Internet Appendix for "A Supply Approach to Valuation"

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Abstract

This appendix furnishes derivations and supplementary results for Belo, Xue, and Zhang (2013, "A supply approach to valuation") to appear at *Review of Financial Studies.*

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A Derivations

Let λ_{it} be the Lagrangian multiplier associated with equation (1). The Lagrangian formulation of the value-maximization problem is as follows:

$$\begin{aligned}
\mathcal{L}_i \;=\;& \ldots + (1-\tau_t)\left[\Pi(K_{it}, X_{it}) - \frac{1}{\nu}\left(\eta\frac{I_{it}}{K_{it}}\right)^{\nu} K_{it}\right] - I_{it} + B_{it+1} - r^B_{it}B_{it} + \tau\delta_{it}K_{it} \\
&+ \tau_t(r^B_{it}-1)B_{it} - \lambda_{it}\left[K_{it+1} - (1-\delta_{it})K_{it} - I_{it}\right] \\
&+ E_t\left[M_{t+1}\left[\begin{array}{c}(1-\tau_{t+1})\left[\Pi(K_{it+1}, X_{it+1}) - \frac{1}{\nu}\left(\eta\frac{I_{it+1}}{K_{it+1}}\right)^{\nu}K_{it+1}\right] - I_{it+1} \\ +B_{it+2} - r^B_{it+1}B_{it+1} + \tau\delta_{it+1}K_{it+1} + \tau_{t+1}(r^B_{it+1}-1)B_{it+1} \\ -\lambda_{it+1}\left[K_{it+2} - (1-\delta_{it+1})K_{it+1} - I_{it+1}\right]\end{array}\right] + \ldots\right]
\end{aligned} \tag{A.1}$$

Setting $\partial\mathcal{L}_i/\partial I_{it} = 0$, $\partial\mathcal{L}_i/\partial K_{it+1} = 0$, and $\partial\mathcal{L}_i/\partial B_{it+1} = 0$ yields, respectively,

$$\lambda_{it} = 1 + (1-\tau_t)\eta^{\nu}\left(\frac{I_{it}}{K_{it}}\right)^{\nu-1} \tag{A.2}$$

$$\lambda_{it} = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left[\kappa\frac{Y_{it+1}}{K_{it+1}} - \frac{\nu-1}{\nu}\left(\eta\frac{I_{it+1}}{K_{it+1}}\right)^{\nu}\right] + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1})\lambda_{it+1}\right]\right] \tag{A.3}$$

$$E_t[M_{t+1}[r^B_{it+1} - (r^B_{it+1}-1)\tau_{t+1}]] = 1 \tag{A.4}$$

Combining equations (A.2) and (A.3) yields equation (6).

To prove equation (5), we start with $P_{it} + D_{it} = V_{it}$ and expand V_{it} using equations (3) and (4):

$$\begin{aligned}
&P_{it} + (1-\tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it}) - r^B_{it}B_{it}] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t\delta_{it}K_{it} = \\
&\quad (1-\tau_t)\left[\Pi(K_{it}, X_{it}) - \frac{\partial\Phi(I_{it}, K_{it})}{\partial I_{it}}I_{it} - \frac{\partial\Phi(I_{it}, K_{it})}{\partial K_{it}}K_{it} - r^B_{it}B_{it}\right] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t\delta_{it}K_{it} \\
&\quad - \lambda_{it}[K_{it+1} - (1-\delta_{it})K_{it} - I_{it}] + E_t[M_{t+1}((1-\tau_t)\left[\Pi(K_{it+1}, X_{it+1}) - \frac{\partial\Phi(I_{it+1}, K_{it+1})}{\partial I_{it+1}}I_{it+1}\right. \\
&\quad \left. - \frac{\partial\Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}}K_{it+1} - r^B_{it+1}B_{it+1}\right] - \tau_{t+1}B_{it+1} - I_{it+1} + B_{it+2} \\
&\quad + \tau_{t+1}\delta_{it+1}K_{it+1} - \lambda_{it+1}[K_{it+2} - (1-\delta_{it+1})K_{it+1} - I_{it+1}] + \ldots\,)]
\end{aligned} \tag{A.5}$$

Recursively substituting equations (A.2), (A.3), and (A.4) yields:

$$
\begin{aligned}
& P_{it} + (1-\tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it}) - r^B_{it} B_{it}] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} = \\
& (1-\tau_t)\left[\Pi(K_{it}, X_{it}) - \frac{\partial \Phi(I_{it}, K_{it})}{\partial K_{it}} K_{it} - r^B_{it} B_{it}\right] - \tau_t B_{it} + \lambda_{it}(1-\delta_{it})K_{it} + \tau_t \delta_{it} K_{it} \qquad \text{(A.6)}
\end{aligned}
$$

Simplifying further and using the linear homogeneity of $\Phi(I_{it}, K_{it})$ yield:

$$
P_{it} + B_{it+1} = (1-\tau_t)\frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} I_{it} + I_{it} + \lambda_{it}(1-\delta_{it})K_{it} = \lambda_{it} K_{it+1} \qquad \text{(A.7)}
$$

To prove equation (8):

$$
\begin{aligned}
w_{it} r^{Ba}_{it+1} + (1-w_{it}) r^S_{it+1} &= \frac{\left[\begin{array}{c} (1-\tau_{t+1}) r^B_{it+1} B_{it+1} + \tau_{t+1} B_{it+1} + P_{it+1} \\ +(1-\tau_{t+1})[\Pi(K_{it+1}, X_{it+1}) - \Phi(I_{it+1}, K_{it+1}) - r^B_{it+1} B_{it+1}] \\ -\tau_{t+1} B_{it+1} - I_{it+1} + B_{it+2} + \tau_{t+1}\delta_{it+1} K_{it+1} \end{array}\right]}{P_{it} + B_{it+1}} \\
&= \frac{1}{\lambda_{it} K_{it+1}} \left[\begin{array}{c} \lambda_{it+1}(I_{it+1} + (1-\delta_{it+1})K_{it+1}) + (1-\tau_{t+1})[\Pi(K_{it+1}, X_{it+1}) \\ -\Phi(I_{it+1}, K_{it+1})] - I_{it+1} + \tau_{t+1}\delta_{it+1} K_{it+1} \end{array}\right] \\
&= \frac{\lambda_{it+1}(1-\delta_{it+1}) + (1-\tau_{t+1})\left[\frac{\partial \Pi(K_{it+1}, X_{it+1})}{\partial K_{it+1}} - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}}\right] + \tau_{t+1}\delta_{it+1}}{\lambda_{it}} = r^I_{it+1}. \qquad \text{(A.8)}
\end{aligned}
$$

B Parameter Stability

The model's structural parameters describe production and capital adjustment technologies, and in principle should be invariant to optimizing behavior and economic policy per Lucas (1976). However, because of specification and measurement errors, parameter instability is not uncommon (e.g., Oliner, Rudebusch, and Sichel (1996) and Fernández-Villaverde and Rubio-Ramírez (2007)). In this section, we explore the stability issue in three ways, subsample analysis, rolling-window estimation, and estimation with time-varying parameters.

