Investment-Based Expected Stock Returns

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We derive and test q-theory implications for cross-sectional stock returns. Under constant returns to scale, stock returns equal levered investment returns, which are tied directly to firm characteristics. When we use generalized method of moments to match average levered investment returns to average observed stock returns, the model captures the average stock returns of portfolios sorted by earnings surprises, book-to-market equity, and capital investment. When we try to match expected returns and return variances simultaneously, the variances predicted in the model are largely comparable to those observed in the data. However, the resulting expected return errors are large.

I. Introduction

We use the q-theory of investment to derive and test predictions for the cross section of stock returns. Under constant returns to scale, stock

We have benefited from helpful comments of Ilona Babenko, Nick Barberis, David Brown, V. V. Chari, Harry DeAngelo, Du Du, Wayne Ferson, Simon Gilchrist, Rick Green, Burton Hollifield, Liang Hu, Patrick Kehoe, Bob King, Narayana Kocherlakota, Leonid Kogan, Owen Lamont, Sydney Ludvigson, Ellen McGrattan, Antonio Mello, Jianjun Miao, Mark Ready, Bryan Routledge, Martin Schneider, Jun Tu, Masako Ueda, Jiang Wang, and

[*Journal of Political Economy,* 2009, vol. 117, no. 6]

returns equal levered investment returns. The latter returns are tied to firm characteristics via firms' first-order conditions for equity value maximization. We use generalized method of moments (GMM) to match means and variances of levered investment returns with those of stock returns. We conduct the GMM tests using data on portfolios sorted by earnings surprises, book-to-market equity, and capital investment, which are firm characteristics tied closely to cross-sectional patterns in returns. We also compare the performance of the *q*-theory model with the performance of traditional asset pricing models such as the capital asset pricing model (CAPM), the Fama-French (1993) three-factor model, and the standard consumption-CAPM with power utility.

When matching the average returns of the testing portfolios, the *q*-theory model outperforms the traditional models. We estimate a mean absolute error of 0.7 percent per year for 10 equal-weighted portfolios sorted by earnings surprises. This error is lower than those from the CAPM, 5.7 percent, the Fama-French model, 4.0 percent, and the standard consumption-CAPM, 3.6 percent. The error for the return on the portfolio that is long on stocks with high earnings surprises and short on stocks with low earnings surprises (high-minus-low earnings surprise portfolio) is −0.4 percent per year. This error is negligible compared to the errors of 12.6 percent from the CAPM, 14.1 percent from the Fama-French model, and 13.4 percent from the standard consumption-CAPM. Similarly, the *q*-theory model produces an error for the high-minus-low book-to-market portfolio of only 1.2 percent per year, which is smaller than 18.6 percent from the CAPM, 7.3 percent from the Fama-French model, and 12.3 percent from the standard consumption-CAPM. Finally, the *q*-theory model produces an error for the high-minus-low capital investment portfolio of −0.5 percent per year, which is smaller than the error of −6.3 percent from the CAPM, −6.3 percent from the Fama-French model, and −8.4 percent from the standard consumption-CAPM.

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other seminar participants at Boston University, Brigham Young University, Carnegie Mellon University, Emory University, Erasmus University Rotterdam, Federal Reserve Bank of Minneapolis, Florida State University, Hong Kong University of Science and Technology, INSEAD, London Business School, London School of Economics, New York University, Purdue University, Renmin University of China, Shanghai Jiao Tong University, Tilburg University, University of Arizona, University of Lausanne, University of Southern California, University of Vienna, University of Wisconsin–Madison, Yale School of Management, Society of Economic Dynamics annual meetings in 2006, University of British Columbia Phillips, Hager & North Centre for Financial Research Summer Finance Conference in 2006, American Finance Association annual meetings in 2007, the China International Conference in Finance in 2007, and the Centre for Economic Policy Research Asset Pricing Week in Gerzensee in 2009. Liu acknowledges financial support from Hong Kong University of Science and Technology (grant SBI07/08.BM04). We especially thank Monika Piazzesi (the editor) and four anonymous referees for helpful comments. Online App. D contains unabridged tables and all the robustness tests not reported in the paper. This paper supersedes our NBER Working Paper no. 13024 titled "Regularities."

When we use the q-theory model to match the average returns and variances of the testing portfolios simultaneously, the variances predicted by the model are largely comparable to stock return variances. The average stock return volatility across the earnings surprise portfolios is 21.1 percent per year, which is close to the average levered investment return volatility of 20.4 percent. The average realized and predicted volatilities also are close for the book-to-market portfolios, 25.0 percent versus 23.6 percent, and for the capital investment portfolios, 24.8 percent versus 24.4 percent. However, the model falls short in two ways. First, while we find no discernible relation between volatilities and firm characteristics in the data, the model predicts a positive relation between volatilities and book-to-market. Second, the resulting expected return errors vary systematically with earnings surprises and capital investment and are comparable in magnitude to those from the traditional models.

Although q-theory originates in Brainard and Tobin (1968) and Tobin (1969), our work is built more directly on Cochrane (1991), which first uses q-theory to study stock market returns, as well as on Cochrane (1996), which uses aggregate investment returns to parameterize the stochastic discount factor in cross-sectional tests. Several more recent articles model cross-sectional returns based on firms' dynamic optimization problems (e.g., Berk, Green, and Naik 1999; Zhang 2005). We differ by doing structural estimation of closed-form Euler equations. Our work is also connected to the literature that estimates investment Euler equations using aggregate or firm-level investment data (e.g., Shapiro 1986; Whited 1992). Our work differs because we use this framework to study cross-sectional returns rather than investment dynamics or financing constraints. Most important, our q-theory approach to understanding cross-sectional returns represents a fundamental departure from the traditional consumption-based approach (e.g., Hansen and Singleton 1982; Lettau and Ludvigson 2001) in that we do not make any preference assumptions.

II. The Model of the Firms

Time is discrete and the horizon infinite. Firms use capital and costlessly adjustable inputs to produce a homogeneous output. Firms choose these latter inputs each period, while taking their prices as given, to maximize operating profits. The operating profits are defined as revenues minus the expenditures on these inputs. Taking operating profits as given, firms choose optimal investment and debt to maximize the market value of equity.

Let $\Pi(K_{it}, X_{it})$ denote the maximized operating profits of firm i at time t. The profit function depends on capital, K_{it}, and a vector of exogenous aggregate and firm-specific shocks, X_{it}. We assume that firm

i has a Cobb-Douglas production function with constant returns to scale. The assumption of constant returns means that $\Pi(K_{it}, X_{it}) = K_{it}\partial\Pi(K_{it}, X_{it})/\partial K_{it}$. The Cobb-Douglas functional form means that the marginal product of capital is given by $\partial\Pi(K_{it}, X_{it})/\partial K_{it} = \alpha Y_{it}/K_{it}$, in which $\alpha > 0$ is capital's share and Y_{it} is sales. This parameterization assumes that shocks to operating profits, X_{it}, are reflected in sales.

End-of-period capital equals investment plus beginning-of-period capital net of depreciation: $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$, in which capital depreciates at an exogenous proportional rate of δ_{it}, which is firm specific and time varying. Firms incur adjustment costs when investing. The adjustment cost function, denoted $\Phi(I_{it}, K_{it})$, is increasing and convex in I_{it}, is decreasing in K_{it}, and exhibits constant returns to scale in I_{it} and K_{it}: $\Phi(I_{it}, K_{it}) = I_{it}\partial\Phi(I_{it}, K_{it})/\partial I_{it} + K_{it}\partial\Phi(I_{it}, K_{it})/\partial K_{it}$. We use a standard quadratic functional form: $\Phi(I_{it}, K_{it}) = (a/2)(I_{it}/K_{it})^2 K_{it}$, in which $a > 0$.

Firms can finance investment with debt. We follow Hennessy and Whited (2007) and model only one-period debt. At the beginning of time t, firm i can issue an amount of debt, denoted B_{it+1}, which must be repaid at the beginning of period $t + 1$. The gross corporate bond return on B_{it}, denoted r_{it}^B, can vary across firms and over time. Taxable corporate profits equal operating profits less capital depreciation, adjustment costs, and interest expenses: $\Pi(K_{it}, X_{it}) - \delta_{it}K_{it} - \Phi(I_{it}, K_{it}) - (r_{it}^B - 1)B_{it}$, in which adjustment costs are expensed, consistent with treating them as forgone operating profits. Let τ_t denote the corporate tax rate at time t. The payout of firm i equals

$$\begin{aligned} D_{it} \equiv\ & (1 - \tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it})] - I_{it} + B_{it+1} - r_{it}^B B_{it} \\ & + \tau_t\delta_{it}K_{it} + \tau_t(r_{it}^B - 1)B_{it}, \end{aligned} \tag{1}$$

in which $\tau_t\delta_{it}K_{it}$ is the depreciation tax shield and $\tau_t(r_{it}^B - 1)B_{it}$ is the interest tax shield.

Let M_{t+1} be the stochastic discount factor from t to $t + 1$, which is correlated with the aggregate component of X_{it+1}. Taking M_{t+1} as exogenous, firm i maximizes its cum-dividend market value of equity:

$$V_{it} \equiv \max_{\{I_{it+s}, K_{it+s+1}, B_{it+s+1}\}_{s=0}^{\infty}} E_t\left[\sum_{s=0}^{\infty} M_{t+s}D_{it+s}\right], \tag{2}$$

subject to a transversality condition that prevents firms from borrowing an infinite amount to distribute to shareholders: $\lim_{T\to\infty} E_t[M_{t+T}B_{it+T+1}] = 0$.

PROPOSITION 1. Firms' equity value maximization implies that $E_t[M_{t+1} r^I_{it+1}] = 1$, in which r^I_{it+1} is the investment return, defined as

$$r^I_{it+1} \equiv \left\{(1-\tau_{t+1})\left[\alpha \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} \right.$$
$$\left. + (1-\delta_{it+1})\left[1 + (1-\tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]\right\} \Big/ \left[1 + (1-\tau_t)a\left(\frac{I_{it}}{K_{it}}\right)\right]. \quad (3)$$

Define the after-tax corporate bond return as $r^{Ba}_{it+1} \equiv r^B_{it+1} - (r^B_{it+1} - 1)\tau_{t+1}$; then $E_t[M_{t+1} r^{Ba}_{it+1}] = 1$. Define $P_{it} \equiv V_{it} - D_{it}$ as the ex-dividend equity value, $r^S_{it+1} \equiv (P_{it+1} + D_{it+1})/P_{it}$ as the stock return, and $w_{it} \equiv B_{it+1}/(P_{it} + B_{it+1})$ as the market leverage; then the investment return is the weighted average of the stock return and the after-tax corporate bond return:

$$r^I_{it+1} = w_{it} r^{Ba}_{it+1} + (1-w_{it}) r^S_{it+1}. \quad (4)$$

Proof. See Appendix A.

The investment return in equation (3) is the ratio of the marginal benefit of investment at time $t+1$ to the marginal cost of investment at t. Define marginal q as the discounted present value of the future marginal profits from investing in one additional unit of capital (see eq. [A2] in App. A). Optimality means that the marginal cost of investment equals the marginal q. In the numerator of equation (3) the term $(1-\tau_{t+1})\alpha Y_{it+1}/K_{it+1}$ is the marginal after-tax profit produced by an additional unit of capital, the term $(1-\tau_{t+1})(a/2)(I_{it+1}/K_{it+1})^2$ is the marginal after-tax reduction in adjustment costs, the term $\tau_{t+1}\delta_{it+1}$ is the marginal depreciation tax shield, and the last term in the numerator is the marginal continuation value of the extra unit of capital net of depreciation. In addition, the first term in the numerator divided by the denominator is analogous to a dividend yield. The last term in the numerator divided by the denominator is analogous to a capital gain because this ratio is proportional to the growth rate of marginal q.

