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Financially Constrained Stock Returns

DMITRY LIVDAN, HORACIO SAPRIZA, and LU ZHANG*

ABSTRACT

We study the effect of financial constraints on risk and expected returns by extending the investment-based asset pricing framework to incorporate retained earnings, debt, costly equity, and collateral constraints on debt capacity. Quantitative results show that more financially constrained firms are riskier and earn higher expected stock returns than less financially constrained firms. Intuitively, by preventing firms from financing all desired investments, collateral constraints restrict the flexibility of firms in smoothing dividend streams in the face of aggregate shocks. The inflexibility mechanism also gives rise to a convex relation between market leverage and expected stock returns.

A VOLUMINOUS LITERATURE in corporate finance and macroeconomics has studied in depth the impact of financial constraints on firm value, capital investment, and business cycles.[1] In asset pricing, an important open question is how financial constraints affect risk and expected returns. Using the Kaplan and Zingales (1997) index of financial constraints, Lamont, Polk, and Saá-Requejo (2001) report that more constrained firms earn lower average returns than less constrained firms. However, Whited and Wu (2006) use an alternative index and find that more constrained firms earn higher average returns than less constrained firms, although the difference is insignificant.

Conflicting evidence is difficult to interpret without models that explicitly tie the characteristics in question with risk and expected returns. We aim to fill this gap. We study the effect of financial constraints on risk and expected stock returns by extending the neoclassical investment framework to incorporate retained earnings, debt, costly equity, and collateral constraints on debt capacity. In doing so, we fill an important void in the literature. To the best

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[1] An incomplete list includes Fazzari, Hubbard, and Petersen (1988), Bernanke and Gertler (1989), Whited (1992), Bond and Meghir (1994), Gertler and Gilchrist (1994), Kaplan and Zingales (1997), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Gomes (2001), Hennessy (2004), Moyen (2004), Almeida and Campello (2007), and Hennessy and Whited (2005, 2007).

of our knowledge, we are among the first to integrate rich debt dynamics into investment-based asset pricing.

Our framework is built on the dynamic asset pricing model of Zhang (2005) and the dynamic capital structure model of Hennessy and Whited (2005). Firms face both aggregate and firm-specific productivity shocks. The aggregate shock is the source of systematic risk, and the firm-specific shock is the source of firm heterogeneity. Adding to the Hennessy–Whited model with risk neutrality, we follow Zhang in modeling the aggregate shock and parameterizing the stochastic discount factor to study the cross-section of expected returns. Adding to Zhang's model with all equity financing, we follow Hennessy and Whited in allowing firms to borrow up to their debt capacity determined by collateral values. The collateral constraints require that the liquidation value of capital net of depreciation is greater than or equal to the promised debt payment. We also allow firms to retain earnings, but the interest rate when firms save is strictly lower than the interest rate when they borrow. Firms can raise external equity but with equity flotation costs. In this setting, firms choose investment and next-period debt to maximize equity value subject to the collateral constraints. The shadow price of new debt, which is given by the Lagrangian multiplier associated with the collateral constraints, precisely measures the extent to which the financial constraints are binding.

We calibrate our model and study its quantitative properties. We provide several fresh insights. Most important, more constrained firms are riskier and earn higher expected returns than less constrained firms. In univariate sorts on simulated data, the decile with the highest shadow prices of new debt outperforms the decile with the lowest shadow prices. The average return spread varies from 0.05% to 0.45% per month across different parameterizations of the model.

Intuitively, the risk of firms increases with the degree of their inflexibility in adjusting capital investment to mitigate the impact of aggregate shocks on dividend streams. After a positive aggregate shock hits the economy, firms increase investments, meaning that dividends increase less than unit-by-unit with cash flows. In effect, raising investments partially absorbs the impact of the positive shock and acts as a buffer for the dividend streams, thereby decreasing risk. This inflexibility mechanism has been used before to understand the equity premium and the value premium (e.g., Jermann (1998), Zhang (2005), and Cooper (2006)). By preventing firms from financing all desired investments, collateral constraints work against the dividend smoothing mechanism. The shadow price of new debt is an exact measure of the extent to which the constraints are binding. The higher the shadow price, the more inflexible firms are in adjusting investments to smooth dividends, the more the dividends will covary with business cycles, and the higher their risk and expected returns.

Our model prediction on the positive constraints–return relation does not contradict the evidence on the negative relation between financial distress and average stock returns.[2] The crux is that financial constraints and financial

[2] Using different measures of financial distress, Dichev (1998), Griffin and Lemmon (2002), and Campbell, Hilscher, and Szilagyi (2008) document that more distressed firms earn lower average returns than less distressed firms.

distress are different (albeit related) concepts. For example, Lamont et al. (2001, p. 529) observe:

> Do firms face financial constraints that hamper their ability to invest? By "financial constraints," we mean frictions that prevent the firm from funding all desired investments. This inability to fund investment might be due to credit constraints or inability to borrow, inability to issue equity, dependence on bank loans, or illiquidity of assets. We do not use "financial constraints" to mean financial distress, economic distress, or bankruptcy risk, although these things are undoubtedly correlated with financial constraints.

In contrast, Wruck (1990, p. 421) says:

> This paper defines financial distress as a situation where cash flow is insufficient to cover current obligations. These obligations can include unpaid debts to suppliers and employees, actual or potential damages from litigation, and missed principal or interest payments under borrowing agreements (default).

This definition has been adopted by standard corporate finance textbooks, such as Ross, Westerfield, and Jaffe (2008). Most telling, both Kaplan and Zingales (1997) and Lamont et al. (2001) limit their samples to manufacturing firms with positive real sales growth on the ground that "[r]estricting attention to firms with growing sales also helps eliminate distressed firms from the construction of the financial constraints factor, helping ensure that we are measuring constraint and not distress (Lamont et al., p. 532, footnote 1)." The differences between financial constraints and financial distress also can be seen from their respective relations with average returns. While the constraints–return relation is ambiguous, the distress–return relation is, as noted, reliably negative. The collateral constraints in our setting primarily capture financial constraints, not financial distress.

To make our quantitative results more comparable with the evidence of Lamont et al. (2001) and Whited and Wu (2006), we conduct multivariate sorts on size and the shadow price of new debt in our simulated data. We find that, after controlling for size, more constrained firms outperform less constrained firms by 7 to 16 basis points per month, but the average return differences are insignificant. These results are largely consistent with the evidence of Lamont et al. and Whited and Wu. More important, why is the constraints–return relation significant in univariate sorts, but not in bivariate sorts? The reason is that risk and the shadow price are jointly determined with size and book-to-market by underlying state variables in equilibrium. The shadow price predicts returns because it contains information about the state variables, but the information is not entirely independent of that contained by size and book-to-market.

Our model sheds light on the cross-sectional determinants of financial constraints. Firms with smaller capital, lower firm-specific productivity, and higher current debt are more constrained (their shadow prices of new debt are higher). Intuitively, the shadow price for a given firm is determined by its financial deficit, which is the difference between its desired investments and internal funds. The higher the deficit, the higher the shadow price. For firms with small scale of production, internal funds are low, but desired investments are high because of decreasing returns to scale. Further, firm-specific productivity shocks have two offsetting effects. A positive shock raises cash flows, decreasing the deficit, but it also raises desired investments (because the firm-specific productivity is persistent), increasing the deficit. Quantitatively, the first effect dominates, meaning that less profitable firms are more constrained. Finally, because of debt repayments, firms with high current debt have fewer internal funds available to finance investments. These firms are therefore more constrained.

We use our model as a natural laboratory to study quantitatively the empirical determinants of financial constraints. Consistent with the evidence in Kaplan and Zingales (1997) and Whited and Wu (2006), our quantitative results show that firms are more constrained if they have lower cash flow-to-assets, higher debt-to-assets, lower sales and sales growth, lower dividends-to-assets, lower liquid assets or cash-to-assets, and higher Tobin's Q. We also run a horse race between the Kaplan–Zingales index and the Whited–Wu index to evaluate their relative quality as empirical proxies for the shadow price of new debt in the context of our model. We find that, although both indexes are positively correlated with the shadow price, the Whited–Wu index appears to do a better job than the Kaplan–Zingales index. However, in cross-sectional regressions, both indexes leave substantial variation in the shadow price unexplained: Most R^2s are below 10%. This result casts doubt on the quality of empirical proxies for financial constraints currently used in practice.

Finally, the inflexibility mechanism underlying the positive constraints–return relation provides a new channel for leverage to affect risk and expected returns. The standard leverage hypothesis says that, when asset beta is fixed, high market leverage means high equity beta, which in turn means high average equity returns (e.g., Grinblatt and Titman (2001), pp. 381–384). When debt is free of default, the equity beta equals the asset beta times the ratio of the market value of assets divided by the market equity. Although debt is default free in our model, the inflexibility mechanism causes the asset beta to increase with leverage. Intuitively, more levered firms are burdened with more debt and must repay existing debt before financing new investments. More levered firms are more likely to be constrained financially, are less flexible in smoothing dividends, and are more likely to have riskier assets. Therefore, while the standard leverage hypothesis predicts a linear relation between market leverage and expected returns, the inflexibility mechanism predicts a convex relation.

Our model of financial constraints in the form of collateral constraints is more realistic than dividend nonnegativity constraints used in the previous literature (e.g., Gomes, Yaron, and Zhang (2003, 2006) and Whited and Wu (2006)). As noted, our modeling of debt dynamics follows Hennessy and Whited (2005), but we add aggregate shocks and asset pricing dynamics. And we contribute to investment-based asset pricing (e.g., Cochrane (1991, 1996), Berk, Green, and

Naik (1999), Carlson, Fisher, and Giammarino (2004, 2006), and Zhang (2005)) by studying the impact of debt dynamics on risk and expected returns.[3]

Section I describes the model. Section II calibrates and explains the basic properties of the model solution. Section III presents the quantitative results. We conclude in Section IV.

