, *Journal of Finance* 46, 209–237.

Cooper, Michael J., Huseyin Gulen, and Michael J. Schill, 2008, Asset growth and the cross-section of stock returns, *Journal of Finance* 63, 1609–1652.

Cooper, Michael J., Roberto C. Gutierrez Jr., and Allaudeen Hameed, 2004, Market states and momentum, *Journal of Finance* 59, 1345–1365.

Cooper, Ilan, and Richard Priestley, 2016, The expected returns and valuations of private and public firms, *Journal of Financial Economics* 120, 41–57.

Daniel, Kent, and Tobias J. Moskowitz, 2016, Momentum crashes, *Journal of Financial Economics* 122, 221–247.

Doyle, Joanne M., and Toni M. Whited, 2001, Fixed costs of adjustment, coordination, and industry investment, *Review of Economics and Statistics* 83, 628–637.

Easton, Peter D., and Steven J. Monahan, 2005, An evaluation of accounting-based measures of expected returns, *The Accounting Review* 80, 501–538.

Fairfield, Patricia A., Sundaresh Ramnath, and Teri Lombardi Yohn, 2009, Do industry-level analyses improve forecasts of financial performance? *Journal of Accounting Research* 47, 147–178.

Fama, Eugene F., and Kenneth R. French, 1996, Multifactor explanation of asset pricing anomalies, *Journal of Finance* 51, 55–84.

Fama, Eugene F. and Kenneth R. French, 1997, Industry costs of equity, *Journal of Financial Economics* 43, 153–193.

Fama, Eugene F., and James D. MacBeth, 1973, Risk return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Gebhardt, William R., Charles M. C. Lee, and Bhaskaram Swaminathan, 2001, Toward an implied cost of capital, *Journal of Accounting Research* 39, 135–176.

Gilchrist, Simon, and Charles P. Himmelberg, 1998, Investment: Fundamentals and finance, *NBER Macroeconomics Annual* 13, 223–262.

Hansen, Lars Peter, 1982, Large sample properties of generalized methods of moments estimators, *Econometrica* 40, 1029–1054.

Hayashi, Fumio, and Tohru Inoue, 1991, The relation between firm growth and Q with multiple capital goods: Theory and evidence from panel data on Japanese firms, *Econometrica* 59, 731–753.

Hou, Kewei, Chen Xue, and Lu Zhang, 2015, Digesting anomalies: An investment approach, *Review of Financial Studies* 28, 650–705.

Hou, Kewei, Chen Xue, and Lu Zhang, 2018, Replicating anomalies, working paper, The Ohio State University.

Hou, Kewei, Haitao Mo, Chen Xue, and Lu Zhang, 2018a, q^5, working paper, The Ohio State University.

Hou, Kewei, Haitao Mo, Chen Xue, and Lu Zhang, 2018b, Which factors? forthcoming, *Review of Finance.*

Jegadeesh, Narasimhan and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Jermann, Urban J., 2010, The equity premium implied by production, *Journal of Financial Economics* 98, 279–296.

Jones, Christopher S., and Selale Tuzel, 2013, Inventory investment and the cost of capital, *Journal of Financial Economics* 107, 557–579.

Jovanovic, Boyan, and Peter L. Rousseau, 2002, The q-theory of mergers, *American Economic Review* 92, 198–204.

Li, Jun, 2017, Explaining momentum and value simultaneously, forthcoming, *Management Science.*

Liu, Laura Xiaolei, Toni M. Whited, and Lu Zhang, 2009, Investment-based expected stock returns, *Journal of Political Economy* 117, 1105–1139.

Liu, Laura Xiaolei, and Lu Zhang, 2014, A neoclassical interpretation of momentum, *Journal of Monetary Economics* 67, 109–128.

Nickell, Stephen J. 1978, *The Investment Decisions of Firms*, Cambridge University Press.

Opler, Tim, Lee Pinkowitz, René Stulz, and Rohan Williamson, 1999, The determinants and implications of corporate cash holdings, *Journal of Financial Economics* 52, 3–46.

Peters, Ryan H., and Lucian A. Taylor, 2017, Intangible capital and the investment-q relation, *Journal of Financial Economics* 123, 251–272.

Petersen, Mitchell A., and Raghuram G. Rajan, 1997, Trade credit: Theories and evidence, *Review of Financial Studies* 10, 661–691.

Schaller, Huntley, 1990, A re-examination of the q theory of investment using U.S. firm data, *Journal of Applied Econometrics* 5, 309–325.

Tuzel, Selale, 2010, Corporate real estate holdings and the cross section of stock returns, *Review of Financial Studies* 23, 2268–2302.

Wildasin, David E., 1984, The q theory of investment with many capital goods, *American Economic Review* 74, 203–210.

Whited, Toni M., 1992, Debt, liquidity constraints, and corporate investment: Evidence from panel data, *Journal of Finance* 47, 1425–1460.

Wu, Jin (Ginger), Lu Zhang, and X. Frank Zhang, 2010, The q-theory approach to understanding the accrual anomaly, *Journal of Accounting Research* 48, 177–223.

Table 1 : Descriptive Properties of Testing Deciles, January 1967–June 2017

For each decile and the high-minus-low decile (H−L), we report its monthly average return in excess of the 1-month Treasury bill rate, $\overline{R}$, and its t-value adjusted for heteroscedasticity and autocorrelations, $t_{\overline{R}}$. Testing deciles are formed with NYSE breakpoints and value-weighted returns.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: The book-to-market (Bm) deciles										
$\overline{R}$	0.43	0.53	0.60	0.46	0.53	0.56	0.67	0.63	0.73	0.90	0.47
$t_{\overline{R}}$	1.85	2.74	3.16	2.26	2.89	3.19	3.65	3.40	4.07	3.93	2.15
	Panel B: The momentum (R^{11}) deciles										
$\overline{R}$	−0.03	0.40	0.47	0.48	0.45	0.48	0.46	0.63	0.68	1.08	1.12
$t_{\overline{R}}$	−0.10	1.53	2.16	2.47	2.43	2.54	2.63	3.25	3.25	3.98	3.88
	Panel C: The asset growth (I/A) deciles										
$\overline{R}$	0.69	0.68	0.63	0.52	0.53	0.56	0.59	0.48	0.58	0.33	−0.36
$t_{\overline{R}}$	2.98	3.42	3.84	3.19	3.09	3.13	3.24	2.49	2.42	1.27	−2.20
	Panel D: The return on equity (Roe) deciles										
$\overline{R}$	0.06	0.25	0.42	0.40	0.54	0.44	0.57	0.53	0.57	0.74	0.68
$t_{\overline{R}}$	0.18	1.03	2.03	2.20	2.98	2.24	3.14	2.90	2.97	3.42	3.01

Table 2 : Descriptive Statistics of Firm-level Accounting Variables in the Fundamental Returns, June 1967–December 2016

For all the components in the fundamental returns, we report the time series averages of cross-sectional statistics, including mean, standard deviation (σ), percentiles (5, 25, 50, 75, and 95), and pairwise correlations. The statistics are computed after the 2.5–97.5% winsorization at the portfolio formation for the variables except for the market leverage, w_{it}^B. I_{it}/K_{it} is time-t physical investment-to-capital, $\triangle W_{it}/W_{it}$ is the time-t ratio of working capital investment divided by working capital. Y_{it+1}/K_{it+1} is the sales-to-physical capital in time $t+1$. Y_{it+1}/W_{it+1} is the sales-to-working capital in time $t+1$. $K_{it+1}/(K_{it+1}+W_{it+1})$ is the fraction of physical capital in total capital. δ_{it+1} is the rate of physical capital depreciation. r_{it+1}^B is the pre-tax cost of debt in percent per annum. The sample for the fundamental returns is from June 1967 to December 2016. However, the accounting variables underlying the fundamental returns for June 1967 can come from the fiscal year ending in calendar year as early as 1966, and the accounting variables underlying the fundamental returns for December 2016 as late as 2018.