Equation (4) is exactly the weighted average cost of capital in corporate finance. Without leverage, this equation reduces to the equivalence between stock and investment returns, a relation first established by Cochrane (1991). This relation is an algebraic restatement of the equivalence between marginal q and average q from Hayashi (1982). Solving for r^S_{it+1} from equation (4) gives

$$r^S_{it+1} = r^{Iw}_{it+1} \equiv \frac{r^I_{it+1} - w_{it} r^{Ba}_{it+1}}{1 - w_{it}}, \quad (5)$$

in which r^{Iw}_{it+1} is the levered investment return.

III. Econometric Methodology

A. Moments for GMM Estimation and Tests

To examine whether cross-sectional variation in average stock returns matches cross-sectional variation in firm characteristics, we test the ex ante restriction implied by equation (5): expected stock returns equal expected levered investment returns,

$$E[r^{S}_{it+1} - r^{Iw}_{it+1}] = 0. \tag{6}$$

To examine whether the q-theory model can reproduce empirically plausible stock return volatilities, we also test whether stock return variances equal levered investment return variances:

$$E[(r^{S}_{it+1} - E[r^{S}_{it+1}])^2 - (r^{Iw}_{it+1} - E[r^{Iw}_{it+1}])^2] = 0. \tag{7}$$

As noted by Cochrane (1991), taken literally, equation (5) says that levered investment returns equal stock returns for every stock, every period, and every state of the world. Because no choice of parameters can satisfy these conditions, equation (5) is rejected at any level of significance. However, we can test the weaker conditions in equations (6) and (7), after adding statistical assumptions about the errors that invalidate these two moment conditions (model errors). These errors arise because of either measurement or specification issues. For example, components of investment returns such as the capital stock are difficult to measure, adjustment costs might not be quadratic, and the marginal product of capital might not be proportional to the sales-to-capital ratio.

Specifically, we define the model errors from the moment conditions as

$$e^{q}_{i} \equiv E_T[r^{S}_{it+1} - r^{Iw}_{it+1}] \tag{8}$$

and

$$e^{\sigma^2}_{i} \equiv E_T[(r^{S}_{it+1} - E_T[r^{S}_{it+1}])^2 - (r^{Iw}_{it+1} - E_T[r^{Iw}_{it+1}])^2], \tag{9}$$

in which $E_T[\cdot]$ is the sample mean of the series in brackets. We call e^{q}_{i} the expected return error and $e^{\sigma^2}_{i}$ the variance error. Both errors are assumed to have a mean of zero. While recognizing that measurement and specification errors, unlike forecast errors, do not necessarily have a zero mean, we note that this simple assumption underlies most Euler equation tests.[1]

We estimate the parameters a and α using GMM to minimize a weighted average of e_i^q or a weighted average of both e_i^q and $e_i^{\sigma^2}$. We use the identity weighting matrix in one-stage GMM. By weighting all the moments equally, the identity matrix preserves the economic structure of the testing assets (e.g., Cochrane 1996). After all, we choose testing assets precisely because the underlying characteristics are economically important in providing a wide cross-sectional spread in average stock returns. The identity weighting matrix also gives potentially more robust, albeit less efficient, estimates. The estimates from second-stage GMM are similar to the first-stage estimates. To conduct inferences, we nevertheless need to calculate the optimal weighting matrix. We use a standard Bartlett kernel with a window length of five. The results are insensitive to the window length. To test whether all (or a subset of) model errors are jointly zero, we use a χ^2 test from Hansen (1982, lemma 4.1). Appendix B provides additional econometric details.

We conduct the GMM estimation and tests at the portfolio level. We use portfolios because the stylized facts in cross-sectional returns can always be represented at the portfolio level (e.g., Fama and French 1993). The usage of portfolios therefore befits our economic question. In addition, portfolio returns have lower residual variance than individual stock returns. As such, average return spreads are more reliable statistically across portfolios than across individual stocks. The portfolio approach also has the advantage that portfolio investment data are smooth, whereas firm-level investment data are lumpy because of nonconvex adjustment costs (e.g., Whited 1998).[2]

B. Data

We construct annual levered investment returns to match with annual stock returns. Our sample of firm-level data is from the Center for Research in Security Prices (CRSP) monthly stock file and the annual and quarterly 2005 Standard and Poor's Compustat industrial files. We

[1] Cochrane (1991, 220) articulates this point as follows: "The consumption-based model suffers from the same problems: unobserved preference shocks, components of consumption that enter nonseparably in the utility function (for example, the service flow from durables), and measurement error all contribute to the error term, and there is no reason to expect these errors to obey the orthogonality restrictions that the forecast error obeys. Empirical work on consumption-based models focuses on the forecast error since it has so many useful properties, but the importance in practice of these other sources of error may be part of the reason for its empirical difficulties."

[2] Thomas (2002) shows that aggregation substantially reduces the effect of lumpy investment in equilibrium business cycle models. Hall (2004) finds that nonconvexities are not important for estimating investment Euler equations at the industry level.

select our sample by first deleting any firm-year observations with missing data or for which total assets, the gross capital stock, debt, or sales are either zero or negative. We include only firms with a fiscal year end in December. Firms with primary standard industrial classifications between 4900 and 4999 or between 6000 and 6999 are omitted because q-theory is unlikely to be applicable to regulated or financial firms.

Portfolio Definitions

We use 30 testing portfolios: 10 standardized unexpected earnings (SUE) portfolios as in Chan, Jegadeesh, and Lakonishok (1996), 10 book-to-market (B/M) portfolios as in Fama and French (1993), and 10 corporate investment (CI) portfolios as in Titman, Wei, and Xie (2004). SUE is a measure of earnings surprises or shocks to earnings, B/M is the ratio of accounting value of equity divided by the market value of equity, and CI is a measure of firm-level capital investment. The relations of stock returns with SUE and B/M represent what are arguably the two most important stylized facts in the cross section of returns (e.g., Fama 1998). We use the CI portfolios because our framework characterizes optimal investment behavior. We equal-weight portfolio returns because equal-weighted returns are harder for asset pricing models to capture than value-weighted returns (e.g., Fama 1998). Our basic results are similar if we value-weight portfolio returns.

Ten SUE portfolios.—Following Chan et al. (1996), we define SUE as the change in quarterly earnings (Compustat quarterly item 8) per share from its value 4 quarters ago divided by the standard deviation of the change in quarterly earnings over the prior 8 quarters. We rank all stocks by their most recent SUEs at the beginning of each month t and assign all the stocks to one of 10 portfolios using New York Stock Exchange (NYSE) breakpoints. We calculate average monthly returns over the holding period from month $t+1$ to $t+6$. The sample is from January 1972 to December 2005. The starting point is restricted by the availability of quarterly earnings data.

Ten B/M portfolios.—Following Fama and French (1993), we sort all stocks at the end of June of year t into 10 groups based on NYSE breakpoints for B/M. The sorting variable for June of year t is book equity for the fiscal year ending in calendar year $t-1$ divided by the market value of common equity for December of year $t-1$. Book equity is common equity (Compustat annual item 60) plus balance sheet deferred tax (item 74). The market value of common equity is the closing price per share (item 199) times the number of common shares outstanding (item 25). We calculate equal-weighted annual returns from July of year t to June of year $t+1$ for the resulting portfolios, which are rebalanced at the end of each June. The sample is from January 1963 to December 2005.

Ten CI portfolios.—Following Titman et al. (2004), we define CI_{t-1}, the sorting variable in the portfolio formation year t, as $\mathrm{CE}_{t-1}/[(\mathrm{CE}_{t-2} + \mathrm{CE}_{t-3} + \mathrm{CE}_{t-4})/3]$, in which CE_{t-1} is capital expenditures (Compustat annual item 128) scaled by sales (item 12) for the fiscal year ending in calendar year $t-1$. The prior 3-year moving average of CE aims to measure the benchmark investment level. At the end of June of year t we sort all stocks on CI_{t-1} into 10 portfolios using breakpoints based on NYSE, American Stock Exchange, and Nasdaq stocks. Equal-weighted annual portfolio returns are calculated from July of year t to June of year $t+1$. The sample is from January 1963 to December 2005.

Variable Measurement

Capital, investment, output, debt, leverage, and depreciation.—The capital stock, K_{it}, is gross property, plant, and equipment (Compustat annual item 7), and investment, I_{it}, is capital expenditures minus sales of property, plant, and equipment (the difference between items 128 and 107). We set sales of property, plant, and equipment to be zero when item 107 is missing. Our basic results are similar when we measure the capital stock as the net property, plant, and equipment (item 8) or investment as item 128. Output, Y_{it}, is sales (item 12), and total debt, B_{it}, is long-term debt (item 9) plus short-term debt (item 34). Our basic results are similar when we use the Bernanke and Campbell (1988) algorithm to convert the book value of debt into the market value of debt. We measure market leverage as the ratio of total debt to the sum of total debt and the market value of equity. The depreciation rate, δ_{it}, is the amount of depreciation (item 14) divided by capital stock.

Both stock and flow variables in Compustat are recorded at the end of year t. However, the model requires stock variables subscripted t to be measured at the beginning of year t and flow variables subscripted t to be measured over the course of year t. We take, for example, for the year 1993 any beginning-of-period stock variable (such as K_{i1993}) from the 1992 balance sheet and any flow variable measured over the year (such as I_{i1993}) from the 1993 income or cash flow statement.

We follow Fama and French (1995) in aggregating firm-specific characteristics to portfolio-level characteristics: Y_{it+1}/K_{it+1} is the sum of sales in year $t+1$ for all the firms in portfolio i formed in June of year t divided by the sum of capital stocks at the beginning of $t+1$ for the same firms; I_{it+1}/K_{it+1} in the numerator of r^I_{it+1} is the sum of investment in year $t+1$ for all the firms in portfolio i formed in June of year t divided by the sum of capital stocks at the beginning of $t+1$ for the same firms; I_{it}/K_{it} in the denominator of r^I_{it+1} is the sum of investment in year t for all the firms in portfolio i formed in June of year t divided by the sum of capital stocks at the beginning of year t for the same firms; and δ_{it+1} is the total amount of depreciation for all the firms in portfolio i formed in June of year t divided by the sum of capital stocks at the beginning of $t+1$ for the same firms.

Corporate bond returns.—Firm-level corporate bond data are rather limited, and few or none of the firms in several portfolios have corporate bond ratings. To construct bond returns, r^B_{it+1}, for firms without bond ratings, we follow Blume, Lim, and MacKinlay's (1998) approach for imputing bond ratings not available in Compustat. First, we estimate an ordered probit model that relates categories of credit ratings to observed explanatory variables. We estimate the model using all the firms that have data on credit ratings (Compustat annual item 280). Second, we use the fitted value to calculate the cutoff value for each rating. Third, for firms without credit ratings we estimate their credit scores using the coefficients estimated from the ordered probit model and impute bond ratings by applying the cutoff values for the different credit ratings. Finally, we assign the corporate bond returns for a given credit rating from Ibbotson Associates as the corporate bond returns to all the firms with the same credit rating.