B.1 Subsample Analysis

Table B.1 reports the GMM estimation and tests over two 25-year subsamples, 1963–1987 and 1987–2011. The table shows that our parameter estimates seem (roughly) stable over time. The slope parameter, η, in the adjustment costs function is 3.60 in the first subsample and 4.76 in the second subsample. The curvature estimate, ν, is more stable, 3.95 and 3.62, across the subsamples. The implied adjustment costs-to-sales ratio is somewhat lower in the first subsample, 3.06% versus 6.33%, likely reflecting the difference in the average q across the two periods. Indeed, Tobin's q averaged across the q deciles is lower in the first half than that in the second half: 1.31 versus 1.82 (untabulated). The mean absolute valuation error is stable, 0.06 and 0.10, across the two subsamples, and the χ^2 test fails to reject the model in both samples. Panel B shows small valuation errors for the high-minus-low decile, 0.03 and 0.10, across the two samples. None of the individual deciles have significant valuation errors in either subsample, probably because of the shortened sample length.

B.2 Rolling-window Estimation

Table B.2 provides further evidence on parameter stability by estimating the model with a series of rolling 20-year windows. The first rolling window starts at 1963 and ends at 1982, the second window starts at 1964 and ends at 1983, and so on. We report the time series of the point estimates, which seem stable. The slope adjustment cost parameter, η, is on average 4.01 with a standard deviation of 0.52. The curvature parameter, ν, is on average 3.77 with a standard deviation of 0.19. The ν estimates are all significantly different from two. The implied adjustment costs-to-sales ratio shows more time-variation. The ratio (not a structural parameter per se) has a mean of 4.22% but a standard deviation of 1.58%, likely reflecting the large time-variation in Tobin's q in the data. The mean absolute valuation errors vary from 0.04 to 0.13, and the model is not rejected by the χ^2 test in any of rolling windows. The high-minus-low errors range from -0.03 to 0.17 with an average magnitude of 0.07. None of the high-minus-low errors are significant at the 5% level.

B.3 Time-varying Parameters

We adopt an empirical design from Gomes, Yaron, and Zhang (2006) to explore the stability issue in our setup. In particular, we parameterize the η and ν parameters to be time-varying:

$$\eta_t = \eta_0 + \eta_1 f_t \tag{B.1}$$

$$\nu_t = \nu_0 + \nu_1 f_t, \tag{B.2}$$

in which f_t is the default premium, the output gap, or the NBER recession indicator. The default premium is the yield spread between Baa-rated and Aaa-rated corporate bonds from Federal Reserve Bank of St. Louis. We take the time series averages of the monthly default premiums within a given year to obtain the annual series. We obtain annual real gross domestic product from National Income and Product Accounts at Bureau of Economic Analysis. Output gap is the Hodrick-Prescott (1997) filtered cyclical component of the log real output with a smoothing parameter of 100. (A smoothing parameter of 6.25 per Ravn and Uhlig (2002) yields quantitatively similar results.) For a given calendar year, the NBER recession indicator is the number of months categorized as recessions by NBER within the year divided by 12. To help interpret the magnitude of parameter estimates, we standardize all the f_t series (demean and divide by the standard deviation of f_t).

Table B.3 reports that the slope and the curvature parameters are fairly stable. The η_1 and the ν_1 estimates are universally insignificant and are within 0.7 standard errors from zero. In addition, the mean absolute valuation errors and the high-minus-low errors are virtually identical to those in the benchmark estimation reported in Table 2.

C Specification Tests

Complementing the subsample analysis in Section 4.2, we use another way to incorporate conditioning information such as cash flows and lagged investment. Instead of the benchmark valuation moment (10), we estimate the following alternative valuation moment via one-step GMM:

$$E\left[q_{it} - \left(1 + (1-\tau_t)\eta^{\nu}\left(\frac{I_{it}}{K_{it}}\right)^{\nu-1} + \beta X_{it}\right)\frac{K_{it+1}}{A_{it}}\right] = 0. \tag{C.1}$$

in which βX_{it} is appended to the (unscaled) marginal costs of investment. We consider an array of variables for X_{it}, including cash flows (CF), lagged CF, lagged investment, idiosyncratic volatility, and the SA index of financial constraints.[1] Accordingly, the valuation errors are defined as:

$$\tilde{e}_i^q \equiv E_T \left[q_{it} - \left(1 + (1 - \tau_t)\eta^\nu \left(\frac{I_{it}}{K_{it}} \right)^{\nu - 1} + \beta X_{it} \right) \frac{K_{it+1}}{A_{it}} \right], \tag{C.2}$$

The specification in equation (C.1) is similar to the standard investment regressions in that a conditioning variable is appended to the "right-hand side" to test the benchmark specification.

Table C.1 reports the specification tests. From Panel A, the estimates for the slope of the conditioning variable, β, are universally insignificant with a maximum t-statistic of 1.65. The estimates for the slope and the curvature of the adjustment costs function are close to those in the benchmark estimation in Table 2. The mean absolute valuation errors vary from 0.04 to 0.07, which are less than or equal to 0.07 in the benchmark estimation. From Panel B, the errors for the individual deciles are small. The high-minus-low errors range from 0.02 to 0.14, and the maximum of 0.14 is only 3.11% of the valuation spread across the q deciles. The scatter plots in Figure C.1 further confirm the good fit across all the alternative specifications.

D Descriptive Statistics for the Joint Estimation Sample

Panel A of Table D.1 reports the firm-level descriptive statistics for the joint estimation sample from 1963 to 2011. The average number of firms drops from 2,291 (in the broader sample for matching the valuation moments only) to 1,751. The mean q, investment-to-capital, and next period capital-to-assets are 1.56, 0.30, and 0.38, which are close to 1.72, 0.35, and 0.36 in the broader sample, respectively. However, the joint sample is less skewed. For instance, the skewness of investment-to-capital is 9.42, in contrast to 17.06 in the broader sample.