Panel A: Mean, standard deviation, and percentiles

	Mean	σ	5	25	50	75	95
I_{it}/K_{it}	0.36	0.44	−0.03	0.11	0.23	0.44	1.32
$\triangle W_{it}/W_{it}$	0.13	0.32	−0.30	−0.05	0.07	0.22	0.82
Y_{it+1}/K_{it+1}	9.05	11.59	0.45	2.38	5.24	10.17	35.52
Y_{it+1}/W_{it+1}	3.09	2.00	0.76	1.77	2.61	3.83	7.46
$Y_{it+1}/(K_{it+1}+W_{it+1})$	1.62	0.93	0.30	0.97	1.50	2.11	3.80
$K_{it+1}/(K_{it+1}+W_{it+1})$	0.38	0.25	0.07	0.18	0.32	0.55	0.88
w_{it}^B	0.26	0.22	0.00	0.07	0.22	0.42	0.68
δ_{it+1}	0.19	0.12	0.05	0.11	0.16	0.25	0.49
r_{it+1}^B	8.74	5.77	0.02	5.65	7.98	10.54	24.89

Panel B: Cross-sectional correlations

	$\frac{I_{it+1}}{K_{it+1}}$	$\frac{\triangle W_{it}}{W_{it}}$	$\frac{\triangle W_{it+1}}{W_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}}$	$\frac{Y_{it+1}}{W_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}+W_{it+1}}$	$\frac{K_{it+1}}{K_{it+1}+W_{it+1}}$	w_{it}^B	δ_{it+1}	r_{it+1}^B
I_{it}/K_{it}	0.32	0.30	0.10	0.15	−0.06	0.06	−0.18	−0.18	0.28	0.06
I_{it+1}/K_{it+1}		0.23	0.30	0.36	0.00	0.20	−0.28	−0.29	0.53	0.16
$\triangle W_{it}/W_{it}$			0.04	0.07	−0.04	0.01	−0.06	−0.08	0.05	0.03
$\triangle W_{it+1}/W_{it+1}$				0.09	0.25	0.20	0.08	−0.13	0.07	0.15
Y_{it+1}/K_{it+1}					0.07	0.56	−0.60	−0.18	0.52	0.03
Y_{it+1}/W_{it+1}						0.55	0.46	0.19	−0.19	0.09
$Y_{it+1}/(K_{it+1}+W_{it+1})$							−0.33	−0.08	0.24	0.13
$K_{it+1}/(K_{it+1}+W_{it+1})$								0.37	−0.59	0.00
w_{it}^B									−0.33	0.04
δ_{it+1}										0.06

Table 3 : GMM Estimation and Tests, the Physical Capital Model Estimated at the Portfolio Level and the Benchmark Two-capital Model Estimated at the Firm Level, June 1967–December 2016

This table uses the 40 testing deciles formed on book-to-market (Bm), prior 11-month returns (R^{11}), asset growth (I/A), and return on equity (Roe), separately and jointly (Bm and R^{11}, I/A and Roe, and all 40 deciles together). The testing deciles are formed with NYSE breakpoints and value-weighted returns. d.f. is the degrees of freedom in the GMM test of overidentification. γ_K is the technological parameter on the marginal product of physical capital as a fraction of sales-to-physical capital, Y_{it+1}/K_{it+1}. γ is the technological parameter on the marginal product of total capital as a fraction of sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$. a is the adjustment costs parameter of physical capital. $[\gamma]$, $[\gamma_K]$, and $[a]$ are the standard errors of the point estimates. $\overline{|\alpha|}$ is the mean absolute alpha across the testing portfolios, $\overline{|\alpha_{\text{H–L}}|}$ is the average absolute high-minus-low alpha, and p is the p-value of the overidentification test across a given set of testing portfolios. γ, γ_K, $[\gamma]$, $[\gamma_K]$, and p-values are in percent, and $\overline{|\alpha|}$ and $\overline{|\alpha_{\text{H–L}}|}$ are in percent per annum.

Panel A: The physical capital model estimated at the portfolio level

	d.f.	γ_K	$[\gamma_K]$	a	$[a]$	$\overline{\lvert\alpha\rvert}$	$\overline{\lvert\alpha_{\text{H–L}}\rvert}$	p
Bm	8	16.56	2.40	6.27	1.94	2.52	0.32	0.01
R^{11}	8	12.00	1.14	1.28	0.56	1.34	1.46	8.37
I/A	8	12.20	1.06	1.06	0.40	2.04	0.54	0.00
Roe	8	10.32	0.97	0.00	0.07	3.35	0.21	0.00
Bm-R^{11}	18	13.44	1.21	2.54	0.52	2.90	7.02	0.00
I/A-Roe	18	11.43	0.99	0.71	0.34	2.86	1.64	0.00
Bm-R^{11}-I/A-Roe	38	12.51	1.08	1.74	0.34	2.96	4.12	0.00

Panel B: The benchmark two-capital model estimated at the firm level

	d.f.	γ	$[\gamma]$	a	$[a]$	$\overline{\lvert\alpha\rvert}$	$\overline{\lvert\alpha_{\text{H–L}}\rvert}$	p
Bm	8	17.62	2.07	3.75	0.68	1.34	0.16	0.07
R^{11}	8	13.37	2.84	8.11	0.00	0.82	0.74	85.28
I/A	8	17.44	1.77	1.63	0.70	0.89	2.31	0.31
Roe	8	14.90	3.20	7.63	0.00	0.79	1.16	92.46
Bm-R^{11}	18	17.89	2.03	3.44	0.55	1.27	0.77	0.00
I/A-Roe	18	17.35	1.79	1.65	0.67	1.14	2.15	0.00
Bm-R^{11}-I/A-Roe	38	17.77	1.94	2.84	0.47	1.33	1.73	0.00

Table 4 : Comparative Statics, the Benchmark Two-capital Model Estimated at the Firm Level, June 1967–December 2016

This table reports the investment CAPM alphas from three comparative statics: $\overline{I_{it}/K_{it}}$, $\overline{I_{it+1}/K_{it+1}}$, and $\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$. In the experiment denoted $\overline{I_{it}/K_{it}}$, I_{it}/K_{it} is set to be its cross-sectional average at period t across all the firms. The parameters from the benchmark GMM estimation (with all 40 Bm, R^{11}, I/A, and Roe deciles together) are used to reconstruct the fundamental returns, with all the other characteristics unchanged. The other experiments are designed analogously. The alpha is the average difference between portfolio stock returns and reconstructed fundamental returns. The "Benchmark" rows report the benchmark model's alphas.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: The book-to-market (Bm) deciles										
Benchmark	−1.66	−1.16	−0.17	−1.32	−1.19	2.19	3.74	2.51	1.08	1.44	3.09
$\overline{I_{it}/K_{it}}$	−10.12	−5.82	−2.90	−0.79	1.67	6.91	11.86	14.86	18.02	26.16	36.28
$\overline{I_{it+1}/K_{it+1}}$	5.45	3.36	2.77	−0.86	−1.73	−1.93	−3.48	−7.89	−12.15	−22.34	−27.79
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−0.67	−0.39	0.39	−1.97	−3.33	−1.32	−1.78	−5.00	−6.69	−7.71	−7.04
	Panel B: The momentum (R^{11}) deciles										
Benchmark	0.05	2.22	1.26	0.61	−0.73	−0.63	−1.81	−0.44	−1.45	1.60	1.55
$\overline{I_{it}/K_{it}}$	1.60	2.83	2.62	2.49	1.78	2.09	0.49	0.79	−2.70	−6.06	−7.65
$\overline{I_{it+1}/K_{it+1}}$	−9.70	−1.87	−1.49	−1.57	−2.89	−2.66	−2.82	0.03	1.92	11.00	20.71
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−4.07	−0.46	−1.04	−1.75	−2.87	−2.70	−3.46	−1.65	−1.56	2.76	6.82
	Panel C: The asset growth (I/A) deciles										
Benchmark	−2.00	−1.85	−0.80	−0.34	0.11	0.54	1.43	−0.20	2.82	−2.07	−0.06
$\overline{I_{it}/K_{it}}$	5.76	6.62	7.17	7.30	5.57	3.37	1.50	−3.21	−4.76	−15.99	−21.75
$\overline{I_{it+1}/K_{it+1}}$	−6.02	−6.40	−5.28	−5.51	−4.03	−1.67	1.12	2.08	7.13	7.34	13.36
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−4.17	−3.59	−3.51	−3.87	−2.55	−1.27	0.06	−0.94	2.73	−1.77	2.40
	Panel D: The return on equity (Roe) deciles										
Benchmark	−3.54	0.13	2.16	0.68	2.61	−0.16	0.09	−1.97	−1.19	−1.31	2.23
$\overline{I_{it}/K_{it}}$	−0.55	4.50	8.52	6.99	7.31	1.88	0.80	−3.44	−4.81	−6.88	−6.34
$\overline{I_{it+1}/K_{it+1}}$	−9.38	−6.61	−5.22	−5.24	−2.17	−1.81	−0.13	0.13	3.07	5.45	14.83
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−8.16	−4.93	−3.66	−4.60	−1.59	−2.65	−1.04	−1.47	−0.48	0.56	8.72

Table 5 : GMM Estimation and Tests, the Two-capital Model Estimated at the Portfolio Level and the Physical Capital Model Estimated at the Firm Level, June 1967–December 2016

This table uses the 40 testing deciles formed on book-to-market (Bm), prior 11-month returns (R^{11}), asset growth (I/A), and return on equity (Roe), separately and jointly (Bm and R^{11}, I/A and Roe, and all 40 deciles together). The testing deciles are formed with NYSE breakpoints and value-weighted returns. d.f. is the degrees of freedom in the GMM test of overidentification. γ_K is the technological parameter on the marginal product of physical capital as a fraction of sales-to-physical capital, Y_{it+1}/K_{it+1}. γ is the technological parameter on the marginal product of total capital as a fraction of sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$. a is the adjustment costs parameter of physical capital. $[\gamma]$, $[\gamma_K]$, and $[a]$ are the standard errors of the point estimates. $\overline{|\alpha|}$ is the mean absolute alpha across the testing portfolios, $\overline{|\alpha_{\text{H–L}}|}$ is the average absolute high-minus-low alpha, and p is the p-value of the overidentification test across a given set of testing portfolios. γ, γ_K, $[\gamma]$, $[\gamma_K]$, and p-values are in percent, and $\overline{|\alpha|}$ and $\overline{|\alpha_{\text{H–L}}|}$ are in percent per annum.

