Aggregation, Capital Heterogeneity, and the Investment CAPM

Andrei S. Gonçalves

University of North Carolina at Chapel Hill

Chen Xue

University of Cincinnati

Lu Zhang

The Ohio State University and National Bureau of Economic Research

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Abstract

A detailed treatment of aggregation and capital heterogeneity substantially improves the performance of the investment CAPM. Firm-level predicted returns are constructed from firm-level accounting variables and aggregated to the portfolio level to match with portfolio-level stock returns. Working capital forms a separate productive input besides physical capital. The model fits well the value, momentum, investment, and profitability premiums simultaneously and partially explains positive stock-fundamental return correlations, the procyclical and short-term dynamics of the momentum and profitability premiums, as well as the countercyclical and long-term dynamics of the value and investment premiums. However, the model falls short in explaining momentum crashes. (*JEL* D21, D92, E22, E44, G12, G14, G31, G32, G34)

Aggregation and heterogeneity have long been recognized as thorny problems for empirical studies of the investment behavior. Nickell (1978) identifies three major problems on aggregation and heterogeneity. First, "the question arises as to whether one can construct aggregates for the outputs, the capital good inputs and the labour inputs so that it is possible to define a production function which gives aggregate output as a function of the aggregate capital and aggregate labour inputs. The answer, in any realistic case, is that it is not (p. 229–230)." Second, even if the empirical relations at the firm level are good approximations of reality, "it is difficult to develop structural restrictions on the aggregate relationships corresponding to those which theory imposes on the micro-level equations (p. 230)." This difficulty is especially acute, if the micro-level equations are nonlinear. Third, there are serious problems associated with measuring investment and capital stock. Investment data can be "based on orders, deliveries or payments or some mixture of all three (p. 231)" that are not additions to a firm's capital stock. The key problem of measuring the capital stock is "the evaluation of old capital goods for which there exist no active markets (p. 231)."[1]

This paper provides a careful treatment of aggregation, which is the second major problem in Nickell (1978). We also address, at least to some extent, the problem of capital heterogeneity and the measurement of investment and capital, which are the first and third problem in Nickell, respectively. We do so in the context of the Investment-based Capital

[1] Aggregation and heterogeneity pose even more challenging problems for empirical studies of the consumption behavior. For example, Blundell and Stoker (2007) write: "[I]t is senseless to ascribe behavioral interpretations to estimated relationships among aggregate data without a detailed treatment of the links between individual and aggregate levels (p. 4614)." "Aggregation problems are among the most difficult problems faced in either the theoretical or empirical study of economics. Heterogeneity across individuals is extremely extensive and its impact is not obviously simplified or lessened by the existence of economic interaction via markets or other institutions. The conditions under which one can ignore a great deal of the evidence of individual heterogeneity are so severe as to make them patently unrealistic. There is no quick, easy or obvious fix to dealing with aggregation problems in general (p. 4658)."

Asset Pricing Model (the Investment CAPM, Zhang 2017).

Prior studies implement the investment CAPM via structural estimation at the portfolio level (Liu, Whited, and Zhang 2009). Firm-level accounting variables are aggregated to portfolio-level variables, from which portfolio-level investment returns are constructed to match with portfolio-level stock returns. While a useful first stab, this approach has a couple of important drawbacks. First, on economic grounds, it implicitly assumes that firms in a given portfolio all have the same investment rate. This assumption is clearly unrealistic. Second, on econometric grounds, this approach overlooks a substantial amount of heterogeneity in firm-level variables that can help identify structural parameters. We instead use firm-level variables to construct firm-level investment returns, which are then aggregated to the portfolio level to match with portfolio-level stock returns.

In addition, most studies ignore capital heterogeneity, with physical capital (net property, plant, and equipment) as the single productive input. However, net property, plant, and equipment is only a small fraction of total assets on firms' balance sheet. While many possibilities exist to introduce an additional input, we settle on working capital, with no adjustment costs (an assumption that we verify empirically). Consequently, the resulting 2-capital model is as parsimonious as the baseline, physical capital model with only two parameters, facilitating comparison with prior work.

Our benchmark model with two capital goods estimated at the firm level goes a long way in resolving the empirical difficulties in prior work. Estimating the physical capital model at the portfolio level, Liu, Whited, and Zhang (2009) show that the marginal product and adjustment costs parameters vary greatly across the value and momentum deciles. If the model is well specified, or "structural," the parameter estimates should be mostly invariant to the sorting variables underlying testing portfolios. As a result, the physical capital model fails to explain value and momentum simultaneously. This weakness applies to the investment CAPM literature more broadly. For example, in a prominent asset pricing textbook, Campbell (2018, p. 213) writes: "This problem, that different parameters are needed to fit each anomaly, is a pervasive one in the q-theoretic asset pricing literature."

The parameter estimates in our benchmark model are relatively stable across the testing deciles on value, momentum, asset growth, and return on equity, separately or jointly. When fitting value and momentum deciles together, with or without adding the asset growth and return on equity deciles, the scatter plots of average predicted stock returns versus average realized stock returns are mostly aligned with the 45-degree line. For example, when fitting value-weighted value and momentum deciles jointly, the model implies a value premium of 5.2% per annum, with an alpha of 1.18% ($t = 0.51$), as well as a momentum premium of 14.62%, with an even smaller alpha of 0.35% ($t = 0.12$). However, the model is still rejected by the test of overidentification.

Aggregation is important for the performance. When implemented at the portfolio level per Liu, Whited, and Zhang (2009), the model yields larger pricing errors. In the joint estimation of value and momentum, the value premium is only 2.88% per annum, with an alpha of 3.51% ($t = 1.23$), although the momentum premium is high, 13.97%, with a small alpha of 1% ($t = 0.63$). Working capital is also important. Empirically, the fraction of physical capital in the sum of physical capital and working capital averages only 38%. Accordingly, the average product in the physical capital model is severely misspecified, giving rise to large pricing errors even when estimated at the firm level. Again in the joint estimation of value and momentum, the value premium is 1.64%, with an alpha of 4.75%

($t = 1.8$), and the momentum premium 24.26%, with a large alpha of -9.29% ($t = -2.79$).

We also use the predicted stock return from the benchmark model (dubbed "the fundamental return") to study the dynamics of factor premiums. The model yields significantly positive stock-fundamental return correlations, the short-term dynamics of the momentum and return on equity premiums, as well as the long-term dynamics of the value and investment premiums. The model also partially explains the procyclical variation of the momentum and return on equity premiums as well as the countercyclical variation of the value and investment premiums. However, the model underestimates the volatility, skewness, and kurtosis of factor premiums as well as momentum crashes.

Finally, prior work only examines the in-sample fit. In contrast, we also conduct out-of-sample tests by constructing firm-level 1-period-ahead expected returns from recursively estimating the benchmark 2-capital model. The expected return estimates forecast subsequent returns reliably. By comparison, the out-of-sample performance of factor models such as the q-factor model (which is a reduced form implementation of our structural model) is poor, echoing Fama and French (1997).

Building on Cochrane (1991), Liu, Whited, and Zhang (2009) estimate the physical capital model at the portfolio level with cross-sectional returns. Cooper and Priestley (2016) use the investment framework to study private firms. Several studies feature heterogeneity with additional productive inputs, such as real estate (Tuzel 2010), working capital (Wu, Zhang, and Zhang 2010), and inventory (Belo and Lin 2012; Jones and Tuzel 2013). We instead perform structural estimation. More important, aggregation has been largely overlooked in the prior literature.[2] We fill this gap.[3]

The rest of the paper is organized as follows. Section 1 sets up the model. Section 2 discusses our econometric design. Section 3 presents the estimation results. Finally, Section 4 concludes.

1 The Model of the Firms

We formulate the 2-capital model in Section 1.1 and explain why we include working capital as a productive input in addition to physical capital in Section 1.2.

1.1 Setup

Firms use both short-term working capital and long-term physical capital to produce a homogeneous output. Let $\Pi_{it} \equiv \Pi(K_{it}, W_{it}, X_{it})$ be the operating profits of firm i at time t, in which K_{it} is physical capital, W_{it} working capital, and X_{it} a vector of exogenous aggregate and firm-specific shocks. We assume that Π_{it} exhibits constant returns to scale, i.e., $\Pi_{it} = K_{it}\partial\Pi_{it}/\partial K_{it} + W_{it}\partial\Pi_{it}/\partial W_{it}$, and that firms have a Cobb-Douglas production function. Following Gilchrist and Himmelberg (1998), we parameterize the marginal product of physical capital as $\partial\Pi_{it}/\partial K_{it} = \gamma_K Y_{it}/K_{it}$ and the marginal product of working capital as $\partial\Pi_{it}/\partial W_{it} = \gamma_W Y_{it}/W_{it}$, in which $\gamma_K, \gamma_W \geq 0$ are the shares of physical and working

[2] In subsequent but independent work, Belo et al. (2018) study aggregation in the context of equity valuation. While it is natural to use portfolios in expected return tests (Black, Jensen, and Scholes 1972), it seems unnecessary to work with portfolios in valuation tests. The crux is that firm-level valuation ratios are less noisy than returns.

[3] Outside asset pricing, Wildasin (1984) examines optimal investment with many capital goods. Schaller (1990) shows that aggregation is partially responsible for large adjustment costs from aggregate time series. Hayashi and Inoue (1991) derive a one-to-one relation between the growth rate of the capital aggregate and Tobin's q in an investment model with multiple capital goods and estimate this relation on Japanese firms. Chirinko (1993) estimates the investment model with multiple capital inputs that differ in adjustment technologies. Doyle and Whited (1998) show that smooth industry-level investment results from aggregating asynchronous and lumpy micro-level investment.

capital in sales, Y_{it}, respectively, with $\gamma_K + \gamma_W \leq 1$.[4]

Taking operating profits as given, firms choose investments in working and physical capital stocks to maximize the market equity. Physical capital evolves as $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$, in which I_{it} is investment in physical capital, and δ_{it} the rate of depreciation, which firm i takes as given. We allow δ_{it} to be firm-specific and time-varying. Working capital evolves as $W_{it+1} = \triangle W_{it} + W_{it}$, in which $\triangle W_{it}$ is investment in working capital. We assume that working capital does not depreciate.

Firms incur adjustment costs when investing in physical capital, but not in working capital. The adjustment costs function, denoted $\Phi(I_{it}, K_{it})$, is increasing and convex in I_{it}, decreasing in K_{it}, and of constant returns to scale in I_{it} and K_{it}, i.e., $\Phi(I_{it}, K_{it}) = I_{it}\, \partial\Phi(I_{it}, K_{it})/\partial I_{it} + K_{it}\, \partial\Phi(I_{it}, K_{it})/\partial K_{it}$. We adopt the standard quadratic functional form:

$$\Phi_{it} \equiv \Phi(I_{it}, K_{it}) = \frac{a}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}, \tag{1}$$

[4] The case with $\gamma_K + \gamma_W < 1$ is consistent with constant returns to scale for $\Pi(K_{it}, W_{it}, X_{it})$. The crux is that γ_K and γ_W are shares in output (measured as sales, Y_{it}), not in operating profits, Π_{it}. Let the production function be $Y_{it} \equiv Y(K_{it}, W_{it}, S_{it}, X_{it}) = X_{it} K_{it}^{\gamma_K} W_{it}^{\gamma_W} S_{it}^{1-\gamma_K-\gamma_W}$, in which S_{it} is intermediate inputs, such as energy, purchased services, and costlessly adjustable labor. Y_{it} is of constant returns to scale in physical capital, working capital, and intermediate inputs with their shares given by γ_K, γ_W, and $1 - \gamma_K - \gamma_W$, respectively. Let p_t^S be the factor price for the intermediate inputs taken as given by the firm. The operating profits function solves the static optimization problem:

$$\Pi(K_{it}, W_{it}, X_{it}) \;=\; \max_{\{S_{it}\}}\; X_{it} K_{it}^{\gamma_K} W_{it}^{\gamma_W} S_{it}^{1-\gamma_K-\gamma_W} - p_t^S S_{it},$$

The first-order condition with respect to S_{it} is given by $(1 - \gamma_K - \gamma_W) Y_{it}/S_{it} = p_t^S$. Solving for the optimal S_{it} yields:

$$S_{it} = \left[\frac{(1 - \gamma_K - \gamma_W) X_{it} K_{it}^{\gamma_K} W_{it}^{\gamma_W}}{p_t^S}\right]^{\frac{1}{\gamma_K + \gamma_W}}.$$

Plugging the first-order condition back to $\Pi(K_{it}, W_{it}, X_{it})$ yields $\Pi_{it} = (\gamma_K + \gamma_W) Y_{it}$. Plugging the optimal S_{it} into Y_{it} to rewrite Π_{it} only in terms of K_{it} and W_{it} yields:

$$\Pi(K_{it}, W_{it}, X_{it}) = (\gamma_K + \gamma_W) X_{it}^{\frac{1}{\gamma_K + \gamma_W}} \left(\frac{1 - \gamma_K - \gamma_W}{p_t^S}\right)^{\frac{1-\gamma_K-\gamma_W}{\gamma_K+\gamma_W}} K_{it}^{\frac{\gamma_K}{\gamma_K+\gamma_W}} W_{it}^{\frac{\gamma_W}{\gamma_K+\gamma_W}}.$$

As such, $\Pi(K_{it}, W_{it}, X_{it})$ is of constant returns to scale in K_{it} and W_{it}, and their shares, given by $\gamma_K/(\gamma_K + \gamma_W)$ and $\gamma_W/(\gamma_K + \gamma_W)$, respectively, sum to one. In particular, $\partial\Pi_{it}/\partial K_{it} = [\gamma_K/(\gamma_K + \gamma_W)](\Pi_{it}/K_{it}) = \gamma_K Y_{it}/K_{it}$.

in which $a > 0$ is the physical adjustment costs parameter.

At the beginning of time t, firm i issues debt, B_{it+1}, which must be repaid at the beginning of $t+1$. When borrowing, the firm takes as given the gross cost of debt on B_{it}, denoted r_{it}^B, which varies across firms and over time. Taxable corporate profits equal operating profits minus physical capital depreciation, adjustment costs, and interest expenses, $\Pi_{it} - \delta_{it}K_{it} - \Phi_{it} - (r_{it}^B - 1)B_{it}$. Let τ_t be the corporate tax rate, $\tau_t \delta_{it} K_{it}$ be depreciation tax shield, and $\tau_t(r_i^B - 1)B_{it}$ be interest tax shield. Firm i's net payout is given by $D_{it} \equiv (1-\tau_t)(\Pi_{it} - \Phi_{it}) - I_{it} - \triangle W_{it} + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it} K_{it} + \tau_t (r_{it}^B - 1) B_{it}$.

Taking the stochastic discount factor, M_{t+1}, as given, firm i chooses $I_{it}, K_{it+1}, \triangle W_{it}, W_{it+1}$, and B_{it+1} to maximize its cum-dividend market value of equity, $V_{it} \equiv E_t \left[\sum_{s=0}^{\infty} M_{t+s} D_{it+s}\right]$, subject to $\lim_{T\to\infty} E_t\left[M_{t+T}\, B_{it+T+1}\right] = 0$ (the transversality condition), which prevents the firm from borrowing an infinite amount of debt. The first-order condition for physical investment implies that $E_t[M_{t+1} r_{it+1}^K] = 1$, in which r_{it+1}^K is the physical capital investment return:

$$r_{it+1}^K \equiv \frac{(1-\tau_{t+1})\left[\gamma_K \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1})\left[1 + (1-\tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]}{1 + (1-\tau_t)a\left(\frac{I_{it}}{K_{it}}\right)}. \tag{2}$$

Intuitively, the physical investment return is the marginal benefit of physical investment at $t+1$ divided by its marginal cost at t. In the numerator of equation (2), $(1 - \tau_{t+1})\gamma_K(Y_{it+1}/K_{it+1})$ is the after-tax marginal product of physical capital, $(1-\tau_{t+1})(a/2)(I_{it+1}/K_{it+1})^2$ the after-tax marginal reduction in physical adjustment costs, and $\tau_{t+1}\delta_{it+1}$ the marginal depreciation tax shield. The last term in the numerator is the marginal continuation value of an extra unit of physical capital net of depreciation, in which the marginal continuation value equals the marginal cost of physical investment in the next period, $1 + (1 - \tau_{t+1})a(I_{it+1}/K_{it+1})$. Finally, $E_t[M_{t+1}r^K_{it+1}] = 1$ says that the marginal cost of investment equals the next period marginal benefit discounted to time t.

Similarly, the firm's first-order condition for investment in working capital is $E_t[M_{t+1}r^W_{it+1}] = 1$, in which r^W_{it+1} is the working capital investment return:

$$r^W_{it+1} \equiv 1 + (1 - \tau_{t+1})\gamma_W \frac{Y_{it+1}}{W_{it+1}}. \tag{3}$$

The working capital investment return is again the marginal benefit of working capital investment at $t+1$ divided by its marginal cost at time t. The marginal cost equals one because of no adjustment costs on working capital. For the marginal benefit, $(1 - \tau_{t+1})\gamma_W(Y_{it+1}/W_{it+1})$ is the after-tax marginal product of working capital, and without adjustment costs or depreciation, the marginal continuation value of an extra unit of working capital net of depreciation equals one.

Define the after-tax cost of debt as $r^{Ba}_{it+1} \equiv r^B_{it+1} - (r^B_{it+1} - 1)\tau_{t+1}$. The firm's first-order condition for new debt implies $E_t[M_{t+1}r^{Ba}_{it+1}] = 1$. Define $P_{it} \equiv V_{it} - D_{it}$ as the ex-dividend market value of equity, $r^S_{it+1} \equiv (P_{it+1} + D_{it+1})/P_{it}$ as the stock return, and $w^B_{it} \equiv B_{it+1}/(P_{it} + B_{it+1})$ as the market leverage. Also, the shadow price of physical capital is marginal q, which in the optimum equals the marginal cost of physical investment, $q_{it} = 1 + (1 - \tau_t)a(I_{it}/K_{it})$. The shadow price of working capital equals one. Finally, define $w^K_{it} \equiv q_{it}K_{it+1}/(q_{it}K_{it+1} + W_{it+1})$ as the weight of the firm's market value attributed to physical capital. The weighted average of the two investment returns equals the weighted average of the cost of equity and the after-tax cost of debt (the Internet Appendix):

$$w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W = w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S. \tag{4}$$

Solving for the stock return from equation (4) yields the investment CAPM:

$$r_{it+1}^S = r_{it+1}^F \equiv \frac{w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W - w_{it}^B r_{it+1}^{Ba}}{1 - w_{it}^B}. \tag{5}$$

The "fundamental" return, r_{it+1}^F, is a nonlinear function of firm characteristics (no market prices). If $w_{it}^K = 1$, equation (4) collapses to the equivalence between the physical investment return and the weighted average cost of capital (Liu, Whited, and Zhang 2009). If $w_{it}^K = 1$ and $w_{it}^B = 0$, equation (5) reduces to the equivalence between the stock and physical investment returns (Cochrane 1991).