From Panel B, the valuation spread across the q deciles in the joint sample is 3.78, which is somewhat smaller than 4.50 in the broader sample. The investment-to-capital spread also falls from 0.24 to 0.19. Sorting on q produces a monotonically decreasing pattern in the average returns.

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[1] The timing of CF in the alternative valuation moment (C.1) differs from the timing of CF in the subsample analysis in Table 5. In equation (C.1), CF is contemporaneous with current investment-to-capital, I_{it}/K_{it}. In Table 5, the timing of CF used in the portfolio sorts is the same as lagged investment (and is in effect lagged CF).

Moving from the low decile to the high decile, the average returns decrease from 24.88% per annum to 9.57%, and the spread of 15.32% is more than 3.9 standard errors from zero. In addition, the corporate bond return and the investment-to-capital growth are largely flat across the q deciles. Finally, the high decile has a lower market leverage than the low decile, 0.04 versus 0.49, but a higher sales-to-capital ratio, 3.73 versus 3.27. However, the latter spread is insignificant.

E Alternative Testing Portfolios

We use the Tobin's q portfolios as the key testing assets because sorting on q produces the largest valuation spread by construction. We next conduct the GMM estimation and tests on alternative testing deciles formed on market-to-book equity, asset growth, and return on equity (ROE).

E.1 Portfolio Construction and Timing Alignment

We sort all NYSE-Amex-NASDAQ stocks on market-to-book equity at the end of June of each year t into deciles based on the breakpoints for the fiscal year ending in calendar year $t-1$. Market-to-book equity is the market equity for December of year $t-1$ divided by book equity for the fiscal year ending in $t-1$.[2] Firm-year observations with negative book equity are excluded. We calculate Tobin's q at the end of June of t, q_{it}, following the timing as described in Section 3.2.

Following Cooper, Gulen, and Schill (2008), we sort all NYSE-Amex-NASDAQ stocks at the end of June of each year t into deciles based on the breakpoints of asset growth for the fiscal year ending in calendar year $t-1$. Asset growth is the change in total assets (Compustat annual item AT) divided by lagged total assets. We again calculate q_{it} at the end of June of t following the timing in Section 3.2. The portfolios are rebalanced at the end of each June.

Following Hou, Xue, and Zhang (2012), we sort all NYSE-Amex-NASDAQ stocks at the beginning of each month into deciles based on ROE. We measure ROE as income before extraordinary

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[2] Following Davis, Fama, and French (2000), we measure annual book equity as stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (Compustat annual item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if it is available. If not, we measure stockholders' equity as the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock. The market value of common equity is the closing price per share (item PRCC_F) times the number of common shares outstanding (item CSHO).

items (Compustat quarterly item IBQ) divided by one-quarter-lagged book equity.[3] Earnings and other accounting variables in Compustat quarterly files are used in the monthly sorts in the months immediately after the most recent public earnings announcement dates (item RDQ). We require earnings to be announced within the three months prior to the portfolio formation to remove stale information. The sample is from July 1972 to June 2011. The starting point of the sample is restricted by quarterly earnings data. Finally, the *ROE* deciles are formed monthly, but the accounting variables used in calculating q_{it} are annual. To align the timing, we follow a procedure analogous to what Liu, Whited, and Zhang (2009) design for deciles on earnings surprises.[4]

E.2 Estimation Results

From Panel A of Table E.1, the valuation spread between the extreme market-to-book deciles is 2.81. Although not small, this spread is lower than 4.5 across the q deciles (Table 1). The investment-to-capital spread is 0.20, but the next period capital-to-sales spread is small, -0.04. Unlike q or market-to-book, asset growth does not contain the market equity as an explicit component. As such, sorting on asset growth produces only a modest valuation spread of 0.47, albeit more than seven standard errors from zero. The investment-to-capital spread is 0.14, which is only slightly more than one half of that across the q deciles (Table 1). The *ROE* deciles display a moderate valuation spread of 0.91 but a small investment-to-capital spread of 0.06, albeit significant ($t = 3.01$).

From Panel B, the benchmark model does a good job in fitting the valuation moments across the market-to-book deciles. The slope and the curvature estimates are close to those in the benchmark estimation with the q deciles (Table 2). The mean absolute valuation error is only 0.07, which is about 5.38% of the average q across the deciles. The model is not rejected by the χ^2 test. The first

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[3] Quarterly book equity is shareholders' equity, plus balance sheet deferred taxes and investment tax credit (Compustat quarterly item TXDITCQ) if available, minus the book value of preferred stock. Depending on availability, we use stockholders equity (item SEQQ), or common equity (item CEQQ) plus the carrying value of preferred stock (item PSTKQ), or total assets (item ATQ) minus total liabilities (item LTQ) in that order as shareholders equity. We use redemption value (item PSTKRQ) if available, or carrying value for the book value of preferred stock.

[4] Consider the *ROE* deciles formed from July of year t to June of year $t+1$ (the July-to-June timing is the same as in the other two sets of deciles). To construct the corresponding annual Tobin's q, q_{it}, consider the 12 low *ROE* deciles formed in each month from July of year t to June of $t+1$. For each month we compute the portfolio-level characteristics of the market equity, capital, and total assets (with the timing described in Section 3.2) by aggregating these variables over the firms in the low *ROE* decile formed in that month. Because the portfolio composition changes from month to month, these portfolio-level characteristics also change from month to month. Accordingly, we average these portfolio characteristics over the 12 monthly low *ROE* portfolios and use these averages to construct portfolio-level q_{it}. This procedure is then repeated for the remaining nine *ROE* deciles.

two rows in Panel C report small valuation errors for the individual deciles, with the high-minus-low error a small -0.03. Panel A of Figure E.1 shows an almost perfect alignment with the 45-degree line for the scatter points of predicted q versus realized q across the market-to-book deciles.

From the second row of Panel B, the benchmark model also does a good job in fitting the valuation moments across the asset growth deciles. The mean absolute valuation error is 0.07, which represents 5.83% of the average q across the deciles. The curvature estimate is only 2.07, which is not significantly different from two, likely because of the small valuation spread of only 0.47. The individual valuation errors in Panel C (the third and the four rows) are mostly small and insignificant. However, the high-minus-low decile has a valuation error of -0.27, which is more than 2.5 standard errors from zero. Despite this (relatively) large high-minus-low error, the scatter points of predicted versus realized q are still largely aligned with the 45-degree line in Panel B of Figure E.1.