The explanatory variables in the ordered probit model are interest coverage defined as the ratio of operating income after depreciation (Compustat annual item 178) plus interest expense (item 15) to interest expense, the operating margin as the ratio of operating income before depreciation (item 13) to sales (item 12), long-term leverage as the ratio of long-term debt (item 9) to assets (item 6), total leverage as the ratio of long-term debt plus debt in current liabilities (item 34) plus short-term borrowing (item 104) to assets, and the natural log of the market value of equity deflated to 1973 by the consumer price index (item 24 times item 25). Following Blume et al. (1998), we also include the market beta and residual volatility from the market regression. For each calendar year we estimate the beta and residual volatility for each firm with at least 200 daily returns. Daily stock returns and value-weighted market returns are from CRSP. We adjust for nonsynchronous trading with one leading and one lagged value of the market return.

The corporate tax rate.—We measure τ_t as the statutory corporate income tax rate. From 1963 to 2005, the tax rate is on average 42.3 percent. The statutory rate starts at around 50 percent in the beginning years of our sample, drops from 46 percent to 40 percent in 1987 and further to 34 percent in 1988, and stays at that level afterward. The source is the Commerce Clearing House, annual publications.[3]

[3] We have experimented with firm-specific tax rates using the trichotomous variable approach of Graham (1996). The trichotomous variable is equal to (i) the statutory cor-

Timing Alignment

To match levered investment returns with stock returns, we need to align their timing. As noted, we use the Fama-French portfolio approach in forming B/M and CI portfolios at the end of June of each year t. Portfolio stock returns are calculated from July of year t to June of year $t+1$. To calculate the matching investment returns, we use stock variables at the beginning of years t and $t+1$ and flow variables for the years t and $t+1$. As such, the timing of the investment returns approximately matches with the timing of stock returns. Appendix C contains further details including the timing for the monthly rebalanced SUE portfolios and for the after-tax corporate bond returns.

IV. Empirical Results

Subsection A reports tests of the CAPM, the Fama-French model, and the standard consumption-CAPM on our portfolios. Subsection B reports tests of the q-theory model in matching expected returns, and subsection C reports tests in matching expected returns and variances simultaneously.

A. Testing Traditional Asset Pricing Models

To test the CAPM, we regress annual portfolio returns in excess of the risk-free rate on market excess returns. The risk-free rate, denoted r_{ft+1}, is the annualized return on the 1-month Treasury bill from Ibbotson Associates. The regression intercept measures the model error from the CAPM. To test the Fama-French model, we regress annual portfolio excess returns on annual returns of the market factor, a size factor, and a book-to-market factor (the factor returns data are from Kenneth French's Web site). The intercept measures the error of the Fama-French model. We also estimate the standard consumption-CAPM with the pricing kernel $M_{t+1} = \beta(C_{t+1}/C_t)^{-\gamma}$, in which β is time preference, γ is risk aversion, and C_t is annual per capita consumption of nondurables and services from the Bureau of Economic Analysis. We use one-

porate income tax rate if the taxable income defined as pretax income (Compustat annual item 170) minus deferred taxes (item 50) divided by the statutory tax rate is positive and net operating loss carryforward (item 52) is nonpositive; (ii) one-half of the statutory rate if one and only one condition in part i is violated; and (iii) zero otherwise. The trichotomous variable does not vary much across our testing portfolios. The portfolio-level tax rate is on average 36.0 percent for the low SUE portfolio, 37.9 percent for the high SUE portfolio, 34.8 percent for the low CI portfolio, and 37.4 percent for the high CI portfolio. The spread across the B/M portfolios is slightly larger: the tax rate is 40.2 percent in the low B/M portfolio and 35.1 percent in the high B/M portfolio. As such, we use time-varying but portfolio-invariant tax rates for simplicity. The results are largely similar using portfolio-specific tax rates.

TABLE 1
Descriptive Statistics of Testing Portfolio Returns

	Low	5	High	H−L	m.a.e.	[p]
	A. 10 SUE Portfolios					
$\bar{r}_i^S$	10.9	19.0	23.4	12.5		
σ_i^S	22.4	22.5	21.1	8.5		
e_i	−1.7	6.6	10.9	12.6	5.7	[.0]
	[−.7]	[2.2]	[5.0]	[12.7]		
e_i^{FF}	−4.6	2.0	9.5	14.1	4.0	[.0]
	[−2.2]	[1.0]	[6.7]	[8.1]		
e_i^C	−8.1	−.0	5.3	13.4	3.6	[.0]
	[−1.3]	[.0]	[1.4]	[.6]		
	B. 10 B/M Portfolios					
$\bar{r}_i^S$	8.7	17.9	25.8	17.1		
σ_i^S	27.9	24.9	27.0	20.5		
e_i	−4.9	5.2	13.7	18.6	6.3	[.0]
	[−1.8]	[2.3]	[3.8]	[6.0]		
e_i^{FF}	−.5	1.8	6.8	7.3	2.8	[.0]
	[−.2]	[1.8]	[2.6]	[2.5]		
e_i^C	−5.4	.3	6.9	12.3	2.4	[.0]
	[−.7]	[.1]	[2.4]	[.2]		
	C. 10 CI Portfolios					
$\bar{r}_i^S$	22.1	18.1	15.2	−7.0		
σ_i^S	32.4	22.3	26.7	11.4		
e_i	8.2	5.9	1.9	−6.3	5.7	[.0]
	[2.4]	[2.6]	[.7]	[−4.5]		
e_i^{FF}	6.5	1.5	.1	−6.3	2.2	[.0]
	[2.9]	[1.6]	[.1]	[−6.5]		
e_i^C	4.0	.5	−4.3	−8.4	1.8	[.0]
	[.7]	[.1]	[−.8]	[−.4]		

Note.—For testing portfolio i, we report in annual percent the average stock return, $\bar{r}_i^S$, the stock return volatility, σ_i^S, the intercept from the CAPM regression, e_i, the intercept from the Fama-French three-factor regression, e_i^{FF}, and the model error from the standard consumption-CAPM, e_i^C. In each panel we report results for only three (low, 5, and high) out of 10 portfolios to save space. The H–L portfolio is long in the high portfolio and short in the low portfolio. The heteroscedasticity- and autocorrelation-consistent t-statistics for the model errors are reported in brackets beneath the corresponding errors. m.a.e. is the mean absolute error in annual percent for a given set of 10 testing portfolios. For the CAPM and the Fama-French model, the p-values in brackets in the last column in each panel are for the Gibbons et al. (1989) tests of the null hypothesis that the intercepts for a given set of 10 portfolios are jointly zero. For the consumption-CAPM the p-values are for the χ^2 test from one-stage GMM that the moment restrictions for all 10 portfolios are jointly zero. In panel A for the consumption-CAPM the estimate of the time preference coefficient is $\beta = 2.8$ (standard error 0.9) and the estimate of risk aversion is $\gamma = 127.6$ (54.9). In panel B $\beta = 3.3$ (1.2) and $\gamma = 142.1$ (58.5). In panel C $\beta = 3.3$ (1.2) and $\gamma = 143.3$ (57.6).

stage GMM with the identity weighting matrix to estimate the moments $E[M_{t+1}(r_{it+1}^S - r_{ft+1})] = 0$. We also include $E[M_{t+1}r_{ft+1}] = 1$ as an additional moment condition to identify β. The error of the standard consumption-CAPM is calculated as $E_T[M_{t+1}(r_{it+1}^S - r_{ft+1})]/E_T[M_{t+1}]$.

The SUE, B/M, and CI effects cannot be captured by the traditional models. Panel A of table 1 shows that from the low SUE to the high SUE portfolio the average return increases monotonically from 10.9 percent to 23.4 percent per year. The portfolio volatilities are largely flat at around 22 percent. The CAPM error of the high-minus-low SUE portfolio is 12.6 percent per year ($t = 12.7$), and the (annualized) mean absolute error, denoted m.a.e., is 5.7 percent. The Gibbons, Ross, and Shanken (1989) statistic, which tests the null hypothesis that all the 10 individual intercepts are jointly zero, rejects the CAPM. (The intercepts do not add up to zero because we equal-weight the portfolio returns.) The performance of the Fama-French model is similar: the m.a.e. is 4.0 percent per year and the Gibbons et al. test rejects the model. The error of the high-minus-low SUE portfolio from the Fama-French model is 14.1 percent per year ($t = 8.1$). The consumption-CAPM error increases from −8.1 percent for the low SUE portfolio to 5.3 percent per year for the high SUE portfolio. Although the errors are not individually significant, probably because of large measurement errors in consumption data, the χ^2 test rejects the null hypothesis that the pricing errors are jointly zero at the 1 percent significance level. In addition, the parameter estimates are high: the time preference estimate is 2.8, and the risk aversion estimate is 127.6.

Panel B of table 1 shows that value stocks with high B/M ratios earn higher average stock returns than growth stocks with low B/M ratios, 25.8 percent versus 8.7 percent per year. The difference of 17.1 percent is significant ($t = 5.5$). There is no discernible relation between B/M and stock return volatility: both the value and the growth portfolios have volatilities around 27 percent. The CAPM error increases monotonically from −4.9 percent for growth stocks to 13.7 percent for value stocks. The average magnitude of the errors is 6.3 percent per year, and the Gibbons et al. test strongly rejects the CAPM. Even the Fama-French model fails to capture the equal-weighted returns: the high-minus-low portfolio has an error of 7.3 percent ($t = 2.5$). The consumption-CAPM error increases from −5.4 percent for growth stocks to 6.9 percent for value stocks with an average magnitude of 2.4 percent, and the model is rejected by the χ^2 test.

From panel C, high CI stocks earn lower average stock returns than low CI stocks: 15.2 percent versus 22.1 percent per year, and the difference is more than four standard errors from zero. The high-minus-low CI portfolio has an error of −6.3 percent ($t = -4.5$) from the CAPM and an error of −6.3 percent ($t = -6.5$) from the Fama-French model. Both models are rejected by the Gibbons et al. test. The consumption-CAPM error decreases from 4.0 percent for the low CI portfolio to −4.3 percent for the high CI portfolio with an average magnitude of 1.8 percent, and the χ^2 test rejects the model.

TABLE 2
Parameter Estimates and Tests of Overidentification

	SUE	B/M	CI
	A. Matching Expected Returns		
a	7.7	22.3	1.0
	[1.7]	[25.5]	[.3]
α	.3	.5	.2
	[.0]	[.3]	[.0]
χ^2	4.4	6.0	6.5
d.f.	8	8	8
p	.8	.7	.6
m.a.e.	.7	2.3	1.5
	B. Matching Expected Returns and Variances		
a	28.9	11.5	16.2
	[16.3]	[4.8]	[5.5]
α	.6	.4	.4
	[.3]	[.1]	[.1]
$\chi^2_{(2)}$	5.1	6.2	6.1
d.f.(2)	8	8	8
$p(2)$	.7	.6	.6
m.a.e.(2) × 100	2.5	4.1	2.2
$\chi^2_{(1)}$	5.2	4.4	4.8
d.f.(1)	8	8	8
$p(1)$	.7	.8	.8
m.a.e.(1)	3.5	2.6	2.2
χ^2	5.5	6.2	6.6
d.f.	18	18	18
p	1.0	1.0	1.0

Note.—Results are from one-stage GMM with an identity weighting matrix. In panel A the moment conditions are $E[r^S_{it+1} - r^{Iw}_{it+1}] = 0$. a is the adjustment cost parameter, and α is capital's share. Their standard errors are in brackets beneath the estimates. χ^2, d.f., and p are the statistic, the degrees of freedom, and the p-value testing that the moment conditions are jointly zero, respectively. m.a.e. is the mean absolute error in annual percent, $E_T[r^S_{it+1} - r^{Iw}_{it+1}]$, in which E_T is the sample mean, across a given set of testing portfolios. In panel B the moment conditions are $E[r^S_{it+1} - r^{Iw}_{it+1}] = 0$ and $E[(r^S_{it+1} - E[r^S_{it+1}])^2 - (r^{Iw}_{it+1} - E[r^{Iw}_{it+1}])^2] = 0$. $\chi^2_{(2)}$, d.f.(2), and $p(2)$ are the statistic, degrees of freedom, and p-value for the χ^2 test that the variance errors, defined as $E_T[(r^S_{it+1} - E_T[r^S_{it+1}])^2 - (r^{Iw}_{it+1} - E_T[r^{Iw}_{it+1}])^2]$, are jointly zero. m.a.e.(2) is the mean absolute variance error. $\chi^2_{(1)}$, d.f.(1), and $p(1)$ are the statistic, degrees of freedom, and p-value for the χ^2 test that the expected return errors are jointly zero. m.a.e.(1) is the mean absolute expected return error in annual percent. χ^2, d.f., and p are the statistic, degrees of freedom, and p-value of the test that the expected return errors and the variance errors are jointly zero.