Panel A: The two-capital model estimated at the portfolio level

	d.f.	γ	$[\gamma]$	a	$[a]$	$\overline{\lvert\alpha\rvert}$	$\overline{\lvert\alpha_{\text{H–L}}\rvert}$	p
Bm	8	22.60	2.73	5.47	2.06	1.60	0.79	0.04
R^{11}	8	19.41	2.19	2.69	1.03	1.00	2.98	10.07
I/A	8	18.71	1.79	1.42	0.64	1.06	2.19	0.03
Roe	8	16.34	1.97	0.26	1.13	1.69	4.86	0.01
Bm-R^{11}	18	20.57	2.00	3.39	0.85	1.52	2.26	0.00
I/A-Roe	18	17.92	1.77	1.22	0.52	1.49	3.28	0.00
Bm-R^{11}-I/A-Roe	38	19.36	1.85	2.43	0.56	1.62	3.01	0.00

Panel B: The physical capital model estimated at the firm level

	d.f.	γ_K	$[\gamma_K]$	a	$[a]$	$\overline{\lvert\alpha\rvert}$	$\overline{\lvert\alpha_{\text{H–L}}\rvert}$	p
Bm	8	6.86	0.94	3.41	0.42	1.89	0.30	0.09
R^{11}	8	7.17	0.64	0.72	0.47	1.37	0.65	3.89
I/A	8	7.26	0.65	1.38	0.36	2.72	0.20	0.00
Roe	8	5.04	1.26	5.66	0.00	1.21	4.53	97.60
Bm-R^{11}	18	7.44	0.80	2.67	0.35	2.43	7.02	0.00
I/A-Roe	18	7.39	0.66	1.35	0.35	2.59	1.07	0.00
Bm-R^{11}-I/A-Roe	38	7.53	0.72	1.88	0.24	2.60	4.52	0.00

Table 6 : Correlations between Stock Returns and Fundamental Returns, June 1967–December 2016

Panel A reports the firm-level and portfolio-level correlations between the stock returns of various leads and lags and fundamental returns, r^F_{it}. The column denoted r^S_{it} reports contemporaneous correlations, and the column r^S_{it-3} the correlations between three-month-lagged stock returns and fundamental returns. Other columns are defined analogously. Portfolio-level correlations are calculated with the 40 portfolios formed on book-to-market, prior 11-month returns, asset growth, and return on equity with NYSE breakpoints and value-weighted returns. The correlations are time series averages of cross-sectional correlations, and their p-values are calculated as the Fama-MacBeth p-values adjusted for autocorrelations of up to 12 lags. Panel B reports for each of the 40 deciles and the high-minus-low decile, the time series contemporaneous correlations between the stock and fundamental returns. The p-values are those of the slopes from regressing the stock returns on the contemporaneous fundamental returns, adjusted for autocorrelations of up to 12 lags. The correlations that are significant at the 1%, 5%, and 10% levels are denoted with three stars, two stars, and one star, respectively. The results are based on the parameter values from estimating the benchmark model on all the 40 value-weighted testing deciles jointly.

	Panel A: Correlations of the stock returns with the fundamental returns, r^F_{it}										
	r^S_{it-60}	r^S_{it-36}	r^S_{it-24}	r^S_{it-12}	r^S_{it-3}	r^S_{it}	r^S_{it+3}	r^S_{it+12}	r^S_{it+24}	r^S_{it+36}	r^S_{it+60}
Firms	$-0.02^{\star\star\star}$	$-0.03^{\star\star\star}$	$-0.03^{\star\star\star}$	$0.02^{\star\star\star}$	$0.10^{\star\star\star}$	$0.11^{\star\star\star}$	$0.12^{\star\star\star}$	$0.05^{\star\star\star}$	0.00	0.01	$-0.01^{\star}$
Portfolios	$0.05^{\star}$	$0.09^{\star\star\star}$	$0.05^{\star}$	$0.09^{\star\star\star}$	$0.17^{\star\star\star}$	$0.19^{\star\star\star}$	$0.20^{\star\star\star}$	$0.12^{\star\star\star}$	$0.08^{\star\star\star}$	$0.12^{\star\star\star}$	$0.11^{\star\star\star}$
	Panel B: Contemporaneous correlations between the stock and fundamental returns across the testing deciles										
	L	2	3	4	5	6	7	8	9	H	H−L
Bm	0.13	0.19	0.12	0.04	$0.13^{\star\star}$	$0.20^{\star}$	0.00	0.00	0.05	0.15	$0.26^{\star\star\star}$
R^{11}	$0.20^{\star\star}$	0.09	0.06	−0.05	−0.03	0.04	0.01	0.08	0.10	$0.22^{\star\star\star}$	$0.14^{\star}$
I/A	$0.19^{\star\star}$	0.11	0.10	−0.03	0.12	−0.02	0.02	−0.02	0.11	$0.30^{\star\star\star}$	$0.42^{\star\star\star}$
Roe	0.19	0.18	0.11	$0.14^{\star}$	−0.02	0.01	0.09	0.01	−0.02	0.09	0.16

Table 7 : Market States and Factor Premiums, June 1967–December 2016

For each month t, we categorize the market state as Up (Down) if the value-weighted market returns from month $t-N$ to $t-1$, with $N = 12, 24$, or 36, are nonnegative (negative). We report the high-minus-low decile returns averaged across Up (Down) states. r^S denotes the stock return, and r^F the fundamental returns. Both are in percent per annum. The t-values are adjusted for heteroscedasticity and autocorrelations of up to 12 lags. The results are based on the parameter values from estimating the benchmark model on the 40 value-weighted testing deciles jointly.

N	MKT	r^S	t_S	r^F	t_F	r^S	t_S	r^F	t_F
		Panel A: Book-to-market, Bm				Panel B: Momentum, R^{11}			
12	Down	11.81	4.17	3.31	0.62	−0.20	−0.02	15.75	6.30
12	Up	4.80	1.65	3.24	1.49	19.50	7.69	12.73	11.27
24	Down	13.60	2.62	14.09	2.56	−7.28	−0.62	13.11	3.64
24	Up	5.14	1.90	1.33	0.63	18.91	7.44	13.48	11.88
36	Down	17.19	3.32	17.43	2.88	−9.49	−0.99	9.78	4.71
36	Up	4.47	1.75	0.70	0.33	19.36	7.61	14.08	11.07
		Panel C: Asset growth, I/A				Panel D: Return on equity, Roe			
12	Down	−10.94	−4.28	−5.54	−2.24	2.06	0.46	4.42	1.78
12	Up	−3.35	−1.87	−4.86	−3.45	10.47	3.86	6.69	4.60
24	Down	−11.57	−5.11	−5.62	−1.94	−3.10	−0.51	3.21	1.03
24	Up	−3.95	−2.08	−4.91	−3.34	10.60	4.15	6.69	4.57
36	Down	−7.62	−3.05	−4.09	−1.42	−5.88	−1.23	1.44	0.63
36	Up	−4.64	−2.35	−5.19	−3.50	11.13	4.37	7.02	4.69

Table 8 : Higher Moments of the Stock and Fundamental Returns, June 1967–December 2016

For each decile, we report volatility, σ, skewness, S_k, and kurtosis, K_u, of its stock returns, r^S, and fundamental returns, r^F. For each high-minus-low decile, the volatility and skewness significantly different from zero and the kurtosis significantly different from three at the 1%, 5%, and 10% levels are denoted with three, two stars, and one star, respectively. The significance is based on 5,000 block bootstrapped samples (each with 60 months). The results are based on parameters from estimating the benchmark model on the 40 value-weighted deciles jointly.