The model's performance in fitting the valuation moments is less impressive across the ROE levels. From Panel B, the mean absolute valuation error is 0.13, which is about 11.17% of the average q across the deciles. The curvature is estimated to be 3.66, which is significantly different from two, reflecting the large valuation spread relative to the investment-to-capital spread across the deciles. However, Panel C reports large errors for the individual deciles. In particular, the high-minus-low decile has an error of 0.70. Albeit insignificant ($t = 1.75$), this error is about 77% of the valuation spread. From the scatter points in Panel C of Figure E.1, the large errors for the two extreme deciles are clearly visible, but the errors for the remaining eight are small.

References

Cooper, Michael J., Huseyin Gulen, and Michael J. Schill, 2008, Asset growth and the cross-section of stock returns, *Journal of Finance* 63, 1609–1652.

Davis, James L., Eugene F. Fama, and Kenneth R. French, 2000, Characteristics, covariances, and average returns: 1929 to 1997, *Journal of Finance* 55, 389–406.

Fernández-Villaverde, Jesús, and Juan F. Rubio-Ramírez, 2007, How structural are structural parameters? *NBER Macroeconomic Annual* 83–137.

Gomes, João F., Amir Yaron, and Lu Zhang, 2006, Asset pricing implications of firms' financing constraints, *Review of Financial Studies* 19, 1321–1356.

Hodrick, Robert J., and Edward C. Prescott, 1997, Postwar U.S. business cycles: An empirical investigation, *Journal of Money, Credit, and Banking* 29, 1–16.

Hou, Kewei, Chen Xue, and Lu Zhang, 2012, Digesting anomalies: An investment approach, working paper, The Ohio State University.

Liu, Laura Xiaolei, Toni M. Whited, and Lu Zhang, 2009, Investment-based expected stock returns, *Journal of Political Economy* 117, 1105–1139.

Liu, Laura Xiaolei, and Lu Zhang, 2013, A model of momentum, working paper, The Ohio State University.

Lucas, Robert E. Jr., 1976, Econometric policy evaluation: A critique, *Carnegie Rochester Conference Series on Public Policy* 1, 19–46.

Oliner, Stephen D., Glenn D. Rudebusch, and Daniel Sichel, 1996, The Lucas critique revisited: Assessing the stability of empirical Euler equations for investment, *Journal of Econometrics* 70, 291–316.

Ravn, Morten O., and Harald Uhlig, 2002, On adjusting the Hodrick-Prescott filter for the frequency of observations, *Review of Economics and Statistics* 84, 371–380.

Table B.1 : GMM Estimation and Tests for the Tobin's q Deciles, Subsample Analysis

This table reports the GMM tests for two equal-length subsamples (1963–1987 and 1987–2011) on the valuation moments given by equation (10), using the Tobin's q deciles. η is the slope and ν is the curvature of the adjustment costs function. The t-statistics, denoted $[t]$, test that a given point estimate equals zero. $\mathrm{p}_{\nu=2}$ is the p-value associated with the Wald statistic testing $\nu = 2$ (quadratic adjustment costs). Φ/Y is the ratio in percent of the implied capital adjustment costs over sales. $\overline{|e_i^q|}$ is the mean absolute valuation error. χ^2, d.f., and p_{χ^2} are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero. Panel B reports for each individual decile and the high-minus-low decile the valuation errors, e_i^q, in equation (11) and their t-statistics.

Panel A: Point estimates and the χ^2 tests

	η	$[t]$	ν	$[t]$	$\mathrm{p}_{\nu=2}$	Φ/Y	$\overline{\lvert e_i^q\rvert}$	χ^2	d.f.	p_{χ^2}
1963–1987	3.60	16.88	3.95	14.59	0.00	3.06	0.06	6.06	8	0.64
1987–2011	4.76	22.82	3.62	13.31	0.00	6.33	0.10	5.92	8	0.66

Panel B: Valuation errors for individual deciles

		Low	2	3	4	5	6	7	8	9	High	H−L
1963–1987	e_i^q	−0.07	−0.10	−0.06	−0.05	−0.06	0.01	−0.02	−0.01	0.19	−0.04	0.03
	$[t]$	−1.23	−1.61	−1.20	−1.33	−1.31	0.28	−0.57	−0.29	1.33	−1.41	0.81
1987–2011	e_i^q	−0.16	−0.14	−0.04	0.03	−0.03	−0.13	0.07	−0.08	0.28	−0.06	0.10
	$[t]$	−1.57	−1.70	−0.67	0.40	−0.30	−1.71	0.90	−0.65	1.20	−1.17	1.32

Table B.2 : GMM Estimation and Tests from Rolling 20-year Windows, 1963–2011

This table estimates the model's parameters via one-step GMM using a series of rolling 20-year windows. The first rolling window starts from 1963 and ends at 1982. T denotes the terminal year at which each rolling window ends. The moment conditions are the valuation moments given by equation (10) on the Tobin's q deciles. There are two parameters: η is the slope, and ν is the curvature of the adjustment costs function. The t-statistics, denoted $[t]$, test that a given point estimate equals zero. $\mathrm{p}_{\nu=2}$ is the p-value associated with the Wald statistic testing $\nu = 2$ (quadratic adjustment costs). Φ/Y is the ratio in percent of the implied capital adjustment costs over sales. $\overline{|e_i^q|}$ is the mean absolute valuation error. p_{χ^2} is the p-value for the χ^2 test on the null that all the errors are jointly zero. The table also reports the valuation errors for the high-minus-low decile, denoted $\triangle e_i^q$, and their t-statistics.