I. The Model

This section describes our theoretical framework.

A. Technology

The production function is given by

$$y_{jt} = e^{x_t + z_{jt}} k_{jt}^{\alpha}, \tag{1}$$

where y_{jt} and k_{jt} are the output and capital stock of firm j in period t, respectively. The production technology exhibits decreasing returns to scale with $0 < \alpha < 1$.

Production is subject to both an aggregate shock, x_t, and a firm-specific shock, z_{jt}. The aggregate shock, x_t, evolves according to a stationary and monotone Markov transition function, denoted $Q_x(x_{t+1} \,|\, x_t)$, as follows:

$$x_{t+1} = \bar{x}(1 - \rho_x) + \rho_x x_t + \sigma_x\, \epsilon^x_{t+1}. \tag{2}$$

In equation (2) ϵ^x_{t+1} is an i.i.d. standard normal shock, which serves as the driving force of economic fluctuations and systematic risk. The firm-specific productivity shocks, denoted z_{jt}, are uncorrelated across firms, indexed by j, and evolve according to a common stationary and monotone Markov transition function, denoted $Q_z(z_{jt+1} \,|\, z_{jt})$, as follows:

$$z_{jt+1} = \rho_z z_{jt} + \sigma_z\, \epsilon^z_{jt+1}. \tag{3}$$

In equation (3) ϵ^z_{jt+1} is an i.i.d. standard normal shock, and is the ultimate driving force of firm heterogeneity. When $i \neq j$, ϵ^z_{jt+1} and ϵ^z_{it+1} are uncorrelated, and ϵ^x_{t+1} is independent of ϵ^z_{jt+1} for all j.

The operating profit function for firm j with capital stock k_{jt}, idiosyncratic productivity z_{jt}, and aggregate productivity x_t is

$$\pi(k_{jt}, z_{jt}, x_t) = e^{x_t + z_{jt}} k_{jt}^{\alpha} - f, \tag{4}$$

where $f > 0$ is the nonnegative fixed costs of production, which must be paid every period.

[3] Since circulating our draft that contains an asset pricing model with debt dynamics in October 2006 (see Livdan, Sapriza, and Zhang (2006)), we have become aware of several related papers (e.g., Obreja (2006), Garlappi and Yan (2007), Gomes and Schmid (2007), and Garlappi, Shu, and Yan (2008)). Our work differs from Garlappi and Yan, and Garlappi et al. who model financial distress, and from Obreja and Gomes and Schmid, who only focus on the relation between leverage and average returns.

B. Stochastic Discount Factor

Following Zhang (2005), we use the partial equilibrium neoclassical investment framework to study asset pricing. This choice is reasonable because we focus on the link between corporate policies and asset prices. We hope that the omission of consumption can be adequately compensated by detailed firm dynamics that are absent from consumption-based asset pricing. We parameterize the stochastic discount factor, denoted m_{t+1}, as follows:

$$\log m_{t+1} = \log \eta + \gamma_t (x_t - x_{t+1}) \tag{5}$$

$$\gamma_t = \gamma_0 + \gamma_1 (x_t - \bar{x}), \tag{6}$$

where $1 > \eta > 0$, $\gamma_0 > 0$, and $\gamma_1 < 0$ are constant parameters, and γ_t decreases in $x_t - \bar{x}$ to capture the time-variation in the price of risk.

C. Investment Costs

The capital stock evolves according to

$$k_{jt+1} = (1 - \delta) k_{jt} + i_{jt}, \tag{7}$$

where δ is the rate of capital depreciation and i_{jt} is investment. When investing, firms incur purchase costs and capital adjustment costs. The total investment cost function, denoted $\phi(i_{jt}, k_{jt})$, is assumed to be asymmetric and quadratic

$$\phi(i_{jt}, k_{jt}) \equiv i_{jt} + \frac{a_P \mathbf{1}^i_{jt} + a_N \left(1 - \mathbf{1}^i_{jt}\right)}{2} \left(\frac{i_{jt}}{k_{jt}}\right)^2 k_{jt}, \tag{8}$$

where $\mathbf{1}^i_{jt} \equiv \mathbf{1}_{\{i_{jt} \geq 0\}}$ and $\mathbf{1}_{\{\cdot\}}$ is the indicator that equals one if the event described in $\{\cdot\}$ is true and zero otherwise. We assume $a_N > a_P > 0$ to capture costly reversibility (e.g., Abel and Eberly (1994, 1996) and Hall (2001)), meaning that firms face higher costs of adjustment in cutting than expanding capital.[4]

D. Collateral Constraints

We follow Hennessy and Whited (2005) and model only single-period debt. Let b_{jt+1} represent the face value of one-period debt chosen by firm j at the beginning of period t with payment due at the beginning of period $t + 1$. Positive values of b_{jt+1} mean that the firm is borrowing and negative values mean that the firm is retaining earnings (saving).

When borrowing, firms face collateral constraints, which require that the liquidation value of capital net of depreciation is greater than or equal to the promised debt payment. Formally,

[4] Zhang (2005) uses asymmetric adjustment costs to address the value premium (the stylized fact that value firms with high book-to-market equity earn higher returns on average than growth firms with low book-to-market equity). In contrast, we focus on the relation between financial constraints and expected returns.

$$b_{jt+1} \leq s_0(1-\delta)k_{jt+1}, \tag{9}$$

where $0 < s_0 < 1$ is a constant parameter. In the event of liquidation, capital can only be sold at a depressed price, $s_0 < 1$ (the price of new capital is normalized to be one). The amount of $(1-s_0)(1-\delta)k_{jt+1}$ is lost in the liquidation process as liquidation costs.

To capture the fact that recovery rates are lower and liquidation costs are higher in recessions (e.g., Shleifer and Vishny (1992), Altman, Resti, and Sironi (2004), Altman, Brady, Resti, and Sironi (2005), and Acharya, Bharath, and Srinivasan (2007)), we implement an alternative parametrization of the collateral constraints, namely,

$$b_{jt+1} \leq s_0 e^{(x_t-\bar{x})s_1}(1-\delta)k_{jt+1}, \tag{10}$$

where $s_1 > 0$. If $s_1 = 0$, we are back to the benchmark case in equation (9). But when $s_1 > 0$, the debt capacity will be lower in bad times when $x_t < \bar{x}$.

Because the collateral constraints guarantee that lenders always get repaid in full, all corporate debts are riskless and their interest rates equal to the risk-free rate r_{ft}. Thus, by committing the repayment of b_{jt+1} at the beginning of $t+1$, firm j obtains cash inflow b_{jt+1}/r_{ft} at the beginning of t.

E. Retained Earnings

Because of the collateral constraints, firms are not indifferent between savings and cash distributions. If the interest rate earned by corporate savings, denoted r_{st}, equals the risk-free borrowing rate, r_{ft}, firms will save all the free cash flow and never distribute. In practice, firms do distribute cash to shareholders because of costs associated with holding cash. Graham (2000) reports that cash retentions are tax-disadvantaged because their tax rates generally exceed tax rates on interest income for bondholders. To capture this effect, we follow Hennessy, Levy, and Whited (2007) and assume that the saving rate is strictly less than the borrowing rate, namely,

$$r_{st} = r_{ft} - \kappa, \tag{11}$$

where $\kappa > 0$ is a constant wedge between borrowing and saving rates. Cooley and Quadrini (2001) further justify $r_{st} < r_{ft}$. Suppose the two rates are equal. With financial frictions, firms would strictly prefer to reinvest profits, generating an excessive supply of loanable funds that subsequently reduce the saving rate to a level below the borrowing rate.

For notational simplicity, let $\mathbf{1}^b_{jt+1} \equiv \mathbf{1}_{\{b_{jt+1}\geq 0\}}$ be the indicator function that equals one if firm j borrows at time t and zero otherwise. Because b_{jt+1} is a choice variable, $\mathbf{1}^b_{jt+1}$ is known at the beginning of t. Further, we let

$$\iota_{jt} \equiv \mathbf{1}^b_{jt+1} r_{ft} + \left(1-\mathbf{1}^b_{jt+1}\right) r_{st} \tag{12}$$

denote the interest rate applicable to firm j from time t to $t+1$, known at the beginning of t.

F. Costly External Equity

When the sum of the investment costs, $\phi(i_{jt}, k_{jt})$, and promised debt repayment, b_{jt}, exceeds the sum of internal funds, π_{jt}, and cash inflows from issuing new debt, b_{jt+1}/ι_{jt}, the firm can raise new equity capital, e_{jt}, to compensate for the financial deficit:

$$e_{jt} \equiv \max\left\{\phi(i_{jt}, k_{jt}) + b_{jt} - \pi(k_{jt}, z_{jt}, x_t) - \frac{b_{jt+1}}{\iota_{jt}}, 0\right\}. \tag{13}$$

Motivated by empirical evidence (e.g., Smith (1977), Lee et al. (1996), and Altinkilic and Hansen (2000)), we assume that there are flotation costs of issuing external equity. These flotation costs are important within the model. If firms can raise new equity without incurring any costs, the collateral constraints will never be binding.

We use a flexible functional form for the equity flotation costs

$$\lambda(e_{jt}, k_{jt}) = \lambda_0 \mathbf{1}^e_{jt} + \frac{\lambda_1}{2}\left(\frac{e_{jt}}{k_{jt}}\right)^2 k_{jt}, \tag{14}$$

where $\lambda_0 > 0$, $\lambda_1 > 0$, and $\mathbf{1}^e_{jt} \equiv \mathbf{1}_{\{e_{jt}>0\}}$ is the indicator function that equals one if firm j issues equity and zero otherwise. The first term in the right-hand side of equation (14) captures the fixed costs of issuing equity, and the second term captures the convex and variable costs. Also, to capture the idea that equity issuance costs can be countercyclical à la Covas and Den Haan (2007), we specify an alternative flotation costs function

$$\lambda(e_{jt}, k_{jt}) = \lambda_0 \mathbf{1}^e_{jt} + \frac{\lambda_1 e^{-(x_t-\bar{x})\lambda_2}}{2}\left(\frac{e_{jt}}{k_{jt}}\right)^2 k_{jt}, \tag{15}$$

where setting $\lambda_2 > 0$ makes the cost of issuing equity countercyclical.