B. *The* q-*Theory Model: Matching Expected Returns*

Point Estimates and Overall Model Performance

We estimate only two parameters in our parsimonious model: the adjustment cost parameter, a, and capital's share, α. Panel A of table 2 provides estimates of α ranging from 0.2 to 0.5. These estimates are largely comparable to the approximate 0.3 figure for capital's share in Rotemberg and Woodford (1992). The estimates of a are not as stable across the different sets of testing portfolios. We find significant estimates of 7.7 and 1.0 for the SUE and CI portfolios, respectively. The estimate is 22.3 for the B/M portfolios but with a high standard error of 25.5. These estimates fall within the wide range of estimates from studies using quantity data. The evidence implies that firms' optimization problem has an interior solution: the positive estimates of a mean that the adjustment cost function is increasing and convex in I_{it}.

Panel A of table 2 also reports two measures of overall model performance: the mean absolute error, m.a.e., and the χ^2 test. The model does a good job in accounting for the average returns of the 10 SUE portfolios. The m.a.e. is 0.7 percent per year, which is lower than those from the CAPM, 5.7 percent, the Fama-French model, 4.0 percent, and the standard consumption-CAPM, 3.6 percent. Unlike the traditional models that are rejected using the SUE portfolios, the q-theory model is not rejected by the χ^2 test. The overall performance of the model is more modest in capturing the average B/M portfolio returns. Although the model is not formally rejected by the χ^2 test, the m.a.e. is 2.3 percent per year, which is comparable to that from the Fama-French model, 2.8 percent, and that from the standard consumption-CAPM, 2.4 percent, but is lower than that from the CAPM, 6.3 percent. The model does better in pricing the 10 CI portfolios. The m.a.e. is 1.5 percent per year, which is lower than those from the CAPM, 5.7 percent, the Fama-French model, 2.2 percent, and the standard consumption-CAPM, 1.8 percent. The q-theory model is again not rejected by the χ^2 test.

Euler Equation Errors

The m.a.e.'s and χ^2 tests indicate only overall model performance. To provide a more complete picture, we report each individual portfolio error, e_i^q, defined in equation (8), in which levered investment returns are constructed using the estimates from panel A of table 2. We also report the t-statistic, described in Appendix B, testing that an individual error equals zero.

The magnitude of the individual errors varies from 0.1 percent to 1.7 percent per year across 10 SUE portfolios, and none of the errors are significant. In particular, panel A of table 3 shows that the high-minus-low SUE portfolio has an error of -0.4 percent per year ($t = -0.4$). This error is negligible compared to the large errors from the traditional models: 12.6 percent for the CAPM, 14.1 percent for the Fama-French model, and 13.4 percent for the standard consumption-CAPM. Figure 1 offers a visual presentation of the fit. Figure 1a plots the average levered investment returns of the 10 SUE portfolios against their average stock returns. If the model performs perfectly, all the observations should lie on the 45-degree line. From figure 1a, the scatter plot from the q-theory model is largely aligned with the 45-degree line. The re-

TABLE 3
EULER EQUATION ERRORS

	10 SUE Portfolios				10 B/M Portfolios				10 CI Portfolios			
	Low	5	High	H−L	Low	5	High	H−L	Low	5	High	H−L
	A. Euler Equation Errors from Matching Expected Returns											
e_i^q	.3	1.7	−.2	−.4	−3.9	2.4	−2.7	1.2	−1.0	2.7	−1.5	−.5
	[.6]	[1.7]	[−.1]	[−.4]	[−1.8]	[1.4]	[−1.4]	[.8]	[−.5]	[1.7]	[−1.2]	[−.4]
	B. Euler Equation Errors from Matching Expected Returns and Variances											
$e_i^{\sigma^2} \times 100$	−4.5	1.8	3.1	7.6	9.5	1.5	−10.4	−20.0	1.0	3.4	−6.3	−7.3
	[−1.9]	[1.0]	[1.5]	[1.8]	[2.4]	[.5]	[−2.0]	[−2.4]	[.3]	[1.3]	[−1.8]	[−1.4]
e_i^q	−7.0	2.6	5.4	12.4	−6.5	1.7	−.6	5.9	1.3	3.5	−5.3	−6.6
	[−2.2]	[1.9]	[2.0]	[2.5]	[−1.9]	[.9]	[−.2]	[1.1]	[.5]	[2.0]	[−2.0]	[−2.0]

NOTE.—Results are from one-stage GMM estimation with an identity weighting matrix. In panel A the moment conditions are $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$. The expected return errors are defined as $e_i^q \equiv E_T[r_{it+1}^S - r_{it+1}^{Iw}]$, in which $E_T[\cdot]$ is the sample mean of the series in brackets. In panel B the moment conditions are $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$ and $E[(r_{it+1}^S - E[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E[r_{it+1}^{Iw}])^2] = 0$. The variance errors are defined as $e_i^{\sigma^2} \equiv E_T[(r_{it+1}^S - E_T[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E_T[r_{it+1}^{Iw}])^2]$. The expected return errors are defined as in panel A. In both panels the expected return errors are in annual percent. In each set of 10 portfolios we report results for only three (low, 5, and high) out of the 10 portfolios to save space. The column H−L reports the difference in the expected return errors and the difference in the variance errors between portfolios high and low, as well as their t-statistics (in brackets).

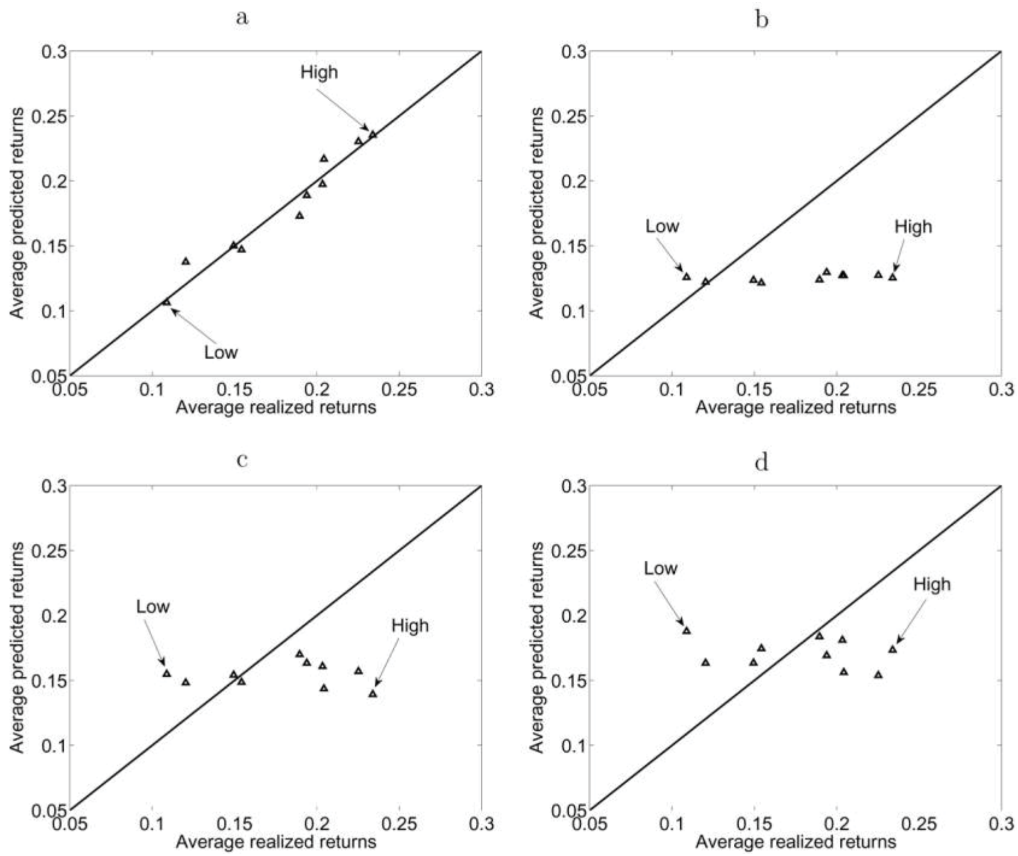

Fig. 1.—Average predicted stock returns versus average realized stock returns, 10 standardized unexpected earnings (SUE) portfolios, matching only expected stock returns. Figures 1*a*, 1*b*, 1*c*, and 1*d* report the results from the *q*-theory model, the CAPM, the Fama-French model, and the standard consumption-CAPM, respectively. High denotes the high SUE decile and low denotes the low SUE decile.

maining panels contain analogous plots for the CAPM, the Fama-French model, and the standard consumption-CAPM. In all three cases the scatter plot is largely horizontal, meaning that the traditional models fail to predict the average returns across the SUE portfolios.

Panel A of table 3 reports large errors for the B/M portfolios in the *q*-theory model. The growth portfolio has an error of −3.9 percent per year, and the value portfolio has an error of −2.7 percent. However, the errors do not vary systematically with B/M. The high-minus-low B/M portfolio has an error of only 1.2 percent, which is smaller than 18.6 percent in the CAPM, 7.3 percent in the Fama-French model, and 12.3 percent in the standard consumption-CAPM. The scatter plots in figure 2 show that, although the errors from the *q*-theory model are largely similar in magnitude to those from the Fama-French model and the standard consumption-CAPM, the average return spread between the extreme B/M portfolios from the *q*-theory model is larger than those from the traditional models.