Equation (5) clearly shows that even without adjustment costs, working capital helps characterize the cross section of expected stock returns more accurately. In this aspect, working capital differs from labor, which does not appear on firms' balance sheet as assets. Firms hire, but do not own, workers. As a result, without adjustment costs on labor hiring, the labor input can be absorbed into the operating profits function and does not affect the cost of equity directly (footnote 4).

1.2 Why Working Capital?

Short-term working capital is essential for firms' operations. The main components of working capital include cash, account receivables, and inventory (Berk and DeMarzo 2017). Firms hold cash to save transaction costs of raising funds and to avoid liquidation of asses to make payments. Also, firms use cash to finance their day-to-day operations and long-term investments if other financing sources are either unavailable or excessively costly (Opler et al. 1999).

Trade credit, in the form of accounts receivable and payable, is an important source of short-term external finance among firms (Petersen and Rajan 1997). Suppliers extend trade credit to their customers in the form of accounts receivable to increase sales against their competitors. Relative to financial institutions, suppliers are more inclined to lend to financially constrained firms because of their comparative advantage in obtaining information on the buyers, their ability to liquidate buyers' assets more efficiently, and their implicit equity stake in the buyers.

Inventory is also indispensable in the production process. Inventory helps avoid stock-outs, in which a firm runs out of its store of commodities and loses sales, or a firm exhausts its store of materials and delays production. In addition, inventory helps ensure a more efficient production cycle to meet seasonal demand. Sales can be highly seasonal with upward spikes in the fourth quarter. In contrast, a smooth production process is more desirable to avoid excessive wear and tear on equipment and overtime worker salaries (Berk and DeMarzo 2017).

For parsimony, we do not model adjustment costs of working capital. The Internet Appendix shows that the adjustment costs on working capital in an extended model are mostly small and insignificant, especially in the joint estimation with value and momentum. The extended model's performance is also quantitatively close to the benchmark model without the extra adjustment costs.

While working capital as a separate productive input seems straightforward to motivate, we should clarify why we do not include other inputs such as labor and intangibles. The crux is measurement difficulties. Working capital can be accurately measured on firms' balance sheet. In contrast, in our sample (detailed in Section 2.3), about 80.1% of wages data (Compustat annual item XLR, total staff expense) are missing at the firm level. More important, measurement errors are likely even more severe for intangibles. For instance, Peters and Taylor (2017) assume a fixed depreciation rate of 20% for organizational capital and a fixed proportion of 30% of selling, general, and administrative expenses as intangible investments. Both rates are assumed to be constant over time and across firms. While these ad hoc assumptions are perhaps unavoidable when measuring intangibles, we hesitate to introduce such measurement errors into our structural estimation.

We should clarify further that our primary focus is on the expected stock return, as opposed to the equity valuation level. In principle, the theoretical structure for explaining the expected return should be identical to that for the valuation level. However, in practice, expected return moments and equity valuation moments contain different identifying information and require different data and econometric specifications (Belo, Xue, and Zhang 2013). While labor and intangibles are likely indispensable for pinning down the valuation level, these ingredients are not necessary for determining the expected return (the first-difference of the equity valuation). After all, our more parsimonious model without labor or intangibles already performs well in expected return tests.

2 Econometric Design

We describe our structural estimation methodology in Section 2.1, the aggregation procedure in Section 2.2, as well as our sample construction and descriptive statistics in Section

2.3.

2.1 Generalized Method of Moments (GMM)

We use GMM to test the ex ante restriction implied by equation (5):

$$E[r^S_{pt+1} - r^F_{pt+1}] = 0, \tag{6}$$

in which r^S_{pt+1} is the stock return of testing portfolio p, and r^F_{pt+1} is the portfolio's fundamental return given by the right hand side of equation (5). In particular, the pricing error (alpha) from the investment CAPM is defined as $\alpha_p \equiv E_T[r^S_{pt+1} - r^F_{pt+1}]$, in which $E_T[\cdot]$ is the sample mean.

2.1.1 Why focusing on the first moment?. Interpreted literally, equation (5) predicts that the stock return equals the fundamental return period by period and state by state. When taking the model to the data, we choose to estimate the structural parameters from the first moment restriction in equation (6), which says that the expected stock return equals the expected fundamental return. We focus on the first moment because the anomalies literature is primarily about the expected return. Why do stocks with high book-to-market, high price momentum, low investment, and high profitability earn higher average returns than stocks with low book-to-market, low price momentum, high investment, and low profitability, respectively? These important questions are all about the first moment.

The first moment restriction is also likely to be more reliable in the data. Although equation (5) predicts ex post equivalence between the stock and fundamental returns, it is straightforward to introduce some residuals to break the ex post equivalence. For example, the marginal product of physical capital, specified as $\gamma_K(Y_{it+1}/K_{it+1})$, might not be exactly proportional to sales-to-physical capital, but with an additive, zero-mean measurement error, ϵ^K_{it+1}. With such an error, the stock return, which accounts for the error, and the (measured) fundamental return, which does not account for the error, would be equivalent only ex ante, but not ex post.

Finally, although we estimate the structural parameters only from the first moment restriction, we push the econometric model as far as possible to explain the second, third, and fourth moments, as well as cross correlations and tail risk, as separate diagnostics of the model (Section 3.4).

2.1.2 Identification, estimation, and tests. Although the model has three parameters (γ_K, γ_W, and a), γ_K and γ_W enter the moment condition (6) only in the form of $\gamma \equiv \gamma_K + \gamma_W$. To see this point, we use equations (2) and (3) to rewrite:

$$\begin{aligned} w^K_{it} r^K_{it+1} + (1 - w^K_{it}) r^W_{it+1} \quad = \quad & \frac{(1-\tau_{t+1})(\gamma_K + \gamma_W) Y_{it+1}/(K_{it+1} + W_{it+1})}{q_{it} K_{it+1}/(K_{it+1} + W_{it+1}) + W_{it+1}/(K_{it+1} + W_{it+1})} + \\ & w^K_{it} \frac{(1-\tau_{t+1})(a/2)\left(I_{it+1}/K_{it+1}\right)^2 + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1}) q_{it+1}}{q_{it}} + (1 - w^K_{it}). \quad (7) \end{aligned}$$

As such, γ_K and γ_W are not separately identifiable, and only their sum, γ, can be estimated. With two parameters, γ and a, the 2-capital model is as parsimonious as the physical capital model.

In addition, the numerator of the first term in the right hand side of equation (7) shows that the marginal product in the 2-capital model should be measured as proportional to sales over the sum of physical capital and working capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, as opposed to sales-to-physical capital, Y_{it+1}/K_{it+1}, in the physical capital model. Finally, the denominator of the first term can be interpreted as the weighted average of the marginal q of physical capital and that of working capital (one), with the weights given by $K_{it+1}/(K_{it+1}+W_{it+1})$ and $W_{it+1}/(K_{it+1}+W_{it+1})$, respectively.

Formally, let $\mathbf{c} \equiv (\gamma, a)$ denote the model's parameters, and $\mathbf{g}_T$ the sample moments. The GMM objective function is a weighted sum of squares of the alphas across a set of testing portfolios, $\mathbf{g}_T'\mathbf{W}\mathbf{g}_T$, in which we set $\mathbf{W} = \mathbf{I}$, the identity matrix (Cochrane 1996). Let $\mathbf{D} = \partial \mathbf{g}_T/\partial \mathbf{c}$ and $\mathbf{S}$ be a consistent estimate of the variance-covariance matrix of the sample alphas, $\mathbf{g}_T$. The $\mathbf{S}$ estimate accounts for autocorrelations of up to 12 lags. The estimate of $\mathbf{c}$, denoted $\hat{\mathbf{c}}$, is asymptotically normal with the variance-covariance matrix given by $\text{var}(\hat{\mathbf{c}}) = (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}/T$. To construct the standard errors for the pricing errors of individual portfolios, we use the variance-covariance matrix for $\mathbf{g}_T$, $\text{var}(\mathbf{g}_T) = \left[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\right]\mathbf{S}\left[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\right]'/T$. Finally, we form a χ^2 test on the null hypothesis that all the alphas are jointly zero, $\mathbf{g}_T'\left[\text{var}(\mathbf{g}_T)\right]^{+}\mathbf{g}_T \sim \chi^2$, in which χ^2 is the chi-square distribution with the degrees of freedom given by the number of moments minus the number of parameters, and the superscript $^{+}$ is pseudo-inversion (Hansen 1982).

2.2 Aggregation

Prior studies estimate the physical capital model with accounting data aggregated to the portfolio level. Portfolio-level fundamental returns are constructed from portfolio-level characteristics to match with portfolio-level stock returns. Formally, the prior studies estimate:

$$E\left[\sum_{i=1}^{N_{pt}} w_{ipt} r^S_{ipt+1} - r^F_{pt+1}\left(\gamma_K, a; Y_{pt+1}, K_{pt+1}, I_{pt+1}, \delta_{pt+1}, I_{pt}, K_{pt}, r^{Ba}_{pt+1}, w^B_{pt}\right)\right] = 0, \quad (8)$$

in which N_{pt} is the number of firms in portfolio p at the beginning of period t, w_{ipt} is the weight of stock i in portfolio p at the beginning of period t, r^S_{ipt+1} is the return of stock i in portfolio p over period t, and r^F_{pt+1} is the fundamental return for portfolio p. For equal-weighted portfolios, $w_{ipt} = 1/N_{pt}$, and for value-weighted portfolios, w_{ipt} is the market value-weights at the beginning of period t. r^F_{pt+1} is constructed from portfolio-level characteristics aggregated from firm-level characteristics, and its functional form does not change with w_{ipt}. To aggregate accounting variables to the portfolio level, $I_{pt+1} = \sum_{i=1}^{N_{pt}} I_{ipt+1}$, in which I_{ipt+1} is investment of firm i in portfolio p over period $t+1$, $K_{pt+1} = \sum_{i=1}^{N_{pt}} K_{ipt+1}$, in which K_{ipt+1} is physical capital of firm i at the beginning of $t+1$, and $w^B_{pt} = \sum_{i=1}^{N_{pt}} B_{ipt+1} / \sum_{i=1}^{N_{pt}} (P_{ipt} + B_{ipt+1})$. Other portfolio-level variables are aggregated analogously.

Working with this aggregation procedure, Liu, Whited, and Zhang (2009) show that the physical capital model explains value and momentum separately, but the parameter estimates vary greatly across the two sets of deciles. In addition, Liu and Zhang (2014) document that when forced to use the same parameter values in the joint estimation, the physical capital model manages to capture the momentum premium but fails to explain the value premium altogether.

We explore a new, exact aggregation scheme. We first construct firm-level fundamental returns from firm-level accounting variables and then aggregate to portfolio-level fundamental returns to match with portfolio-level stock returns. Formally, we estimate:

$$E\left[\sum_{i=1}^{N_{pt}} w_{ipt} r^S_{ipt+1} - \sum_{i=1}^{N_{pt}} w_{ipt} r^F_{ipt+1}\left(\gamma, a; Y_{ipt+1}, K_{ipt+1}, W_{ipt+1}, I_{ipt+1}, \delta_{ipt+1}, I_{ipt}, K_{ipt}, r^{Ba}_{ipt+1}, w^B_{ipt}\right)\right] = 0, \tag{9}$$

in which r^F_{ipt+1} is the fundamental return for firm i. As such, aggregating r^S_{ipt+1} and r^F_{ipt+1} is symmetric, and the portfolio-level fundamental return, $r^F_{pt+1} \equiv \sum_{i=1}^{N_{pt}} w_{ipt} r^F_{ipt+1}$, varies with w_{ipt}.

2.3 Data

We obtain firm-level data from Center for Research in Security Prices (CRSP) monthly stock files and annual Standard and Poor's Compustat industrial files. We exclude firms with primary standard industrial classifications between 6000 and 6999 (financial firms), firms with negative book equity, and firms with nonpositive total assets, net property, plant, and equipment, or sales at the portfolio formation. These data items are needed to calculate firm-level fundamental returns.

2.3.1 Testing portfolios. We use 40 testing deciles formed on book-to-market, momentum, asset growth, and return on equity, either separately or jointly, in the moment condition (6). Value and momentum are classic anomalies. We also include asset growth and return on equity, both of which feature prominently in the recent asset pricing literature. Although we construct the fundamental returns at the firm level, our structural estimation relies on the cross-sectional variation of average returns to identify the model parameters. To the extent that forming portfolios on value, momentum, asset growth, and return on equity yields economically large and statistically reliable average return spreads, using these testing deciles facilitates the identification (Black, Jensen, and Scholes 1972).

More generally, sorting on the key components of the fundamental return, such as investment and profitability, is the basic idea behind the q-factor model, from which we include the asset growth and return on equity deciles (Hou, Xue, and Zhang 2015). Sales-to-total capital, $Y_{it}/(K_{it}+W_{it})$, in the fundamental return is economically related to return on equity. Both are measures of profitability. While return on equity accounts for operating costs, sales do not. Investment-to-physical capital, I_{it}/K_{it}, is economically related to asset growth, in which investment is measured as the change in total assets (including both short-term and long-term investments). Finally, market leverage, w_{it}^B, is related to book-to-market (both have the market equity in the denominator), and momentum to return on equity (shocks to earnings are positively correlated with shocks to stock prices).

To control for microcaps (stocks smaller than the 20th percentile of NYSE market equity), we form testing deciles with NYSE breakpoints and value-weighted returns. The Internet Appendix also furnishes the results with testing deciles with all-but-micro breakpoints and equal-weighted returns. We first exclude microcaps, sort the remaining stocks into deciles, and calculate equal-weighted returns. The results are robust with equal-weighted deciles (and are overall stronger).

To form the book-to-market (Bm) deciles, at the end of June of each year t, we sort stocks on Bm, which is the book equity for the fiscal year ending in calendar year $t-1$ divided by the market equity (from CRSP) at the end of December of $t-1$. For firms with more than one share class, we merge the market equity for all share classes before computing Bm. Monthly decile returns are calculated from July of year t to June of $t+1$, and the deciles are rebalanced in June of $t+1$.[5]

To form the momentum (R^{11}) deciles, we split all stocks at the beginning of each month

[5]Following Davis, Fama, and French (2000), we measure book equity as stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (Compustat annual item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if it is available. If not, we measure stockholders' equity as the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock.

t based on their prior 11-month returns from month $t-12$ to $t-2$. Skipping month $t-1$, we calculate monthly decile returns for month t and rebalance the deciles at the beginning of month $t+1$ (Fama and French 1996). Liu and Zhang (2014) follow Jegadeesh and Titman (1993), sort on the prior 6-month return, skip one month, and hold the deciles for the subsequent 6-month period. To simplify the portfolio construction, we avoid the resulting six overlapping sets of momentum deciles with only 1-month holding period. We emphasize that the momentum profits from the R^{11} deciles are higher than those in Liu and Zhang, raising the hurdle for the structural model to explain.

To form the asset growth (I/A) deciles, at the end of June of each year t, we sort stocks on I/A, defined as total assets (Compustat annual item AT) for the fiscal year ending in calendar year $t-1$ divided by total assets for the fiscal year ending in $t-2$ (Cooper, Gulen, and Schill 2008). Monthly decile returns are from July of year t to June of $t+1$, and the deciles are rebalanced in June of $t+1$.

Return on equity (Roe) is income before extraordinary items (Compustat quarterly item IBQ) divided by 1-quarter-lagged book equity.[6] At the beginning of each month t, we sort stocks into deciles on their most recent past Roe. Before 1972, we use the most recent Roe computed with quarterly earnings from the fiscal quarter ending at least four months ago. From 1972 onward, we use Roe computed with quarterly earnings from the most recent quarterly earnings announcement date (item RDQ). For a firm to enter the portfolio formation, we require the end of the fiscal quarter corresponding to its most recent Roe to

[6] From 1972 onward, quarterly book equity is shareholders' equity, plus balance sheet deferred taxes and investment tax credit (item TXDITCQ) if available, minus the book value of preferred stock (item PSTKQ). Depending on availability, we use stockholders' equity (item SEQQ), or common equity (item CEQQ) plus the book value of preferred stock, or total assets (item ATQ) minus total liabilities (item LTQ) in that order as shareholders' equity. Prior to 1972, we expand the sample coverage by using book equity from Compustat annual files and imputing quarterly book equity with clean surplus accounting (Hou, Xue, and Zhang 2019).

be within six months prior to the portfolio formation and its earnings announcement date to be after the corresponding fiscal quarter end. Monthly decile returns are calculated for the current month t, and the deciles are rebalanced at the beginning of month $t + 1$.

Table 1 shows the descriptive statistics of the 40 testing deciles and the high-minus-low deciles from January 1967 to June 2017. The value, momentum, investment, and Roe premiums, measured as the average returns of the high-minus-low Bm, R^{11}, I/A, and Roe deciles, are 0.47%, 1.12%, −0.36%, and 0.68% per month (t = 2.15, 3.88, −2.2, and 3.01), respectively.

2.3.2 Measurement. In the model, time-t stock variables are at the beginning of period t, and time-t flow variables are over the course of period t. However, in Compustat, both stock and flow variables are recorded at the end of period t. As such, for the year $t = 2010$, for example, we take time-t stock variables from the 2009 balance sheet, and time-t flow variables from the 2010 income or cash flow statement.

We measure output, Y_{it}, as sales (Compustat annual item SALE) and short-term working capital as current assets (item ACT). Total debt, B_{it+1}, is long-term debt (item DLTT, zero if missing) plus short-term debt (item DLC, zero if missing). The market leverage, w_{it}^{B}, is the ratio of total debt to the sum of total debt and market equity (from CRSP). The tax rate, τ_t, is the statutory corporate income tax rate from the Commerce Clearing House's annual publications.