When the sum of investment costs and debt repayments is lower than the sum of internal funds and cash inflows from new debt, firms distribute the difference back to shareholders. Firms do not incur any costs when distributing cash. We do not model specific forms of the payout (dividend vs. share repurchases), meaning that we only pin down the total amount of payout.

G. The Market Value of Equity

Define the effective payout accrued to the shareholders as

$$o_{jt} \equiv \pi(k_{jt}, z_{jt}, x_t) - \phi(i_{jt}, k_{jt}) + \frac{b_{jt+1}}{\iota_{jt}} - b_{jt} - \lambda(e_{jt}, k_{jt}). \tag{16}$$

In words, o_{jt} is profits minus investment costs plus cash inflows from new debt net of repayment of old debt minus equity flotation costs. When the new equity is positive, o_{jt} can be negative. Empirically, o_{jt} corresponds to total net payouts, which are dividends plus repurchases less equity issuances (e.g., Boudoukh et al. (2007)).

Let $v(k_{jt}, b_{jt}, z_{jt}, x_t)$ denote the market value of equity for firm j. Using Bellman's Principle of Optimality, we can formulate the firm's dynamic equity value maximization problem as

$$v(k_{jt}, b_{jt}, z_{jt}, x_t) = \max_{\{i_{jt}, b_{jt+1}\}} \{o_{jt} + E_t[m_{t+1} v(k_{jt+1}, b_{jt+1}, z_{jt+1}, x_{t+1})]\}, \quad (17)$$

subject to the capital accumulation equation (7) and the collateral constraints (equation (9) or (10)).

H. The Shadow Price of New Debt

Let $\nu_{jt} \equiv \nu(k_{jt}, b_{jt}, z_{jt}, x_t)$ be the Lagrange multiplier associated with the collateral constraints. We can interpret ν_{jt} as the shadow price of new debt. The higher ν_{jt}, the more financially constrained firm j is. Appendix A shows that

$$\nu_{jt} = \frac{1}{r_{ft}} \lambda_e(e_{jt}, k_{jt}) \mathbf{1}^e_{jt} - E_t\left[m_{t+1} \lambda_e(e_{jt+1}, k_{jt+1}) \mathbf{1}^e_{jt+1}\right], \quad (18)$$

where $\lambda_e(e_{jt}, k_{jt})$ is the first derivative of λ with respect to e_{jt} when $e_{jt} > 0$.

The interpretation of equation (18) is intuitive. Because debt and equity are two sources of external funds, the shadow price of new debt depends on the debt-equity tradeoff. On the one hand, one additional unit of debt saves firm j an amount that equals the marginal cost of equity finance, $\lambda_e(e_{jt}, k_{jt}) \mathbf{1}^e_{jt}$. This marginal benefit of new debt must be discounted by r_{ft} because the firm only raises $1/r_{ft}$ at the beginning of t by agreeing to pay one additional unit of debt, b_{jt+1}, at the beginning of $t+1$. On the other hand, there are costs associated with borrowing one additional unit of debt because it must be repaid. Having to repay the debt at the beginning of $t+1$ means that the firm must incur the marginal equity flotation costs $\lambda_e(e_{jt+1}, k_{jt+1}) \mathbf{1}^e_{jt+1}$. This (stochastic) amount must be discounted back to the beginning of t, as shown in the second term in the right-hand side of equation (18).

I. Risk and Expected Returns

Evaluating the value function in equation (17) at the optimum yields

$$v_{jt} = o_{jt} + E_t[m_{t+1} v_{jt+1}]. \quad (19)$$

Moving o_{jt} to the left-hand side and dividing both sides by $v_{jt} - o_{jt}$, we obtain $1 = E_t[m_{t+1} r_{jt+1}]$, where $r_{jt+1} \equiv v_{jt+1}/(v_{jt} - o_{jt})$ is the stock return. Note that v_{jt} is the cum-dividend equity value because it is measured before the effective dividends are paid out. We further rewrite $1 = E_t[m_{t+1} r_{jt+1}]$ as the beta-pricing form given by

$$E_t[r_{jt+1}] - r_{ft} = \beta_{jt} \zeta_{mt}, \quad (20)$$

where $r_{ft} \equiv 1/E_t[m_{t+1}]$ is the real risk-free rate from period t to $t+1$, risk is given by

$$\beta_{jt} \equiv \frac{-\mathrm{Cov}_t[r_{jt+1}, m_{t+1}]}{\mathrm{Var}_t[m_{t+1}]}, \tag{21}$$

and the price of risk is given by $\zeta_{mt} \equiv \mathrm{Var}_t[m_{t+1}]/E_t[m_{t+1}]$.

J. Discussion

In the context of financial constraints, our model is among the first to incorporate debt dynamics into investment-based asset pricing. Cooper and Ejarque (2003), Gomes, Yaron, and Zhang (2003, 2006), and Whited and Wu (2006) capture financial constraints as dividend nonnegativity constraints in standard investment-based models. But these constraints are extremely restrictive because firms cannot issue equity, borrow, or retain earnings. Whited (1992), Bond and Meghir (1994), and Hennessy and Whited (2005, 2007) model debt dynamics but with risk neutrality, meaning that all firms earn exactly the risk-free rate ex ante.

Because our setting is richer than most existing investment-based models (e.g., Berk et al. (1999), Carlson et al. (2004), and Zhang (2005)), we can link risk and expected returns to characteristics such as the shadow price of new debt and leverage not captured by most existing studies. To foreshadow our new results on the determinants of risk, we find that high leverage ratios reduce the flexibility of firms in smoothing cash flows accrued to shareholders via capital investment. Reflecting the relative inflexibility, the shadow price of new debt increases with risk and expected returns.

Although our model breaks new ground, we leave salient features of debt outside our framework. We do not model defaultable bonds. The collateral constraints make all bonds risk-free in our framework. While default is indispensable in modeling financial distress, we view collateral constraints as a reasonable approach for capturing financial constraints (e.g., Kiyotaki and Moore (1997), Hennessy and Whited (2005), and Almeida and Campello (2007)). As noted, financial constraints and financial distress are related but separate concepts. Both Kaplan and Zingales (1997) and Lamont et al. (2001) restrict their samples to manufacturing firms with positive real sales growth to eliminate distressed firms. Using collateral constraints therefore befits our economic question, namely, how financial constraints affect risk and expected returns.

II. Qualitative Analysis

We calibrate the model in Section II.A, present the basic properties of the model solution in Section II.B, and discuss the underlying intuition in Section II.C. Appendix B details the solution algorithm.

Table I

Benchmark Parameter Values

This table lists the benchmark parameter values used to solve and simulate our model with the collateral constraints.

Parameter	Value	Description
α	0.65	Curvature in the production function
δ	0.01	Monthly rate of capital depreciation
ρ_x	$0.95^{1/3}$	Persistence coefficient of aggregate productivity
σ_x	0.007/3	Conditional volatility of aggregate productivity
η	0.994	Time preference coefficient
γ_0	50	Constant price of risk parameter
γ_1	−1,000	Time-varying price of risk parameter
a_P	15	Adjustment cost parameter when investment is positive
a_N	150	Adjustment cost parameter when investment is negative
ρ_z	0.96	Persistence coefficient of firm-specific productivity
σ_z	0.10	Conditional volatility of firm-specific productivity
f	0.015	Fixed costs of production
s_0	0.85	Liquation value per unit of capital net of (acyclical) bankruptcy cost
s_1	0	Countercyclical liquidation cost parameter
λ_0	0.08	Fixed flotation cost parameter
λ_1	0.025	Convex (acyclical) flotation cost parameter
λ_2	0	Countercyclical flotation cost parameter
κ	0.50%/12	Monthly wedge between the borrowing and saving rates of interest

A. Calibration

We calibrate all model parameters at the monthly frequency to be consistent with the empirical literature. Table I reports the parameter values that we use to solve and simulate the model.

We set the curvature parameter in the production production, α, to be 0.65, which is roughly the average of the estimates provided by Cooper and Ejarque (2001, 2003), Cooper and Haltiwanger (2006), and Hennessy and Whited (2005, 2007). The monthly depreciation rate δ is 0.01, which implies an annual rate of 12%. The persistence of the aggregate productivity process, ρ_x, is $0.95^{1/3} = 0.983$, and its conditional volatility, σ_x, is $0.007/3 = 0.0023$. With the first-order autoregressive specification for x_t, these monthly values correspond to the quarterly values of 0.95 and 0.007, respectively, consistent with Cooley and Prescott (1995). The long-run average of x_t, $\bar{x}$, only affects the scale of the economy, and we choose $\bar{x} = -3.75$. Further, we set the three parameters governing the stochastic discount factor $\eta = 0.994$, $\gamma_0 = 50$, and $\gamma_1 = -1000$ to generate an average Sharpe ratio of 0.41, an average annual real interest rate of 2.20%, and an annual volatility of real interest rate of 2.90% all of which are similar to those in the data.

The adjustment cost parameters, a_P and a_N, can be interpreted as the periods required to expand and cut capital given a one unit change in the marginal q, respectively. We set $a_P = 15$ and $a_N = 150$ months, which are close to the average estimates in the empirical investment literature (e.g., Shapiro (1986) and

Whited (1992)). For the persistence ρ_z and conditional volatility σ_z in the firm-specific productivity, we set $\rho_z = 0.96$ and $\sigma_z = 0.10$. These values are chosen to generate a plausible amount of dispersion in the cross-sectional distribution of firms. In particular, the average annual cross-sectional volatility of individual stock returns is around 27%.