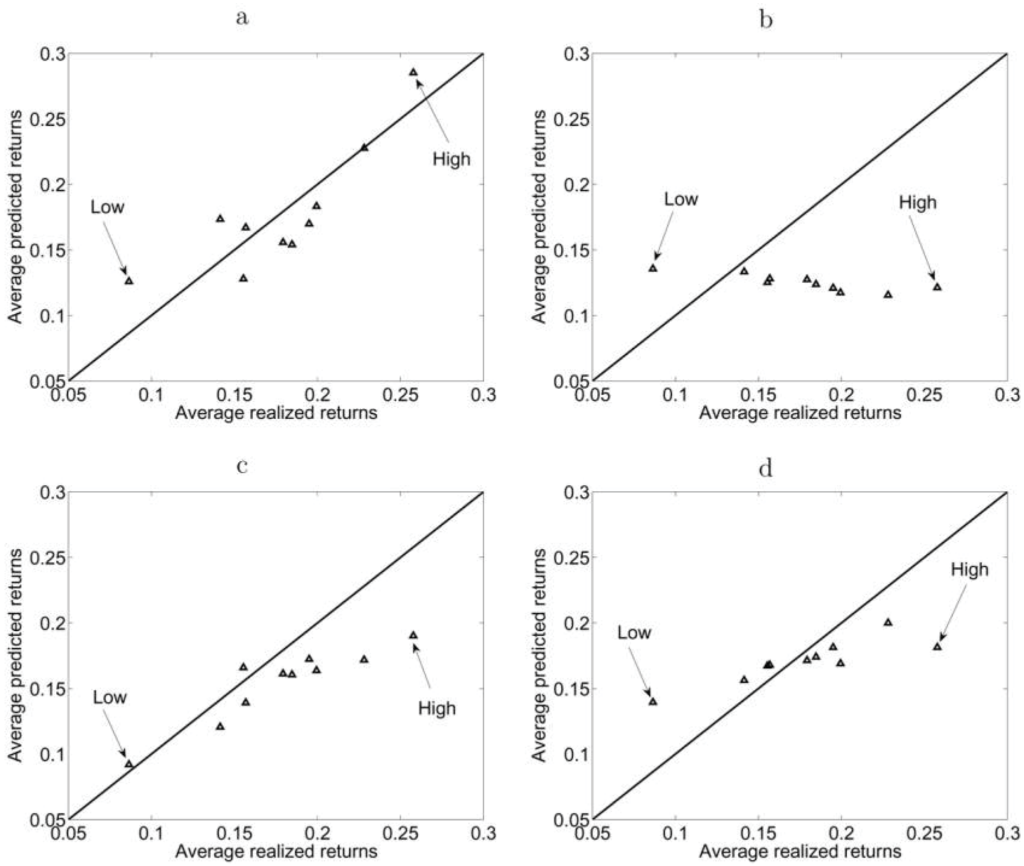

Fig. 2.—Average predicted stock returns versus average realized stock returns, 10 book-to-market (B/M) portfolios, matching only expected stock returns. Figures 2*a*, 2*b*, 2*c*, and 2*d* report the results from the *q*-theory model, the CAPM, the Fama-French model, and the standard consumption-CAPM, respectively. High denotes the high B/M decile and low denotes the low B/M decile.

From panel A of table 3, the errors from the CI portfolios are larger than those from the SUE portfolios but are smaller than those from the B/M portfolios. The high-minus-low CI portfolio has an error of −0.5 percent per year ($t = -0.4$), meaning that the *q*-theory model generates a large average return spread across the two extreme CI portfolios. The scatter plot in figure 3*a* confirms this observation. In contrast, none of the traditional models are able to reproduce the average return spread, as shown in the rest of figure 3.

Economic Mechanisms Behind Expected Stock Returns

The intuition behind our estimation results comes from the investment return equation (3) and the levered investment return equation (5). The equations suggest several economic mechanisms that underlie the cross-sectional variation of average stock returns. Each mechanism corresponds to a specific component of the levered investment return. The

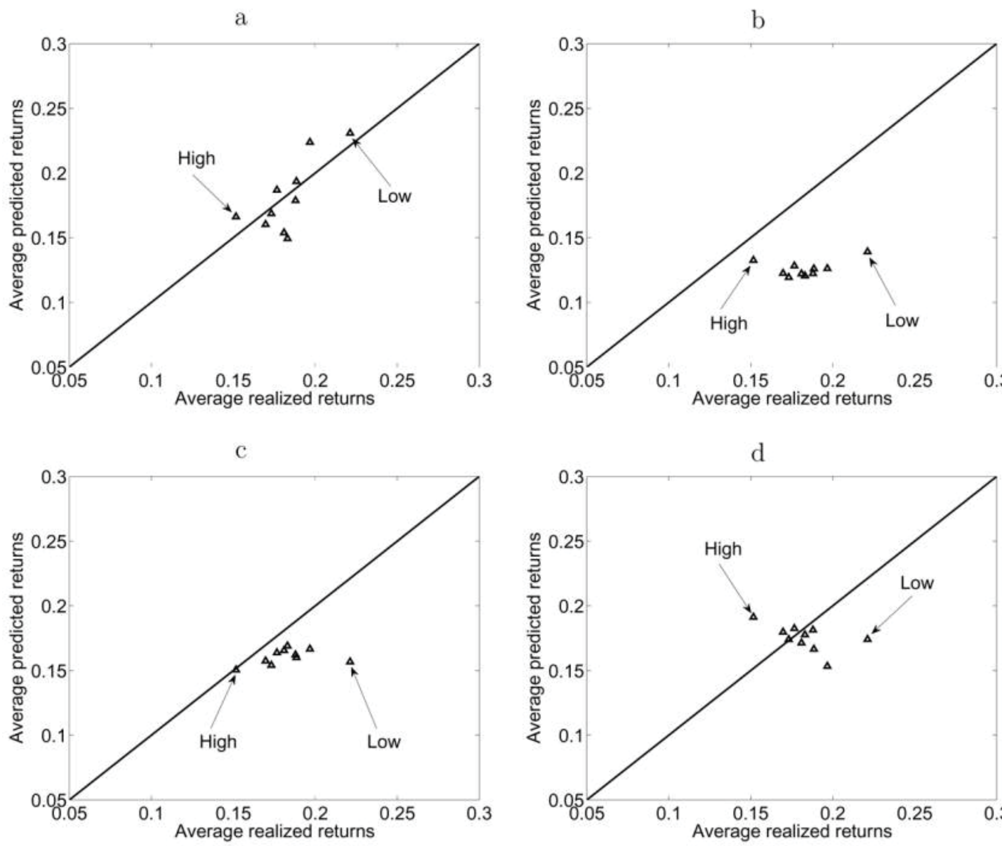

FIG. 3.—Average predicted stock returns versus average realized stock returns, 10 corporate investment (CI) portfolios, matching only expected stock returns. Figures 3*a*, 3*b*, 3*c*, and 3*d* report the results from the *q*-theory model, the CAPM, the Fama-French model, and the standard consumption-CAPM, respectively. High denotes the high CI decile and low denotes the low CI decile.

first component is the marginal benefit of investment, which is primarily the marginal product of capital at $t+1$ in the numerator of the investment return. The second component is roughly proportional to the growth rate of investment, which corresponds to the "capital gain" component of the investment return. Investment-to-capital is an increasing function of marginal q, denoted q_{it}, which is related to firm i's stock price.

The third economic mechanism works through the component I_{it}/K_{it} in the denominator of the investment return. Because investment today increases with the net present value of one additional unit of capital and because the net present value decreases with the cost of capital, a low cost of capital means high net present value and high investment. As such, investment today and average stock returns are negatively correlated. Relatedly, because investment is an increasing function of marginal q and because marginal q is in turn inversely related to book-to-market equity, expected stock returns and book-to-market equity are positively correlated. The fourth component is the rate of depreciation, δ_{it+1}. Collecting terms involving δ_{it+1} in the numerator of equation (3) yields $-(1-\tau_{t+1})[1+a(I_{it+1}/K_{it+1})]\delta_{it+1}$, meaning that high rates of depreciation tomorrow imply lower average returns. The fifth component is market leverage: taking the first-order derivative of equation (5) with respect to w_{it} shows that expected stock returns should increase with market leverage today.

In short, all else equal, firms should earn lower average stock returns if they have high investment-to-capital today, low expected investment growth, low sales-to-capital tomorrow, high rates of depreciation tomorrow, or low market leverage today.

Expected Returns Accounting

To understand our estimation results, table 4 presents averages of the different components of levered investment returns across testing portfolios. From panel A, the average I_{it}/K_{it}, δ_{it+1}, and the bond returns, r^B_{it+1}, are largely flat across 10 SUE portfolios. The average $(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$ (future investment growth) and Y_{it+1}/K_{it+1} both increase from the low SUE portfolio to the high SUE portfolio, going in the right direction to capture average stock returns. However, going in the wrong direction, market leverage decreases from the low SUE portfolio to the high SUE portfolio.

For the 10 B/M portfolios, I_{it}/K_{it} decreases from 0.2 to 0.1 per year from the low to the high B/M portfolio. The low B/M firms also have lower market leverage (0.1 vs. 0.5) than the high B/M firms. Both characteristics go in the right direction to match average stock returns. However, going in the wrong direction, the low B/M firms have higher average Y_{it+1}/K_{it+1} (2.0 vs. 1.4) than the high B/M firms. The depreciation rate, corporate bond returns, and investment growth are largely flat. Sorting on CI produces a spread in I_{it}/K_{it} of 0.1. Compared to the high CI firms, the low CI firms have higher future investment growth (1.3 vs. 0.8) and higher market leverage (0.4 vs. 0.3). All three patterns go in the right direction to match expected stock returns.

The observed patterns in characteristics shed light on the differences in the parameter estimates across the different sets of portfolios. Intuitively, GMM fits the model to the data by minimizing the differences between average levered investment returns and average stock returns. If the cross-sectional variation in the main components of the investment returns (sales-to-capital, investment-to-capital, and investment growth) is not matched in the same way with the cross-sectional variation in average stock returns across the different sets of portfolios, our estimation necessarily produces different parameter estimates. As noted, the sales-to-capital ratio goes in the right direction to match the expected

TABLE 4

EXPECTED RETURNS ACCOUNTING

A. CHARACTERISTICS IN LEVERED INVESTMENT RETURNS

	10 SUE Portfolios					10 B/M Portfolios					10 CI Portfolios				
	Low	5	High	H−L	[t_{H-L}]	Low	5	High	H−L	[t_{H-L}]	Low	5	High	H−L	[t_{H-L}]
I_{it}/K_{it}	.1	.1	.1	.0	[.7]	.2	.1	.1	−.1	[−8.0]	.1	.1	.2	.1	[11.1]
$(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$	.9	1.0	1.1	.2	[4.1]	1.0	1.0	1.0	.0	[.7]	1.3	1.0	.8	−.4	[−7.2]
Y_{it+1}/K_{it+1}	1.5	1.5	1.8	.3	[5.2]	2.0	1.5	1.4	−.6	[−6.8]	1.8	1.6	1.9	.1	[.4]
δ_{it+1}	.1	.1	.1	.0	[.6]	.1	.1	.1	−.0	[−5.0]	.1	.1	.1	.0	[−.5]
w_{it}	.3	.3	.2	−.1	[−5.8]	.1	.3	.5	.4	[12.4]	.4	.3	.3	−.1	[−2.6]
r^B_{it+1}	9.4	9.8	9.4	−.1	[−.3]	8.2	8.1	8.5	.4	[1.1]	8.5	8.3	8.4	−.0	[−.2]

B. EXPECTED RETURN ERRORS FROM COMPARATIVE STATIC EXPERIMENTS

	10 SUE Portfolios					10 B/M Portfolios					10 CI Portfolios				
	Low	5	High	H−L	m.a.e.	Low	5	High	H−L	m.a.e.	Low	5	High	H−L	m.a.e.
$\overline{I_{it}/K_{it}}$	−2.5	4.5	−4.3	−1.8	2.4	−42.1	4.7	48.2	90.2	21.3	2.9	3.5	−5.7	−8.5	2.3
$\underline{q_{it+1}/q_{it}}$	−5.2	1.8	3.6	8.9	2.6	−1.9	2.1	−4.1	−2.1	1.9	.7	3.0	−3.9	−4.6	1.7
$\underline{Y_{it+1}/K_{it+1}}$	−.8	.4	3.5	4.3	1.3	.2	.9	−6.3	−6.5	1.9	.6	−.4	.1	−.5	.4
$\overline{w_{it}}$	.1	1.9	−1.5	−1.6	.9	−6.0	2.2	5.6	11.6	3.6	1.8	2.6	−.9	−2.7	1.4