		L	2	3	4	5	6	7	8	9	H	H−L
		Panel A: Book-to-market, Bm										
σ	r^S	0.20	0.18	0.18	0.19	0.17	0.16	0.17	0.17	0.17	0.22	0.20⋆⋆⋆
	r^F	0.05	0.06	0.06	0.07	0.08	0.10	0.07	0.11	0.13	0.18	0.18⋆⋆⋆
S_k	r^S	−0.24	0.03	−0.08	−0.04	−0.16	−0.07	−0.20	−0.48	−0.14	0.12	0.42
	r^F	−0.96	−1.26	1.05	0.57	0.81	−1.57	0.67	1.27	0.73	0.63	0.36
K_u	r^S	3.04	3.12	2.75	3.43	3.20	3.57	3.52	4.36	3.94	4.47	3.28
	r^F	3.97	6.24	8.33	5.36	4.81	8.13	2.95	6.62	4.29	4.64	4.03
		Panel B: Momentum, R^{11}										
σ	r^S	0.30	0.24	0.20	0.18	0.16	0.17	0.16	0.18	0.19	0.26	0.28⋆⋆⋆
	r^F	0.12	0.08	0.08	0.07	0.07	0.06	0.07	0.07	0.07	0.07	0.13⋆⋆⋆
S_k	r^S	1.47	0.94	0.19	0.42	−0.10	−0.14	−0.23	−0.16	−0.11	−0.03	−1.78⋆
	r^F	−0.56	−0.03	0.33	0.38	0.57	0.69	1.01	0.62	0.13	−0.41	0.30⋆
K_u	r^S	9.92	8.05	3.91	4.07	3.70	3.58	3.02	3.07	3.57	3.19	11.59⋆⋆⋆
	r^F	6.58	4.10	6.00	4.81	5.51	5.24	6.73	5.06	4.07	3.91	5.29⋆⋆
		Panel C: Asset growth, I/A										
σ	r^S	0.22	0.18	0.16	0.15	0.16	0.16	0.17	0.17	0.21	0.23	0.14⋆⋆⋆
	r^F	0.09	0.07	0.08	0.07	0.06	0.07	0.06	0.05	0.07	0.08	0.11⋆⋆⋆
S_k	r^S	0.36	−0.01	−0.01	−0.16	−0.25	−0.18	−0.20	−0.15	−0.30	−0.22	0.13
	r^F	0.22	0.88	0.41	1.00	0.40	0.03	−0.27	0.43	−0.29	−0.60	0.08
K_u	r^S	4.13	3.67	3.18	3.48	3.55	3.19	3.22	3.07	3.33	3.15	3.44
	r^F	2.71	4.60	2.95	5.17	3.01	3.43	4.48	4.15	3.59	5.03	3.18
		Panel D: Return on equity, Roe										
σ	r^S	0.27	0.22	0.19	0.16	0.17	0.18	0.16	0.17	0.17	0.20	0.20⋆⋆⋆
	r^F	0.14	0.12	0.09	0.08	0.08	0.07	0.07	0.05	0.05	0.05	0.14⋆⋆⋆
S_k	r^S	0.20	0.23	−0.03	−0.02	−0.25	−0.38	−0.39	−0.14	−0.20	−0.06	−0.84⋆
	r^F	0.46	0.38	0.58	0.38	0.50	1.31	0.07	−0.38	−0.15	−0.09	−0.38
K_u	r^S	3.69	3.94	4.13	3.36	3.12	3.66	3.14	2.90	3.35	2.70	5.75⋆⋆⋆
	r^F	4.99	5.45	6.73	4.53	4.85	6.56	4.19	3.88	2.98	3.08	4.45⋆⋆⋆

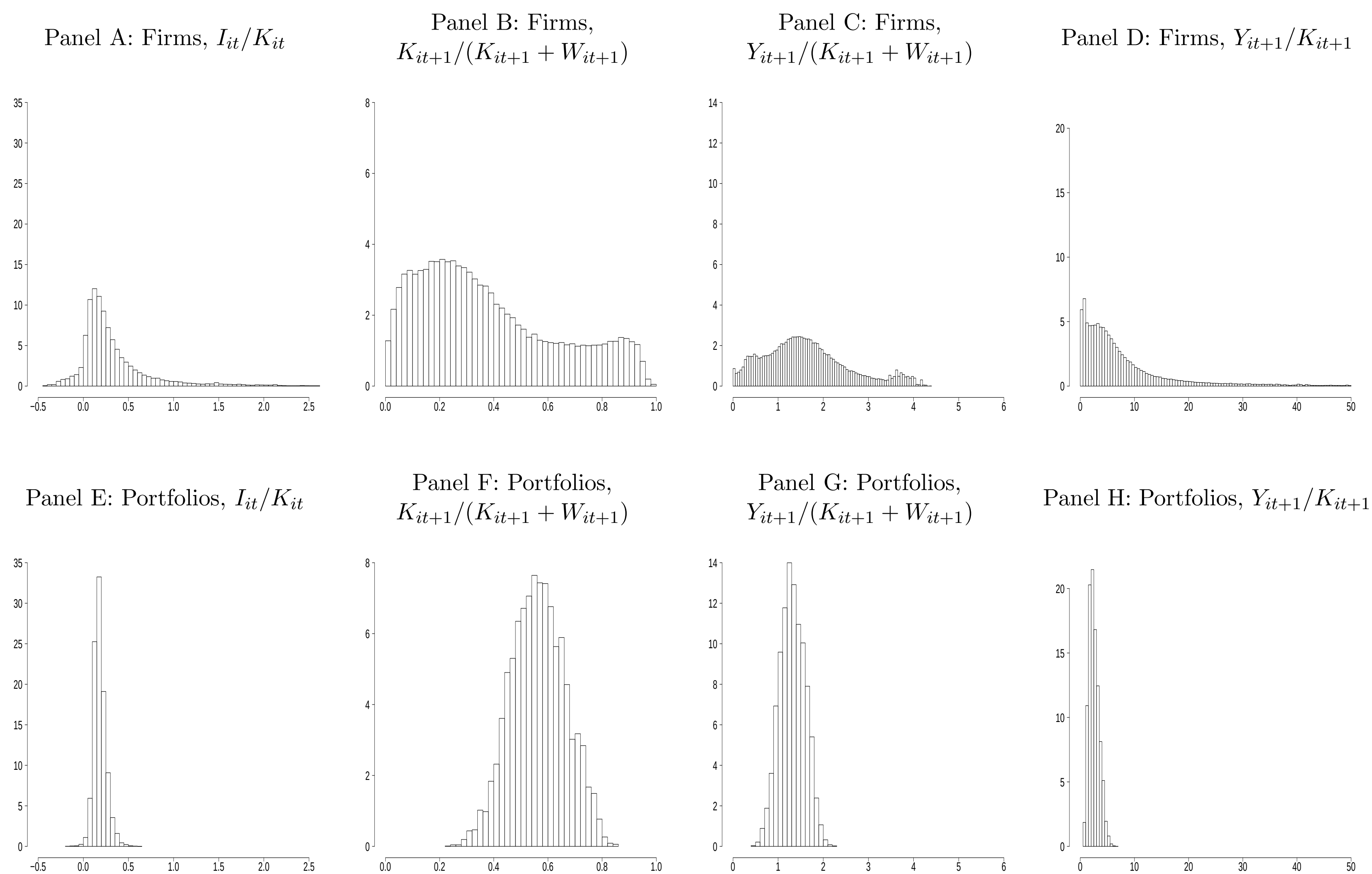
Table 9 : Deciles Formed on the Expected Return Estimates, July 1985–December 2017

This table reports the average excess return of a given expected return decile for the h-month holding period, in which $h = 1$, 6, and 12. The t-values adjusted for heteroscedasticity and autocorrelations are reported in the rows beneath the corresponding estimates. The deciles are formed on the expected return estimates with NYSE breakpoints and value-weighted returns.

h	L	2	3	4	5	6	7	8	9	H	H−L
Panel A: The two-capital model estimated at the firm level											
1	0.48	0.75	0.75	0.73	0.81	0.73	0.61	0.71	0.76	0.92	0.44
	1.50	3.03	2.79	3.11	3.67	3.11	2.79	3.28	3.72	3.78	2.32
6	0.48	0.80	0.81	0.73	0.79	0.71	0.64	0.73	0.71	0.84	0.36
	1.59	3.31	3.07	3.04	3.62	2.90	2.91	3.67	3.47	3.46	2.10
12	0.55	0.79	0.80	0.70	0.78	0.74	0.67	0.71	0.68	0.79	0.24
	1.90	3.36	3.08	2.94	3.43	3.11	3.05	3.62	3.32	3.18	1.44
Panel B: The physical capital model estimated at the portfolio level											
1	0.60	0.60	0.51	0.72	0.60	0.73	0.87	0.69	0.81	0.77	0.17
	2.00	2.36	1.84	3.00	2.21	3.02	3.80	2.72	3.37	3.71	0.80
6	0.64	0.61	0.64	0.60	0.68	0.75	0.81	0.73	0.76	0.77	0.13
	2.32	2.43	2.79	2.48	2.84	2.99	3.55	3.33	3.39	3.57	0.72
12	0.68	0.63	0.63	0.65	0.70	0.78	0.81	0.76	0.79	0.72	0.04
	2.65	2.58	2.80	2.70	2.89	3.27	3.53	3.52	3.70	3.29	0.28
Panel C: The q-factor model											
1	0.72	0.74	0.76	0.67	0.87	0.77	0.70	0.79	0.70	0.86	0.14
	2.30	3.00	3.42	2.99	4.61	4.09	3.17	3.68	3.05	2.61	0.61
6	0.70	0.84	0.78	0.73	0.83	0.75	0.70	0.70	0.69	0.83	0.13
	2.16	3.50	3.60	3.45	4.29	4.00	3.27	3.26	3.02	2.68	0.52
12	0.66	0.76	0.84	0.76	0.79	0.77	0.74	0.66	0.72	0.88	0.22
	2.06	3.38	4.00	3.61	4.14	3.92	3.61	3.15	3.17	2.82	0.88

Figure 1 : Histograms of Firm-level versus Portfolio-level Accounting Variables in the Fundamental Returns, June 1967–December 2016

The histograms are produced after the 2.5–97.5% winsorization at the firm level at each portfolio formation for all the accounting variables except for the market leverage, w_{it}^B, which is already bounded in $[0,1]$. I_{it}/K_{it} is physical investment-to-capital, $K_{it+1}/(K_{it+1}+W_{it+1})$ the fraction of physical capital in total capital, $Y_{it+1}/(K_{it+1}+W_{it+1})$ the ratio of sales over total capital, Y_{it+1}/K_{it+1} sales-to-physical capital, Y_{it+1}/W_{it+1} sales-to-working capital, δ_{it+1} the rate of physical capital depreciation, w_{it}^B the market leverage, and r_{it+1}^B the pre-tax cost of debt. Portfolio-level histograms are across the 40 testing deciles. While the sample for the fundamental returns is from January 1967 to December 2016, the accounting variables underlying the fundamental returns can come from the fiscal year ending in as early as 1966 and as late as 2018.