We set the fixed costs of production, f, to be 0.015. Because f reduces the profits in equation (4), it directly reduces the market value of equity. Following Gomes (2001) and Zhang (2005), we choose $f > 0$ such that the average aggregate book-to-market equity in the model economy is 0.71, which roughly matches that in the data, 0.67, reported by Pontiff and Schall (1999). A positive f also generates the operating leverage effect à la Carlson et al. (2004) (see also Zhang (2005), Table IV) that helps explain the cross-section of expected returns.

The calibration of α and f is connected. A lower α means steeper curvature in the operating profit function and lower book-to-market equity from, for example, monopoly power à la Cooper and Ejarque (2001). Thus, a higher f is required to increase the average aggregate book-to-market to an empirically plausible level. As a technical matter, when $f > 0$, firms with extremely small values of capital (close to zero) and extremely low realizations of the firm-specific productivity z cannot survive. The equity value of these firms can be negative because we do not allow firms to exit the economy. Fortunately, negative equity values rarely occur in our model simulations. As noted, the long-run average capital for a given firm is around one in our simulations.

For the liquidation cost parameters, we set $s_0 = 0.85$, which implies proportional liquidation costs of 15%, consistent with empirical studies. For example, Altman (1984) estimates the average liquidation costs to be 12% of the firm value 3 years prior to the petition date and 16.7% at the petition date. Hennessy and Whited (2007) estimate liquidation costs to be 10.4% of the value of the assets. In the benchmark calibration, we set $s_1 = 0$. For the countercyclical liquidation costs case, we set $s_1 = 10.79$, such that $s_0 e^{(x_t - \bar{x})s_1} = 0.70$ when x_t is one unconditional standard deviation below its long-run average $\bar{x}$. Also, $s_0 e^{(x_t - \bar{x})s_1} = 1.03$ when x_t is one unconditional standard deviation above $\bar{x}$: Capping $s_0 e^{(x_t - \bar{x})s_1}$ to be below one across all realizations of x_t yields similar results (not reported).

For the equity flotation costs, we calibrate the fixed costs parameter, λ_0, to be 0.08 and the flow costs parameter, λ_1, to be 0.025. These parameter values are from Gomes (2001), who estimates these parameters based on Smith (1977). In the benchmark calibration, we set $\lambda_2 = 0$. For the case with countercyclical equity issuance costs, we set $\lambda_2 = 38.51$, which means $\lambda_1 e^{-(x_t - \bar{x})\lambda_2} = 0.05$ when x_t is one unconditional standard deviation below its long-run average $\bar{x}$, and $\lambda_1 e^{-(x_t - \bar{x})\lambda_2} = 0.0125$ when x_t is one unconditional standard deviation above $\bar{x}$.

Armed with these parameter values, we use value function iteration techniques to solve the model. It is worthwhile to point out that solving the model is technically challenging because of "the curse of dimensionality" (e.g., Judd (1998), p. 430). To be (relatively) realistic, our model has in total four state variables (capital stock k_{jt}, current-period debt b_{jt}, firm-specific productivity

z_{jt}, and aggregate productivity x_t). Further complicating the solution algorithm are the two control variables (next-period capital k_{jt+1} and next-period debt b_{jt+1}). By way of contrast, Hennessy and Whited (2005) have two controls and three states, and Hennessy and Whited (2007) have two controls and two states. Also, Hennessy and Whited calibrate and solve their models at the annual frequency, but our asset pricing applications require the monthly frequency. The higher frequency lowers the convergence speed of our solution algorithm by an order of magnitude. Another useful comparison is with Zhang (2005), who solves his model with four states at the monthly frequency, but he has only one control. Despite the curse of dimensionality, we opt to use the value function iteration algorithm because of its well-known stability and precision.

B. Properties of the Model Solution

Using the benchmark parametrization, we discuss how key endogenous variables such as the shadow price of new debt and risk are determined by the underlying state variables.

B.1. Market Equity-to-Capital and Optimal Investment-to-Capital

From Panels A and B in Figure 1, firms with small capital and high firm-specific productivity have high market equity-to-capital ratios, v_{jt}/k_{jt}, consistent with the evidence in Fama and French (1992, 1995). Also, firms have high market equity-to-capital when the aggregate productivity is high, consistent with the evidence on time-series predictability with aggregate valuation ratios (e.g., Kothari and Shanken (1997) and Pontiff and Schall (1999)). Panels E and F show that firms with small capital and high firm-specific productivity also invest more relative to their capital and grow faster, consistent with the evidence in Fama and French (1995). Because investment-to-capital is independent of capital with constant returns to scale, the inverse relation between investment-to-capital and capital is driven by the decreasing returns to scale technology ($\alpha < 1$).

From Panels C and D of Figure 1, market equity-to-capital decreases with the current-period debt, b_{jt}.[5] Intuitively, a high debt burden in the current period lowers the equity value through two channels. First, one more dollar in b_{jt} means one less dollar for the effective dividends in the current period (see equation (16)). Second, a higher b_{jt} imposes higher equity flotation costs on firms to finance investment. Further, Panels G and H show that firms with a large amount of debt invest less than firms with a small or even negative amount of debt (corporate liquidity). Intuitively, firms with more debt must finance at least in part with costly external equity.

[5] This result is expected. Ignoring the fixed costs that make the value function nondifferentiable, we can invoke the Envelope Theorem to obtain $\partial v(k_{jt}, b_{jt}, z_{jt}, x_t)/\partial b_{jt} = -(1 + \lambda_e(e_{jt}, k_{jt})\mathbf{1}^e_{jt}) < 0$.

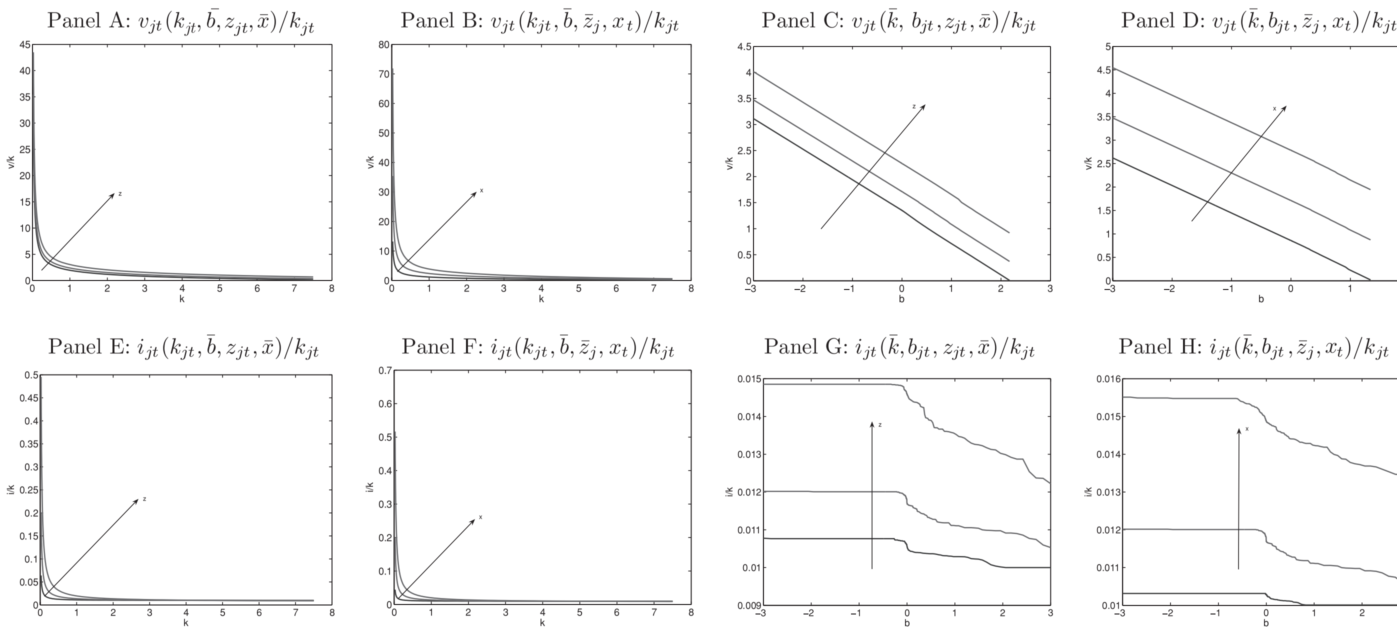

Figure 1. The market equity-to-capital and optimal investment-to-capital ratios against underlying state variables; the benchmark parametrization. We plot the ratio of market value of equity to capital (v_{jt}/k_{jt}, Panels A–D) and the optimal investment-to-capital ratio (i_{jt}/k_{jt}, Panels E–H) as functions of underlying state variables. Panels A and E plot the variables as functions of current-period capital, k_{jt}, and firm-specific productivity, z_{jt}, while fixing aggregate productivity x_t and current-period debt b_{jt} at their respective long-run average levels ($\bar{x}$ and $\bar{b}$, respectively; $\bar{b}$ is determined in model simulations). Panels A and E have a class of curves, corresponding to different values of z_{jt}, and the arrow in each panel indicates the direction along which z_{jt} increases. Panels B and F plot the variables as functions of current-period capital, k_{jt}, and aggregate productivity, x_t, while fixing firm-specific productivity z_{jt} and current-period debt b_{jt} at their respective long-run average levels (zero and $\bar{b}$, respectively). Panels B and F have a class of curves, corresponding to different values of x_t, and the arrow in each panel indicates the direction along which x_t increases. Panels C and G plot the variables as functions of current-period debt, b_{jt}, and firm-specific productivity, z_{jt}, while fixing aggregate productivity x_t and capital stock k_{jt} at their respective long-run average levels ($\bar{x}$ and $\bar{k}$, respectively; $\bar{k}$ is determined in model simulations). Panels C and G have a class of curves, corresponding to different values of z_{jt}, and the arrow in each panel indicates the direction along which z_{jt} increases. Panels D and H plot the variables as functions of current-period debt, b_{jt}, and aggregate productivity x_t, while fixing firm-specific productivity z_{jt} and capital stock k_{jt} at their long-run average levels (zero and $\bar{k}$, respectively). Panels D and H have a class of curves, corresponding to different values of x_t, and the arrow in each panel indicates the direction along which x_t increases.