T	η	$[t]$	ν	$[t]$	$\mathrm{p}_{\nu=2}$	Φ/Y	$\overline{\lvert e_i^q\rvert}$	p_{χ^2}	$\triangle e_i^q$	$[t]$
1982	3.44	14.76	4.20	14.00	0.00	2.56	0.04	0.73	0.01	0.35
1983	3.45	16.70	4.15	14.16	0.00	2.65	0.04	0.75	0.02	0.51
1984	3.43	17.31	4.04	13.52	0.00	2.69	0.04	0.73	0.02	0.45
1985	3.43	17.31	4.03	13.27	0.00	2.65	0.04	0.80	0.03	0.58
1986	3.48	15.88	4.02	12.20	0.00	2.64	0.04	0.73	0.03	0.59
1987	3.53	14.23	3.93	12.61	0.00	2.75	0.05	0.75	0.02	0.38
1988	3.52	13.69	3.87	13.65	0.00	2.73	0.07	0.76	0.01	0.15
1989	3.53	13.30	3.86	13.65	0.00	2.66	0.06	0.84	0.00	−0.10
1990	3.56	13.10	3.83	13.91	0.00	2.73	0.06	0.72	−0.02	−0.40
1991	3.58	12.95	3.83	14.12	0.00	2.77	0.06	0.72	−0.03	−0.66
1992	3.59	12.49	3.78	15.12	0.00	2.87	0.06	0.76	−0.03	−0.83
1993	3.64	12.24	3.74	16.71	0.00	3.01	0.06	0.72	−0.03	−0.75
1994	3.70	12.18	3.66	18.77	0.00	3.22	0.07	0.73	−0.01	−0.35
1995	3.76	12.30	3.60	22.21	0.00	3.50	0.09	0.73	0.00	0.09
1996	3.81	12.53	3.55	26.24	0.00	3.86	0.10	0.75	0.03	0.49
1997	3.86	13.05	3.55	29.12	0.00	4.09	0.10	0.77	0.05	0.74
1998	3.94	13.81	3.55	27.88	0.00	4.41	0.10	0.76	0.06	0.90
1999	4.02	15.87	3.61	20.57	0.00	4.50	0.10	0.70	0.08	1.01
2000	4.14	22.08	3.61	18.85	0.00	4.80	0.12	0.72	0.10	1.23
2001	4.31	49.37	3.78	17.09	0.00	4.72	0.10	0.76	0.14	1.56
2002	4.41	64.65	3.75	15.97	0.00	4.94	0.10	0.75	0.13	1.52
2003	4.45	44.16	3.78	16.32	0.00	4.95	0.09	0.73	0.13	1.44
2004	4.53	31.98	3.90	18.55	0.00	4.97	0.10	0.75	0.13	1.49
2005	4.61	26.72	3.86	17.94	0.00	5.28	0.09	0.76	0.12	1.33
2006	4.65	23.08	3.76	14.94	0.00	5.79	0.09	0.73	0.11	1.25
2007	4.69	20.06	3.63	11.32	0.00	6.46	0.10	0.72	0.10	1.11
2008	4.74	20.81	3.61	11.46	0.00	6.78	0.09	0.73	0.12	1.21
2009	4.80	20.66	3.58	11.85	0.00	6.98	0.10	0.77	0.13	1.35
2010	4.86	20.19	3.55	12.00	0.00	7.14	0.12	0.76	0.16	1.61
2011	4.89	20.06	3.52	12.21	0.00	7.36	0.13	0.75	0.17	1.74

Table B.3 : GMM Estimation and Tests for the Tobin's q Deciles, Tests for Time-varying Parameters, 1963–2011

Panel A reports the results via one-step GMM on the valuation moments given by equation (10), in which the parameters are allowed to be time-varying. In particular, we redefine $\eta = \eta_0 + \eta_1 f_t$ and $\nu = \nu_0 + \nu_1 f_t$, in which f_t is one of three business cycle indicators including the default premium (DP), the output gap (OG), and the NBER recession indicator (REC). DP is the yield spread between Baa-rated and Aaa-rated corporate bonds. The monthly DP data are from Federal Reserve Bank of St. Louis. We take the time series averages of the monthly default premiums within a calendar year to obtain the annual series. Output is annual real gross domestic product from the National Income and Product Accounts at Bureau of Economic Analysis. We calculate OG as the Hodrick-Prescott (1997) filtered cyclical component of the log output with a smoothing parameter of 100. For a given calendar year, REC is the number of months categorized as recessions by NBER within the year divided by 12. To help interpret the magnitudes of the parameter estimates, we standardize all the f_t series (demean and divide by the standard deviation of f_t). The t-statistics, denoted $[t]$, test that a given point estimate equals zero. Φ/Y is the ratio in percent of the implied capital adjustment costs over sales. $\overline{|e_i^q|}$ is the mean absolute valuation error, defined as the average magnitude of the valuation errors given by equation (11). χ^2, d.f., and p_{χ^2} are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero across the q deciles. Panel B reports for each individual decile and the high-minus-low decile the valuation errors, e_i^q, as well as their t-statistics.

Panel A: Point estimates and the χ^2 tests

	η_0	$[t]$	η_1	$[t]$	ν_0	$[t]$	ν_1	$[t]$	Φ/Y	$\overline{\lvert e_i^q\rvert}$	χ^2	d.f.	p_{χ^2}
$f_t =$ DP	4.16	16.43	−0.01	−0.02	3.78	16.93	−0.18	−0.11	4.76	0.07	7.25	6	0.30
$f_t =$ OG	4.06	19.19	0.36	0.62	3.71	5.96	0.10	0.07	4.86	0.07	6.90	6	0.33
$f_t =$ REC	4.07	15.60	0.34	0.46	3.77	5.64	0.01	0.01	4.80	0.07	7.84	6	0.25

Panel B: Valuation errors for individual deciles

		Low	2	3	4	5	6	7	8	9	High	H−L
$f_t =$ DP	e_i^q	−0.10	−0.11	−0.05	−0.04	−0.06	−0.03	0.01	−0.04	0.23	−0.05	0.05
	$[t]$	−2.15	−2.27	−1.28	−1.31	−1.50	−0.83	0.28	−1.17	1.77	−1.80	1.69
$f_t =$ OG	e_i^q	−0.09	−0.11	−0.06	−0.06	−0.08	−0.03	0.02	0.00	0.22	−0.05	0.04
	$[t]$	−2.04	−2.17	−1.47	−1.34	−1.32	−0.76	0.54	−0.17	1.57	−1.64	1.31
$f_t =$ REC	e_i^q	−0.08	−0.12	−0.05	−0.01	−0.04	−0.02	−0.01	−0.07	0.24	−0.05	0.04
	$[t]$	−1.78	−2.47	−1.56	−0.52	−1.21	−0.71	−0.22	−2.02	1.88	−1.91	1.32

Table C.1 : GMM Estimation and Tests for the Tobin's q Deciles, Specification Tests, 1963–2011