To measure the physical capital, K_{it}, some studies, including Liu, Whited, and Zhang (2009), use gross property, plant, and equipment (PPE, Compustat annual item PPEGT). Other studies, such as Liu and Zhang (2014), use net PPE (item PPENT). We use net PPE because it is more appropriate than gross PPE as the measure of K_{it} in the model. In

Compustat, gross PPE is the accumulated historical cost of tangible fixed assets, and net PPE is gross PPE minus accumulated depreciation. Also, net PPE is a component of total assets, but gross PPE is not. In the model's notations, firm i's gross PPE at the beginning of year t is $K_{i0} + \sum_{s=0}^{t-1} I_{is}$, its accumulated depreciation $\sum_{s=0}^{t-1} \delta_{is} K_{is}$, and its net PPE is K_{it}. Clearly, gross PPE should not be used to measure K_{it}.[7]

Many studies measure the depreciate rate of physical capital, δ_{it}, as the amount of depreciation and amortization (Compustat annual item DP) divided by physical capital (item PPENT). We subtract the amortization of intangibles (item AM, zero if missing) from item DP, before scaling the difference by net PPE. This measure is more accurate. In the data, the AM/DP ratio is on average 6.6%, with a standard deviation of 14.3%. The AM/DP distribution has a long right tail. Its median is 0%, but the 75th, 90th, and 95th percentiles are 4.7%, 25.7%, and 41.3%, respectively.

Many studies measure investment, I_{it}, as capital expenditures (Compustat annual item CAPX) minus sales of PPE (item SPPE, zero if missing). However, this investment measure violates the capital accumulation equation, $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$, in the data. The

[7] Write out $K_{is+1} = (1 - \delta_{is})K_{is} + I_{is}$ for $s = 0, 1, \ldots, t-1$, in which year 0 is when firm i first appears in Compustat:

$$
\begin{aligned}
K_{i1} &= (1 - \delta_{i0})K_{i0} + I_{i0} \\
K_{i2} &= (1 - \delta_{i1})K_{i1} + I_{i1} \\
\vdots & \qquad \vdots \\
K_{it} &= (1 - \delta_{it-1})K_{it-1} + I_{it-1}
\end{aligned}
$$

Recursively substituting K_{is} for $s = 0, 1, \ldots, t-1$ and collecting terms, we obtain:

$$
K_{it} = \left(K_{i0} + \sum_{s=0}^{t-1} I_{is} \right) - \sum_{s=0}^{t-1} \delta_{is} K_{is},
$$

which corresponds to the definition of net PPE as gross PPE minus accumulated depreciation in financial accounting. Hulten (1991, p. 126) also shows that it is the net stock of capital that is consistent with the production function, and the gross stock of capital is consistent only when assets retain full efficiency until falling apart completely.

differences between CAPX−SPPE and $K_{it+1} - (1 - \delta_{it})K_{it}$ are more than 10.28%, 31.5%, and 57.45% of physical capital, K_{it}, in magnitude, for 25%, 10%, and 5% of the firm-level observations, respectively (the Internet Appendix). A possible reason is that item SPPE is not available before 1971 in Compustat and is missing for 23.43% of the observations from 1971 onward in our sample. Also, item SPPE only records cash inflows from asset sales, but not equity inflows (Slovin, Sushka, and Polanchek 2005).

More important, mergers and acquisitions (M&As) play a role in explaining the deviations. We identify M&As by combining the Securities Data Company (SDC) dataset and Compustat (item AQC, acquisitions). M&As are prevalent. The subsample that contains only the observations with M&As of any size accounts for 38.63% of the observations in the full sample. The capital accumulation deviations are substantially larger in this subsample than in the full sample. The deviations are more than 19.28%, 53.35%, and 94.59% of physical capital in magnitude for 25%, 10%, and 5% of the observations, respectively, in the subsample with only M&As.[8] Because our estimation procedure requires the construction of the firm-level fundamental returns, in which investment-to-physical capital is a key component, we opt to measure I_{it} directly as $K_{it+1} - (1 - \delta_{it})K_{it}$.

We emphasize that M&As are not random corporate events. Firms with M&As are more likely to be growth firms, momentum winners, high investment firms, and high profitability firms than firms without M&As. As such, we retain firms with M&As in our sample to facilitate identification. Our results are robust if we follow Whited (1992) in excluding

[8]However, M&As do not fully explain the capital accumulation deviations. In the subsample that contains only observations without M&As, the deviations account for more than 7.09%, 23.08%, and 43.23% of physical capital for 25%, 10%, and 5% of the observations, respectively (the Internet Appendix). As such, the deviations are more general than M&As. Possible reasons include capital retirements, gains and losses from sales of long-term assets, restructuring charges and impairment losses, and foreign currency translations (Wahlen, Baginski, and Bradshaw 2018).

observations with sizeable M&As, in which the target's assets are at least 15% of the acquirer's assets (the Internet Appendix).[9]

Finally, to measure the cost of debt in a broad sample, prior studies impute credit ratings for firms with no credit ratings data in Compustat and then assign the corporate bond returns for a given credit rating to all the firms with the same credit rating. This imputed measure captures heterogeneity in the cost of debt only across a few categories of credit ratings and likely contains estimation errors. We instead compute the pre-tax cost of debt as the ratio of total interest and related expenses (Compustat annual item XINT) scaled by total debt, B_{it+1}. This measure increases the sample coverage by 12.7% and also facilitates our goal of accounting for heterogeneity.[10]

2.3.3 Timing alignment. We follow Liu and Zhang (2014) in aligning the timing of stock returns and accounting variables. Due to the large number of data items required to construct the firm-level fundamental return, we work with the Compustat annual files, as opposed to the quarterly files, because of their limited coverage for many of the data items. We construct monthly fundamental returns from annual accounting variables to match with monthly stock returns. For each month, we take firm-level accounting variables from the fiscal year end that is closest to the month in question to measure (flow) variables dated t in the model and take accounting variables from the subsequent fiscal year end to measure (flow) variables dated $t+1$ in the model. Because the portfolio composition

[9] We should acknowledge that measuring investment as $K_{it+1} - (1 - \delta_{it})K_{it}$ implicitly assumes that internal growth in physical capital and external growth via M&As face the same adjustment costs technology. This assumption is for parsimony only because treating M&As separately would take us too far afield and complicate the econometric specification. However, the basic idea of the investment theory also applies to M&As (Jovanovic and Rousseau 2002).

[10] Our results are robust if we instead use the imputed cost of debt measure (the Internet Appendix). The crux is that the identifying information in the structural estimation comes mostly from the cross section of the cost of equity. Relative to the cost of equity, the dispersion in the cost of debt is economically small.

can change monthly (and firms end a given fiscal year and update accounting variables in different months), the portfolio fundamental returns aggregated from the firm level also change monthly.

While portfolio stock returns are in monthly terms and in monthly frequency, portfolio fundamental returns are in monthly frequency but in annual terms, constructed from annual accounting variables. To align the units, Liu and Zhang (2014) annualize monthly portfolio stock returns in a given month to match with portfolio fundamental returns for the month in question. This procedure creates timing mismatch, as the portfolio stock returns are for a given month, but the matching fundamental returns are based on annual accounting variables both prior to and after the month.

To better align the timing, we compound the portfolio stock returns within a 12-month rolling window with the end of the month in question in the middle of the window. We multiply simple gross returns from month $t-5, t-4, \ldots, t, t+1, \ldots$, and $t+6$ to match with the fundamental returns constructed in month t. Applying this rolling procedure to the monthly returns of testing deciles (January 1967–June 2017) yields the monthly observations of annual stock returns from June 1967 to December 2016 to match with the fundamental returns constructed over the same sample period.

2.3.4 Descriptive properties of the accounting variables. We report descriptive statistics for firm-level accounting variables in the fundamental returns. As noted, the sample for the fundamental returns is from June 1967 to December 2016 to align with the portfolio stock returns from the 12-month rolling procedure. However, it is important to note that the accounting variables underlying the fundamental returns for June 1967 can come from the fiscal year ending in calendar time as early as December 1965, and the accounting variables underlying the fundamental returns for December 2016 can come from as late as May 2018. In all, our guiding principle in the sample construction is to maximize the data coverage both across firms and over time.

To mitigate the impact of outliers, we winsorize 5% of the extreme observations at the portfolio formation. We winsorize the unbounded variables such as physical investment-to-capital, I_{it}/K_{it}, at the 2.5–97.5% level. For variables that are bounded below at zero, such as sales-to-total capital, $Y_{it+1}/(K_{it+1}+W_{it+1})$, and the depreciation rate of physical capital, δ_{it+1}, we winsorize at the 0–95% level. Finally, we do not winsorize the fraction of physical capital in total capital, $K_{it+1}/(K_{it+1}+W_{it+1})$, or the market leverage, w_{it}^B, because both are bounded between zero and one.

Table 2 reports the descriptive statistics of firm-level accounting variables in the fundamental returns, while Figure 1 reports the histograms of the variables both at the firm level and the portfolio level. From Table **??**, the mean firm-level physical investment-to-capital, I_{it}/K_{it}, is 0.36, with a standard deviation of 0.44. The mean is more than 50% higher than the median of 0.23, indicating a skewed distribution of the firm-level I_{it}/K_{it}. From Panel A of Figure 1, the histogram of the firm-level I_{it}/K_{it} distribution is highly asymmetric, with a long right tail.

The relatively high mean I_{it}/K_{it} is due to our more accurate measure of K_{it} as net PPE. If instead we scale investment by gross PPE, the mean I_{it}/K_{it} is only 0.19, with a standard deviation of 0.23 and a median of 0.12 (untabulated). If we measure investment as capital expenditures (Compustat annual item CAPX) minus sales of PPE (item SPPE) but still use gross PPE as the scalar, the mean I_{it}/K_{it} is 0.17, with a standard deviation of 0.17 and a median of 0.11. Finally, if we scale the difference between CAPX and SPPE by net PPE, the mean I_{it}/K_{it} goes back to 0.3, with a standard deviation of 0.3 and a median of 0.21. The mean and standard deviation are close to the estimates of 0.29 and 0.27, respectively, reported in Belo and Lin (2012) in their 1965–2009 sample.

From Table 2, the mean working capital investment rate, $\triangle W_{it}/W_{it}$, is 0.13, with a standard deviation of 0.32. Disinvestment in working capital is much more frequent than disinvestment in physical capital, as the 5th percentile of $\triangle W_{it}/W_{it}$ is -0.3 but -0.03 for I_{it}/K_{it}. On average, physical capital accounts for only 38% of the sum of physical capital and working capital, and the 25th and 75th percentiles of this fraction are 18% and 55%, respectively. This evidence indicates the importance of accounting for capital heterogeneity in the data.

The ratio of sales to total capital, $Y_{it+1}/(K_{it+1}+W_{it+1})$, is on average 1.62, which is close to the median of 1.5, and its standard deviation is only 0.93. In contrast, sales-to-physical capital, Y_{it+1}/K_{it+1}, has a mean of 9.05, a median of 5.24, and a standard deviation of 11.59. As such, Y_{it+1}/K_{it+1} is much more volatile and more skewed than $Y_{it+1}/(K_{it+1} + W_{it+1})$. The evidence indicates that $Y_{it+1}/(K_{it+1} + W_{it+1})$ is a more appropriate measure of the average product of capital than Y_{it+1}/K_{it+1} in the model and in the data. The rate of capital depreciation is on average 19%, with a standard deviation of 12%. The market leverage, w^B_{it}, is on average 0.26, with a standard deviation of 0.22. For the pre-tax cost of debt, the mean is 8.74%, and the standard deviation 5.77%.

Table 2 also reports pairwise correlations of the accounting variables. The investment rate in physical capital, I_{it}/K_{it}, and the investment rate in working capital, $\triangle W_{it}/W_{it}$, have a positive correlation of 0.3. I_{it}/K_{it} has an autocorrelation of 0.32. In contrast, $\triangle W_{it}/W_{it}$ has an autocorrelation of only 0.04, which accords well with our assumption of zero adjustment costs on working capital. I_{it+1}/K_{it+1} has positive correlations of 0.36 and 0.2 with sales-to-physical capital, Y_{it+1}/K_{it+1}, and sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, respectively, but a zero correlation with Y_{it+1}/W_{it+1}. Similarly, $\triangle W_{it+1}/W_{it+1}$ have positive correlations of 0.25 and 0.2 with Y_{it+1}/W_{it+1} and $Y_{it+1}/(K_{it+1} + W_{it+1})$, respectively, but a small correlation of 0.09 with Y_{it+1}/K_{it+1}.

Finally, from Figure 1, aggregating firm-level variables to the portfolio level eliminates a great deal of heterogeneity. Firm-level investment-to-physical capital, I_{it}/K_{it}, varies from -0.5 to 2.5, but the portfolio-level between -0.5 and 1.0, while centering about 0.25. Firm-level sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, varies from 0.0 to 4.5, whereas the portfolio-level from 0.4 to 2.5. The firm-level Y_{it+1}/K_{it+1} distribution is much more dispersed, ranging from 0.0 to 50, whereas the portfolio-level Y_{it+1}/K_{it+1} ranges from 0.0 to 7.0. The firm-level pre-tax cost of debt, r^B_{it+1}, varies from 0.0 to 0.4, whereas the portfolio-level r^B_{it+1} mostly from 0.0 to 0.12. The firm-level distribution of r^B_{it+1} has a spike at zero because we treat zero-debt firms as having zero cost of debt.

3 Estimation Results

We first replicate the key findings in the prior studies that estimate the physical capital model at the portfolio level in Section 3.1. In Section 3.2, we report the results from the benchmark 2-capital model estimated at the firm level. In Section 3.3, we quantify the impact of aggregation and capital heterogeneity by estimating the 2-capital model at the portfolio level and the physical capital model at the firm level, respectively. In Section 3.4, we use the fundamental returns implied from the benchmark 2-capital model estimated at the firm level to examine the dynamics of factor premiums. Finally, in Section 3.5, we examine the model's out-of-sample performance.

3.1 Replicating the Prior Studies

Panel A of Table 3 reports the GMM estimation and tests for the physical capital model estimated directly at the portfolio level, without first constructing firm-level fundamental returns. Consistent with prior studies, the physical capital model does a good job in accounting for value and momentum separately but fails to do so jointly. The failure is reflected in, for example, the large average absolute high-minus-low alpha in the joint value and momentum estimation, 7.02% per annum, which is substantially higher than 0.32% and 1.46% in the separate value and momentum estimation, respectively. The parameters also appear unstable across the testing deciles when estimated separately. The marginal product parameter, γ_K, is 0.166 with the book-to-market deciles but 0.12 with the momentum deciles. For the adjustment costs parameter, a, the contrast is between 6.27 and 1.28.[11]

Figure 2 reports the alphas of individual deciles by plotting average predicted stock returns against average realized stock returns across the value and momentum deciles as well as across all the 40 testing deciles in the joint estimation. The physical capital model manages to explain the momentum premium but fails entirely for the value premium. Panel A shows that with value and momentum jointly, the model predicts a *negative* value premium of -2.46% per annum, in contrast to 6.39% in the data. The high-minus-low alpha is economically large, 8.85%, and statistically significant ($t = 2.76$). The model also pre-

[11] Prior studies use equal-weighted deciles. The Internet Appendix shows that the joint estimation failure is more severe with equal-weighted deciles (Table A.3). The marginal product parameter, γ_K, is estimated to be 0.251 and the adjustment costs parameter, a, 15.03 with the book-to-market deciles, but 0.128 and 1.34, respectively, with the momentum deciles. In the joint value and momentum estimation, the γ_K estimate is 0.142, and a 3.19. As a result, the average absolute high-minus-low alpha in the joint estimation is 12.49% per annum, which is substantially larger than 3.25% and 0.12% in the separate value and momentum estimation, respectively.

dicts a momentum premium of 20.17%, overshooting the data moment of 14.97% with a high-minus-low alpha of -5.2% ($t = -2.63$).[12]

From Panel B, adding the asset growth and Roe deciles exacerbates the model's failure in explaining the value premium in the joint estimation. With all 40 testing deciles together, the model predicts a value premium of -4.72% per annum, giving rise to a large alpha of 11.11% ($t = 3.89$). The model does well in predicting a momentum premium of 16.17%, with a small alpha of -1.2% ($t = -0.48$), and an investment premium of -6.88%, with an alpha of 1.57% ($t = 0.79$). Finally, the model predicts an Roe premium of 11.02%, with an alpha of -2.59% ($t = -1.05$).

3.2 The Benchmark 2-capital Specification

From Panel B of Table 3, our benchmark 2-capital model estimated at the firm level succeeds in explaining value and momentum simultaneously. A first indication is that the parameter estimates are more stable across the testing deciles. The marginal product parameter, γ, is 0.176 with the book-to-market deciles and 0.134 with the momentum deciles. For the adjustment costs parameter, a, the contrast is between 3.75 and 8.11. In terms of pricing errors, with value and momentum jointly, the average absolute high-minus-low alpha is only 0.77% per annum, which is an order of magnitude smaller than 7.02% from the physical capital model estimated at the portfolio level. The mean absolute alpha is also smaller in the benchmark model than in the physical capital model, 1.27% versus 2.9%. However, the benchmark model is still rejected by the overidentification test. Finally, adding the asset

[12] The failure in fitting the equal-weighted deciles is again more severe. The Internet Appendix (Figure A.1) shows that the model predicts a large, negative value premium of -7.52% per annum, in contrast to an observed value premium of 8.89%. The high-minus-low alpha is massive, 16.41% ($t = 5.05$). The model implied momentum premium is 24.8%, relative to the data moment of 16.24%, giving rise to a high-minus-low alpha of -8.57% ($t = -4.28$).

growth and Roe deciles does not materially change the results.[13]

Figure 3 plots average predicted stock returns from the benchmark estimation against average realized stock returns across the testing deciles. The model performs well, and the scatter points are mostly aligned with the 45-degree line. In particular, Panel A shows that when fitting value and momentum deciles jointly, the model predicts a value premium of 5.2% per annum (6.39% in the data), giving rise to a small alpha of 1.18% ($t = 0.51$). The model also predicts a momentum premium of 14.62% (14.97% in the data), with an even smaller alpha of 0.35% ($t = 0.12$).[14]

Panel B shows that the scatter plots continue to align largely along the 45-degree line even after adding the asset growth and Roe deciles, although the alphas increase somewhat in magnitude. The model predicts a value premium of 3.29% per annum, with an alpha of 3.09% ($t = 1.37$), and a momentum premium of 13.42%, with an alpha of 1.55% ($t = 0.5$). The investment premium is -5.05% in the model (-5.11% in the data), giving rise to a tiny alpha of -0.06% ($t = -0.04$). Finally, the Roe premium is 6.2% in the model (8.43% in the data), with an alpha of 2.23% ($t = 0.89$). Overall, although the alpha for the value premium, 3.29%, is not small, the improvement of the benchmark model estimated at the firm level over the physical capital model estimated at the portfolio level (which yields an alpha of 11.11% for the value premium) is substantial.