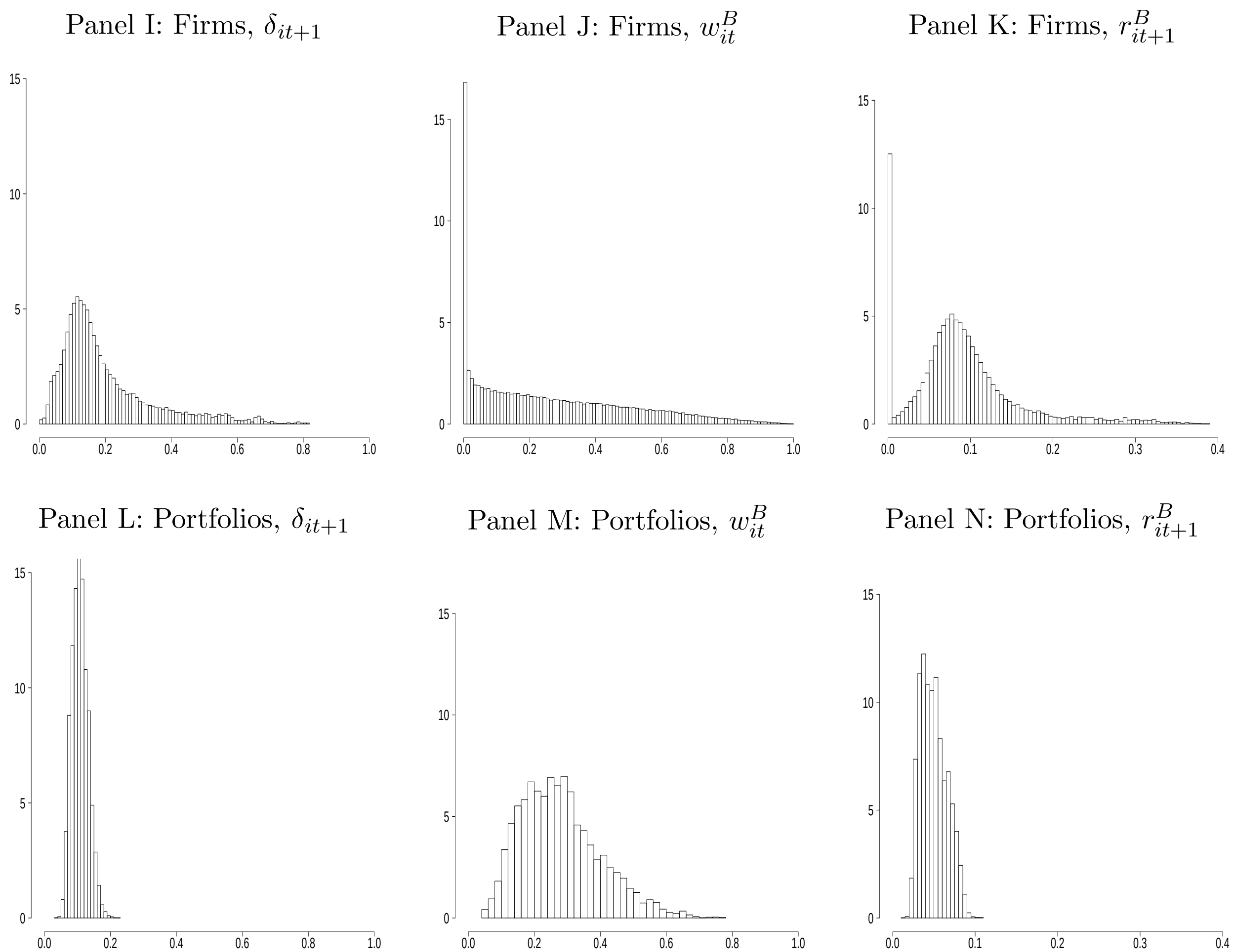
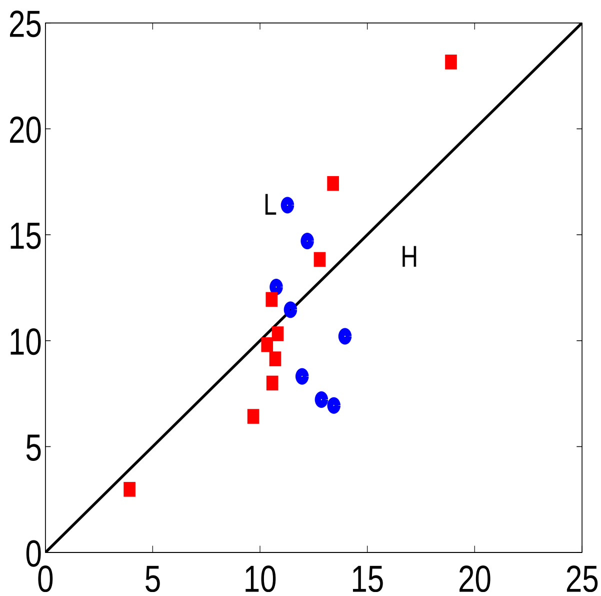

Panel I: Firms, δ_{it+1}
Panel J: Firms, w_{it}^{B}
Panel K: Firms, r_{it+1}^{B}
Panel L: Portfolios, δ_{it+1}
Panel M: Portfolios, w_{it}^{B}
Panel N: Portfolios, r_{it+1}^{B}
0
5
10
15
0.0
0.2
0.4
0.6
0.8
1.0
0.1
0.3

Figure 2 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Physical Capital Model Estimated at the Portfolio Level, June 1967–December 2016

Both average predicted and realized stock returns are in percent per annum. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H." Panel A fits the Bm and R^{11} deciles jointly, and Panel B fits all the 40 value-weighted deciles together.

Panel A: Bm-R^{11}



Panel B: Bm-R^{11}-I/A-Roe

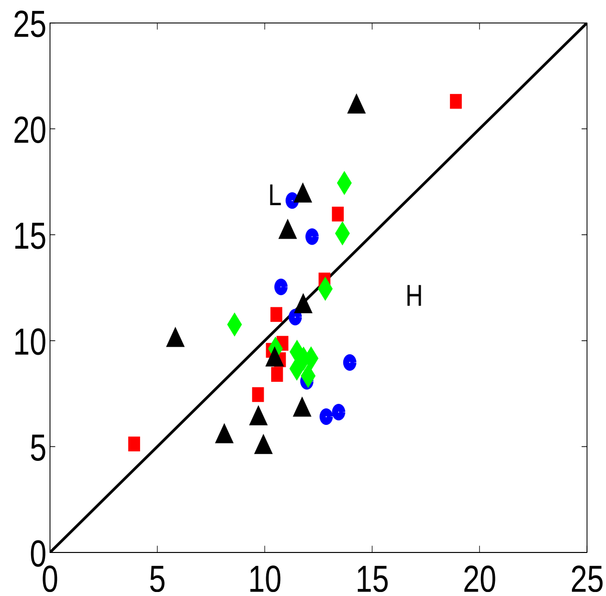


Figure 3 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Benchmark Two-capital Model Estimated at the Firm Level, June 1967–December 2016

Both average predicted and realized stock returns are in percent per annum. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H." Panel A fits the Bm and R^{11} deciles jointly, and Panel B fits all the 40 value-weighted deciles together.

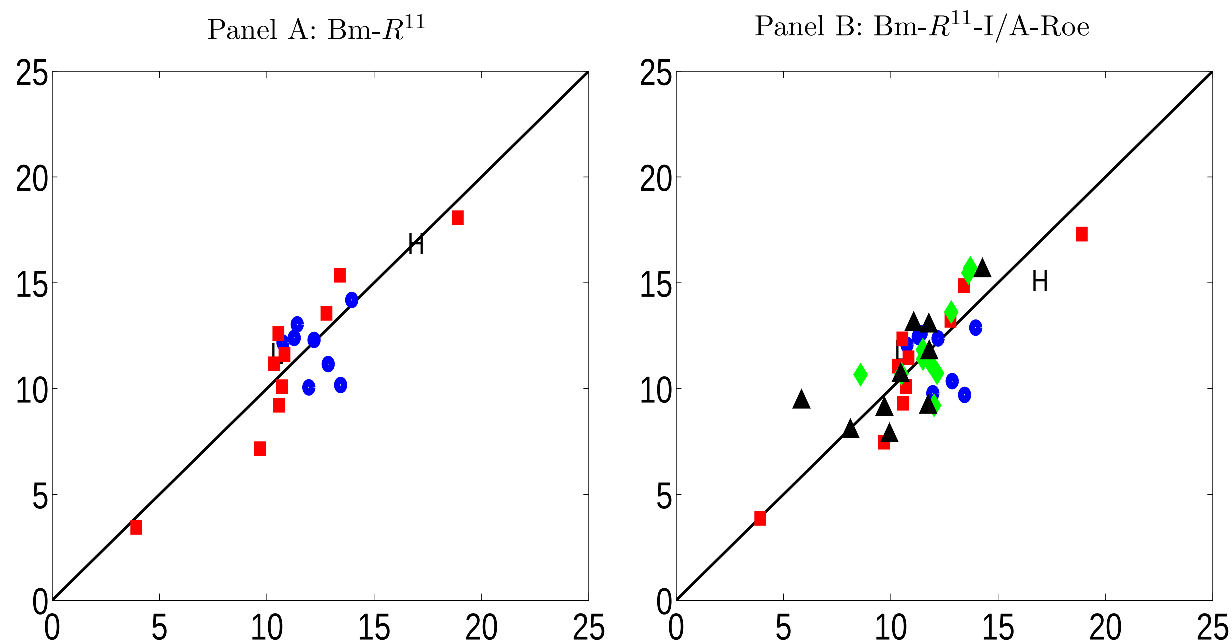


Figure 4 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Two-capital Model Estimated at the Portfolio Level, June 1967–December 2016

Both average predicted and realized stock returns are in percent per annum. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H." Panel A fits the Bm and R^{11} deciles jointly, and Panel B fits all the 40 value-weighted deciles together.