B.2. Optimal Next-Period Debt-to-Capital and the Shadow Price of New Debt

Panels A to D in Figure 2 plot the optimal next-period debt-to-capital, b_{jt+1}/k_{jt}, against the underlying state variables. Several intuitive patterns arise. Firms with a small scale of production and low firm-specific productivity borrow more (Panel A). Also, the debt-to-capital ratio is persistent because firms with more current-period debt borrow more and firms with more corporate savings retain more earnings (Panels C and D). These predictions are largely consistent with the empirical evidence (e.g., Titman and Wessels (1988), Smith and Watts (1992), and Rajan and Zingales (1995)). The relation between debt-to-capital and aggregate productivity (x_t) is ambiguous. Fixing debt at its long-run average, we see that b_{jt+1}/k_{jt} increases with x_t (Panel B). But fixing capital at its long-run average, we see that b_{jt+1}/k_{jt} decreases with x_t when $b_{jt} > 0$ (Panel D). Accordingly, we use simulations in Section III.C to sort out the cyclical properties of leverage ratios.

From Panels E and F of Figure 2, the shadow price of new debt, ν_{jt}, decreases in capital stock and in firm-specific productivity. Financial constraints are therefore more binding for small and less profitable firms, consistent with the available evidence.[6] Further, Panels G and H show that firms with positive retained earnings or low current-period debt are unconstrained financially (the shadow price of new debt is zero). And firms with high current-period debt are more constrained (the shadow price is positive).

B.3. Risk and Expected Excess Returns

Figure 3 plots expected excess returns and risk against the underlying state variables. From Panels A, B, E, and F, firms with a small scale of production and low firm-specific productivity are riskier and earn higher expected returns. These results verify those of Carlson et al. (2004) and Zhang (2005). More important, the figure also sheds light on how the current-period debt, b_{jt}, affects risk and expected returns. From Panels C, D, G, and H, all else equal, firms with high current-period debt are riskier and earn higher expected returns than firms with low current-period debt (or with corporate savings). The positive relation between the current-period debt and risk and expected returns is even more dramatic for less profitable firms (Panels C and G). Further, as shown in Figure 2, firms with small capital, low firm-specific productivity, and high current-period debt are more financially constrained. Collectively, our model suggests that more constrained firms are riskier and earn higher expected returns than less constrained firms.

However, size and book-to-market equity are determined jointly and endogenously with the shadow price of new debt by the underlying state variables. To quantify the marginal effects of the shadow price on risk and expected returns independent of market equity and book-to-market, we use multivariate

[6] See, e.g., Chan and Chen (1991), Gertler and Gilchrist (1994), Perez-Quiros and Timmermann (2000), Lamont et al. (2001), and Whited and Wu (2006).

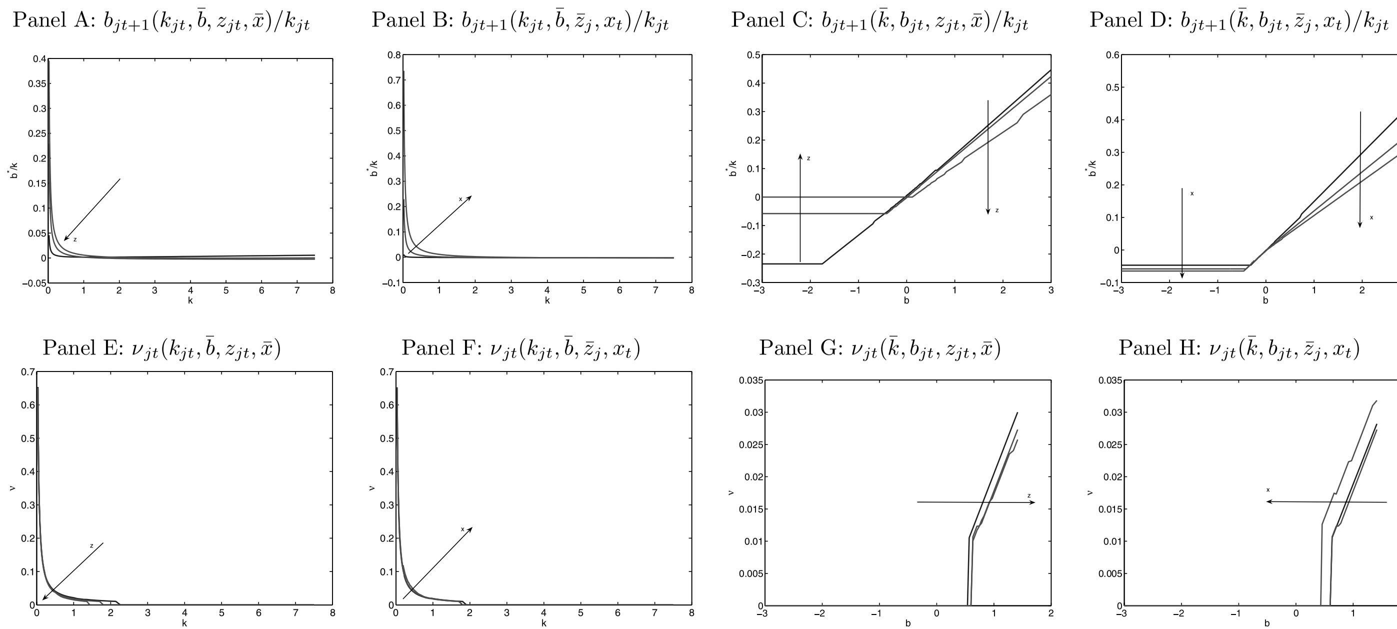

Figure 2. The optimal debt-to-capital ratio and the shadow price of new debt against underlying state variables; the benchmark parametrization. We plot the optimal next-period debt-to-capital ratio (b_{jt+1}/k_{jt}, Panels A–D) and the shadow price of new debt for the collateral constraints in equation (9) (ν_{jt}, Panels E–H) as functions of underlying state variables. Panels A and E plot the variables as functions of current-period capital, k_{jt}, and firm-specific productivity, z_{jt}, while fixing aggregate productivity x_t and current-period debt b_{jt} at their respective long-run average levels ($\bar{x}$ and $\bar{b}$, respectively; $\bar{b}$ is determined in model simulations). Panels A and E have a class of curves, corresponding to different values of z_{jt}, and the arrow in each panel indicates the direction along which z_{jt} increases. Panels B and F plot the variables as functions of current-period capital, k_{jt}, and aggregate productivity, x_t, while fixing firm-specific productivity z_{jt} and current-period debt b_{jt} at their respective long-run average levels (zero and $\bar{b}$, respectively). Panels B and F have a class of curves, corresponding to different values of x_t, and the arrow in each panel indicates the direction along which x_t increases. Panels C and G plot the variables as functions of current-period debt, b_{jt}, and firm-specific productivity, z_{jt}, while fixing aggregate productivity x_t and capital stock k_{jt} at their respective long-run average levels ($\bar{x}$ and $\bar{k}$, respectively; $\bar{k}$ is determined in model simulations). Panels C and G have a class of curves, corresponding to different values of z_{jt}, and the arrow in each panel indicates the direction along which z_{jt} increases. Panels D and H plot the variables as functions of current-period debt, b_{jt}, and aggregate productivity x_t, while fixing firm-specific productivity z_{jt} and capital stock k_{jt} at their long-run average levels (zero and $\bar{k}$, respectively). Panels D and H have a class of curves, corresponding to different values of x_t, and the arrow in each panel indicates the direction along which x_t increases.