[13]The improvement relative to the physical capital model estimated at the portfolio level is more visible in the equal-weighted deciles. The Internet Appendix (Table A.3) shows that the marginal product parameter, γ, is 0.167 with the book-to-market deciles and 0.165 with the momentum deciles. For the adjustment costs parameter, a, the contrast is between 3.93 and 3.02. As a result, the average absolute high-minus-low alpha in the joint value and momentum estimation is only 1.23% per annum, which is an order of magnitude smaller than 12.49% in the physical capital model. The mean absolute alpha is also much smaller, 0.83% versus 4.06%.

[14]For equal-weighted deciles, the Internet Appendix (Figure A.2) shows that the model predicts a large, positive value premium of 7.48% per annum (8.89% in the data), with a relatively small alpha of 1.41% ($t = 0.67$). The model implied momentum premium is 17.28% (16.24% in the data), giving rise to a small alpha of -1.04% ($t = -0.34$).

3.2.1 Intuition: Current investment, expected investment, and expected returns. What are the economic mechanisms behind the value, momentum, investment, and Roe premiums in the benchmark model? In a 2-period setting, with $I_{it+1} = 0$, equation (2) reduces to:

$$r_{it+1}^K = \frac{(1-\tau_{t+1})\gamma_K(Y_{it+1}/K_{it+1}) + (\tau_{t+1}-1)\delta_{it+1} + 1}{1 + (1-\tau_t)a(I_{it}/K_{it})}. \tag{10}$$

All else equal, stocks with low current investment, I_{it}/K_{it}, should earn higher expected returns than stocks with high current investment, and stocks with high profitability, Y_{it+1}/K_{it+1}, should earn higher expected returns than stocks with low profitability. Intuitively, given expected profitability, high costs of capital give rise to low net present values of new projects and low investment. Given investment, high expected profitability imply high costs of capital, which are necessary to induce low net present values of new projects to keep investment constant (Hou, Xue, and Zhang 2015).

In the multiperiod model, equation (2) implies that the cost of capital is also linked to the next period investment, I_{it+1}/K_{it+1}. The intuition is analogous to the positive profitability-expected return relation. The term, $1+(1-\tau_t)a(I_{it+1}/K_{it+1})$, in the numerator of equation (2) is the marginal cost of investment next period, which, per the first principle of investment, equals the marginal q_{it+1} next period. The marginal q_{it+1}, as the present value of all future cash flows generated from one extra unit of physical capital next period, represents an important part of the marginal benefit of current investment. As such, high next period investment (relative to current investment) must imply high current costs of capital to offset the high next period marginal benefit of current investment.

3.2.2 Comparative statics. To quantify these mechanisms, we conduct comparative statics on the current investment-to-physical capital, I_{it}/K_{it}, the next period investment-to-physical capital, I_{it+1}/K_{it+1}, and the next period sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, which measures profitability in the 2-capital model. Other variables also matter, but their quantitative impact is not nearly as important.

In the experiment on I_{it}/K_{it}, we set I_{it}/K_{it} to be its cross-sectional median at period t across all firms. We use the parameter estimates from the benchmark estimation with all the 40 deciles jointly to reconstruct the fundamental returns and recalculate the model's alphas as the average portfolio stock-minus-fundamental returns. If the resulting alphas are large relative to those from the benchmark estimation, we can infer that the I_{it}/K_{it} spread is quantitatively important to explain the average return spreads. The other comparative statics are designed analogously.

Table 4 shows that the current investment, I_{it}/K_{it}, and the next period investment, I_{it+1}/K_{it+1}, are the two most important drivers of expected stock returns. I_{it}/K_{it} is more important than I_{it+1}/K_{it+1} for the value and investment premiums, but I_{it+1}/K_{it+1} is more important than I_{it}/K_{it} for the momentum and Roe premiums. Intuitively, I_{it}/K_{it} and I_{it+1}/K_{it+1} are locked in a "tug of war." When I_{it}/K_{it} overpowers I_{it+1}/K_{it+1}, the model predicts the value and investment premiums. Otherwise, the model predicts the momentum and Roe premiums. Most surprisingly, the premiums of value and momentum, as well as investment and profitability, are all driven by closely related, if not identical, mechanisms. $Y_{it+1}/(K_{it+1} + W_{it+1})$ also plays a role, especially for the Roe premium.

From Panel A, I_{it}/K_{it} is essential for the value premium. Removing its cross-sectional variation gives rise to a large, positive alpha of 36.28% per annum for the value premium. Intuitively, firms that invest more are growth firms with high marginal q, which equals the marginal cost of physical investment, $q_{it} = 1 + (1 - \tau)a(I_{it}/K_{it})$. Firms that invest less are value firms with low marginal q. In the data, the average cross-sectional correlation between I_{it}/K_{it} and book-to-market is -0.23. Because the marginal cost of investment is in the denominator of equation (2), growth firms with high I_{it}/K_{it} have lower fundamental returns than value firms with low I_{it}/K_{it}. Fixing I_{it}/K_{it}, we hold the denominator of equation (2) constant to shut down this mechanism, yielding a large alpha.

The next period investment-to-physical capital, I_{it+1}/K_{it+1}, is the countervailing force of current investment, I_{it}/K_{it}. Fixing I_{it+1}/K_{it+1} across firms yields a large, negative alpha of -27.79% per annum for the value premium. Intuitively, growth firms also invest more and have higher marginal q next period than value firms. In the data, the average cross-sectional correlation between I_{it+1}/K_{it+1} and book-to-market is -0.19. Because I_{it+1}/K_{it+1} appears in the numerator of equation (2), the I_{it+1}/K_{it+1} spread implies that growth firms should have higher expected returns than value firms, countervailing the I_{it}/K_{it} spread from the denominator. On net, I_{it}/K_{it} dominates I_{it+1}/K_{it+1} across the book-to-market deciles, allowing the model to yield a positive value premium.

Panel B shows that the next period investment-to-physical capital, I_{it+1}/K_{it+1}, is the most important driver of momentum, and the current I_{it}/K_{it} is the countervailing force. Fixing I_{it+1}/K_{it+1} across firms yields a large, positive alpha of 20.71% per annum for the momentum premium. Intuitively, winners are expected to have higher marginal q_{it} and investment next period than losers. In the data the average cross-sectional correlation between I_{it+1}/K_{it+1} and prior 11-month returns, R^{11}, is 0.19. This expected investment mechanism implies that winners should have higher expected returns than losers. The current I_{it}/K_{it} is the offsetting force, but weaker. Fixing its cross-sectional variation yields a negative alpha of -7.65%, but its magnitude is substantially smaller than 20.71% from fixing I_{it+1}/K_{it+1}. In the data the average cross-sectional correlation between I_{it}/K_{it} and R^{11} is lower, 0.09. On net, I_{it+1}/K_{it+1} dominates I_{it}/K_{it}, allowing the model to explain momentum.

Not surprisingly, Panel C shows that current investment, I_{it}/K_{it}, is the most important driver for the investment premium. Fixing I_{it}/K_{it} across firms yields an alpha of -21.75% per annum for the investment premium. In the data the average cross-sectional correlation between I_{it}/K_{it} and asset growth is 0.18. The next period I_{it+1}/K_{it+1} is the countervailing force. Fixing its cross-sectional variation yields an alpha of 13.36%, but its magnitude is smaller than that of -21.75% from fixing I_{it}/K_{it}. The average cross-sectional correlation between asset growth and I_{it+1}/K_{it+1} is 0.09. The economic mechanism for the investment premium is similar to that for the value premium.

Finally, from Panel D, the next period investment-to-physical capital, I_{it+1}/K_{it+1}, is the most important driver of the Roe premium. Fixing I_{it+1}/K_{it+1} across firms yields an alpha of 14.83% per annum for the Roe premium. Sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$, reinforces I_{it+1}/K_{it+1} in the numerator of equation (2). Removing its dispersion yields an alpha of 8.72%. The current I_{it}/K_{it} is the countervailing force. Fixing its cross-sectional variation yields an alpha of -6.34%. On net, the combined effect from I_{it+1}/K_{it+1} and $Y_{it+1}/(K_{it+1} + W_{it+1})$ in the numerator dominates the I_{it}/K_{it} effect in the denominator, allowing the model to yield a positive Roe premium.

3.3 Alternative Econometric Specifications

To shed light on the sources of the improvement of the benchmark specification relative to prior studies, we quantify the impact of aggregation and capital heterogeneity in this subsection.

3.3.1 Aggregation. Panel A of Table 5 estimates the 2-capital model at the portfolio level. Instead of constructing firm-level fundamental returns, we aggregate firm-level accounting variables to the portfolio level and then construct fundamental returns directly at the portfolio level. The portfolio-level estimation yields larger alphas. For example, with value and momentum jointly, the mean absolute alpha is 1.52% per annum, and the average absolute high-minus-low alpha 2.26%. Both are larger than 1.27% and 0.77%, respectively, from the firm-level estimation (Panel B of Table 3).

Figure 4 shows the scatter plots of average predicted stock returns from the portfolio-level estimation of the 2-capital model versus average realized stock returns. The model struggles to fit the value premium in the joint estimation. With value and momentum jointly (Panel A), the value premium is only 2.88% in the model, with an alpha of 3.51%, albeit insignificant ($t = 1.23$). With asset growth and Roe added to the joint estimation (Panel B), the value premium drops further to 1.45% in the model, with an alpha of 4.94% ($t = 1.93$). Intuitively, the amount of heterogeneity in the accounting variables is substantial at the firm level (Figure 1). This heterogeneity is dampened greatly once the variables are aggregated to the portfolio level. As such, estimating the 2-capital model at the firm level is more "structural" (and more accurate) than at the portfolio level.

3.3.2 Capital heterogeneity. To quantify the impact of working capital as a separate input in the benchmark 2-capital model, we estimate the physical capital model at the firm level. Panel B of Table 5 shows that without working capital, the physical capital model with the new, exact aggregation yields a mean absolute alpha of 2.43% per annum and an average absolute high-minus-low alpha of 7.02%. These alphas are much larger than 1.27% and 0.77%, respectively, from the benchmark 2-capital specification.

The γ_K estimates in Panel B of Table 5 are lower than those from the portfolio-level estimation (Panel A of Table 3). The crux is that the firm-level distribution of sales-to-capital, Y_{it+1}/K_{it+1}, is highly skewed, but the portfolio-level Y_{it+1}/K_{it+1} distribution is substantially less dispersed (Figure 1). The lower γ_K estimates reflect the different Y_{it+1}/K_{it+1} distribution at the firm level. The γ_K estimates are also lower than the γ estimates in the 2-capital model at the firm level. The key is that sales-to-total capital, $Y_{it+1}/(K_{it+1}+W_{it+1})$, is much less dispersed and less skewed than sales-to-physical capital, Y_{it+1}/K_{it+1}. As noted, physical-to-total capital, $K_{it+1}/(K_{it+1}+W_{it+1})$, is on average only 0.38 (Table 2). As such, incorporating working capital more accurately characterizes the firm-level distributions of the average product of capital and the fundamental returns.

Figure 5 shows the scatter plots of average predicted stock returns from the firm-level estimation of the physical capital model versus average realized stock returns. The model struggles to explain the average returns across the testing deciles. With value and momentum jointly (Panel A), the value premium is 1.64% per annum in the model, with an alpha of 4.75% ($t = 1.8$). The model also exaggerates the momentum premium to 20.17%, yielding a large, negative alpha of -9.29% ($t = -2.79$). With asset growth and Roe added to the joint estimation (Panel B), the value premium deteriorates further to -2.14% per annum in the model, giving rise to a large alpha of 8.52% ($t = 3.41$). The momentum premium becomes 21.36%, with an alpha of -6.39% ($t = -1.89$).

3.4 Diagnostics: The Dynamics of Factor Premiums

In this subsection we use the fundamental returns implied from the benchmark 2-capital model estimated at the firm level to study the dynamics of factor premiums. Because the parameters are estimated from only matching the average returns across the testing portfolios, the dynamics are economically important as separate diagnostics on the model's performance. We examine both calendar- and event-time dynamics. Finally, to construct the fundamental returns, we always use the parameter estimates from the joint estimation of all the 40 value-weighted testing deciles.

3.4.1 Correlations between stock and fundamental returns. Equation (5) implies that the stock and fundamental returns are equal ex post. However, Liu, Whited, and Zhang (2009) document a correlation puzzle that the contemporaneous correlations between the stock and fundamental returns are weakly *negative*. We match the fundamental returns for a given month (value-weighted to the portfolio level) with portfolio stock returns compounded across the 12-month rolling window surrounding the month in question. This rolling procedure better aligns the timing of stock and fundamental returns and helps resolve the correlation puzzle.

Table 6 shows that the contemporaneous correlations between stock and fundamental returns from the benchmark model are significantly positive. From Panel A, the time series average of cross-sectional correlations of the two types of returns is 0.11 across all firms and 0.19 across the 40 testing deciles. Both correlations are significant at the 1% level. At the firm level, the lead-lag correlations are all positive within the 12-month horizon but turn negative at longer horizons. At the portfolio level, the lead-lag correlations are all positive across the horizons within 60 months.

Panel B shows the time series correlation between the stock and fundamental returns for each testing decile. The correlations are positive and mostly significant for the extreme deciles and high-minus-low deciles. In particular, the correlations are 0.26 for the value premium and 0.42 for the investment premium. Both are significant at the 1% level. The correlations are 0.14 for the momentum premium and 0.16 for the Roe premium, but both are only marginally significant. Finally, we emphasize that equation (5) predicts perfect stock-fundamental return correlations across firms (and portfolios). The correlations in Table 6, while mostly positive, are far from perfect.[15]

3.4.2 Persistence of factor premiums. Fama and French (1995) show that the value premium subsists for three to five years after the portfolio formation, whereas Chan, Jegadeesh, and Lokonishok (1996) show that momentum profits are more short-lived, positive within the 12-month horizon but negative afterward. Liu and Zhang (2014) show that the physical capital model estimated at the portfolio level explains the short-lived dynamics of momentum. We show that the 2-capital model estimated at the firm level retains this success and also extend the evidence to the value, investment and Roe premiums.

Figure 6 reports the event-time dynamics of stock and fundamental returns of the high and low deciles during 36 months after the portfolio formation. From Panels A–D, in the data the value premium persists even after three years, whereas the momentum premium converges to zero after about ten months. The investment premium lasts about two years, and the Roe premium converges to zero within ten months. Panels E–H show that the

[15] In addition to Liu, Whited, and Zhang (2009), evidence against the perfect stock-fundamental return correlations is also presented in Delikouras and Dittmar (2018). While the former directly reports a weakly negative correlation between stock and fundamental returns, the latter indirectly examines this correlation by showing that a stochastic discount factor formed with (and constructed to price) fundamental returns cannot price stock returns, and vice versa. Measurement and specification errors in fundamental returns most likely drive these results, by breaking the perfect correlations. For this reason, we focus on the predictions that are more immune to measurement and specification errors in equation (5).

benchmark model succeeds in explaining the short-lived nature of the momentum and Roe premiums as well as the long-lived nature of the value and investment premiums. The fundamental returns mimic the stock returns in event-time dynamics.[16]

3.4.3 Market states and factor premiums. Cooper, Gutierrez, and Hameed (2004) show that momentum is large and positive following nonnegative prior 36-month market returns (Up markets) but negative following negative prior 36-month market returns (Down markets). Liu and Zhang (2014) show that the physical capital model estimated at the portfolio level fails to explain this evidence in that it predicts weakly countercyclical momentum profits. The benchmark 2-capital model helps resolve this difficulty. We also extend the evidence to the value, investment, and Roe factor premiums.

Panel A of Table 7 shows that the value premium is stronger following Down than Up markets identified with prior 36-month market returns, 17.19% versus 4.47% per annum. The model succeeds in explaining the countercyclical variation, 17.43% versus 0.7%. From Panel B, the momentum premium is stronger following Up than Down markets. With the market states again identified with prior 36-month market returns, the momentum premium is 19.36% following Up markets but −9.49% following Down markets. The contrast is 14.08% versus 9.78% in the model.

Panel C shows that the investment premium is stronger following Down than Up markets. With prior 12-month market returns defining the market states, the investment premium is −10.94% per annum following Down markets but −3.35% following Up markets. In the model the contrast is only −5.54% versus −4.86%, albeit going in the right direction.

[16] The Internet Appendix (Figure A.6) shows that the marginal q_{it} growth for physical capital exhibits the same short- and long-term dynamics as the fundamental returns. The marginal q_{it} growth reflects the "tug of war" between current investment, I_{it}/K_{it}, and future investment, I_{it+1}/K_{it+1}. The evidence shows that when I_{it}/K_{it} dominates I_{it+1}/K_{it+1}, the impact is long-lasting. However, when I_{it+1}/K_{it+1} overpowers I_{it}/K_{it}, the impact is short-lived.

Finally, from Panel D, the Roe premium is stronger following Up than Down markets. With prior 36-month market returns identifying the market states, the Roe premium is 11.13% following Up markets but −5.88% following Down markets. In the model the contrast is between 7.02% and 1.44%.

Although the dynamics in the model are weaker than those in the data, the benchmark model reproduces the procyclicality of the momentum and Roe premiums as well as the countercyclicality of the value and investment premiums. Intuitively, the 12-month rolling procedure (described in Section 2.3.3) allows us to better align the timing between stock and fundamental returns. In contrast, the procedure in Liu and Zhang (2014) creates a timing mismatch between stock and fundamental returns, messing up the cross correlations between the model's factor premiums and stock market returns.

3.4.4 Higher moments. Table 8 compares higher moments including volatility, skewness, and kurtosis of stock returns with those of fundamental returns. Several patterns emerge. First, the fundamental returns are less volatile, echoing Cochrane (1991) at the aggregate level. The stock return volatilities of the value, momentum, investment, and Roe premiums are 20%, 28%, 14%, and 20% per annum, in contrast to the fundamental return volatilities of 18%, 13%, 11%, and 14%, respectively. For individual deciles, the fundamental return volatilities are often less than one half of their stock return volatilities.

Second, the benchmark model largely fails to explain the negative skewness of momentum. Daniel and Moskowitz (2016) show that momentum tends to experience infrequent and persistent negative returns. Such crashes yield a negative skewness for the momentum premium. Panel B replicates their evidence. The momentum premium has a skewness of −1.78, albeit significant only at the 10% level. In contrast, the fundamental momentum premium shows a positive but small skewness of 0.3. Panel D extends the Daniel-Moskowitz evidence to the Roe premium. Its skewness is −0.84, which is significant at the 10% level, and the model predicts a skewness of −0.38.