NOTE.—Panel A reports the averages of investment-to-capital, I_{it}/K_{it}, future investment growth, $(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$, sales-to-capital, Y_{it+1}/K_{it+1}, the depreciation rate, δ_{it+1}, market leverage, w_{it}, and corporate bond returns in annual percent, r^B_{it+1}. We report results for only three (low, 5, and high) out of a given set of 10 portfolios to save space. The column H−L reports the average differences between portfolios high and low, and the column [t_{H-L}] reports the heteroscedasticity- and autocorrelation-consistent t-statistics for the test that the differences equal zero. Panel B performs four comparative static experiments denoted $\overline{I_{it}/K_{it}}$, $\overline{q_{it+1}/q_{it}}$, $\overline{Y_{it+1}/K_{it+1}}$, and $\overline{w_{it}}$, in which $q_{it+1}/q_{it} = [1 + (1-\tau_{t+1})a(I_{it+1}/K_{it+1})]/[1 + (1-\tau_t)a(I_{it}/K_{it})]$. In the experiment denoted $\overline{Y_{it+1}/K_{it+1}}$, we set Y_{it+1}/K_{it+1} for a given set of 10 portfolios, indexed by i, to be its cross-sectional average in $t+1$. We then use the parameters reported in panel A of table 2 to reconstruct the levered investment returns, while keeping all the other characteristics unchanged. The other three experiments are designed analogously. We report the expected return errors defined as $e^q_i \equiv E_T[r^S_{it+1} - r^{Iw}_{it+1}]$ in annual percent for the testing portfolios, the high-minus-low portfolios, and the mean absolute value of e^q_i (m.a.e.) across a given set of 10 testing portfolios.

returns of the SUE portfolios but goes in the wrong direction to match the expected returns of the B/M portfolios. The different estimates imply different economic mechanisms underlying the cross section of expected returns across the different sets of portfolios.

To quantify the role of each component of the investment return in matching expected returns, we conduct the following accounting exercises. We set a given component equal to its cross-sectional average in each year. We then use the parameter estimates in panel A of table 2 to reconstruct levered investment returns, while keeping all the other characteristics unchanged. In the case of investment growth, we hold constant the capital gain component of the investment return, which is given by

$$\frac{1+(1-\tau_{t+1})a(I_{it+1}/K_{it+1})}{1+(1-\tau_t)a(I_{it}/K_{it})} = \frac{q_{it+1}}{q_{it}}. \tag{10}$$

We focus on the resulting change in the magnitude of the expected return errors: a large change would suggest that the component in question is quantitatively important.

Panel B of table 4 reports several insights. First, the most important component for the SUE portfolio returns is q_{it+1}/q_{it}: eliminating its cross-sectional variation makes the q-theory model underpredict the average stock return of the high-minus-low SUE portfolio by 8.9 percent per year. In contrast, this error is only -0.4 percent in the benchmark estimation. Without the cross-sectional variation of Y_{it+1}/K_{it+1}, the error of the high-minus-low SUE portfolio becomes 4.3 percent. Second, investment and leverage are both important for the B/M portfolios. Fixing I_{it}/K_{it} to its cross-sectional average produces an error of 90.2 percent per year for the high-minus-low B/M portfolio. This huge error reflects the large estimate of the parameter a for the B/M portfolios. Setting w_{it} to its cross-sectional average produces an error of 11.6 percent for the high-minus-low B/M portfolio. The terms Y_{it+1}/K_{it+1} and q_{it+1}/q_{it} are less important. Third, the dominating force in driving the average stock returns across the CI portfolios is I_{it}/K_{it}. Eliminating its cross-sectional variation gives rise to an error of -8.5 percent per year for the high-minus-low CI portfolio. Fixing q_{it+1}/q_{it} produces an error of 4.6 percent, and fixing w_{it} produces an error of 2.7 percent per year. The effect of Y_{it+1}/K_{it+1} is negligible.

C. *Matching Expected Returns and Variances Simultaneously*

Point Estimates and Overall Model Performance

Panel B of table 2 reports the point estimates and overall model performance when we use the q-theory model to match both the expected returns and variances of the testing portfolios. Capital's share, α, is estimated from 0.4 to 0.6, and all estimates are significant. The estimates of the adjustment cost parameter, a, are on average higher than those reported in panel A. The estimates are 11.5 and 16.2 for the B/M and CI portfolios, and both are significant. The estimate of a for the SUE portfolios is 28.9, but with a large standard error of 16.3.

As explained in Erickson and Whited (2000), it can be misleading to interpret the parameter a in terms of adjustment costs or speeds. We follow their suggestion of gauging the economic magnitude of this parameter in terms of the elasticity of investment with respect to marginal q. Evaluated at the sample mean, this elasticity is given by $1/a$ times the ratio of the mean of q_{it} to the mean of I_{it}/K_{it}. The estimates in panel B imply elasticities that range from 0.4 to 0.7. A similar inelastic response of 0.1 is implied by the estimate of a for the B/M portfolios in panel A. However, the implied elasticity for the SUE portfolios is greater than one, and that for the CI portfolios is over 10. Although this last estimate seems large, the others fall in a reasonable range between zero and 1.3. The general inference is that investment responds to q inelastically.

Panel B of table 2 reports three tests of overall model performance: $\chi^2_{(2)}$ is the χ^2 test that all the variance errors are jointly zero, $\chi^2_{(1)}$ is the χ^2 test that all the expected return errors are jointly zero, and the statistic labeled χ^2 tests that all the model errors are jointly zero. The $\chi^2_{(2)}$ tests do not reject the model, and the mean absolute variance errors, denoted m.a.e.(2), are small. To better interpret their economic magnitude, we use the parameter estimates from panel B of table 2 to calculate the average levered investment return volatility (instead of variance). At 20.4 percent, this average predicted volatility is close to the average realized volatility, 21.1 percent, across the 10 SUE portfolios. For the 10 B/M portfolios, the average stock return volatility is 25.0 percent, and the average levered investment return volatility is 23.6 percent. Finally, for the 10 CI portfolios the average stock return volatility is 24.8 percent, and their average levered investment return volatility is 24.4 percent.

Cochrane (1991) reports that the aggregate investment return volatility is only about 60 percent of the value-weighted stock market volatility. Our results complement Cochrane's in several ways. First, we account for leverage, whereas Cochrane does not. Second, we use portfolios as testing assets, in which firm-specific shocks are unlikely to be diversified away entirely, whereas Cochrane studies the stock market portfolio. Third, we formally choose parameters to match variances, whereas Cochrane calibrates his parameters to match expected returns exactly but allows variances to vary.

Although the $\chi^2_{(1)}$ tests on the expected return errors do not reject the model, the mean absolute expected return errors, denoted m.a.e.(1), are large. The m.a.e.(1) for the SUE portfolios is 3.5 percent per year, up from 0.7 percent when matching only expected returns. The m.a.e.(1) for the B/M portfolios increases from 2.3 percent to 2.6 percent, whereas that for the CI portfolios goes up from 1.5 percent to 2.2 percent. This increase is to be expected because we are asking more of the model by matching more moments.

Euler Equation Errors

Panel B of table 3 reports individual variance errors, defined as in equation (9), and expected return errors, defined as in equation (8), in which levered investment returns, r^{Iw}_{it+1}, are constructed using the estimates from panel B of table 2. The *t*-statistics of the errors, described in Appendix B, are calculated using the variance-covariance matrix from one-stage GMM.

Panel B of table 3 shows that the magnitude of the variance errors is small relative to stock return variances. Most variance errors are insignificant. The left panels in figure 4 plot levered investment return volatilities against stock return volatilities for the testing portfolios. (To facilitate interpretation, we plot volatilities instead of variances.) The points in the scatter plot are generally aligned with the 45-degree line. However, while there is no discernible relation between stock return volatilities and the characteristics in the data, the model predicts a negative relation between levered investment return volatilities and SUE (fig. 4*a*) and a positive relation between the predicted volatilities and B/M (fig. 4*c*). Panel B of table 3 also shows that the variance errors increase with SUE and decrease with B/M. The difference in the variance errors is 7.6/100 ($t = 1.8$) between the high and low SUE portfolios and is $-20/100$ ($t = -2.4$) between the high and low B/M portfolios.

Panel B of table 3 shows that the expected return errors vary systematically with SUE, increasing from -7.0 percent per year for the low SUE portfolio to 5.4 percent for the high SUE portfolio. The difference of 12.4 percent ($t = 2.5$) is similar in magnitude to those from the traditional models. Figure 4*b* plots the average levered investment returns against the average stock returns. The pattern is largely horizontal, similar to those from the traditional models.

The expected return errors for the B/M portfolios in panel B of table 3 also are larger than those in panel A from matching only expected returns. However, the model still predicts an average return spread of 11.3 percent per year between the extreme B/M portfolios. The expected return error for the high-minus-low B/M portfolio is 5.9 percent per year in the *q*-theory model, which is lower than 7.3 percent from the Fama-French model. The CAPM and the standard consumption-CAPM produce even higher errors, 18.6 percent and 12.3 percent, re-