Panel A reports the one-step GMM results on the alternative valuation moment (C.1), in which βX_{it} is appended to the (unscaled) marginal costs of investment. X_{it} is one of four variables: Cash flows, CF; lagged CF; lagged investment, I_{it-1}/K_{it-1}; idiosyncratic volatility, IVOL; and the size-age index of financial constraints, SA. For the calendar year t, CF is earnings before extraordinary items (Compustat annual item IB) plus depreciation and amortization (item DP) for the fiscal year ending in calendar year t, scaled by capital (item PPENT) for the fiscal year ending in year $t-1$. Lagged investment is capital expenditures (item CAPX) minus sales of property, plant, and equipment if available (item SPPE) for the fiscal year ending in year $t-1$, scaled by capital for the fiscal year ending in year $t-2$. To measure IVOL for each firm, we regress the firm's weekly excess returns in the prior year from July of year $t-1$ to June of year t on the value-weighted market excess returns and on the value-weighted industry excess returns per the Fama-French (1997) 30-industry classification. IVOL is the logarithm of the volatility of the residual returns. The timing for the SA index is at the fiscal year ending in calendar year $t-1$. To help interpret the magnitudes of the parameter estimates, we standardize IVOL and SA series (demean and divide by the standard deviation). (CF and lagged investment are in the magnitude of the unscaled marginal costs of investment.) The t-statistics, denoted $[t]$, test that a given point estimate equals zero. Φ/Y is the ratio in percent of the implied capital adjustment costs over sales. $\overline{|\tilde{e}_i^q|}$ is the mean absolute valuation error, defined as the average magnitude of the valuation errors given by equation (C.2). χ^2, d.f., and p_{χ^2} are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that all the errors are jointly zero across the q deciles. Panel B reports for each individual decile and the high-minus-low decile the valuation errors, $\tilde{e}_i^q$, as well as their t-statistics.

Panel A: Point estimates and the χ^2 tests

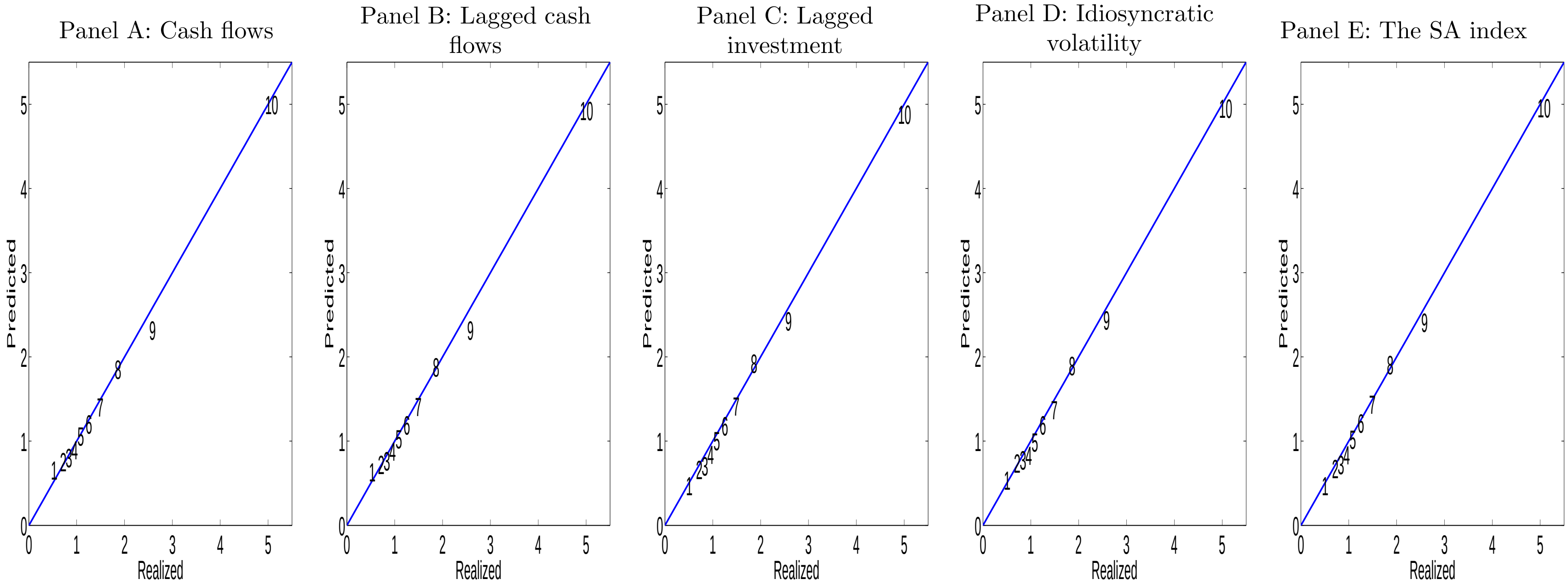
	η	$[t]$	ν	$[t]$	$\mathrm{p}_{\nu=2}$	β	$[t]$	Φ/Y	$\overline{\lvert\tilde{e}_i^q\rvert}$	χ^2	d.f.	p_{χ^2}
X_{it} = CF	4.40	5.28	3.57	5.70	0.01	−1.61	−0.33	6.50	0.07	7.33	7	0.40
X_{it} = lagged CF	4.65	4.88	3.57	5.38	0.02	−3.21	−0.53	7.70	0.06	7.77	7	0.35
$X_{it} = \frac{I_{it-1}}{K_{it-1}}$	4.82	8.12	3.38	7.45	0.00	−5.89	−1.28	9.27	0.05	7.80	7	0.35
X_{it} = IVOL	4.50	10.14	3.46	8.36	0.00	0.44	1.14	7.19	0.05	7.69	7	0.36
X_{it} = SA	4.63	11.48	3.28	8.63	0.00	0.67	1.65	8.51	0.04	5.16	7	0.64