Third, the model does better in explaining kurtosis. For the value premium, the kurtosis is 3.28 for stock returns and 4.03 for fundamental returns. The fundamental returns also match the kurtosis of the stock returns for the investment premium, 3.44 versus 3.18. However, the model falls far short for momentum, 11.59 versus 5.29, but comes close for the Roe premium, 5.75 versus 4.45.

Figure 7 plots the time series of stock and fundamental factor premiums. The fundamental returns track the stock returns well, reflecting the economically large and statistically significant correlations in Table 6. However, the fundamental returns clearly fall short in explaining the extreme movements in the momentum and Roe premiums. In particular, the momentum premium experiences a crash of −168% in the 12 months around August 2009, but its fundamental return falls no more than 50%. The Roe premium experiences a crash of −110% in the 12 months around August 1999, but its fundamental return is positive, 8.7%. Overall, unlike the first moment, the benchmark model's performance in explaining higher moments of stock returns leaves much to be desired. Intuitively, as noted in Section 2.1.1, measurement errors in the fundamental returns tend to be averaged out when matching the first moment. However, these errors do impact on higher moments.

3.5 Out-of-sample Performance

We study the out-of-sample performance in two ways. First, we recursively estimate the model's parameters and evaluate the fit with 1-period-ahead alphas (Section 3.5.1). Second, we construct the cross-sectional forecasts of 1-period-ahead sales growth and investment-to-physical capital, combine the forecasts with the recursively estimated parameters to form expected return estimates, and sort portfolios on these estimates to evaluate the model's ability to predict subsequent returns (Section 3.5.2). For comparison with the benchmark 2-capital model estimated at the firm level, we also report the out-of-sample tests for the physical capital model estimated at the portfolio level, as well as the Hou-Xue-Zhang (2015) q-factor model and the Fama-French (2015) 5-factor model. Both are directly connected to the fundamental return equation (5).

3.5.1 Recursive estimation. At the beginning of each month from July 1980 to December 2017, we recursively estimate the model parameters from an expanding window that starts in June 1967. The starting point, June 1967, is identical to that of the in-sample estimation in that the accounting variables underlying the fundamental returns for June 1967 can come from as early as December 1965. However, crucially differing from the in-sample estimation, the latest accounting variables in the first recursive estimation must come from the fiscal year ending at least four months prior to the beginning of July 1980 (not later than February 1980).[17] We impose this 4-month lag to ensure no look-ahead bias. We expand the recursive windows one month at a time until December 2017.

With the recursive parameters, we calculate the 1-month-ahead fundamental returns with the next month's out-of-sample accounting variables and compare these fundamental returns to the 1-month-ahead stock returns. The differences between the 1-month-ahead

[17]The stock returns in the first recursive estimation window end much earlier than February 1980 (March 1979, in effect). The reason is that the fundamental return for September 1978 (which is matched to the stock return cumulated over the 12-month rolling window ending in March 1979) is the latest fundamental return that uses accounting information no later than February 1980. In particular, the fundamental return for October 1978 uses time-t investment, I_{it}, from March 1979 for firms with March fiscal year end as well as next period investment, I_{it+1}, from March 1980.

stock and fundamental returns are defined as the 1-month-ahead alphas. This procedure, which combines the recursive parameters with the next month's realized accounting variables (instead of their forecasts), is in the same spirit as in Fama and French (1997). We tackle the forecasting problem in Section 3.5.2.

For the q-factor model and the Fama-French 5-factor model, we use the 60-month rolling window to estimate the factor loadings of testing deciles and combine the loadings with the next month's realized factor premiums to generate the 1-month-ahead predicted decile returns. The predicted returns are in monthly terms for a given month. To ease comparison with the structural model, we use the same 12-month rolling procedure (described in Section 2.3.3) to convert the monthly to annual predicted returns, which we compare with the annual stock returns from the same rolling procedure.

Figure 8 reports the 1-period-ahead fits of the 40 testing deciles. From Panel A, the scatter plots of average predicted against average realized stock returns for the 2-capital model estimated at the firm level are mostly aligned with the 45-degree line. The 1-period-ahead alpha of the value premium is 3.87% per annum ($t = 0.63$), which is only slightly larger than 3.29% ($t = 1.37$) from the in-sample fit (Figure 3). The 1-period-ahead t-value is smaller because of the shorter sample for the 1-period-ahead evaluation. The 1-period-ahead alpha of the momentum premium is -3.17% ($t = -0.52$), which is larger in magnitude than the in-sample alpha of 1.55% ($t = 0.5$). For the investment premium, the contrast is between 0.4% ($t = 0.14$) and -0.06% ($t = -0.04$), and for the Roe premium, between -0.44% ($t = -0.12$) and 2.23% ($t = 0.89$). Finally, the average absolute high-minus-low alpha and mean absolute alpha across the 40 deciles in the 1-period-ahead fit are 1.97% and 1.58%, which are slightly higher than 1.73% and 1.33% from the in-sample fit (Table

3), respectively.

Panel B shows the poor 1-period-ahead fit for the physical capital model estimated at the portfolio level. The value premium is 5.78% per annum in the data but -9.86% in the model, yielding a massive 1-period-ahead alpha of 15.64% ($t = 2.81$). The model also overshoots the momentum premium, which is 11.98% in the data but 20.78% in the model, with an alpha of -8.8% ($t = -1.49$). The average absolute high-minus-low alpha is 8.56%, and the mean absolute alpha 4.13%. Both are larger than 4.12% and 2.96% from the in-sample fit of the physical capital model as well as 1.97% and 1.58% from the 1-period-ahead fit of the benchmark 2-capital model, respectively.

The q-factor model performs well (Panel C). The average absolute high-minus-low alpha and mean absolute alpha are 1.09% and 1.43% per annum, respectively. The alpha of the value premium is 2.7% ($t = 0.56$), and that of the momentum premium 1.19% ($t = 0.03$). Finally, Panel D shows that the Fama-French 5-factor model accounts for the value premium, with an alpha of -2.65% ($t = -1.77$), but fails to fit the momentum premium, with a massive alpha of 16.11% ($t = 3.91$).

3.5.2 Expected return estimates. Equation (5) provides a detailed, theoretical description of the 1-period-ahead expected stock return, $E_t[r^F_{it+1}]$. To construct $E_t[r^F_{it+1}]$, we must form expectations for the stochastic variables in the equation, including sales-to-total capital, $Y_{it+1}/(K_{it+1}+W_{it+1})$, investment-to-physical capital, I_{it+1}/K_{it+1}, the after-tax cost of debt, r^{Ba}_{it+1}, the tax rate, τ_{t+1}, and the depreciation rate, δ_{it+1}. To reduce estimation errors, we set the expected r^{Ba}_{it+1}, τ_{t+1}, and δ_{it+1} values to their current values from the most recent fiscal year ending at least four months ago. Because the tax rate is already known at the beginning of a calendar year, our assumption on τ_{t+1} only takes effect when the next fiscal year ends in the next calendar year. Finally, due to the 1-period time-to-build in the model, although dated $t+1$, the two capital goods, K_{it+1} and W_{it+1} are known at the beginning of time t.

The key is to forecast I_{it+1} and Y_{it+1}. We forecast I_{it+1}/K_{it+1} on lagged Tobin's Q_{it}, sales-to-total capital, $Y_{it}/(K_{it}+W_{it})$, and investment-to-physical capital, I_{it}/K_{it}.[18] To form $E_t[Y_{it+1}]$, we forecast annual sales growth, Y_{it+1}/Y_{it}, on the year-over-year quarterly sales growth rates of prior four quarters.[19] We winsorize the sales growth rates at the 2.5–97.5% level. To estimate the forecasting specifications, we perform monthly Fama-MacBeth (1973) cross-sectional regressions. To accord with value-weighting, we use weighted least squares with a firm's market equity as the weight.[20]

At the beginning of each month t from July 1980 to December 2017, we use the prior 120-month rolling window to estimate the I_{it+1}/K_{it+1} and Y_{it+1}/Y_{it} cross-sectional forecasting regressions. The I_{it+1} and Y_{it+1} data are from the most recent fiscal year ending at least four months prior to month t, and the predictors in the forecasting regressions are further lagged accordingly. We then combine the regression coefficients with the latest known predictors (which are lagged by at least four months as of month t) to compute $E_t[I_{it+1}/K_{it+1}]$ and $E_t[Y_{it+1}/Y_{it}]$, from which we calculate $E_t[Y_{it+1}/(K_{it+1}+W_{it+1})]$. Finally, we plug all the expectations, data items, and recursive parameters as of month t into equation (5) to

[18] Hou et al. (2019a) use a similar specification to forecast investment-to-assets changes when constructing their expected investment growth factor. Also, to reduce estimation errors, we do not separately forecast $(I_{it+1}/K_{it+1})^2$ in the numerator of equation (2). We instead compute $E_t[(I_{it+1}/K_{it+1})^2]$ as $(E_t[I_{it+1}/K_{it+1}])^2$. The quadratic term, $(I_{it+1}/K_{it+1})^2$, is economically small, meaning that the ignored Jensen's inequity term is even smaller.

[19] Fairfield, Ramnath, and Yohn (2009) use a similar specification to forecast sales growth in panel regressions.

[20] The Internet Appendix reports the forecasting regressions in the full sample. Tobin's Q_{it}, sales-to-total capital, $Y_{it}/(K_{it}+W_{it})$, and investment-to-physical capital, I_{it}/K_{it}, all forecast I_{it+1}/K_{it+1} with significantly positive slopes, with an average R^2 of 28.34%. The year-over-year quarterly sales growth rates of prior four quarters all forecast annual sales growth with significantly positive slopes, with an average R^2 of 67.45%.

construct the 1-period-ahead expected stock return, $E_t[r^F_{it+1}]$.

With the $E_t[r^F_{it+1}]$ estimates in hand at the beginning of month t, we use their NYSE breakpoints to split NYSE, Amex, and NASDAQ stocks into deciles. We calculate the monthly decile returns for three different holding periods (1-, 6-, and 12-month), over the current month t, from month t to $t+5$, and from month t to $t+11$. The 6-month horizon means that for a given decile in each month, there exist six subdeciles, each initiated in a different month in the prior six months. We take the simple average of the subdecile returns as the monthly return for the decile.

Panel A of Table 9 shows that $E_t[r^F_{it+1}]$ from the 2-capital model estimated at the firm level forecasts subsequent returns reliably. At the 1-month horizon, the high-minus-low decile earns an average return of 0.48% per month ($t = 2.52$). The average return spread declines somewhat to 0.39% ($t = 2.21$) at the 6-month and further to 0.28% ($t = 1.66$) at the 12-month. This evidence is potentially important. A voluminous literature in finance and accounting shows that the expected returns from accounting-based valuation models do not forecast 1-period-ahead realized returns (Easton and Monahan 2005). Intuitively, accounting models estimate the internal rate of return, which, as a constant, should not forecast returns in the time series (Hou et al. 2019b). In contrast, $E_t[r^F_{it+1}]$ is the 1-period-ahead expected return, which can vary both over time and across firms.

Panel B shows that $E_t[r^F_{it+1}]$ from the physical capital model estimated at the portfolio level also forecasts subsequent returns. The high-minus-low decile earns on average 0.41% per month ($t = 2.43$) at the 1-month horizon, which declines to 0.33% ($t = 2.07$) at the 6-month and further to 0.26% ($t = 1.77$) at the 12-month horizon. Although weaker than the benchmark model, this out-of-sample performance of the physical capital model contrasts with its poor in-sample fit. Intuitively, from equation (2), $E_t[r^F_{it+1}]$ from the physical capital model is essentially a nonlinear function of firm-level investment and profitability, both of which forecast returns reliably out of sample.

The expected return estimates from the q-factor model do not forecast returns (Panel C). At the beginning of each month t, we estimate the q-factor loadings for a given stock from the prior 60-month rolling window (36 months minimum) and then combine the loadings with the factor premiums averaged over the expanding window from January 1967 to month $t-1$ to calculate the stock's expected risk premium. The high-minus-low decile earns insignificant average returns of only 0.2%, 0.19%, and 0.25% per month ($t = 0.94, 0.93,$ and 1.22) at the 1-, 6-, and 12-month, respectively. (The estimates from the Fama-French 5-factor model are quantitatively similar.) This evidence contrasts with the better performance in Panel C of Figure 8 because instead of using subsequently realized factor premiums, we estimate them with prior information known at the beginning of month t.

The weak out-of-sample performance is generic to all factor models. Fama and French (1997) show that industry costs of equity based on their 3-factor model are very imprecise, and firm-level estimates are surely even less accurate. As such, we view the main application of factor models as describing the common variation of returns to facilitate risk management, portfolio optimization, and performance attribution for investment managers (Bodie, Kane, and Marcus 2014, Chapter 8). In contrast, in the same spirit as accounting-based valuation models (in terms of inferring discount rates from firm-level variables), but allowing for time-varying and cross-sectionally varying expected returns, our economic model seems more promising for estimating expected returns.

4 Conclusion

Aggregation and capital heterogeneity are thorny challenges for empirical investment studies. This paper provides a detailed treatment of aggregation, and to a lesser extent, heterogeneity in the context of the investment CAPM. We use firm-level variables to construct firm-level fundamental returns, which are then aggregated to the portfolio level to match with portfolio-level stock returns. We also introduce working capital as a separate productive input from physical capital to deal with capital heterogeneity. Both innovations make the empirical specification of the fundamental returns more "structural," stabilize parameter estimates, and more accurately describe the cross-sectional stock return distribution. The benchmark 2-capital model estimated at the firm level largely succeeds in explaining the value, investment, momentum, and profitability premiums simultaneously.

Figure Legends

Figure 1

Histograms of firm-level versus portfolio-level accounting variables in the fundamental returns, June 1967–December 2016

I_{it}/K_{it} is physical investment-to-capital, $K_{it+1}/(K_{it+1}+W_{it+1})$ the fraction of physical capital in total capital, $Y_{it+1}/(K_{it+1}+W_{it+1})$ the ratio of sales over total capital, Y_{it+1}/K_{it+1} sales-to-physical capital, w_{it}^B the market leverage, and r_{it+1}^B the pre-tax cost of debt. We winsorize 5% of the firm-level extreme observations at the portfolio formation. For the unbounded I_{it}/K_{it}, we use the 2.5–97.5% winsorization. For $Y_{it+1}/(K_{it+1}+W_{it+1})$, Y_{it+1}/K_{it+1}, δ_{it+1}, and r_{it+1}^B that are bounded below at zero, we use the 0–95% winsorization. We do not winsorize $K_{it+1}/(K_{it+1}+W_{it+1})$, or the market leverage, w_{it}^B. Both are bounded between zero and one. Portfolio-level histograms are across the 40 testing deciles. The sample for the fundamental returns is from January 1967 to December 2016, but the underlying variables can come from the fiscal year ending in 1966 and 2018.

Panel A: Firm-level I_{it}/K_{it}.

Panel B: Portfolio-level I_{it}/K_{it}.

Panel C: Firm-level $K_{it+1}/(K_{it+1}+W_{it+1})$.

Panel D: Portfolio-level $K_{it+1}/(K_{it+1}+W_{it+1})$.

Panel E: Firm-level $Y_{it+1}/(K_{it+1}+W_{it+1})$.

Panel F: Portfolio-level $Y_{it+1}/(K_{it+1}+W_{it+1})$.

Panel G: Firm-level Y_{it}/K_{it}.

Panel H: Portfolio-level Y_{it}/K_{it}.

Panel I: Firm-level w^B_{it}.

Panel J: Portfolio-level w^B_{it}.

Panel K: Firm-level r^B_{it+1}.

Panel L: Portfolio-level r^B_{it+1}.

Figure 2

Average predicted stock returns versus average realized stock returns, the physical capital model estimated at the portfolio level, June 1967–December 2016

Both average predicted and realized stock returns are in percent per annum. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H." Panel A fits the Bm and R^{11} deciles jointly, and Panel B fits all the 40 value-weighted deciles together.

Panel A: Bm-R^{11}.

Panel B: Bm-R^{11}-I/A-Roe.

Figure 3

Average predicted stock returns versus average realized stock returns, the benchmark 2-capital model estimated at the firm level, June 1967–December 2016

Both average predicted and realized stock returns are in percent per annum. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H." Panel A fits the Bm and R^{11} deciles jointly, and Panel B fits all the 40 value-weighted deciles together.

Panel A: Bm-R^{11}.

Panel B: Bm-R^{11}-I/A-Roe.

Figure 4

Average predicted stock returns versus average realized stock returns, the 2-capital model estimated at the portfolio level, June 1967–December 2016

Both average predicted and realized stock returns are in percent per annum. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H." Panel A fits the Bm and R^{11} deciles jointly, and Panel B fits all the 40 value-weighted deciles together.

Panel A: Bm-R^{11}.

Panel B: Bm-R^{11}-I/A-Roe.

Figure 5

Average predicted stock returns versus average realized stock returns, the physical capital model estimated at the firm level, June 1967–December 2016

Both average predicted and realized stock returns are in percent per annum. The book-to-market (Bm) deciles (except for the two extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H." Panel A fits the Bm and R^{11} deciles jointly, and Panel B fits all the 40 value-weighted deciles together.

Panel A: Bm-R^{11}.

Panel B: Bm-R^{11}-I/A-Roe.

Figure 6

Event-time dynamics of the stock and fundamental returns of the high and low deciles, June 1967–December 2016

For 36 months after the portfolio formation, we plot the stock returns, r^S_{it+1}, and the fundamental returns, r^F_{it+1}, for the high and low deciles formed on book-to-market, prior 11-month returns, asset growth, and return on equity. Both stock and fundamental returns are in percent per annum. The blue solid lines represent the low deciles, and the red broken lines the high deciles. The fundamental returns are based on the parameters from estimating the 2-capital model at the firm level on the 40 value-weighted deciles jointly.

Panel A: Book-to-market, r^S_{it+1}.

Panel B: Momentum, r^S_{it+1}.

Panel C: Asset growth, r^S_{it+1}.

Panel D: Return on equity, r^S_{it+1}.

Panel E: Book-to-market, r^F_{it+1}.

Panel F: Momentum, r^F_{it+1}.

Panel G: Asset growth, r^F_{it+1}.

Panel H: Return on equity, r^F_{it+1}.

Figure 7

Time series of the stock and fundamental returns of the factor premiums, June 1967–December 2016

The blue solid lines represent the value-weighted stock returns of the high-minus-low deciles, and the red broken lines the corresponding fundamental returns. Both returns are in percent per annum. Stock returns outliers are indicated with their values and the corresponding months.

Panel A: Book-to-market, Bm.

Panel B: Momentum, R^{11}.

Panel C: Asset growth, I/A.

Panel D: Return on equity, Roe.