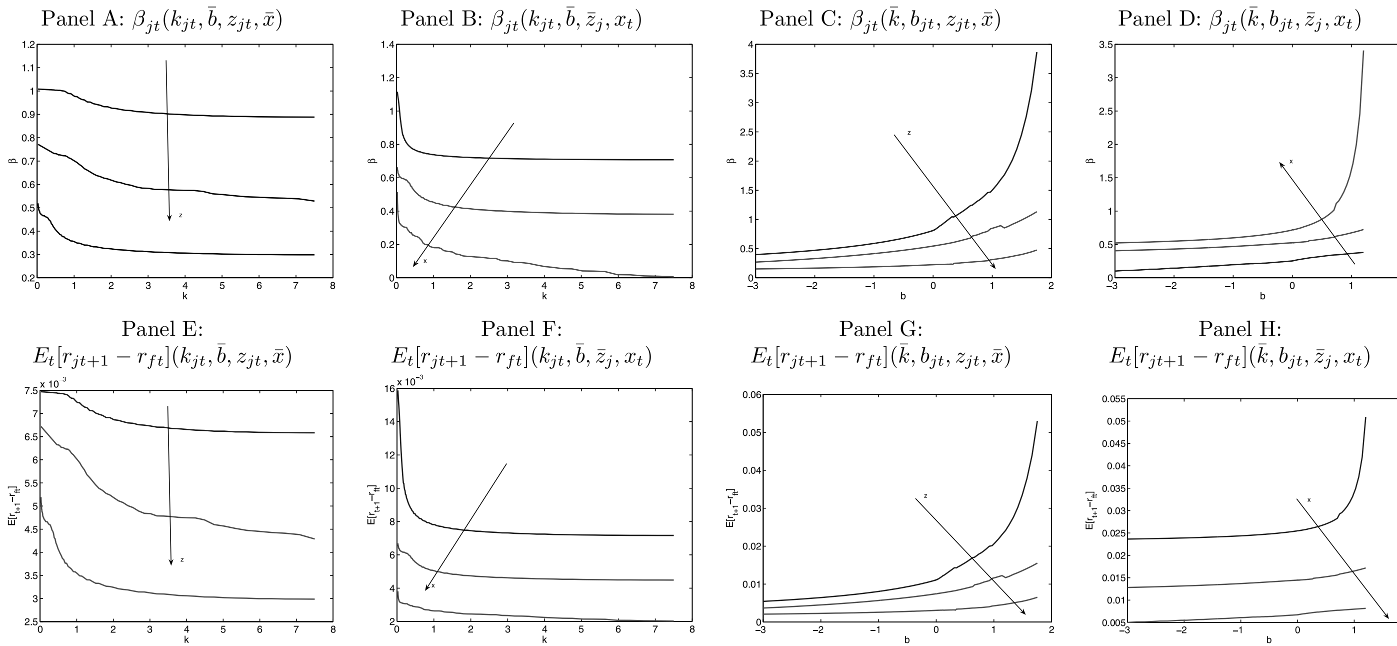

Figure 3. Risk and expected excess returns against underlying state variables; the benchmark parametrization. We plot risk (β_{jt}, Panels A–D) and the expected excess return ($E_t[r_{jt+1}] - r_{ft}$, Panels E–H) as functions of underlying state variables. Panels A and E plot the variables as functions of current-period capital, k_{jt}, and firm-specific productivity, z_{jt}, while fixing aggregate productivity x_t and current-period debt b_{jt} at their respective long-run average levels ($\bar{x}$ and $\bar{b}$, respectively; $\bar{b}$ is determined in model simulations). Panels A and E have a class of curves, corresponding to different values of z_{jt}, and the arrow in each panel indicates the direction along which z_{jt} increases. Panels B and F plot the variables as functions of current-period capital, k_{jt}, and aggregate productivity, x_t, while fixing firm-specific productivity z_{jt} and current-period debt b_{jt} at their respective long-run average levels (zero and $\bar{b}$, respectively). Panels B and F have a class of curves, corresponding to different values of x_t, and the arrow in each panel indicates the direction along which x_t increases. Panels C and G plot the variables as functions of current-period debt, b_{jt}, and firm-specific productivity, z_{jt}, while fixing aggregate productivity x_t and capital stock k_{jt} at their respective long-run average levels ($\bar{x}$ and $\bar{k}$, respectively; $\bar{k}$ is determined in model simulations). Panels C and G have a class of curves, corresponding to different values of z_{jt}, and the arrow in each panel indicates the direction along which z_{jt} increases. Panels D and H plot the variables as functions of current-period debt, b_{jt}, and aggregate productivity x_t, while fixing firm-specific productivity z_{jt} and capital stock k_{jt} at their long-run average levels (zero and $\bar{k}$, respectively). Panels D and H have a class of curves, corresponding to different values of x_t, and the arrow in each panel indicates the direction along which x_t increases.

sorts or cross-sectional regressions in simulation-based experiments (see Section III.A).

C. Intuition: Risk as Inflexibility

What drives the relation between financial constraints and risk? The key word is *inflexibility*. In production economies, the risk of firms increases with the degree of their inflexibility in adjusting capital investment to smooth dividend streams in the face of exogenous aggregate shocks. The less flexible firms are, the riskier their returns will be. Rouwenhorst (1995) and Jermann (1998), among others, show that explaining the equity premium puzzle is more difficult in a general equilibrium production economy than in an endowment economy. After a positive aggregative shock in an endowment economy, dividends will increase unit-by-unit with exogenous cash flows. In a production economy, firms can increase investment (because of the persistence in the aggregate productivity), meaning that dividends will not covary as much with business cycles as in an endowment economy. Higher investments serve as a buffer for the dividend streams to absorb some of the impact from the positive aggregate shock, thereby decreasing risk.

By preventing firms from financing all desired investments, collateral constraints work against the dividend smoothing mechanism. The shadow price of new debt precisely measures the extent to which collateral constraints are binding. The higher the shadow price, the more inflexible firms are in adjusting investment, the more dividends will covary with business cycles, and the higher their risk and expected returns.

This inflexibility mechanism is similar in nature to the mechanism in Zhang (2005). By restricting investment flexibility, adjustment costs also work against the dividend smoothing mechanism and drive up risk. Further, adjustment costs are higher for cutting than expanding capital. In bad times, firms want to scale down, especially value firms that are less profitable than growth firms. Because cutting capital is more costly, value firms are less flexible, are more correlated with economic downturns, and are riskier than growth firms. Although related, our inflexibility mechanism stems from collateral constraints, which are different from the adjustment costs studied by Zhang.

III. Quantitative Results

We use simulation-based experiments to study two key issues: the relation between financial constraints and average returns and the cross-sectional determinants of financial constraints. We also examine the leverage–return relation.

Our experiment design follows that of Kydland and Prescott (1982) and Berk et al. (1999). We simulate 100 artificial panels, each of which has 3,000 firms and 480 months. The sample size is similar to that used in empirical studies based on the Center for Research in Security Prices (CRSP)-COMPUSTAT merged data set. We implement a variety of empirical procedures on each artificial panel and report the cross-simulation averaged results. Whenever possible, we compare model moments with those in the data.

A. Financial Constraints and Average Stock Returns

We first examine the quantitative relations between the shadow price of new debt and average stock returns. Using the Fama and French (1993) portfolio approach, we construct portfolios by sorting on the shadow price, with and without controlling for size. Because the shadow price is the precise measure of financial constraints in our model, our quantitative results help interpret the evidence in Lamont et al. (2001) and Whited and Wu (2006).

A.1. Univariate Sorts

Table II reports the average monthly stock returns for 10 deciles from an annual one-way sort on the shadow price of new debt in simulated panels.

Table II

Average Monthly Percent Stock Returns for Portfolios Based on One-Way Sorts on the Shadow Price of New Debt in Model Simulations

This table reports average stock returns of 10 value-weighted portfolios from one-way sorts on the shadow price of new debt on the collateral constraints. We report average returns in percent per month for each portfolio as well as the average high-minus-low portfolios and its t-statistics. We sort all firms based on their shadow prices of new debt at the beginning of each year and then hold the portfolios for the whole year. For each model, we simulate 100 artificial panels, each of which has 3,000 firms and 480 monthly observations, and we then report the across-simulation average results. We report the simulation results from the model using the benchmark parameters reported in Table I (Benchmark parametrization). We also report the results from four comparative static experiments ($s_0 = 0.70$, $\lambda_0 = 0.02$, $s_1 > 0$, and $\lambda_2 > 0$).

Low	2	3	4	5	6	7	8	9	High	*FC*	t_{FC}
Panel A: Benchmark Parametrization											
0.33	0.37	0.43	0.44	0.46	0.50	0.54	0.57	0.61	0.67	0.34	3.54
Panel B: High Liquidation Costs, $s_0 = 0.70$											
0.14	0.14	0.15	0.15	0.16	0.16	0.18	0.18	0.18	0.19	0.05	2.23
Panel C: Low Equity Flotation Costs, $\lambda_0 = 0.02$											
0.37	0.40	0.45	0.50	0.52	0.53	0.58	0.61	0.62	0.62	0.25	4.63
Panel D: Countercyclical Liquidation Costs, $s_1 > 0$											
0.11	0.12	0.14	0.15	0.16	0.16	0.17	0.18	0.18	0.20	0.09	2.70
Panel E: Countercyclical Equity Flotation Costs, $\lambda_2 > 0$											
0.34	0.40	0.44	0.49	0.53	0.60	0.63	0.70	0.75	0.79	0.45	4.53

Beside the benchmark parametrization, we also report results from four comparative static experiments. Panel B considers the case with high liquidation costs, in which the liquidation value per unit of capital, s_0, is reset to be 0.70 from its benchmark value of 0.85. We consider this case because Hennessy and Whited (2005) estimate this parameter to be 0.59 but with a high p-value of 0.35. Panel C considers the case with low fixed flotation costs, in which the fixed flotation cost parameter, λ_0, is reset to be 0.02 from its benchmark value of 0.08. Panel E reports the case with countercyclical liquidation costs with $s_1 > 0$ and Panel F reports the case with countercyclical equity flotation costs with $\lambda_2 > 0$.

From Panel A of Table II, the one-way sort on the shadow price of new debt, ν_{jt}, generates a positive relation between the shadow price and average returns. The average return increases monotonically from 0.33% per month for the lowest-shadow-price (least constrained) portfolio to 0.67% for the highest-shadow-price (most constrained) portfolio. The average return spread between the two extreme deciles is 0.34% per month (t = 3.54). The magnitude of the spread varies from 0.05% to 0.45% per month across the four comparative static experiments, and is significant in all four cases.

A.2. Multivariate Sorts

Using their respective measures of financial constraints, Lamont et al. (2001) and Whited and Wu (2006) document that, after controlling for size, the average return spread between the most constrained and the least constrained firms is insignificantly different from zero.

We ask whether our model is consistent with this finding. Specifically, using artificial panels we conduct two-way sorts on the shadow price of new debt and market capitalization measured as the ex-dividend market value of equity ($v_{jt} - o_{jt}$). Following Lamont et al. (2001) and Whited and Wu (2006), we define small-caps (S), mid-caps (M), and large-caps (L) as firms in the bottom 40%, middle 20%, and top 40% of the sample sorted on market capitalization, respectively. Similarly, low-, middle-, and high-shadow-price portfolios contain firms in the bottom 40% (L), middle 20% (M), and top 40% (H) of the sample sorted on the shadow price of new debt, respectively. We also define the average high-shadow-price portfolio as $HIGHFC \equiv (BH + MH + SH)/3$, and the average low-shadow-price portfolio as $LOWFC \equiv (BL + ML + SL)/3$, and the financial constraints factor as $FC \equiv HIGHFC - LOWFC$.