Panel B: Valuation errors for individual deciles

		Low	2	3	4	5	6	7	8	9	High	H−L
X_{it} = CF	$\tilde{e}_i^q$	−0.19	−0.10	−0.03	−0.01	−0.04	−0.01	0.02	−0.05	0.20	−0.06	0.14
	$[t]$	−2.53	−1.94	−1.15	−0.29	−1.08	−0.38	0.50	−1.01	1.74	−1.97	2.63
X_{it} = lagged CF	$\tilde{e}_i^q$	−0.17	−0.07	0.00	0.01	−0.01	0.00	0.02	−0.08	0.20	−0.05	0.12
	$[t]$	−2.49	−1.53	0.14	0.27	−0.18	−0.07	0.39	−1.44	1.26	−1.29	2.42
$X_{it} = \frac{I_{it-1}}{K_{it-1}}$	$\tilde{e}_i^q$	−0.11	−0.01	0.07	0.05	0.01	0.01	0.01	−0.13	0.09	−0.01	0.10
	$[t]$	−2.06	−0.33	1.93	1.25	0.31	0.19	0.22	−1.46	1.07	−0.64	1.85
X_{it} = IVOL	$\tilde{e}_i^q$	−0.09	−0.09	0.00	0.06	0.03	−0.01	0.06	−0.10	0.08	−0.01	0.08
	$[t]$	−1.71	−1.95	−0.19	1.00	0.64	−0.11	1.10	−1.16	1.38	−1.96	1.59
X_{it} = SA	$\tilde{e}_i^q$	−0.03	−0.02	0.05	0.06	0.00	−0.03	−0.01	−0.11	0.11	−0.01	0.02
	$[t]$	−0.79	−0.48	1.79	1.30	−0.06	−0.60	−0.20	−1.41	1.32	−1.19	0.51

Figure C.1 : Average Predicted q versus Average Realized q for the Tobin's q Deciles, Specification Tests, 1963–2011

The results are one-step GMM on the alternative valuation moment (C.1). The Tobin's q deciles are labeled in an ascending order.


Panel A: Cash flows

Panel B: Lagged cash flows

Panel C: Lagged investment

Panel D: Idiosyncratic volatility

Panel E: The SA index

Table D.1 : Firm-level and Portfolio-level Descriptive Statistics of the Sample for Matching the Valuation Moment and the Expected Return Moment Jointly (and for Matching the Valuation Moment and the Investment Euler Equation Moment Jointly), 1963–2011

Panel A reports firm-level statistics such as mean, standard deviation (Std), skewness, and the 5, 25, 50, 75, and 95 percentiles for Tobin's q, q_{it}); investment-to-capital, I_{it}/K_{it}; capital-to-assets, K_{it+1}/A_{it}; the stock return, r^S_{it+1}; the corporate bond return, r^B_{it+1}; the investment-to-capital growth, $\frac{I_{it+1}/K_{it+1}}{I_{it}/K_{it}}$; the market leverage, $\frac{B_{it+1}}{P_{it}+B_{it+1}}$; and sales-to-capital, Y_{it+1}/K_{it+1}. The returns r^S_{it+1} and r^B_{it+1} are in percent. Panel B reports the averages of these variables across the Tobin's q deciles, the differences between the extreme deciles, and their t-statistics.

Panel A: Firm-level descriptive statistics

	Mean	Std	Skewness	5%	25%	50%	75%	95%
q_{it}	1.56	1.92	7.26	0.49	0.76	1.05	1.65	4.14
$\frac{I_{it}}{K_{it}}$	0.30	1.29	9.42	0.01	0.11	0.20	0.33	0.81
$\frac{K_{it+1}}{A_{it}}$	0.38	0.29	3.71	0.08	0.19	0.31	0.51	0.88
r^S_{it+1}	16.66	58.41	3.63	−46.06	−15.74	6.99	34.49	109.34
r^B_{it+1}	8.15	2.02	−0.03	5.49	7.05	8.17	9.54	10.51
$\frac{I_{it+1}/K_{it+1}}{I_{it}/K_{it}}$	4.57	167.78	1.72	−0.05	0.59	0.96	1.43	3.63
$\frac{B_{it+1}}{P_{it}+B_{it+1}}$	0.27	0.22	0.80	0.00	0.08	0.22	0.41	0.70
$\frac{Y_{it+1}}{K_{it+1}}$	10.26	56.37	22.65	0.61	2.39	4.84	8.94	30.22

Panel B: Portfolio-level descriptive statistics across the Tobin's q deciles

	Mean	Low	2	3	4	5	6	7	8	9	High	H−L	[t]
q_{it}	1.43	0.47	0.65	0.76	0.87	0.98	1.13	1.34	1.65	2.24	4.25	3.78	13.38
$\frac{I_{it}}{K_{it}}$	0.21	0.16	0.16	0.16	0.17	0.18	0.19	0.21	0.23	0.28	0.35	0.19	15.09
$\frac{K_{it+1}}{A_{it}}$	0.45	0.36	0.40	0.45	0.46	0.49	0.48	0.49	0.48	0.44	0.42	0.06	3.68
r^S_{it+1}	16.67	24.88	21.38	19.53	18.22	16.99	15.65	13.99	13.97	12.49	9.57	−15.32	−3.92
r^B_{it+1}	8.15	8.24	8.21	8.16	8.18	8.13	8.13	8.09	8.09	8.07	8.20	−0.03	−0.19
$\frac{I_{it+1}/K_{it+1}}{I_{it}/K_{it}}$	1.00	1.00	0.98	1.01	1.01	0.99	1.03	1.01	0.99	0.98	0.97	−0.03	−1.11
$\frac{B_{it+1}}{P_{it}+B_{it+1}}$	0.29	0.49	0.47	0.42	0.38	0.32	0.27	0.22	0.16	0.10	0.04	−0.45	−21.38
$\frac{Y_{it+1}}{K_{it+1}}$	2.86	3.27	2.84	2.49	2.54	2.36	2.57	2.64	2.84	3.30	3.73	0.46	1.74

Table E.1 : GMM Estimation and Tests, Alternative Testing Deciles

For each decile, Panel A reports the averages of Tobin's q, q_{it}; investment-to-capital, $\frac{I_{it}}{K_{it}}$; and next period capital-to-assets, $\frac{K_{it+1}}{A_{it}}$. In Panel B, η is the slope, and ν is the curvature of the adjustment costs function. The t-statistics, denoted $[t]$, test that a given point estimate equals zero. $\mathrm{p}_{\nu=2}$ is the p-value associated with the Wald statistic testing $\nu = 2$. Φ/Y is the ratio in percent of the implied capital adjustment costs over sales. $\overline{|e_i^q|}$ is the mean absolute valuation error. χ^2, d.f., and p_{χ^2} are the statistic, the degrees of freedom, and the p-value for the χ^2 test on the null that the errors are jointly zero across all the deciles. Panel C reports individual valuation errors, e_i^q, and their t-statistics.