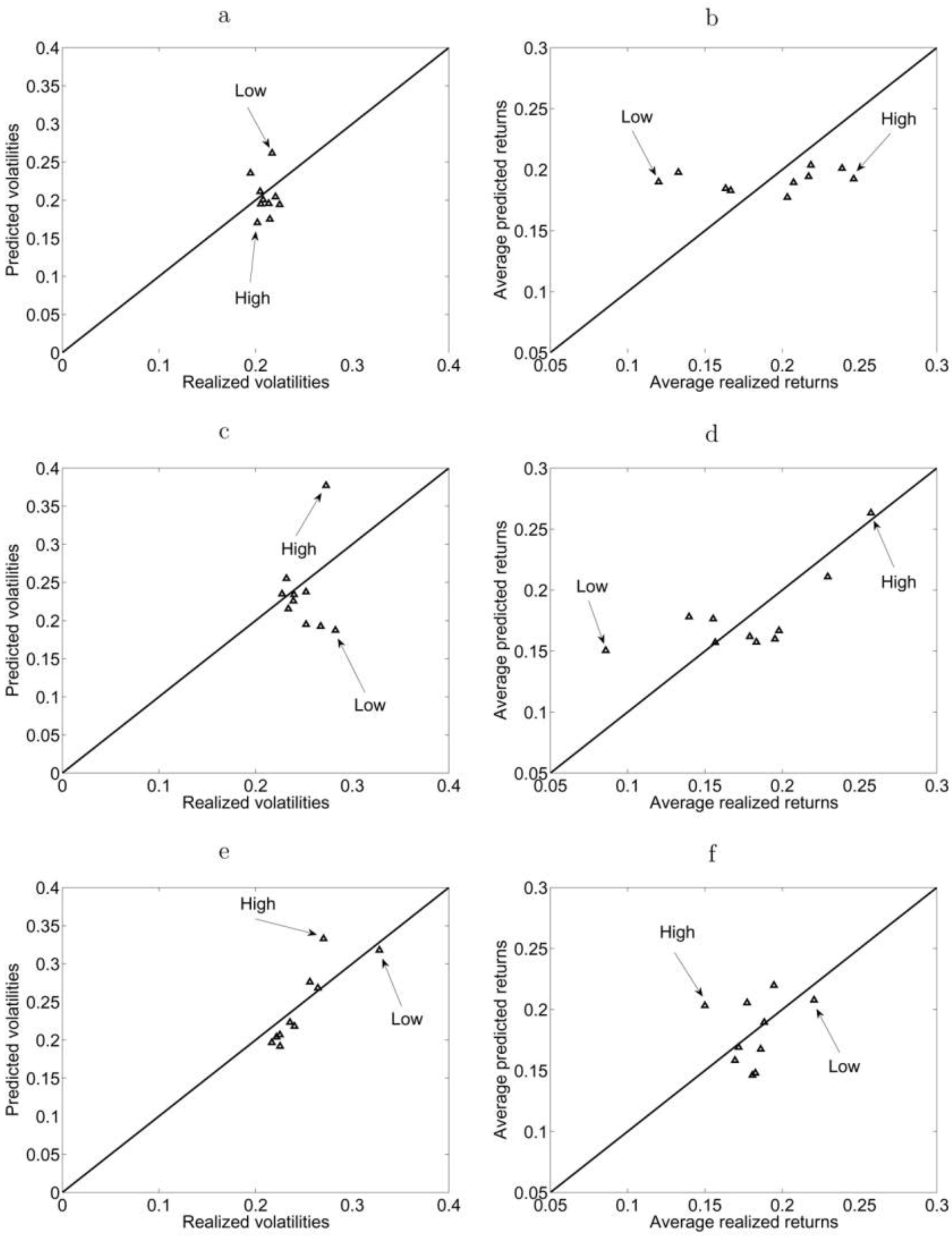

FIG. 4.—Predicted stock return volatilities versus realized stock return volatilities, average predicted stock returns versus average realized stock returns, the q-theory model, matching expected returns and variances simultaneously. Figures 4*a*, 4*c*, and 4*e* report the volatility plots for the 10 SUE portfolios, the 10 B/M portfolios, and the 10 CI portfolios, respectively. Figures 4*b*, 4*d*, and 4*f* report the expected return plots for the 10 SUE portfolios, the 10 B/M portfolios, and the 10 CI portfolios, respectively.

spectively. The q-theory model's performance in reproducing the average returns of the CI portfolios deteriorates to the same level as in the traditional models. The difference in the expected return errors between the extreme CI portfolios is -6.6 percent, which is similar to those from the CAPM and the Fama-French model. From figure 4*f*, the scatter plots of average returns from the q-theory model are largely horizontal.

The evidence shows that the q-theory model does a poor job of matching expected returns and variances simultaneously in the SUE and CI portfolios but a somewhat better job in the B/M portfolios. For the SUE and CI portfolios, when we match only expected returns, the predicted investment return variances are lower than observed stock return variances because investment and output are not as volatile as stock returns. As such, to minimize model errors, the joint estimation of expected returns and variances produces empirically plausible variances by picking large estimates of the adjustment cost parameter and of capital's share. These large estimates in turn cannot produce small expected return errors. For the B/M portfolios, when we match only expected returns, the predicted variances are no longer low because of the high adjustment cost parameter estimate required to match expected returns. As such, the mean absolute expected return error does not deteriorate as much as it does in the case of the SUE and CI portfolios when we do the joint estimation.

A Correlation Puzzle

As noted, equation (5), taken literally, predicts that stock returns should equal levered investment returns at every data point. We have so far examined the first and second moments of returns that are the focus of much work in financial economics. We can explore yet another, even stronger, prediction of the model: stock returns should be perfectly correlated with levered investment returns.

Table 5 reports that the contemporaneous time-series correlations between stock and levered investment returns are weakly negative, whereas those between one-period-lagged stock returns and levered investment returns are positive. When we pool all the observations in the SUE portfolios together, the contemporaneous correlation is $-.1$, which is significant at the 5 percent level. However, the correlation between one-period-lagged stock returns and levered investment returns is .2, which is significant at the 1 percent level. Replacing levered investment returns with investment growth yields similar results, meaning that the correlations are insensitive to the investment return specifications.

Investment lags (lags between the decision to invest and the actual investment expenditure) can temporally shift the correlations between

TABLE 5
CORRELATIONS

	A. 10 SUE Portfolios				B. 10 B/M Portfolios				C. 10 CI Portfolios			
	Low	5	High	All	Low	5	High	All	Low	5	High	All
$\rho(r^S_{it+1}, r^{Iw}_{it+1})$	−.3	−.2	−.3	−.1**	−.2	−.2	−.1	−.1**	.2	−.3**	−.3*	−.1
$\rho(r^S_{it}, r^{Iw}_{it+1})$	.2	.0	.1	.2***	.1	.2	.3***	.2***	.4***	.2	.3*	.2***
$\rho(r^S_{it+1}, I_{it+1}/I_{it})$	−.3	−.2	−.2	−.1	−.1	−.1	−.1	−.2***	−.3*	−.3**	−.1	−.0
$\rho(r^S_{it}, I_{it+1}/I_{it})$	.2	.0	−.0	.1**	.1	.2	.3*	.1***	.2	.1	.3	.2***

NOTE.—We report time-series correlations of stock returns (contemporaneous, r^S_{it+1}, and one-period-lagged, r^S_{it}) with levered investment returns, r^{Iw}_{it+1}, and with investment growth, I_{it+1}/I_{it}. In each panel we report results for only three (low, 5, and high) out of 10 portfolios to save space. $\rho(\cdot,\cdot)$ denotes the correlation between the two series in the parentheses. In the last column of each panel (all) we report the correlations and their significance by pooling all the observations for a given set of 10 testing portfolios. The levered investment returns are constructed using the parameters in panel A of table 2.

* Significant at the 10 percent level.
** Significant at the 5 percent level.
*** Significant at the 1 percent level.

investment growth and stock returns (e.g., Lamont 2000). Lags prevent firms from adjusting investment immediately in response to discount rate changes. Consider a 1-year lag. A discount rate fall in year t increases investment only in year $t+1$. When stock returns rise in year t (because of the discount rate fall), investment growth rises in year $t+1$: lagged stock returns should be positively correlated with investment growth. The discount rate fall in year t also means low average stock returns in year $t+1$, coinciding with high investment growth in year $t+1$. As such, the contemporaneous correlation between stock returns and investment growth should be negative. These lead-lag correlations are consistent with the evidence in table 5.

V. Conclusion

We use GMM to estimate a structural model of cross-sectional stock returns derived from the q-theory of investment. The model is parsimonious with only two parameters. We construct empirical first- and second-moment conditions based on the q-theory prediction that stock returns equal levered investment returns. The latter can be constructed from firm characteristics. When matching the first moments only, the model captures the average stock returns of portfolios sorted by earnings surprises, book-to-market equity, and capital investment. When matching the first and the second moments simultaneously, the volatilities from the model are empirically plausible, but the resulting expected returns errors are large. Finally, the model also falls short in reproducing the correlation structure between stock returns and investment growth. We conclude that, on average, portfolios of firms do a good job of aligning investment policies with their costs of capital and that this alignment drives many stylized facts in cross-sectional returns. However, because we do not parameterize the stochastic discount factor, our work is silent about why average return spreads across characteristics-sorted portfolios are not matched with spreads in covariances empirically.

Appendix A

Proof of Proposition 1

Let q_{it} be the Lagrangian multiplier associated with $K_{it+1} = I_{it} + (1-\delta_{it})K_{it}$. The optimality conditions with respect to I_{it}, K_{it+1}, and B_{it+1} from maximizing equation (2) are, respectively,

$$q_{it} = 1 + (1-\tau_t)\frac{\partial\Phi(I_{it},\ K_{it})}{\partial I_{it}}, \tag{A1}$$

$$q_{it} = E_t\left[M_{t+1}\left\{(1-\tau_{t+1})\left[\frac{\partial\Pi(K_{it+1},\ X_{it+1})}{\partial K_{it+1}} - \frac{\partial\Phi(I_{it+1},\ K_{it+1})}{\partial K_{it+1}}\right]\right.\right.$$

$$\left.\left. + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1})q_{it+1}\right\}\right], \tag{A2}$$

and

$$1 = E_t[M_{t+1}[r^B_{it+1} - (r^B_{it+1} - 1)\tau_{t+1}]]. \tag{A3}$$

Equation (A1) equates the marginal purchase and adjustment costs of investing to the marginal benefit, q_{it}. Equation (A2) is the investment Euler condition, which describes the evolution of q_{it}. The term $(1-\tau_{t+1})\partial\Pi(K_{it+1},\ X_{it+1})/\partial K_{it+1}$ captures the marginal after-tax profit generated by an additional unit of capital at $t+1$, the term $-(1-\tau_{t+1})\partial\Phi(I_{it+1},\ K_{it+1})/\partial K_{it+1}$ captures the marginal after-tax reduction in adjustment costs, the term $\tau_{t+1}\delta_{it+1}$ is the marginal depreciation tax shield, and the term $(1-\delta_{it+1})q_{it+1}$ is the marginal continuation value of an extra unit of capital net of depreciation. Discounting these marginal profits of investment dated $t+1$ back to t using the stochastic discount factor yields q_{it}.

Dividing both sides of equation (A2) by q_{it} and substituting equation (A1), we obtain $E_t[M_{t+1}r^I_{it+1}] = 1$, in which r^I_{it+1} is the investment return, defined as

$$r^I_{it+1} \equiv \left\{(1-\tau_{t+1})\left[\frac{\partial\Pi(K_{it+1},\ X_{it+1})}{\partial K_{it+1}} - \frac{\partial\Phi(I_{it+1},\ K_{it+1})}{\partial K_{it+1}}\right] + \tau_{t+1}\delta_{it+1}\right.$$

$$\left. + (1-\delta_{it+1})\left[1 + (1-\tau_{t+1})\frac{\partial\Phi(I_{it+1},\ K_{it+1})}{\partial I_{it+1}}\right]\right\}\Bigg/\left[1 + (1-\tau_t)\frac{\partial\Phi(I_{it},\ K_{it})}{\partial I_{it}}\right]. \tag{A4}$$

The investment return is the ratio of the marginal benefit of investment at time $t+1$ to the marginal cost of investment at t. Substituting $\partial\Pi(K_{it+1},\ X_{it+1})/\partial K_{it+1} = \alpha Y_{it+1}/K_{it+1}$ and $\Phi(I_{it},\ K_{it}) = (a/2)(I_{it}/K_{it})^2 K_{it}$ into equation (A4) yields the investment return equation (3).

Equation (A3) says that $E_t[M_{t+1}r^B_{it+1}] = 1 + E_t[M_{t+1}(r^B_{it+1} - 1)\tau_{t+1}]$. Intuitively, because of the tax benefit of debt, the unit price of the pretax bond return, $E_t[M_{t+1}r^B_{it+1}]$, is higher than one. The difference is precisely the present value of the tax benefit. Because we define the after-tax corporate bond return, $r^{Ba}_{it+1} \equiv r^B_{it+1} - (r^B_{it+1} - 1)\tau_{t+1}$, equation (A3) says that the unit price of the after-tax corporate bond return is one: $E_t[M_{t+1}r^{Ba}_{it+1}] = 1$.