Figure 8

The 1-period-ahead model fits via recursive estimation, July 1980–December 2016

Both average fundamental returns (y-axis) and stock returns (x-axis) are in percent. The book-to-market (Bm) deciles (except for the extreme deciles) are in blue circles, the momentum (R^{11}) deciles in red squares, the asset growth (I/A) deciles in green diamonds, and the return on equity (Roe) deciles in black triangles. The low Bm decile is denoted "L," and the high Bm decile "H."

Panel A: The 2-capital model estimated at the firm level.

Panel B: The physical capital model estimated at the portfolio level.

Panel C: The Hou-Xue-Zhang q-factor model.

Panel D: The Fama-French 5-factor model.

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Table 1

Descriptive properties of testing deciles, January 1967–June 2017

For each decile, we report the monthly average return in excess of the 1-month Treasury bill rate, $\overline{R}$, and its t-value adjusted for heteroscedasticity and autocorrelations, $t_{\overline{R}}$. Testing deciles are formed with NYSE breakpoints and value-weighted returns. L denotes the low decile, H the high decile, and H−L the high-minus-low decile.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: Book-to-market, Bm										
$\overline{R}$	0.43	0.53	0.60	0.46	0.53	0.56	0.67	0.63	0.73	0.90	0.47
$t_{\overline{R}}$	1.85	2.74	3.16	2.26	2.89	3.19	3.65	3.40	4.07	3.93	2.15
	Panel B: Momentum, R^{11}										
$\overline{R}$	−0.03	0.40	0.47	0.48	0.45	0.48	0.46	0.63	0.68	1.08	1.12
$t_{\overline{R}}$	−0.10	1.53	2.16	2.47	2.43	2.54	2.63	3.25	3.25	3.98	3.88
	Panel C: Asset growth, I/A										
$\overline{R}$	0.69	0.68	0.63	0.52	0.53	0.56	0.59	0.48	0.58	0.33	−0.36
$t_{\overline{R}}$	2.98	3.42	3.84	3.19	3.09	3.13	3.24	2.49	2.42	1.27	−2.20
	Panel D: Return on equity, Roe										
$\overline{R}$	0.06	0.25	0.42	0.40	0.54	0.44	0.57	0.53	0.57	0.74	0.68
$t_{\overline{R}}$	0.18	1.03	2.03	2.20	2.98	2.24	3.14	2.90	2.97	3.42	3.01

Table 2

Descriptive statistics of firm-level accounting variables in the fundamental returns, June 1967–December 2016

This table reports the time series averages of cross-sectional statistics, including mean, standard deviation (σ), percentiles (5th, 25th, 50th, 75th, and 95th), and pairwise correlations. I_{it}/K_{it} is period-t physical investment-to-physical capital, $\triangle W_{it}/W_{it}$ the period-t ratio of working capital investment over working capital, Y_{it+1}/K_{it+1} the sales-to-physical capital in period $t+1$, Y_{it+1}/W_{it+1} the sales-to-working capital in period $t+1$, $K_{it+1}/(K_{it+1}+W_{it+1})$ the fraction of physical capital in total capital, δ_{it+1} the rate of physical capital depreciation, and r^B_{it+1} the pre-tax cost of debt in percent per annum. The sample for the fundamental returns is from June 1967 to December 2016. However, the accounting variables underlying the fundamental returns for June 1967 can come from the fiscal year ending in calendar year as early as 1966, and the accounting variables underlying the fundamental returns for December 2016 as late as 2018. The descriptive statistics are computed after winsorizing 5% of the extreme observations at the portfolio formation. We winsorize unbounded variables, including I_{it}/K_{it}, I_{it+1}/K_{it+1}, $\triangle W_{it}/W_{it}$, and $\triangle W_{it+1}/W_{it+1}$ at the 2.5–97.5% level. For variables bounded below at zero, including Y_{it+1}/K_{it+1}, Y_{it+1}/W_{it+1}, $Y_{it+1}/(K_{it+1}+W_{it+1})$, δ_{it+1}, and r^B_{it+1}, we use the 0–95% winsorization. Finally, we do not winsorize $K_{it+1}/(K_{it+1}+W_{it+1})$, or the market leverage, w^B_{it}, both of which are bounded between zero and one.

Panel A: Mean, standard deviation, and percentiles

	Mean	σ	5th	25th	50th	75th	95th
I_{it}/K_{it}	0.36	0.44	−0.03	0.11	0.23	0.44	1.32
$\triangle W_{it}/W_{it}$	0.13	0.32	−0.30	−0.05	0.07	0.22	0.82
Y_{it+1}/K_{it+1}	9.05	11.59	0.45	2.38	5.24	10.17	35.52
Y_{it+1}/W_{it+1}	3.09	2.00	0.76	1.77	2.61	3.83	7.46
$Y_{it+1}/(K_{it+1}+W_{it+1})$	1.62	0.93	0.30	0.97	1.50	2.11	3.80
$K_{it+1}/(K_{it+1}+W_{it+1})$	0.38	0.25	0.07	0.18	0.32	0.55	0.88
w^B_{it}	0.26	0.22	0.00	0.07	0.22	0.42	0.68
δ_{it+1}	0.19	0.12	0.05	0.11	0.16	0.25	0.49
r^B_{it+1}	8.74	5.77	0.02	5.65	7.98	10.54	24.89

Panel B: Cross-sectional correlations

	$\frac{I_{it+1}}{K_{it+1}}$	$\frac{\triangle W_{it}}{W_{it}}$	$\frac{\triangle W_{it+1}}{W_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}}$	$\frac{Y_{it+1}}{W_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}+W_{it+1}}$	$\frac{K_{it+1}}{K_{it+1}+W_{it+1}}$	w_{it}^B	δ_{it+1}	r_{it+1}^B
I_{it}/K_{it}	0.32	0.30	0.10	0.15	−0.06	0.06	−0.18	−0.18	0.28	0.06
I_{it+1}/K_{it+1}		0.23	0.30	0.36	0.00	0.20	−0.28	−0.29	0.53	0.16
$\triangle W_{it}/W_{it}$			0.04	0.07	−0.04	0.01	−0.06	−0.08	0.05	0.03
$\triangle W_{it+1}/W_{it+1}$				0.09	0.25	0.20	0.08	−0.13	0.07	0.15
Y_{it+1}/K_{it+1}					0.07	0.56	−0.60	−0.18	0.52	0.03
Y_{it+1}/W_{it+1}						0.55	0.46	0.19	−0.19	0.09
$Y_{it+1}/(K_{it+1}+W_{it+1})$							−0.33	−0.08	0.24	0.13
$K_{it+1}/(K_{it+1}+W_{it+1})$								0.37	−0.59	0.00
w_{it}^B									−0.33	0.04
δ_{it+1}										0.06

Table 3

GMM estimation and tests, the physical capital model estimated at the portfolio level and the benchmark 2-capital model estimated at the firm level, June 1967–December 2016

This table uses the 40 testing deciles formed on book-to-market (Bm), prior 11-month returns (R^{11}), asset growth (I/A), and return on equity (Roe), separately and jointly (Bm and R^{11}, I/A and Roe, and all 40 deciles together). The testing deciles are formed with NYSE breakpoints and value-weighted returns. d.f. is the degrees of freedom in the GMM test of overidentification. γ_K is the technological parameter on the marginal product of physical capital as a fraction of sales-to-physical capital, Y_{it+1}/K_{it+1}. γ is the technological parameter on the marginal product of total capital as a fraction of sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$. a is the adjustment costs parameter of physical capital. $[\gamma]$, $[\gamma_K]$, and $[a]$ are the standard errors of the point estimates. $\overline{|\alpha|}$ is the mean absolute alpha across the testing portfolios, $\overline{|\alpha_{\text{H–L}}|}$ is the average absolute high-minus-low alpha, and p is the p-value of the overidentification test across a given set of testing portfolios. γ, γ_K, $[\gamma]$, $[\gamma_K]$, and p-values are in percent, and $\overline{|\alpha|}$ and $\overline{|\alpha_{\text{H–L}}|}$ are in percent per annum.

Panel A: The physical capital model estimated at the portfolio level

	d.f.	γ_K	$[\gamma_K]$	a	$[a]$	$\overline{\vert\alpha\vert}$	$\overline{\vert\alpha_{\text{H–L}}\vert}$	p
Bm	8	16.56	2.40	6.27	1.94	2.52	0.32	0.01
R^{11}	8	12.00	1.14	1.28	0.56	1.34	1.46	8.37
I/A	8	12.20	1.06	1.06	0.40	2.04	0.54	0.00
Roe	8	10.32	0.97	0.00	0.07	3.35	0.21	0.00
Bm-R^{11}	18	13.44	1.21	2.54	0.52	2.90	7.02	0.00
I/A-Roe	18	11.43	0.99	0.71	0.34	2.86	1.64	0.00
Bm-R^{11}-I/A-Roe	38	12.51	1.08	1.74	0.34	2.96	4.12	0.00

Panel B: The benchmark 2-capital model estimated at the firm level

	d.f.	γ	$[\gamma]$	a	$[a]$	$\overline{\vert\alpha\vert}$	$\overline{\vert\alpha_{\text{H–L}}\vert}$	p
Bm	8	17.62	2.07	3.75	0.68	1.34	0.16	0.07
R^{11}	8	13.37	2.84	8.11	0.00	0.82	0.74	85.28
I/A	8	17.44	1.77	1.63	0.70	0.89	2.31	0.31
Roe	8	14.90	3.20	7.63	0.00	0.79	1.16	92.46
Bm-R^{11}	18	17.89	2.03	3.44	0.55	1.27	0.77	0.00
I/A-Roe	18	17.35	1.79	1.65	0.67	1.14	2.15	0.00
Bm-R^{11}-I/A-Roe	38	17.77	1.94	2.84	0.47	1.33	1.73	0.00

Table 4

Comparative statics, the benchmark 2-capital model estimated at the firm level, June 1967–December 2016

This table reports the investment CAPM alphas from three comparative statics: $\overline{I_{it}/K_{it}}$, $\overline{I_{it+1}/K_{it+1}}$, and $\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$. In the experiment denoted $\overline{I_{it}/K_{it}}$, I_{it}/K_{it} is set to be its cross-sectional median at period t across all the firms. The parameters from the benchmark GMM estimation (with all 40 Bm, R^{11}, I/A, and Roe deciles together) are used to reconstruct the fundamental returns, with all the other characteristics unchanged. The other experiments are designed analogously. The alpha is the average difference between portfolio stock returns and reconstructed fundamental returns. The "Benchmark" rows report the benchmark model's alphas.

	L	2	3	4	5	6	7	8	9	H	H−L
	Panel A: Book-to-market, Bm										
Benchmark	−1.66	−1.16	−0.17	−1.32	−1.19	2.19	3.74	2.51	1.08	1.44	3.09
$\overline{I_{it}/K_{it}}$	−10.12	−5.82	−2.90	−0.79	1.67	6.91	11.86	14.86	18.02	26.16	36.28
$\overline{I_{it+1}/K_{it+1}}$	5.45	3.36	2.77	−0.86	−1.73	−1.93	−3.48	−7.89	−12.15	−22.34	−27.79
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−0.67	−0.39	0.39	−1.97	−3.33	−1.32	−1.78	−5.00	−6.69	−7.71	−7.04
	Panel B: Momentum, R^{11}										
Benchmark	0.05	2.22	1.26	0.61	−0.73	−0.63	−1.81	−0.44	−1.45	1.60	1.55
$\overline{I_{it}/K_{it}}$	1.60	2.83	2.62	2.49	1.78	2.09	0.49	0.79	−2.70	−6.06	−7.65
$\overline{I_{it+1}/K_{it+1}}$	−9.70	−1.87	−1.49	−1.57	−2.89	−2.66	−2.82	0.03	1.92	11.00	20.71
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−4.07	−0.46	−1.04	−1.75	−2.87	−2.70	−3.46	−1.65	−1.56	2.76	6.82
	Panel C: Asset growth, I/A										
Benchmark	−2.00	−1.85	−0.80	−0.34	0.11	0.54	1.43	−0.20	2.82	−2.07	−0.06
$\overline{I_{it}/K_{it}}$	5.76	6.62	7.17	7.30	5.57	3.37	1.50	−3.21	−4.76	−15.99	−21.75
$\overline{I_{it+1}/K_{it+1}}$	−6.02	−6.40	−5.28	−5.51	−4.03	−1.67	1.12	2.08	7.13	7.34	13.36
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−4.17	−3.59	−3.51	−3.87	−2.55	−1.27	0.06	−0.94	2.73	−1.77	2.40
	Panel D: Return on equity, Roe										
Benchmark	−3.54	0.13	2.16	0.68	2.61	−0.16	0.09	−1.97	−1.19	−1.31	2.23
$\overline{I_{it}/K_{it}}$	−0.55	4.50	8.52	6.99	7.31	1.88	0.80	−3.44	−4.81	−6.88	−6.34
$\overline{I_{it+1}/K_{it+1}}$	−9.38	−6.61	−5.22	−5.24	−2.17	−1.81	−0.13	0.13	3.07	5.45	14.83
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−8.16	−4.93	−3.66	−4.60	−1.59	−2.65	−1.04	−1.47	−0.48	0.56	8.72

Table 5
GMM estimation and tests, the 2-capital model estimated at the portfolio level and the physical capital model estimated at the firm level, June 1967–December 2016

This table uses the 40 testing deciles formed on book-to-market (Bm), prior 11-month returns (R^{11}), asset growth (I/A), and return on equity (Roe), separately and jointly (Bm and R^{11}, I/A and Roe, and all 40 deciles together). The testing deciles are formed with NYSE breakpoints and value-weighted returns. d.f. is the degrees of freedom in the GMM test of overidentification. γ_K is the technological parameter on the marginal product of physical capital as a fraction of sales-to-physical capital, Y_{it+1}/K_{it+1}. γ is the technological parameter on the marginal product of total capital as a fraction of sales-to-total capital, $Y_{it+1}/(K_{it+1} + W_{it+1})$. a is the adjustment costs parameter of physical capital. $[\gamma]$, $[\gamma_K]$, and $[a]$ are the standard errors of the point estimates. $\overline{|\alpha|}$ is the mean absolute alpha across the testing portfolios, $\overline{|\alpha_{\text{H–L}}|}$ is the average absolute high-minus-low alpha, and p is the p-value of the overidentification test across a given set of testing portfolios. γ, γ_K, $[\gamma]$, $[\gamma_K]$, and p-values are in percent, and $\overline{|\alpha|}$ and $\overline{|\alpha_{\text{H–L}}|}$ are in percent per annum.

Panel A: The 2-capital model estimated at the portfolio level

	d.f.	γ	$[\gamma]$	a	$[a]$	$\overline{\vert\alpha\vert}$	$\overline{\vert\alpha_{\text{H–L}}\vert}$	p
Bm	8	22.60	2.73	5.47	2.06	1.60	0.79	0.04
R^{11}	8	19.41	2.19	2.69	1.03	1.00	2.98	10.07
I/A	8	18.71	1.79	1.42	0.64	1.06	2.19	0.03
Roe	8	16.34	1.97	0.26	1.13	1.69	4.86	0.01
Bm-R^{11}	18	20.57	2.00	3.39	0.85	1.52	2.26	0.00
I/A-Roe	18	17.92	1.77	1.22	0.52	1.49	3.28	0.00
Bm-R^{11}-I/A-Roe	38	19.36	1.85	2.43	0.56	1.62	3.01	0.00

Panel B: The physical capital model estimated at the firm level

	d.f.	γ_K	$[\gamma_K]$	a	$[a]$	$\overline{\vert\alpha\vert}$	$\overline{\vert\alpha_{\text{H–L}}\vert}$	p
Bm	8	6.86	0.94	3.41	0.42	1.89	0.30	0.09
R^{11}	8	7.17	0.64	0.72	0.47	1.37	0.65	3.89
I/A	8	7.26	0.65	1.38	0.36	2.72	0.20	0.00
Roe	8	5.04	1.26	5.66	0.00	1.21	4.53	97.60
Bm-R^{11}	18	7.44	0.80	2.67	0.35	2.43	7.02	0.00
I/A-Roe	18	7.39	0.66	1.35	0.35	2.59	1.07	0.00
Bm-R^{11}-I/A-Roe	38	7.53	0.72	1.88	0.24	2.60	4.52	0.00

Table 6

Correlations between stock returns and fundamental returns, June 1967–December 2016

Panel A reports the firm- and portfolio-level correlations between the stock returns of various leads and lags and fundamental returns, r_{it}^F. The column denoted "r_{it}^S" reports contemporaneous correlations, and the column "r_{it-3}^S" the correlations between three-month-lagged stock returns and fundamental returns. Other columns are denoted analogously. Portfolio-level correlations are calculated with the 40 portfolios formed on book-to-market, prior 11-month returns, asset growth, and return on equity with NYSE breakpoints and value-weighted returns. The correlations are time series averages of cross-sectional correlations, and their p-values are calculated as the Fama-MacBeth p-values adjusted for autocorrelations of up to 12 lags. Panel B reports for each of the 40 deciles and the high-minus-low decile, the time series contemporaneous correlations between the stock and fundamental returns. The p-values are those of the slopes from regressing the stock returns on the contemporaneous fundamental returns, adjusted for autocorrelations of up to 12 lags. The correlations that are significant at the 1%, 5%, and 10% levels are denoted with three stars, two stars, and one star, respectively. The results are based on the parameter values from estimating the benchmark model on all the 40 value-weighted testing deciles jointly.

Panel A: Correlations of the stock returns with the fundamental returns, r_{it}^F

	r_{it-60}^S	r_{it-36}^S	r_{it-24}^S	r_{it-12}^S	r_{it-3}^S	r_{it}^S	r_{it+3}^S	r_{it+12}^S	r_{it+24}^S	r_{it+36}^S	r_{it+60}^S
Firms	−0.02***	−0.03***	−0.03***	0.02***	0.10***	0.11***	0.12***	0.05***	0.00	0.01	−0.01*
Portfolios	0.05*	0.09***	0.05*	0.09***	0.17***	0.19***	0.20***	0.12***	0.08***	0.12***	0.11***

Panel B: Contemporaneous correlations between the stock and fundamental returns across the testing deciles

	L	2	3	4	5	6	7	8	9	H	H−L
Bm	0.13	0.19	0.12	0.04	0.13**	0.20*	0.00	0.00	0.05	0.15	0.26***
R^{11}	0.20**	0.09	0.06	−0.05	−0.03	0.04	0.01	0.08	0.10	0.22***	0.14*
I/A	0.19**	0.11	0.10	−0.03	0.12	−0.02	0.02	−0.02	0.11	0.30***	0.42***
Roe	0.19	0.18	0.11	0.14*	−0.02	0.01	0.09	0.01	−0.02	0.09	0.16

Table 7
Market states and factor premiums, June 1967–December 2016
For each month t, we categorize the market state as Up (Down) if the value-weighted market returns from month $t-N$ to $t-1$, with $N = 12, 24$, or 36, are nonnegative (negative). We report the high-minus-low decile returns averaged across Up (Down) states. r^S denotes the stock returns, and r^F the fundamental returns. Both are in percent per annum. The t-values are adjusted for heteroscedasticity and autocorrelations of up to 12 lags. The results are based on the parameter values from estimating the benchmark model on the 40 value-weighted testing deciles jointly.