Table III reports the model-implied average returns of the two-way sorted portfolios in excess of the risk-free rate, and compares the model moments with the data moments. From the last two columns of the table, Lamont et al. (2001) and Whited and Wu (2006) estimate the average return of FC to be −0.13% per month (t = −1.17) and 0.18% (t = 0.95), respectively. (The t-statistic for the average FC return from Lamont et al. is calculated from the information reported in their Table 5.) The column denoted "Benchmark" shows that the average FC return in our model is 0.12% per month (t = 1.11) with the benchmark parametrization. Also, the magnitude of the average excess return for the nine

Table III

Average Monthly Percentage Excess Returns for Portfolios Sorted on the Shadow Price of New Debt and Market Capitalization in Model Simulations

We report average returns in monthly percent in excess of the risk-free rate for nine value-weighted portfolios sorted on market capitalization and the shadow price of new debt in model simulations. The rankings are performed annually and independently such that each portfolio contains firms in both a given size category and a given financial constraints category. Following Lamont, Polk, and Saá-Requejo (2001) and Whited and Wu (2006), we define small-caps (S) as firms that are in the bottom 40%, mid-caps (M) are firms in the middle 20%, and large-caps (B) are firms in the top 40% of the sample sorted on market capitalization. Similarly, low-, middle-, and high-shadow-price portfolios consist of firms in the bottom 40% (L), middle 20% (M), and top 40% (H) of the sample sorted on the shadow price of new debt, respectively. We also define the average high-*FC* portfolio as $HIGH\,FC \equiv (BH + MH + SH)/3$, and average low-*FC* portfolio as $LOW\,FC \equiv (BL + ML + SL)/3$, and the financial constraints factor as $FC \equiv HIGH\,FC - LOW\,FC$. For each model we simulate 100 artificial panels, each of which has 3,000 firms and 480 monthly observations, and we then report the cross-simulation average results. The column denoted "Benchmark" reports the simulation results from the model using the benchmark parameter values in Table I. We also report results from four comparative static experiments. The column denoted "$s_0 = 0.70$" reports the results using the benchmark parameters except that the liquidation value per unit of capital net of liquidation costs, s_0, is reset to be 0.70. The column denoted "$\lambda_0 = 0.02$" reports the results using the benchmark parameters except that the fixed flotation cost of equity, λ_0, is reset to be 0.02. The column denoted "$s_1 > 0$" reports the results from the countercyclical liquidation costs model with $s_1 > 0$ (see equation (10)). The column denoted "$\lambda_2 > 0$" reports the results from the countercyclical equity flotation costs model with $\lambda_2 > 0$ (see equation (15)). The last two columns report those from Table I of Lamont et al. and from Table 4 of Whited and Wu, respectively.

		Benchmark	$s_0 = 0.70$	$\lambda_0 = 0.02$	$s_1 > 0$	$\lambda_2 > 0$	Lamont et al. (2001)	Whited and Wu (2006)
Small-caps								
Low *FC*	*SL*	0.61	0.25	0.68	0.28	0.68	0.45	0.89
Middle *FC*	*SM*	0.64	0.31	0.75	0.34	0.84	0.67	0.66
High *FC*	*SH*	0.75	0.40	0.88	0.38	0.91	0.38	0.83
Mid-caps								
Low *FC*	*ML*	0.45	0.14	0.56	0.16	0.56	0.37	0.65
Middle *FC*	*MM*	0.50	0.16	0.60	0.21	0.74	0.56	0.81
High *FC*	*MH*	0.59	0.16	0.65	0.25	0.84	0.26	0.74
Large-caps								
Low *FC*	*BL*	0.21	0.11	0.37	0.14	0.42	0.47	0.71
Middle *FC*	*BM*	0.30	0.08	0.41	0.15	0.51	0.53	0.96
High *FC*	*BH*	0.37	0.09	0.50	0.18	0.59	0.25	1.23
HIGH FC		0.51	0.22	0.63	0.26	0.74	0.30	0.93
LOW FC		0.39	0.06	0.51	0.16	0.67	0.43	0.75
FC		0.12	0.16	0.12	0.10	0.07	−0.13	0.18
t_{FC}		1.11	0.56	0.98	0.89	0.63	−1.17	0.95

size-financial constraints portfolios is comparable with that in the data. Varying the liquidation costs and flotation costs parameters changes the magnitude of the average excess returns for the size-financial constraints portfolios, but does not affect the *FC* results.

A.3. Cross-sectional Regressions

To further evaluate the marginal effects of the shadow price of new debt on average returns, we perform cross-sectional regressions. Cross-sectional regressions can be powerful in some circumstances because they provide an easy way to control for different characteristics simultaneously. Multiple regression slopes provide direct estimates of marginal effects. In contrast, sorts are clumsy for controlling for multiple characteristics because some of the finely cut portfolios can contain few firms in certain periods.

Table IV reports the Fama-MacBeth (1973) monthly cross-sectional regressions of stock returns, r_{jt+1}, from the beginning of month t to the beginning of $t+1$, on the shadow price of new debt, size, and book-to-market equity, all of which are measured at the beginning of t. Size is defined as the logarithm of the market value of equity, and book-to-market equity is measured as $\ln[(k_{jt} - b_{jt})/(v_{jt} - o_{jt})]$ in simulated data. The table shows that size and book-to-market largely subsume the effects of financial constraints on risk and expected returns. The slopes of the shadow price are all significantly positive in

Table IV
Fama-MacBeth (1973) Monthly Cross-sectional Regressions of Percentage Stock Returns on the Shadow Price of New Debt, Size, and Book-to-Market in Model Simulations

We report the Fama-MacBeth (1973) monthly cross-sectional regressions of stock returns, r_{jt+1}, on the shadow price of new debt, size, and book-to-market, all of which are measured at the beginning of month t. ν_{jt} denotes the shadow price, ln(ME) is the logarithm of the market value of equity, measured as $\ln(v_{jt} - o_{jt})$ in the model, and ln(BE/ME) is the logarithm of the book-to-market equity ratio, measured as $\ln((k_{jt} - b_{jt})/(v_{jt} - o_{jt}))$ in the model. We simulate 100 artificial panels, each of which has 3,000 firms and 480 monthly observations, and then report the across-simulation average slopes and t-statistics. We report the results from the model with the benchmark parametrization in Table I (Benchmark). We also report four comparative static experiments ($s_0 = 0.70$, $\lambda_0 = 0.02$, $s_1 > 0$, and $\lambda_2 > 0$).

Benchmark			High Liquidation Cost ($s_0 = 0.70$)		
ν_{jt}	ln (ME)	ln (B/M)	ν_{jt}	ln (ME)	ln (B/M)
1.69			1.17		
(3.47)			(2.10)		
2.52	−1.96	3.67	0.63	−3.01	2.03
(0.79)	(−2.55)	(3.07)	(1.59)	(−2.26)	(1.03)

Low Fixed Flotation Costs ($\lambda_0 = 0.02$)			Countercyclical Liquidation Costs ($s_1 > 0$)			Countercyclical Flotation Costs ($\lambda_2 > 0$)		
ν_{jt}	ln (ME)	ln (B/M)	ν_{jt}	ln (ME)	ln (B/M)	ν_{jt}	ln (ME)	ln (B/M)
3.97			1.17			1.86		
(2.12)			(2.65)			(3.07)		
−1.96	−3.31	4.01	−0.76	−3.08	2.70	−1.25	−1.22	2.56
(−0.11)	(−4.72)	(3.07)	(−0.53)	(−2.46)	(2.62)	(−1.07)	(−4.78)	(2.64)

univariate regressions. But the slopes become insignificantly positive and even negative in multiple regressions once we control for size and book-to-market.

Why does the relation between financial constraints and average returns appear significant in the one-way sorts and univariate regressions but largely insignificant in the two-way sorts and multiple regressions? The reason is that risk and expected returns are jointly determined with other endogenous variables such as size, book-to-market, and the shadow price of new debt by the underlying state variables. In our model firms differ in three state variables that include capital stock (k_{jt}), firm-specific productivity (z_{jt}), and current-period debt (b_{jt}). The cross-section of risk and expected returns is ultimately determined by these firm-specific state variables, and the whole cross-sectional distribution also varies over time with aggregate productivity (x_t). The shadow price of new debt is correlated with risk and expected returns because it contains information about the state variables that determine risk and expected returns. But this information is not fully independent of the information captured by size and book-to-market because of the joint determination.

B. Cross-sectional Determinants of Financial Constraints

Because the shadow price of new debt is unobservable in the data, researchers are forced to use observable characteristics to serve as proxies for financial constraints. In the model simulations, we can calculate the shadow price as the precise measure of the degree of financial constraints. It is therefore interesting to ask, using our theoretical model as a natural laboratory, how well the characteristics commonly used in practice can proxy for financial constraints. The answer is mixed.

B.1. The Whited and Wu (2006) Characteristics

The first set of characteristics is from Whited and Wu (2006), who use cash flow-to-assets (CF_{jt}, measured as π_{jt}/k_{jt} in our model); debt-to-assets ($TLTD_{jt}$, measured as $b_{jt}\mathbf{1}^b_{jt}/k_{jt}$); the logarithm of assets ($LNTA_{jt}$, measured as $\ln(k_{jt})$); sales growth (SG_{jt}, measured as y_{jt}/y_{jt-1}); and a dividend dummy ($DIVPOS_{jt}$, which takes the value of one if $o_{jt} > 0$ and zero otherwise). The Whited–Wu (WW) index of financial constraints is defined as

$$\begin{aligned} WW_{jt} = &-0.091\, CF_{jt} - 0.062\, DIVPOS_{jt} + 0.021\, TLTD_{jt} \\ &- 0.044\, LNTA_{jt} - 0.035\, SG_{jt}. \end{aligned} \tag{22}$$

(Whited and Wu also use industry sales growth in their index. We do not use this variable in our simulations because our one-sector model provides no cross-sectional variation in industry sales growth. If we include this term in the estimation, it will just be absorbed into the intercept term. Our model can equivalently be interpreted as a multi-sector model by treating firm-specific shocks as industry shocks. But then industry sales growth coincides with firm-level sales growth.)