To prove equation (4), we first show that $q_{it}K_{it+1} = P_{it} + B_{it+1}$ under constant returns to scale. We start with $P_{it} + D_{it} = V_{it}$ and expand V_{it} using equations (1) and (2):

$$
\begin{aligned}
&P_{it} + (1-\tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it}) - r_{it}^B B_{it}] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} \\
&\quad = (1-\tau_t)\left[\Pi(K_{it}, X_{it}) - \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} I_{it} - \frac{\partial \Phi(I_{it}, K_{it})}{\partial K_{it}} K_{it} - r_{it}^B B_{it}\right] \\
&\quad - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} - q_{it}[K_{it+1} - (1-\delta_{it})K_{it} - I_{it}] \\
&\quad + E_t\Bigg[M_{t+1}\Bigg\{(1-\tau_t)\Bigg[\Pi(K_{it+1}, X_{it+1}) - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial I_{it+1}} I_{it+1} \\
&\quad - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} K_{it+1} - r_{it+1}^B B_{it+1}\Bigg] - \tau_{t+1} B_{it+1} - I_{it+1} + B_{it+2} \\
&\quad + \tau_{t+1}\delta_{it+1} K_{it+1} - q_{it+1}[K_{it+2} - (1-\delta_{it+1})K_{it+1} - I_{it+1}] + \cdots\Bigg\}\Bigg].
\end{aligned} \tag{A5}
$$

Recursively substituting equations (A1), (A2), and (A3), and simplifying, we obtain

$$
\begin{aligned}
&P_{it} + (1-\tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it}) - r_{it}^B B_{it}] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} = \\
&(1-\tau_t)\left[\Pi(K_{it}, X_{it}) - \frac{\partial \Phi(I_{it}, K_{it})}{\partial K_{it}} K_{it} - r_{it}^B B_{it}\right] - \tau_t B_{it} + q_{it}(1-\delta_{it})K_{it} + \tau_t \delta_{it} K_{it}.
\end{aligned} \tag{A6}
$$

Simplifying further and using the linear homogeneity of $\Phi(I_{it}, K_{it})$, we obtain

$$
P_{it} + B_{it+1} = (1-\tau_t)\frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} I_{it} + I_{it} + q_{it}(1-\delta_{it})K_{it} = q_{it} K_{it+1}. \tag{A7}
$$

Finally, we are ready to prove equation (4):

$$
\begin{aligned}
& w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^{S} \\
& \quad = \{(1 - \tau_{t+1}) r_{it+1}^{B} B_{it+1} + \tau_{t+1} B_{it+1} + P_{it+1} \\
& \qquad + (1 - \tau_{t+1})[\Pi(K_{it+1},\ X_{it+1}) - \Phi(I_{it+1},\ K_{it+1}) - r_{it+1}^{B} B_{it+1}] \\
& \qquad - \tau_{t+1} B_{it+1} - I_{it+1} + B_{it+2} + \tau_{t+1}\delta_{it+1} K_{it+1}\}/(P_{it} + B_{it+1}) \\
& \quad = \frac{1}{q_{it} K_{it+1}} \{q_{it+1}[I_{it+1} + (1 - \delta_{it+1}) K_{it+1}] \\
& \qquad + (1 - \tau_{t+1})[\Pi(K_{it+1},\ X_{it+1}) - \Phi(I_{it+1},\ K_{it+1})] - I_{it+1} + \tau_{t+1}\delta_{it+1} K_{it+1}\} \\
& \quad = \left\{ q_{it+1}(1 - \delta_{it+1}) + (1 - \tau_{t+1}) \left[\frac{\partial \Pi(K_{it+1},\ X_{it+1})}{\partial K_{it+1}} - \frac{\partial \Phi(I_{it+1},\ K_{it+1})}{\partial K_{it+1}} \right] \right. \\
& \qquad \left. + \tau_{t+1}\delta_{it+1} \right\} \Big/ q_{it} \\
& \quad = r_{it+1}^{I}.
\end{aligned}
\tag{A8}
$$

Appendix B

Estimation Details

Following the standard GMM procedure (e.g., Hansen and Singleton 1982), we estimate the parameters $\mathbf{b} \equiv (a, \alpha)$ to minimize a weighted combination of the sample moments (8) or (8) and (9). Specifically, let $\mathbf{g}_T$ be the sample moments. The GMM objective function is a weighted sum of squares of the model errors across assets, $\mathbf{g}_T'\mathbf{W}\mathbf{g}_T$, in which we use $\mathbf{W} = \mathbf{I}$, the identity matrix. Let $\mathbf{D} = \partial \mathbf{g}_T/\partial \mathbf{b}$ and $\mathbf{S}$ equal a consistent estimate of the variance-covariance matrix of the sample errors $\mathbf{g}_T$. We estimate $\mathbf{S}$ using a standard Bartlett kernel with a window length of five.

The estimate of $\mathbf{b}$, denoted $\hat{\mathbf{b}}$, is asymptotically normal with variance-covariance matrix

$$
\mathrm{Var}\,(\hat{\mathbf{b}}) = \frac{1}{T}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}. \tag{B1}
$$

To construct standard errors for the model errors on individual portfolios or groups of model errors, we use the variance-covariance matrix for the model errors, $\mathbf{g}_T$:

$$
\mathrm{Var}\,(\mathbf{g}_T) = \frac{1}{T}[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}]\mathbf{S}[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}]'. \tag{B2}
$$

In particular, the χ^2 test whether all model errors are jointly zero is given by

$$\mathbf{g}_T'[\mathrm{Var}\,(\mathbf{g}_T)]^+\mathbf{g}_T \sim \chi^2(\text{no. moments} - \text{no. parameters}). \tag{B3}$$

The superscript + denotes pseudo-inversion.

Appendix C

Details of Timing Alignment

Figure C1 illustrates our timing convention. We use the Fama-French portfolio approach to form the B/M and CI portfolios by sorting stocks at the end of June of each year t on the basis of characteristics for the fiscal year ending in calendar year $t-1$. Portfolio stock returns, r^S_{it+1}, are calculated from July of year t to June of year $t+1$. To construct the annual investment returns in equation (3), r^I_{it+1}, we use the tax rate and investment observed at the end of year t (τ_t and I_{it}) and other variables at the end of year $t+1$ (τ_{t+1}, Y_{it+1}, I_{it+1}, and δ_{it+1}). Because stock variables are measured at the beginning of the year and because flow variables are realized over the course of the year, the investment returns go roughly from the middle of year t to the middle of year $t+1$. As such, the investment return timing largely matches the stock return timing.

The changes in stock composition in a given portfolio from portfolio rebalancing raise further subtleties. In the Fama-French portfolio approach, for the

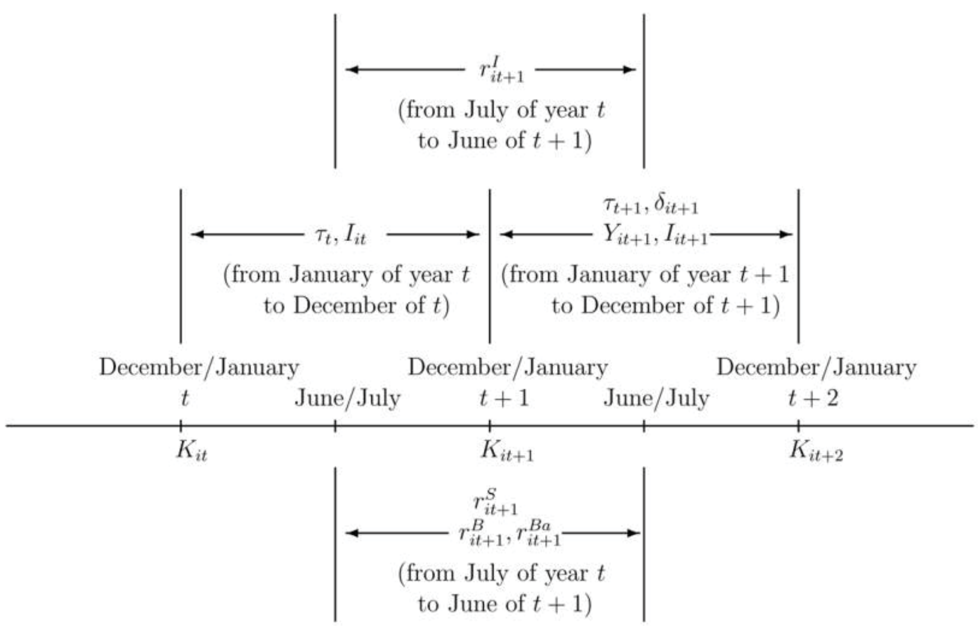

FIG. C1.—Timing alignment between stock returns and investment returns. r^I_{it+1} is the annualized investment return from July of year t to June of year $t+1$. τ_t and I_{it} are the corporate income tax rate and investment for year t, respectively. δ_{it+1} and Y_{it+1} are the rate of depreciation and sales for year $t+1$, respectively. K_{it} is capital at the beginning of year t. r^S_{it+1}, r^B_{it+1}, and r^{Ba}_{it+1} are the stock return, the pretax corporate bond return, and the after-tax corporate bond return, all annualized, from July of year t to June of year $t+1$, respectively.

annually rebalanced B/M and CI portfolios, the set of firms in a given portfolio formed in year t is fixed when we aggregate returns from July of year t to June of $t+1$. The stock composition changes only at the end of June of year $t+1$ when we rebalance. As such, we fix the set of firms in a given portfolio in the formation year t when aggregating characteristics, dated both t and $t+1$, across firms in the portfolio. In particular, to construct the numerator of r^I_{it+1}, we use I_{it+1}/K_{it+1} from the portfolio formation year t, which is different from the I_{it+1}/K_{it+1} from the formation year $t+1$ used to construct the denominator of r^I_{it+2}.

The SUE portfolios are initially formed monthly. We time-aggregate monthly returns of the SUE portfolios from July of year t to June of $t+1$ to obtain annual returns. Constructing the matching annual investment returns, r^I_{it+1}, requires care because the composition of the SUE portfolios changes from month to month. First, consider the 12 low SUE portfolios formed in each month from July of year t to June of $t+1$. For each month we calculate portfolio-level characteristics by aggregating individual characteristics over the firms in the low SUE portfolio. We use the following specific characteristics: I_{it} and τ_t observed at the end of year t, K_{it} at the beginning of year t, K_{it+1} at the beginning of $t+1$, and τ_{t+1}, Y_{it+1}, I_{it+1}, and δ_{it+1} at the end of year $t+1$. Because the portfolio composition changes from month to month, these portfolio-level characteristics also change from month to month. Accordingly, we average these portfolio characteristics over the 12 monthly low SUE portfolios and use these averages to construct r^I_{it+1}, which is in turn matched with the annual r^S_{it+1} from July of t to June of $t+1$. We then repeat this procedure for the remaining SUE portfolios.

The after-tax corporate bond return, r^{Ba}_{it+1}, depends on the tax rate and the pretax bond return, r^B_{it+1}, which we measure as the observed corporate bond returns in the data. The timing of r^B_{it+1} is the same as that of stock returns: after sorting stocks on characteristics for the fiscal year ending in calendar year $t-1$, we measure r^B_{it+1} as the equal-weighted corporate bond return from July of year t to June of $t+1$. However, calculating $r^{Ba}_{it+1} = r^B_{it+1} - (r^B_{it+1} - 1)\tau_{t+1}$ is less straightforward: τ_{t+1} is applicable from January to December of year $t+1$, but r^B_{it+1} is applicable from July of year t to June of $t+1$. We deal with this timing mismatch by replacing τ_{t+1} in the calculation of r^{Ba}_{it+1} with the average of τ_t and τ_{t+1} in the data. This timing mismatch matters little for our results because the tax rate exhibits little time-series variation. In particular, we have experimented with time-invariant tax rates in calculating r^{Ba}_{it+1}, and the results are largely similar.

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