N	MKT	r^S	t_S	r^F	t_F	r^S	t_S	r^F	t_F
		Panel A: Book-to-market, Bm				*Panel B: Momentum, R^{11}*			
12	Down	11.81	4.17	3.31	0.62	−0.20	−0.02	15.75	6.30
12	Up	4.80	1.65	3.24	1.49	19.50	7.69	12.73	11.27
24	Down	13.60	2.62	14.09	2.56	−7.28	−0.62	13.11	3.64
24	Up	5.14	1.90	1.33	0.63	18.91	7.44	13.48	11.88
36	Down	17.19	3.32	17.43	2.88	−9.49	−0.99	9.78	4.71
36	Up	4.47	1.75	0.70	0.33	19.36	7.61	14.08	11.07
		Panel C: Asset growth, I/A				*Panel D: Return on equity, Roe*			
12	Down	−10.94	−4.28	−5.54	−2.24	2.06	0.46	4.42	1.78
12	Up	−3.35	−1.87	−4.86	−3.45	10.47	3.86	6.69	4.60
24	Down	−11.57	−5.11	−5.62	−1.94	−3.10	−0.51	3.21	1.03
24	Up	−3.95	−2.08	−4.91	−3.34	10.60	4.15	6.69	4.57
36	Down	−7.62	−3.05	−4.09	−1.42	−5.88	−1.23	1.44	0.63
36	Up	−4.64	−2.35	−5.19	−3.50	11.13	4.37	7.02	4.69

Table 8
Higher moments of the stock and fundamental returns, June 1967–December 2016

For each decile, we report volatility, σ, skewness, S_k, and kurtosis, K_u, of its stock returns, r^S, and fundamental returns, r^F. For each high-minus-low decile, the volatility and skewness significantly different from zero and the kurtosis significantly different from three at the 1%, 5%, and 10% levels are denoted with three, two stars, and one star, respectively. The significance is based on 5,000 block bootstrapped samples (each with 60 months). The results are based on parameters from estimating the benchmark model on the 40 value-weighted deciles jointly.

		L	2	3	4	5	6	7	8	9	H	H−L
		Panel A: Book-to-market, Bm										
σ	r^S	0.20	0.18	0.18	0.19	0.17	0.16	0.17	0.17	0.17	0.22	0.20***
	r^F	0.05	0.06	0.06	0.07	0.08	0.10	0.07	0.11	0.13	0.18	0.18***
S_k	r^S	−0.24	0.03	−0.08	−0.04	−0.16	−0.07	−0.20	−0.48	−0.14	0.12	0.42
	r^F	−0.96	−1.26	1.05	0.57	0.81	−1.57	0.67	1.27	0.73	0.63	0.36
K_u	r^S	3.04	3.12	2.75	3.43	3.20	3.57	3.52	4.36	3.94	4.47	3.28
	r^F	3.97	6.24	8.33	5.36	4.81	8.13	2.95	6.62	4.29	4.64	4.03
		Panel B: Momentum, R^{11}										
σ	r^S	0.30	0.24	0.20	0.18	0.16	0.17	0.16	0.18	0.19	0.26	0.28***
	r^F	0.12	0.08	0.08	0.07	0.07	0.06	0.07	0.07	0.07	0.07	0.13***
S_k	r^S	1.47	0.94	0.19	0.42	−0.10	−0.14	−0.23	−0.16	−0.11	−0.03	−1.78*
	r^F	−0.56	−0.03	0.33	0.38	0.57	0.69	1.01	0.62	0.13	−0.41	0.30*
K_u	r^S	9.92	8.05	3.91	4.07	3.70	3.58	3.02	3.07	3.57	3.19	11.59***
	r^F	6.58	4.10	6.00	4.81	5.51	5.24	6.73	5.06	4.07	3.91	5.29**
		Panel C: Asset growth, I/A										
σ	r^S	0.22	0.18	0.16	0.15	0.16	0.16	0.17	0.17	0.21	0.23	0.14***
	r^F	0.09	0.07	0.08	0.07	0.06	0.07	0.06	0.05	0.07	0.08	0.11***
S_k	r^S	0.36	−0.01	−0.01	−0.16	−0.25	−0.18	−0.20	−0.15	−0.30	−0.22	0.13
	r^F	0.22	0.88	0.41	1.00	0.40	0.03	−0.27	0.43	−0.29	−0.60	0.08
K_u	r^S	4.13	3.67	3.18	3.48	3.55	3.19	3.22	3.07	3.33	3.15	3.44
	r^F	2.71	4.60	2.95	5.17	3.01	3.43	4.48	4.15	3.59	5.03	3.18
		Panel D: Return on equity, Roe										
σ	r^S	0.27	0.22	0.19	0.16	0.17	0.18	0.16	0.17	0.17	0.20	0.20***
	r^F	0.14	0.12	0.09	0.08	0.08	0.07	0.07	0.05	0.05	0.05	0.14***
S_k	r^S	0.20	0.23	−0.03	−0.02	−0.25	−0.38	−0.39	−0.14	−0.20	−0.06	−0.84*
	r^F	0.46	0.38	0.58	0.38	0.50	1.31	0.07	−0.38	−0.15	−0.09	−0.38
K_u	r^S	3.69	3.94	4.13	3.36	3.12	3.66	3.14	2.90	3.35	2.70	5.75***
	r^F	4.99	5.45	6.73	4.53	4.85	6.56	4.19	3.88	2.98	3.08	4.45***

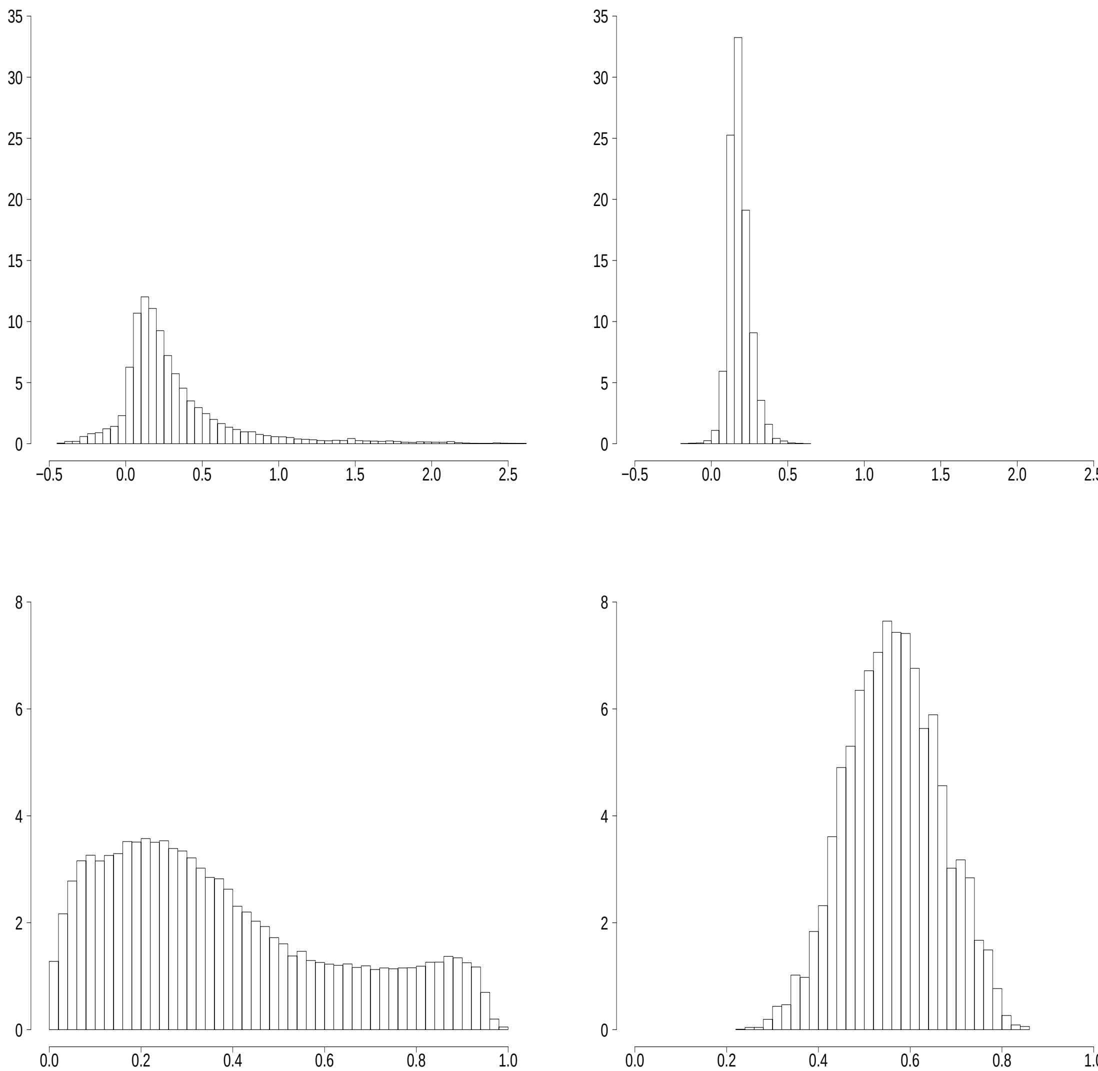
Table 9

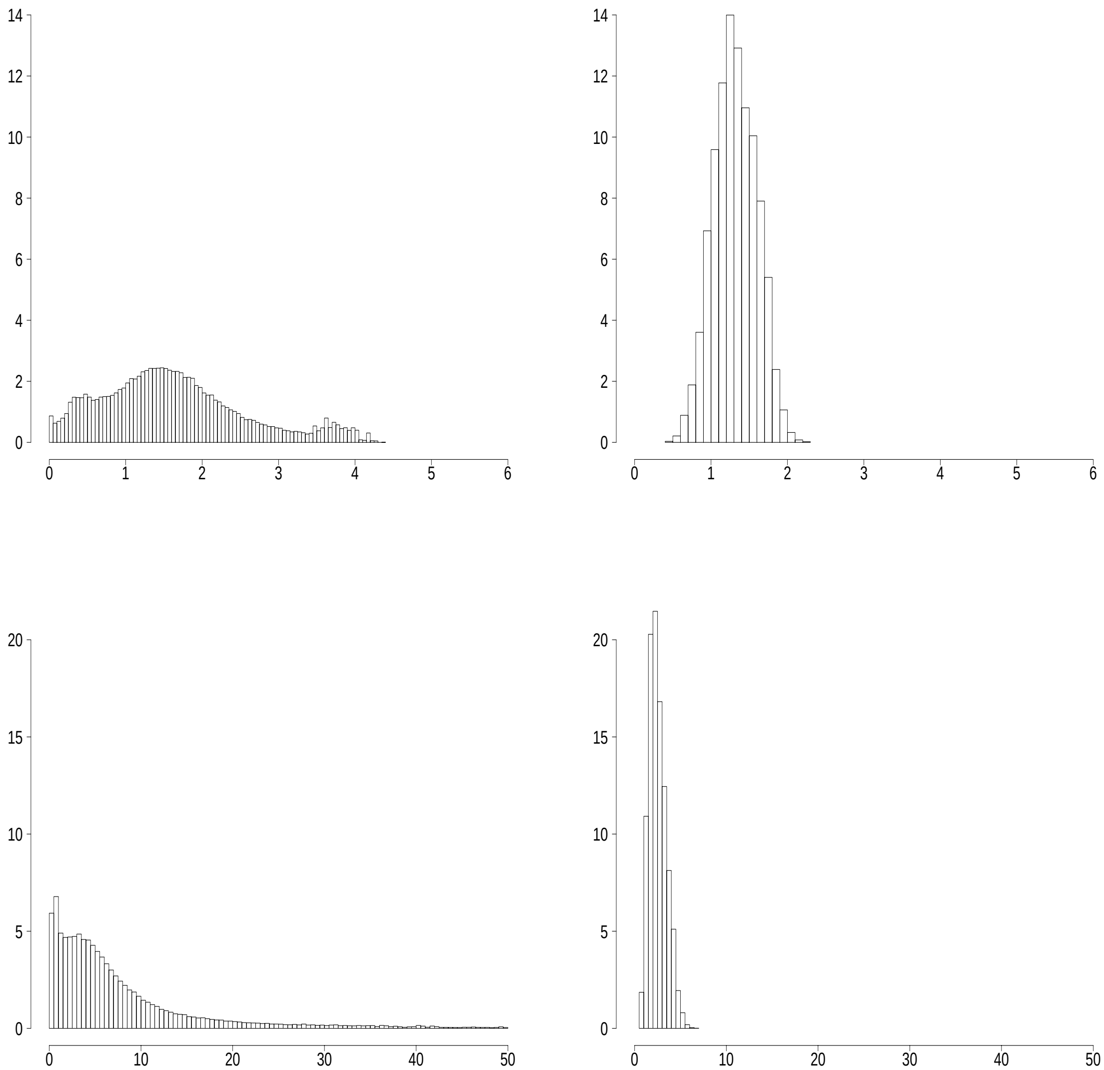
Deciles formed on the expected return estimates, July 1980–December 2017

This table reports the average excess return of a given expected return decile for the h-month holding period, in which $h = 1$, 6, and 12. The t-values adjusted for heteroscedasticity and autocorrelations are reported in the rows beneath the corresponding estimates. The deciles are formed on the expected return estimates with NYSE breakpoints and value-weighted returns.

h	L	2	3	4	5	6	7	8	9	H	H−L
Panel A: The 2-capital model estimated at the firm level											
1	0.39	0.80	0.61	0.75	0.80	0.57	0.80	0.82	0.85	0.86	0.48
	1.34	3.08	2.50	3.13	3.88	2.75	4.13	4.26	4.21	4.06	2.52
6	0.44	0.76	0.69	0.76	0.75	0.60	0.79	0.78	0.81	0.83	0.39
	1.55	2.87	2.86	3.21	3.64	2.93	4.07	4.09	4.04	3.98	2.21
12	0.49	0.77	0.65	0.78	0.75	0.64	0.76	0.74	0.83	0.77	0.28
	1.78	2.99	2.70	3.29	3.62	3.22	3.91	3.88	4.14	3.68	1.66
Panel B: The physical capital model estimated at the portfolio level											
1	0.42	0.75	0.59	0.66	0.72	0.79	0.83	0.82	0.95	0.83	0.41
	1.39	2.82	2.76	2.98	3.39	4.12	3.93	3.60	3.98	3.30	2.43
6	0.46	0.77	0.61	0.63	0.74	0.71	0.87	0.80	0.88	0.79	0.33
	1.55	3.03	2.89	2.80	3.43	3.74	4.20	3.51	3.80	3.11	2.07
12	0.54	0.76	0.61	0.64	0.75	0.68	0.81	0.81	0.87	0.79	0.26
	1.88	3.05	2.90	2.83	3.53	3.55	3.93	3.56	3.83	3.15	1.77
Panel C: The Hou-Xue-Zhang q-factor model											
1	0.64	0.70	0.78	0.65	0.79	0.76	0.74	0.80	0.76	0.83	0.20
	2.18	2.89	3.79	3.23	4.02	4.00	3.50	3.69	3.24	2.79	0.94
6	0.63	0.83	0.74	0.69	0.78	0.75	0.70	0.74	0.75	0.82	0.19
	2.13	3.56	3.67	3.53	4.09	3.95	3.49	3.48	3.19	2.83	0.93
12	0.61	0.75	0.79	0.72	0.75	0.76	0.71	0.73	0.77	0.86	0.25
	2.07	3.33	3.93	3.72	3.99	3.95	3.59	3.49	3.30	2.96	1.22
Panel D: The Fama-French 5-factor model											
1	0.64	0.57	0.66	0.79	0.71	0.76	0.86	0.86	0.79	0.84	0.19
	2.33	2.85	3.51	4.18	3.55	3.46	3.68	3.67	2.88	2.51	0.81
6	0.66	0.56	0.67	0.77	0.71	0.78	0.78	0.84	0.90	0.87	0.20
	2.40	2.86	3.59	4.17	3.62	3.78	3.42	3.57	3.28	2.69	0.85
12	0.65	0.57	0.66	0.75	0.72	0.78	0.80	0.86	0.95	0.90	0.24
	2.36	2.92	3.55	4.05	3.75	3.74	3.54	3.72	3.53	2.87	1.07

Figure 1 : Histograms of firm-level versus portfolio-level accounting variables in the fundamental returns, June 1967–December 2016