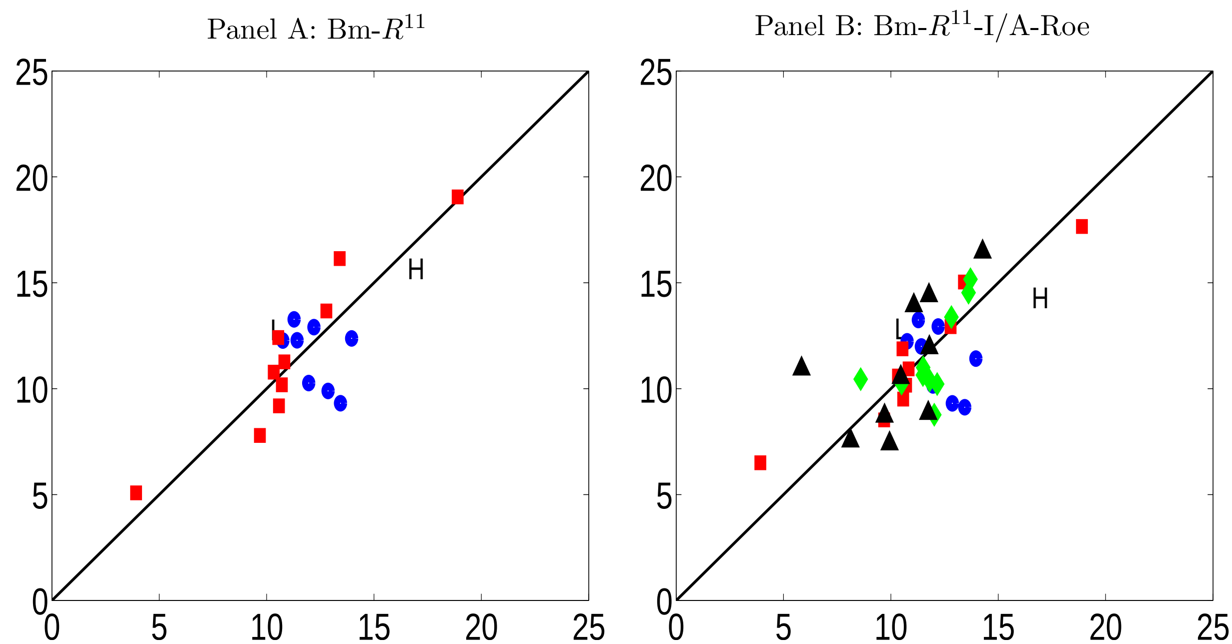
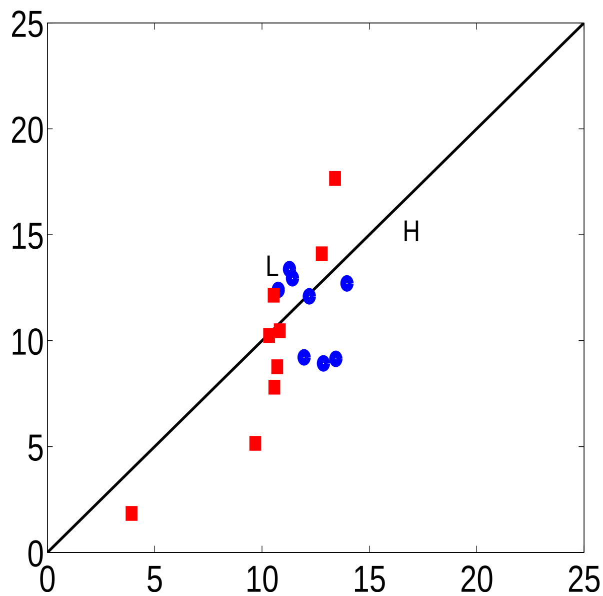


Figure 5 : Average Predicted Stock Returns versus Average Realized Stock Returns, The Physical Capital Model Estimated at the Firm Level, June 1967–December 2016

Both average predicted and realized stock returns are in percent per annum. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H." Panel A fits the Bm and R^{11} deciles jointly, and Panel B fits all the 40 value-weighted deciles together.

Panel A: Bm-R^{11}



Panel B: Bm-R^{11}-I/A-Roe

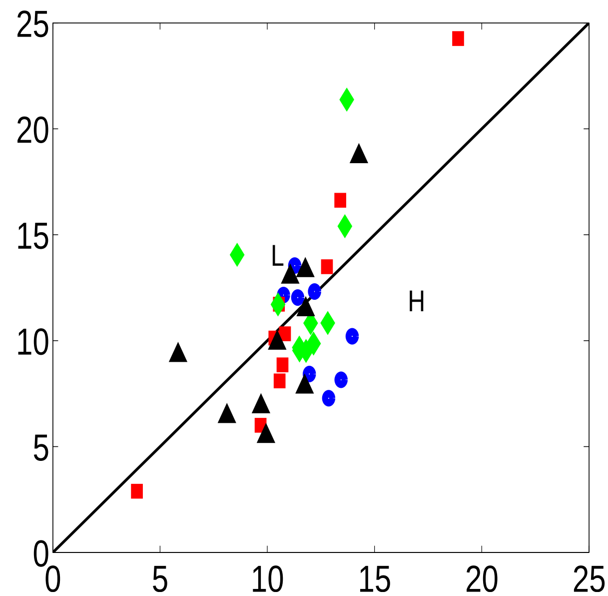


Figure 6 : Event-time Dynamics of the Stock and Fundamental Returns of the High and Low Deciles, June 1967–December 2016

For 36 months after the portfolio formation, we plot the stock returns, r^S_{it+1}, and the fundamental returns, r^F_{it+1}, for the high and low deciles formed on book-to-market, prior 11-month returns, asset growth, and return on equity. Both stock and fundamental returns are in percent per annum. The blue solid lines represent the low deciles, and the red broken lines the high deciles. The fundamental returns are based on the parameters from estimating the two-capital model at the firm level on the 40 value-weighted deciles jointly.