Panel A: Descriptive statistics

	Mean	Low	2	3	4	5	6	7	8	9	High	H−L	$[t]$
Deciles on market-to-book													
q_{it}	1.30	0.56	0.67	0.76	0.84	0.94	1.10	1.23	1.53	2.00	3.38	2.81	11.71
$\frac{I_{it}}{K_{it}}$	0.20	0.12	0.15	0.16	0.18	0.19	0.20	0.21	0.24	0.26	0.32	0.20	10.88
$\frac{K_{it+1}}{A_{it}}$	0.43	0.45	0.45	0.45	0.46	0.46	0.45	0.40	0.42	0.39	0.40	−0.04	−2.53
Deciles on asset growth													
q_{it}	1.20	0.99	0.93	0.95	1.04	1.11	1.25	1.35	1.42	1.53	1.45	0.47	7.36
$\frac{I_{it}}{K_{it}}$	0.20	0.14	0.14	0.15	0.16	0.18	0.21	0.22	0.25	0.26	0.28	0.14	11.58
$\frac{K_{it+1}}{A_{it}}$	0.43	0.35	0.39	0.43	0.47	0.47	0.44	0.44	0.42	0.44	0.42	0.07	3.45
Deciles on *ROE*													
q_{it}	1.11	0.94	0.93	0.91	0.84	0.87	0.94	1.08	1.25	1.53	1.85	0.91	11.76
$\frac{I_{it}}{K_{it}}$	0.19	0.18	0.18	0.17	0.17	0.17	0.18	0.19	0.21	0.23	0.23	0.06	3.01
$\frac{K_{it+1}}{A_{it}}$	0.42	0.39	0.42	0.44	0.44	0.43	0.46	0.44	0.38	0.38	0.38	−0.01	−0.83

Panel B: Point estimates and the χ^2 tests

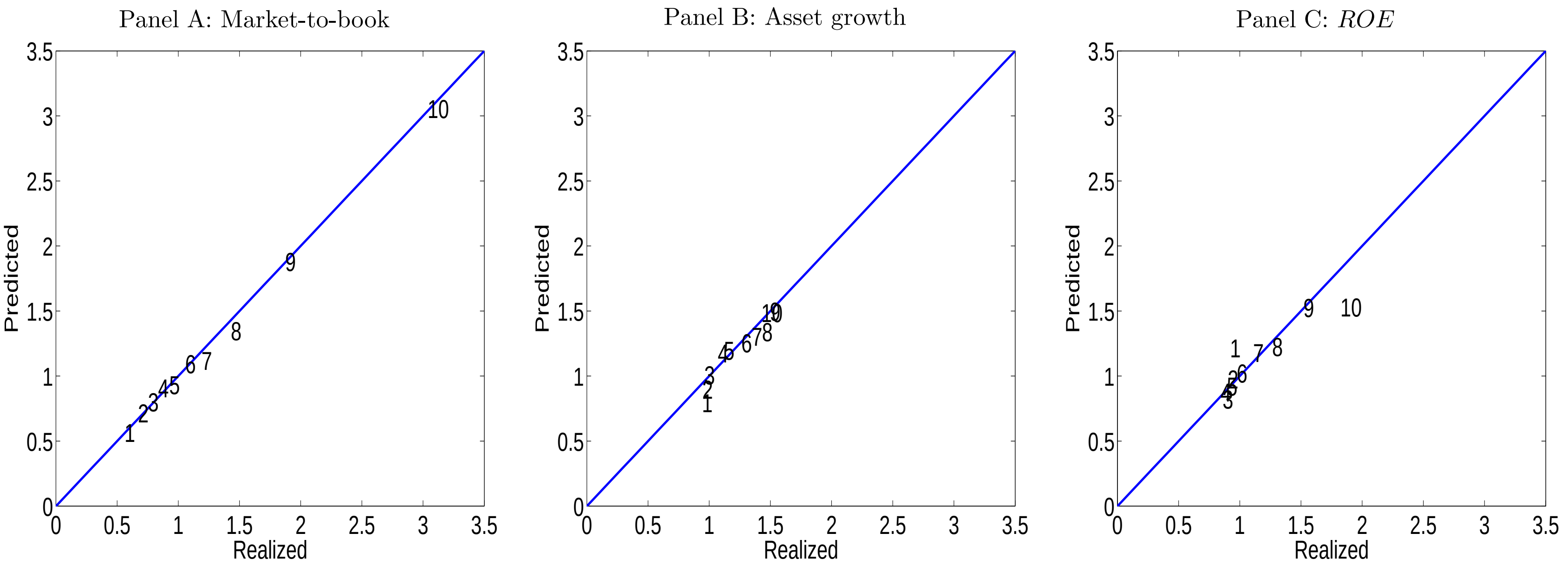
η	$[t]$	ν	$[t]$	$\mathrm{p}_{\nu=2}$	Φ/Y	$\overline{\lvert e_i^q\rvert}$	χ^2	d.f.	p_{χ^2}
Deciles on market-to-book									
4.08	18.11	3.91	15.97	0.00	4.22	0.07	8.99	8	0.34
Deciles on asset growth									
3.98	15.79	2.07	16.76	0.57	10.89	0.07	11.28	8	0.19
Deciles on *ROE*									
4.24	16.43	3.66	7.36	0.00	5.03	0.13	8.59	8	0.38

Panel C: Model errors for individual deciles

	Low	2	3	4	5	6	7	8	9	High	H−L
Deciles on market-to-book											
e_i^q	−0.02	−0.06	−0.05	−0.11	−0.04	−0.04	0.08	−0.04	0.21	−0.05	−0.03
$[t]$	−0.56	−1.31	−1.32	−1.99	−1.31	−0.90	1.49	−0.77	2.32	−2.07	−0.91
Deciles on asset growth											
e_i^q	0.20	0.06	−0.04	−0.11	−0.10	−0.01	0.03	0.06	0.05	−0.06	−0.27
$[t]$	2.93	1.71	−1.60	−2.86	−2.58	−0.40	0.92	1.43	0.86	−1.30	−2.74
Deciles on *ROE*											
e_i^q	−0.31	−0.15	−0.03	−0.06	−0.06	−0.09	−0.07	0.05	0.08	0.39	0.70
$[t]$	−1.51	−1.22	−0.50	−1.56	−1.13	−1.60	−1.65	1.21	0.63	1.96	1.75

Figure E.1 : Average Predicted q versus Average Realized q, The Benchmark Model, Alternative Testing Deciles

The results are from estimating the investment model with nonlinear marginal costs of investment via GMM on the valuation moments given by equation (10). All the deciles are labeled in an ascending order.



Panel A: Market-to-book

Panel B: Asset growth

Panel C: *ROE*