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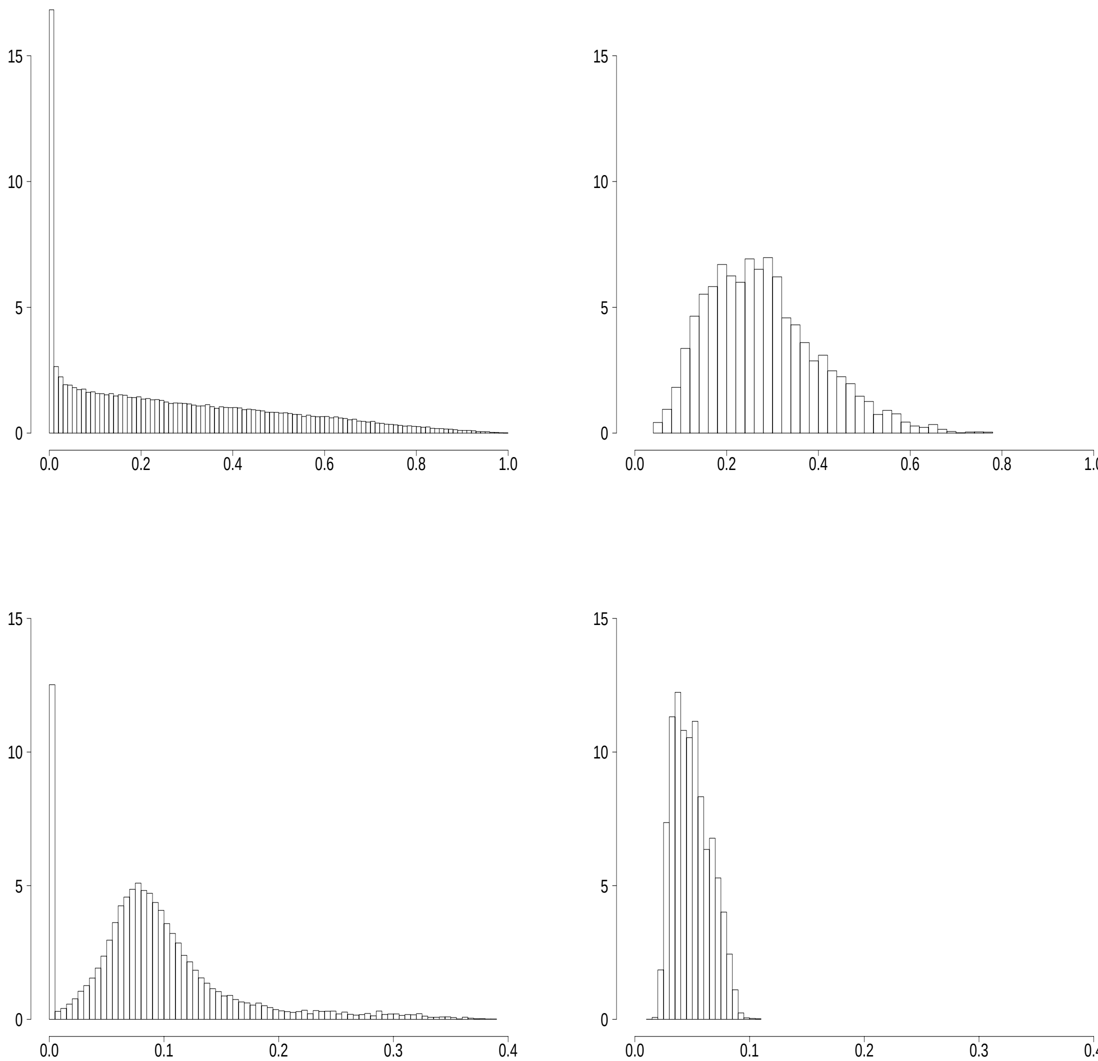
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Figure 2 : Average predicted stock returns versus average realized stock returns, the physical capital model estimated at the portfolio level, June 1967–December 2016

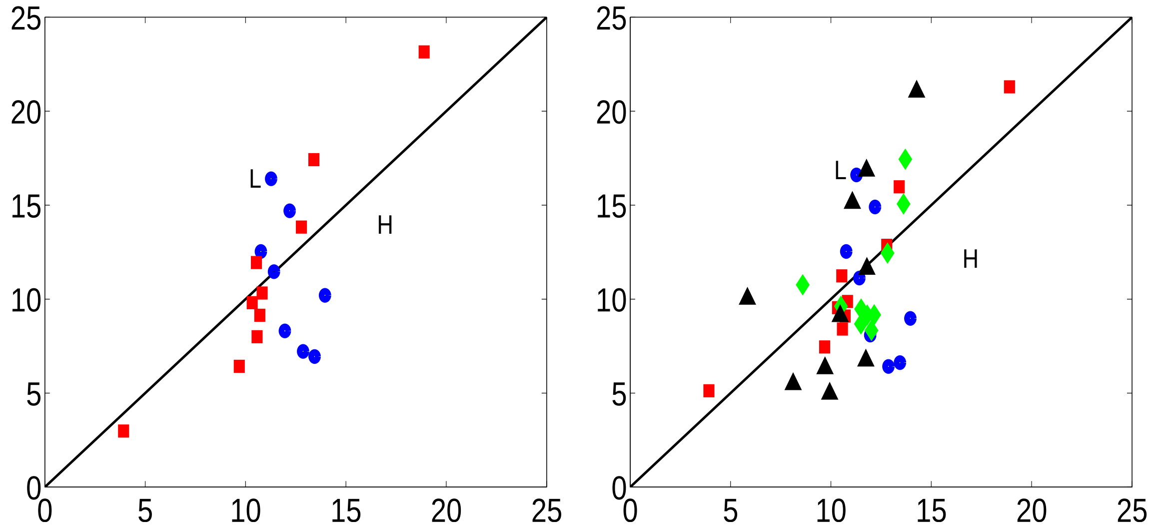

Figure 3 : Average predicted stock returns versus average realized stock returns, the benchmark 2-capital model estimated at the firm level, June 1967–December 2016

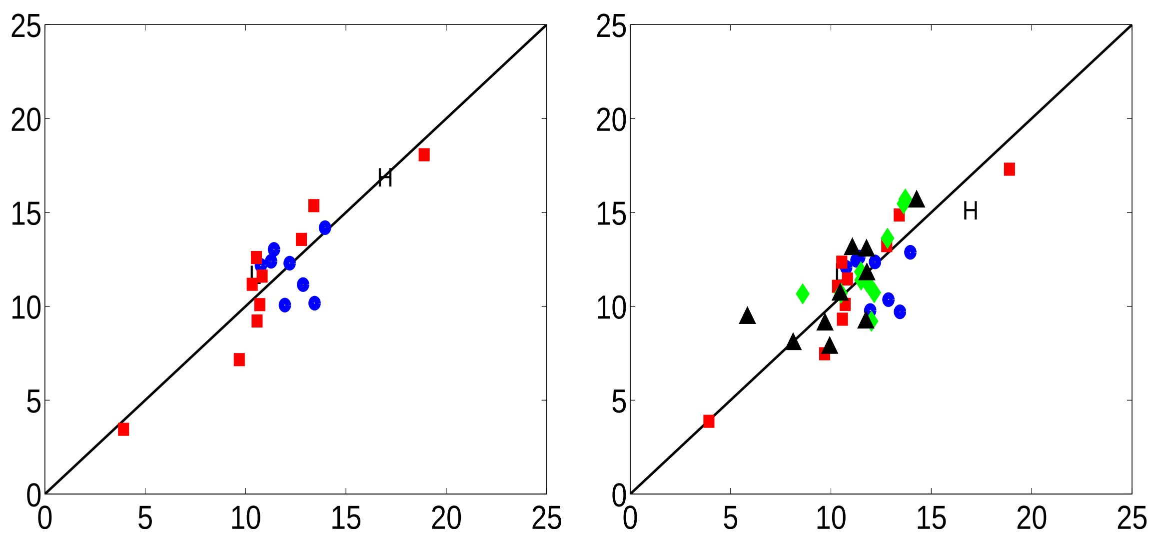

Figure 4 : Average predicted stock returns versus average realized stock returns, the 2-capital model estimated at the portfolio level, June 1967–December 2016

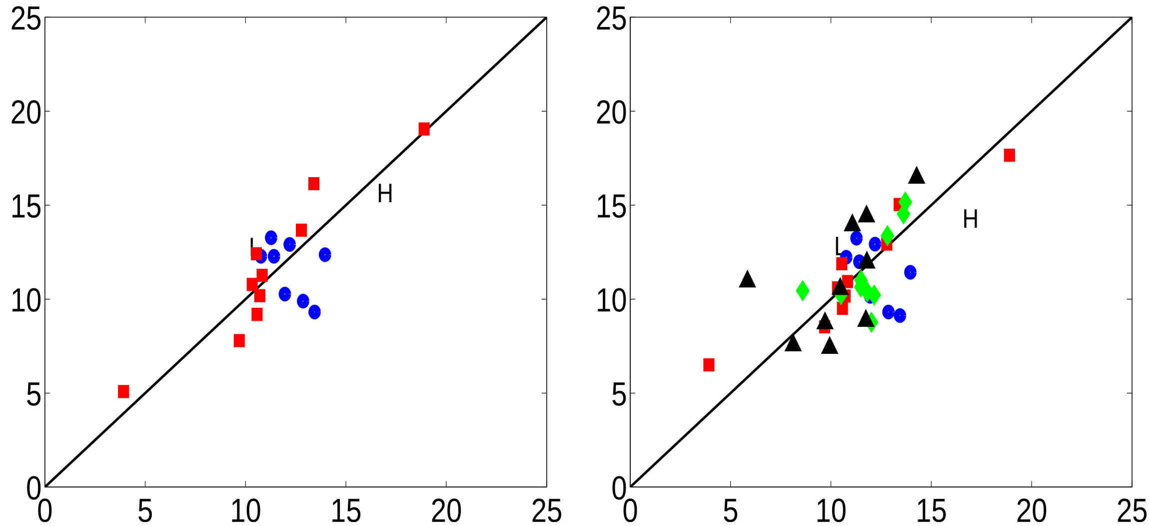

Figure 5 : Average predicted stock returns versus average realized stock returns, the physical capital model estimated at the firm level, June 1967–December 2016

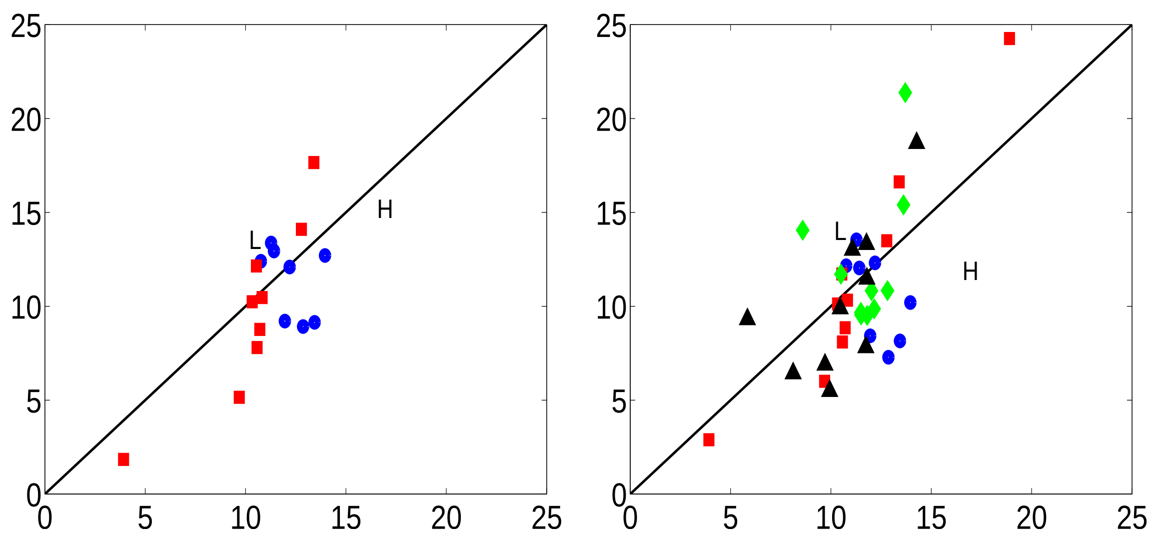

Figure 6 : Event-time dynamics of the stock and fundamental returns of the high and low deciles, June 1967–December 2016

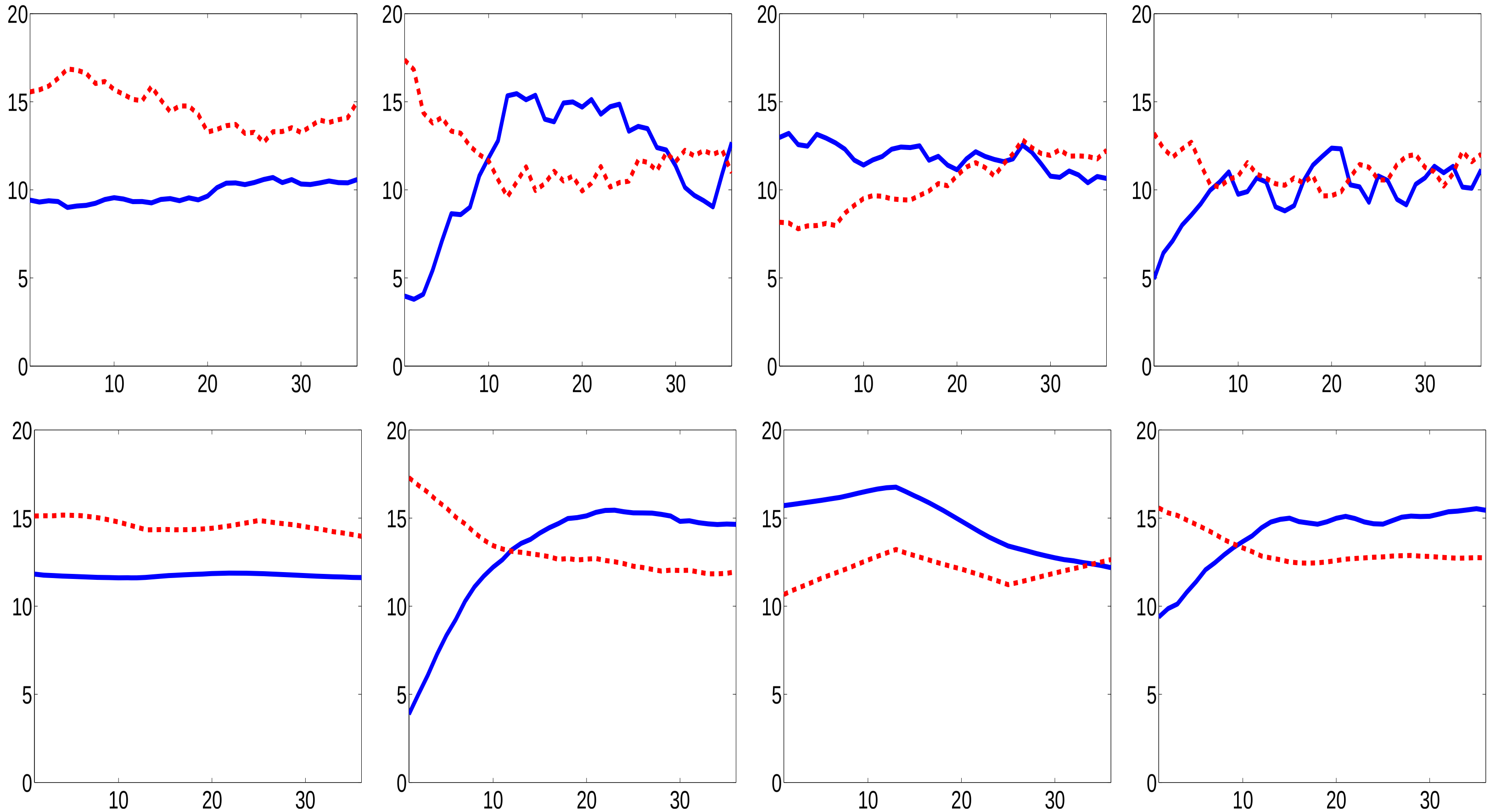

Figure 7 : Time series of the stock and fundamental returns of the factor premiums, June 1967–December 2016

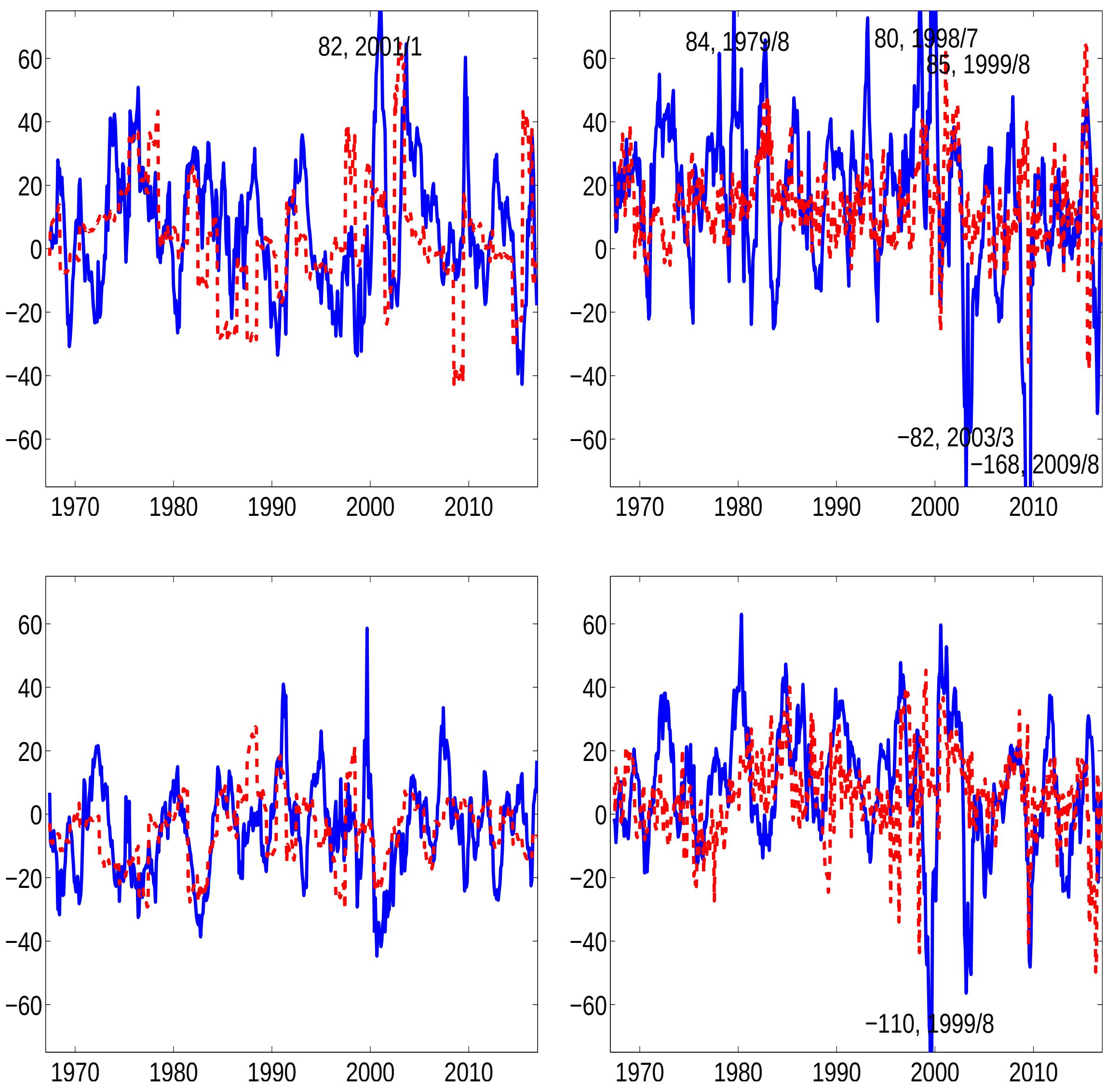

Figure 8 : The 1-period-ahead model fits via recursive estimation, July 1980–December 2016