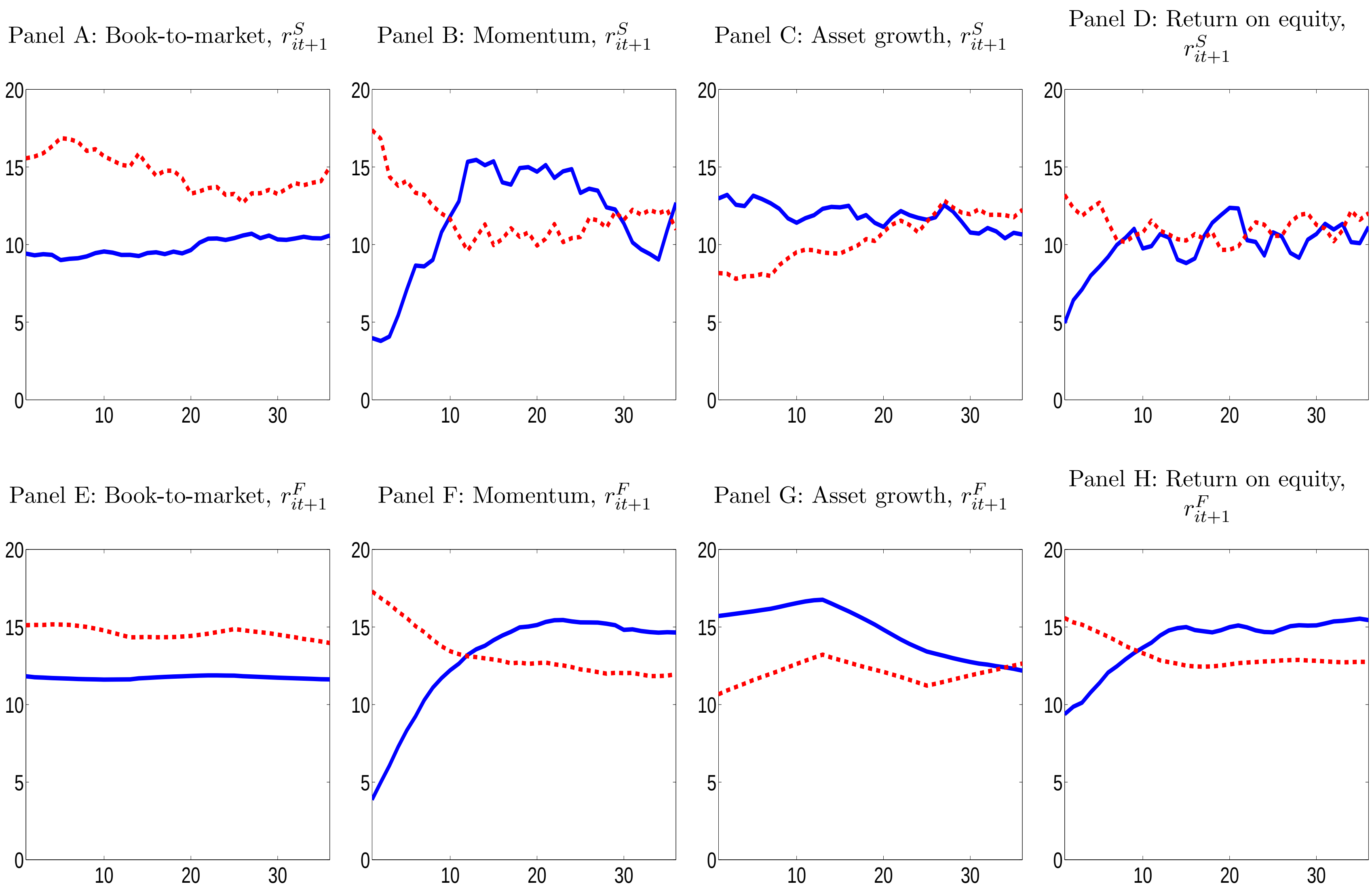


Figure 7 : Time Series of the Stock and Fundamental Returns of the Factor Premiums, June 1967–December 2016

The blue solid lines represent the value-weighted stock returns of the high-minus-low deciles, and the red broken lines the corresponding fundamental returns. Both returns are in percent per annum. Stock returns outliers are indicated with their values and the corresponding months.

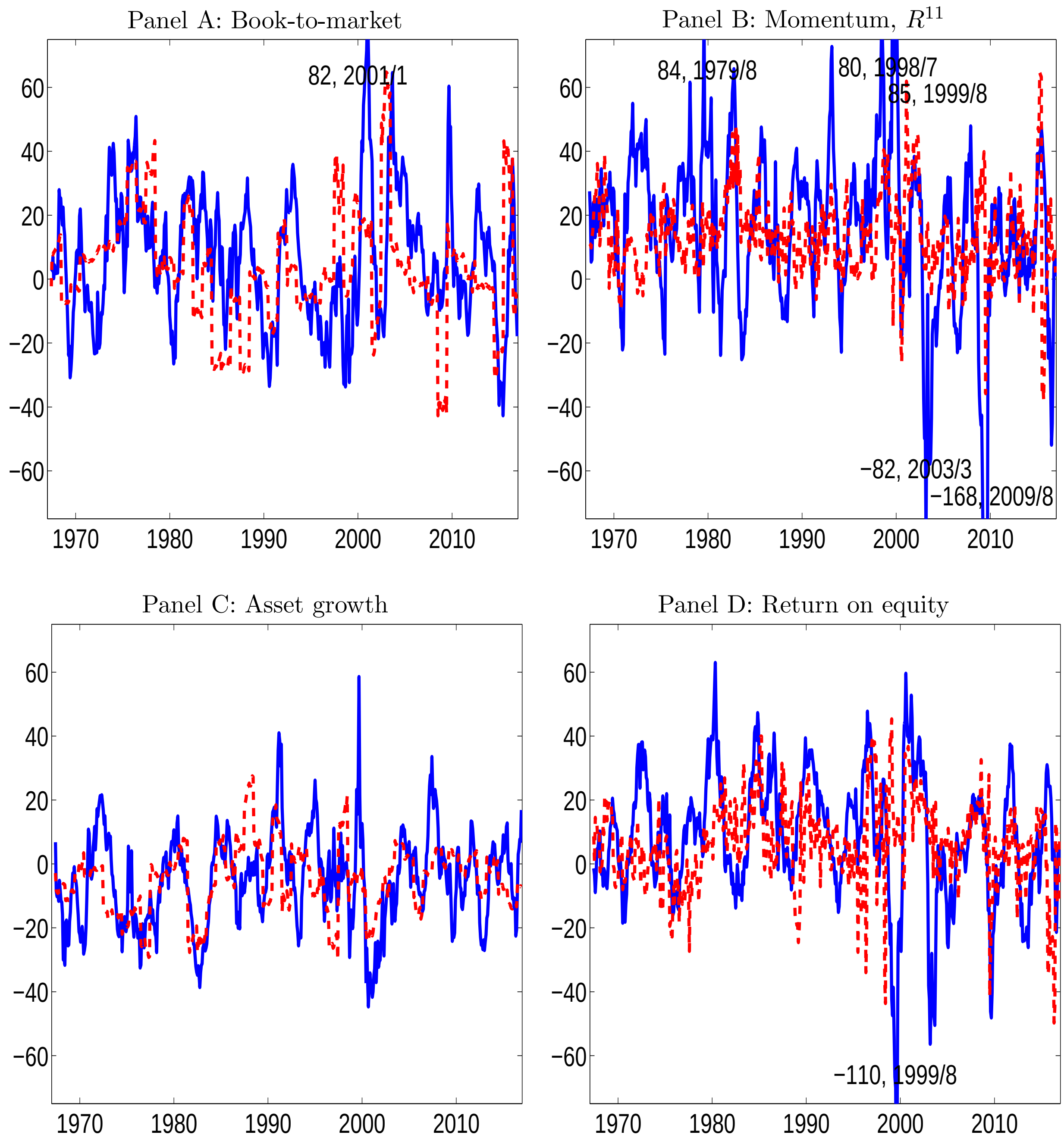


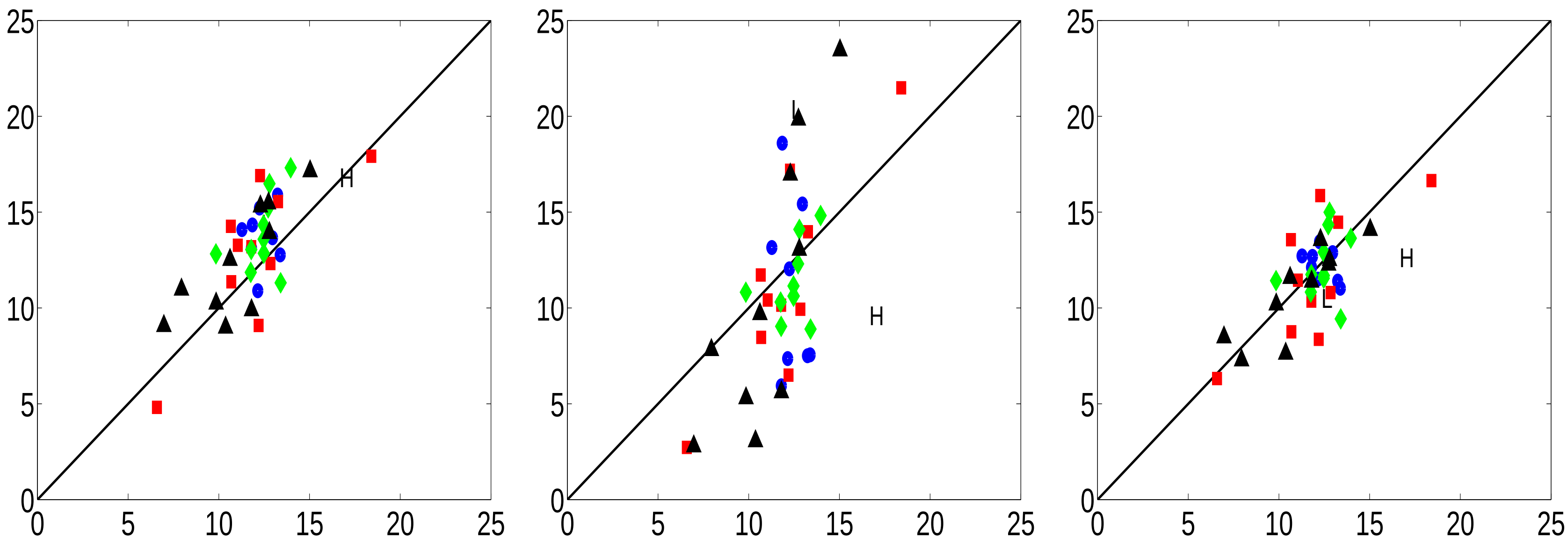
Figure 8 : The 1-period-ahead Model Fits via Recursive Estimation, July 1985–December 2016

Both average fundamental returns (y-axis) and average stock returns (x-axis) are in percent. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H."

Panel A: The two-capital model estimated at the firm level

Panel B: The physical capital model estimated at the portfolio level

Panel C: The q-factor model



A Derivations

Let q_{it} and q_{it}^W be the Lagrangian multipliers associated with $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$ and $W_{it+1} = W_{it} + \triangle W_{it}$, respectively. Form the Lagrangian function:

$$\begin{aligned}\mathcal{L} = \ldots &+ (1-\tau_t)(\Pi_{it} - \Phi_{it}) - I_{it} - \triangle W_{it} + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it} K_{it} + \tau_t (r_{it}^B - 1) B_{it} \\ &- q_{it}(K_{it+1} - (1-\delta_{it})K_{it} - I_{it}) - q_{it}^W (W_{it+1} - W_{it} - \triangle W_{it}) \\ &+ E_t[M_{t+1}[(1-\tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) - I_{it+1} - \triangle W_{it+1} + B_{it+2} - r_{it+1}^B B_{it+1} + \tau_{t+1}\delta_{it+1}K_{it+1} \\ &+ \tau_{t+1}(r_{it+1}^B - 1)B_{it+1} - q_{it+1}(K_{it+2} - (1-\delta_{it+1})K_{it+1} - I_{it+1}) - q_{it+1}^W(W_{it+2} - W_{it+1} - \triangle W_{it+1})]] \\ &+ \ldots \end{aligned} \quad \text{(A1)}$$

Setting the first-order derivatives of $\mathcal{L}$ with respect to $I_{it}, \triangle W_{it}, K_{it+1}, W_{it+1}$, and B_{it+1} to zero yields, respectively:

$$q_{it} = 1 + (1-\tau_t)\frac{\partial \Phi_{it}}{\partial I_{it}} \quad \text{(A2)}$$

$$q_{it}^W = 1 \quad \text{(A3)}$$

$$q_{it} = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left[\frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial K_{it+1}}\right] + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1})q_{it+1}^K\right]\right] \quad \text{(A4)}$$

$$q_{it}^W = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\frac{\partial \Pi_{it+1}}{\partial W_{it+1}} + q_{it+1}^W\right]\right] \quad \text{(A5)}$$

$$1 = E_t[M_{t+1}(r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1})] = E_t[M_{t+1} r_{it+1}^{Ba}] \quad \text{(A6)}$$

Equations (A2) and (A4) yield $E_t[M_{t+1} r_{it+1}^K] = 1$, in which r_{it+1}^K is given by equation (2), and equations (A3) and (A5) yield $E_t[M_{t+1} r_{it+1}^W] = 1$, in which r_{it+1}^W is given by equation (3).

To prove equation (4), we first show $P_{it} + B_{it+1} = q_{it}K_{it+1} + W_{it+1}$. We proceed with a guess-and-verify approach. We first assume that this equation holds for period $t+1$, and then show it also holds for period t. It then follows that the equation must hold for all periods. We start with:

$$P_{it} + B_{it+1} = E_t[M_{t+1}(P_{it+1} + D_{it+1})] + B_{it+1} \quad \text{(A7)}$$

Using $P_{it+1} + B_{it+2} = q_{it+1}K_{it+2} + W_{it+2}$ to rewrite the right hand side yields:

$$P_{it} + B_{it+1} = E_t[M_{t+1}(q_{it+1}K_{it+2} + W_{it+2} - B_{it+2} + D_{it+1})] + B_{it+1} \quad \text{(A8)}$$

Using the definition of $D_{it+1} \equiv (1-\tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) - I_{it+1} - \triangle W_{it+1} + B_{it+2} - r_{it+1}^B B_{it+1} + \tau_{t+1}\delta_{it+1}K_{it+1} + \tau_{t+1}(r_{it+1}^B - 1)B_{it+1}$ to write the right hand side yields:

$$\begin{aligned} P_{it} + B_{it+1} = &\ E_t[M_{t+1}[(1-\tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) + \tau_{t+1}\delta_{it+1}K_{it+1} + q_{it+1}K_{it+2} - I_{it+1}]] \\ &+ E_t[M_{t+1}(W_{it+2} - \triangle W_{it+1})] - B_{it+1}E_t[M_{t+1}[r_{it+1}^B - \tau_{t+1}(r_{it+1}^B - 1)]] + B_{it+1} \end{aligned} \quad \text{(A9)}$$

The constant returns to scale for Π_{it} and equation (A6) then imply:

$$\begin{aligned} P_{it} + B_{it+1} = &\ E_t\left[M_{t+1}\left[K_{it+1}(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}}{K_{it+1}}\right) + \tau_{t+1}\delta_{it+1}K_{it+1} + q_{it+1}[(1-\delta_{it+1})K_{it+1} + I_{it+1}] - I_{it+1}\right]\right] \\ &+ E_t\left[M_{t+1}\left[W_{it+1}(1-\tau_{t+1})\frac{\partial \Pi_{it+1}}{\partial W_{it+1}} + (W_{it+1} + \triangle W_{it+1}) - \triangle W_{it+1}\right]\right] \end{aligned} \quad \text{(A10)}$$

Using the first-order conditions in equations (A2) and (A3) to rewrite the right hand side yields:

$$P_{it} + B_{it+1} = E_t \left[M_{t+1} \left[K_{it+1}(1-\tau_{t+1}) \left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}}{K_{it+1}} + \frac{I_{it+1}}{K_{it+1}} \frac{\partial \Phi_{it+1}}{\partial I_{it+1}} \right) + \tau_{t+1}\delta_{it+1}K_{it+1} + q_{it+1}(1-\delta_{it+1})K_{it+1} \right] \right]$$
$$+ E_t \left[M_{t+1} \left[W_{it+1}(1-\tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial W_{it+1}} + W_{it+1} \right] \right] \qquad \text{(A11)}$$

Constant returns to scale imply that $\Phi_{it} = I_{it}\partial\Phi^K_{it}/\partial I_{it} + K_{it}\partial\Phi^K_{it}/\partial K_{it}$. Equation (A11) becomes:

$$P_{it} + B_{it+1} = K_{it+1}E_t \left[M_{t+1} \left[(1-\tau_{t+1}) \left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial K_{it+1}} \right) + \tau_{t+1}\delta_{it+1} + q_{it+1}(1-\delta_{it+1}) \right] \right]$$
$$+ W_{it+1}E_t \left[M_{t+1} \left[(1-\tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial W_{it+1}} + 1 \right] \right] = q_{it}K_{it+1} + W_{it+1}, \qquad \text{(A12)}$$

in which the last equality follows from equations (A4) and (A5).

Finally, we are ready to prove equation (4),

$$w^B_{it} r^{Ba}_{it+1} + (1-w^B_{it}) r^S_{it+1} = \frac{B_{it+1}}{P_{it}+B_{it+1}} [r^B_{it+1} - (r^B_{it+1}-1)\tau_{t+1}] + \frac{P_{it}}{P_{it}+B_{it+1}} \frac{(P_{it+1}+D_{it+1})}{P_{it}}$$
$$= \frac{B_{it+1}[r^B_{it+1} - (r^B_{it+1}-1)\tau_{t+1}] + q_{it+1}K_{it+2} + W_{it+2} - B_{it+2} + D_{it+1}}{P_{it}+B_{it+1}} \qquad \text{(A13)}$$

Using the definition of D_{it+1} yields:

$$w^B_{it} r^{Ba}_{it+1} + (1-w^B_{it}) r^S_{it+1} = \frac{(1-\tau_{t+1})(\Pi_{it+1} - \Phi_{it+1}) + \tau_{t+1}\delta_{it+1}K_{it+1} + q_{it+1}K_{it+2} + W_{it+2} - I_{it+1} - \triangle W_{it+1}}{P_{it}+B_{it+1}} \qquad \text{(A14)}$$

Using the constant returns to scale for Π_{it+1} yields:

$$w^B_{it} r^{Ba}_{it+1} + (1-w^B_{it}) r^S_{it+1} = \frac{K_{it+1}(1-\tau_{t+1}) \left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\Phi_{it+1}}{K_{it+1}} \right) + \tau_{t+1}\delta_{it+1}K_{it+1} + q_{it+1}(I_{it+1} + (1-\delta_{it+1})K_{it+1}) - I_{it+1}}{P_{it}+B_{it+1}}$$
$$+ \frac{W_{it+1}(1-\tau_{t+1})\frac{\partial \Pi_{it+1}}{\partial W_{it+1}} + (W_{it+1} + \triangle W_{it+1}) - \triangle W_{it+1}}{P_{it}+B_{it+1}} \qquad \text{(A15)}$$

Using the constant returns to scale for Φ_{it+1} and equations (A2) and (A3), we obtain:

$$w^B_{it} r^{Ba}_{it+1} + (1-w^B_{it}) r^S_{it+1} = \frac{K_{it+1}}{q_{it}K_{it+1} + W_{it+1}} \left[(1-\tau_{t+1}) \left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}} - \frac{\partial \Phi_{it+1}}{\partial K_{it+1}} \right) + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1})q_{it+1} \right]$$
$$+ \frac{W_{it+1}}{q_{it}K_{it+1} + W_{it+1}} \left[(1-\tau_{t+1}) \frac{\partial \Pi_{it+1}}{\partial W_{it+1}} + 1 \right] \qquad \text{(A16)}$$

Using equations (A4) and (A5) yields the desired result:

$$w^B_{it} r^{Ba}_{it+1} + (1-w^B_{it}) r^S_{it+1} = \frac{q_{it}K_{it+1}}{q_{it}K_{it+1} + W_{it+1}} r^K_{it+1} + \frac{W_{it+1}}{q_{it}K_{it+1} + W_{it+1}} r^W_{it+1}. \qquad \text{(A17)}$$