Panel A of Table V reports Fama-MacBeth (1973) cross-sectional regressions of the shadow price of new debt on contemporaneous characteristics motivated from Whited and Wu (2006). Consistent with their evidence, our simulations show that firms are more financially constrained if they have lower ratios of cash flow to assets, higher ratios of debt to assets, lower sales, lower contemporaneous sales growth, and zero rather than positive dividend payments. The slopes on these characteristics are also comparable to those in the data. This general pattern holds both for the benchmark parametrization and for four comparative static experiments.

B.2. The Kaplan and Zingales (1997) Characteristics

We also consider the characteristics used by Lamont et al. (2001) to proxy for financial constraints. These characteristics are in turn motivated by Kaplan and Zingales (1997). The list includes cash flow-to-assets (CF_{jt}), debt-to-assets ($TLTD_{jt}$), dividend-to-assets ($TDIV_{jt}$, measured as $o_{jt}(1-\mathbf{1}^{e}_{jt})/k_{jt}$ in the model), liquid assets or cash-to-assets, ($CASH_{jt}$, measured as $-b_{jt}(1-\mathbf{1}^{b}_{jt})/k_{jt}$), and Tobin's Q (Q_{jt}, measured as $(v_{jt}-o_{jt}+b_{jt})/k_{jt}$). Kaplan and Zingales classify firms on a scale from one to four on financial constraints, and perform an ordered logit of the scale on the aforementioned characteristics. Lamont et al. use these logit coefficients to construct the Kaplan–Zingales (KZ) index of financial constraints as follows:

$$\begin{aligned} KZ_{jt} = &-1.002\, CF_{jt} + 3.139\, TLTD_{jt} - 39.368\, TDIV_{jt} \\ &- 1.315\, CASH_{jt} + 0.283\, Q_{jt}. \end{aligned} \tag{23}$$

Panel B of Table V reports Fama-MacBeth (1973) cross-sectional regressions of the shadow price of new debt on characteristics from Kaplan and Zingales (1997). Consistent with their evidence, our simulations show that firms are more financially constrained if they have lower cash flow-to-assets, higher debt-to-assets, lower dividend-to-assets, lower liquid assets or cash-to-assets, and higher Tobin's Q. The slopes from the model are not literally comparable to those from Lamont et al. (2001). The slopes in the data are from ordered logit regressions, and we opt to use more precise ordinary least squares (OLS) regressions because we can precisely calculate the shadow price in simulations.

B.3. A Horse Race

Armed with the "observable" shadow price of new debt in model simulations, we use the model as a laboratory to evaluate the relative quality of the KZ and WW indexes as measures of financial constraints. Specifically, we perform Fama-MacBeth (1973) cross-sectional regressions of the shadow price, ν_{jt}, on the indexes, KZ_{jt} and WW_{jt}, both separately and jointly. We use the relative magnitudes of the slopes and the average cross-sectional R^2s as measures of relative quality for the indexes. To make the magnitudes of their slopes comparable, we

Table V

Cross-sectional Determinants of the Shadow Price of New Debt in Model Simulations

We report Fama-MacBeth (1973) monthly cross-sectional regressions of the shadow price of new debt on firm characteristics. Panel A reports the regression similar to that in column 4 of Table I in Whited and Wu (2006): $\nu_{jt} = b_0 + b_1\, CF_{jt} + b_2\, TLTD_{jt} + b_3\, LNTA_{jt} + b_4\, SG_{jt} + b_5\, DIVPOS_{jt} + \epsilon_{jt}$, where ν_{jt} is the shadow price for firm j in month t. CF_{jt} is the cash flow-to-asset ratio, $TLTD_{jt}$ is the total debt-to-assets ratio, $LNTA_{jt}$ is the logarithm of the total assets, SG_{jt} is the firm-level sales growth, and $DIVPOS_{jt}$ is a dummy variable that equals one if firm j has paid dividends during month t, and equals zero otherwise. Panel B reports the regression similar to that reported in Table 9 in Lamont, Polk, and Saá-Requejo (2001), the regression that is in turn based on Kaplan and Zingales (1997): $\nu_{jt} = b_0 + b_1\, CF_{jt} + b_2\, TLTD_{jt} + b_3\, TDIV_{jt} + b_4\, CASH_{jt} + b_5\, Q_{jt} + \epsilon_{jt}$, where CF_{jt} is the cash flow-to-asset ratio, $TLTD_{jt}$ is the total debt-to-assets ratio, $TDIV_{jt}$ is the dividend-to-assets ratio, $CASH_{jt}$ is the ratio of total amount of liquid assets or cash divided by assets, and Q_{jt} is Tobin's Q. The t-statistics are in parentheses. We report quantitative results from the benchmark parametrization reported in Table I and four comparative static experiments: (i) the high liquidation costs case with $s_0 = 0.70$; (ii) the low fixed flotation costs case with $\lambda_0 = 0.02$; (iii) the countercyclical liquidation costs case with $s_1 > 0$; and (iv) the countercyclical equity flotation costs case with $\lambda_2 > 0$. For each case, we simulate 100 artificial panels, each of which has 3,000 firms and 480 months, and report across-simulation average results. We compare our results to those of Lamont et al. and Whited and Wu (reported under "Data").

Panel A: $\nu_{jt} = b_0 + b_1\, CF_{jt} + b_2\, TLTD_{jt} + b_3\, LNTA_{jt} + b_4\, SG_{jt} + b_5\, DIVPOS_{jt} + \epsilon_{jt}$ from Whited and Wu (2006)

CF		*TLTD*		*LNTA*		*SG*		*DIVPOS*	
Data	Benchmark	Data	Benchmark	Data	Benchmark	Data	Benchmark	Data	Benchmark
−0.09	−0.25	0.02	0.14	−0.04	−0.11	−0.04	−0.02	−0.06	−0.37
(−2.94)	(−10.24)	(1.91)	(4.59)	(−1.91)	(−10.53)	(−1.52)	(−5.88)	(−2.14)	(−3.67)
$s = 0.70$	$\lambda_0 = 0.02$	$s_0 = 0.70$	$\lambda_0 = 0.02$	$s_0 = 0.70$	$\lambda_0 = 0.02$	$s_0 = 0.70$	$\lambda_0 = 0.02$	$s_0 = 0.70$	$\lambda_0 = 0.02$
−0.08	−0.33	0.03	0.02	−0.03	−0.03	−0.00	−0.05	−0.07	−0.13
(−2.36)	(−2.81)	(3.84)	(5.88)	(−6.11)	(−8.29)	(−3.46)	(−1.50)	(−2.23)	(−2.83)
$s_1 > 0$	$\lambda_2 > 0$	$s_1 > 0$	$\lambda_2 > 0$	$s_1 > 0$	$\lambda_2 > 0$	$s_1 > 0$	$\lambda_2 > 0$	$s_1 > 0$	$\lambda_2 > 0$
−0.12	−0.03	0.03	0.01	−0.01	−0.01	−0.00	−0.00	−0.12	−0.03
(−2.78)	(−2.55)	(8.60)	(2.22)	(−0.89)	(−2.83)	(−0.42)	(−0.31)	(−1.79)	(−1.57)

Panel B: $\nu_{jt} = b_0 + b_1\, CF_{jt} + b_2\, TLTD_{jt} + b_3\, TDIV_{jt} + b_4\, CASH_{jt} + b_5\, Q_{jt} + \epsilon_{jt}$ from Kaplan and Zingales (1997)

CF		*TLTD*		*TDIV*		*CASH*		*Q*	
Data	Benchmark	Data	Benchmark	Data	Benchmark	Data	Benchmark	Data	Benchmark
−1.00	−2.50	3.14	1.78	−39.37	−3.61	−1.32	−0.10	0.28	0.20
(−4.28)	(−5.60)	(6.99)	(9.23)	(−6.46)	(−9.95)	(−4.55)	(−7.30)	(3.63)	(7.47)
$s_0 = 0.70$	$\lambda_0 = 0.02$	$s_0 = 0.70$	$\lambda_0 = 0.02$	$s_0 = 0.70$	$\lambda_0 = 0.02$	$s_0 = 0.70$	$\lambda_0 = 0.02$	$s_0 = 0.70$	$\lambda_0 = 0.02$
−0.35	−1.36	1.33	0.23	−3.46	−1.36	−0.00	−0.24	1.70	0.12
(−3.08)	(−2.93)	(9.54)	(5.77)	(−2.07)	(−2.91)	(−7.28)	(−8.10)	(7.27)	(5.70)
$s_1 > 0$	$\lambda_2 > 0$	$s_1 > 0$	$\lambda_2 > 0$	$s_1 > 0$	$\lambda_2 > 0$	$s_1 > 0$	$\lambda_2 > 0$	$s_1 > 0$	$\lambda_2 > 0$
−1.80	−1.88	1.25	1.59	−1.77	−1.19	−0.06	−0.00	0.06	0.01
(−2.66)	(−2.24)	(8.60)	(2.23)	(−2.60)	(−1.27)	(−2.83)	(−2.95)	(7.97)	(3.04)

standardize both indexes by dividing their demeaned values by their respective standard deviations before using them in the regressions.

From Table VI, both the *KZ* index and the *WW* index are positively correlated with the true shadow price. All the univariate slopes are significantly positive. However, the regression R^2s are universally low, with the highest being 11.74%. This result means that both indexes leave much of the cross-sectional variation in the true shadow price unexplained. From the relative magnitude of the slopes, the *WW* index appears to do a better job than the *KZ* index as a

Table VI

Fama-MacBeth (1973) Cross-sectional Regressions of the Shadow Price of New Debt on the Kaplan and Zingales (1997) and the Whited and Wu (2006) Indexes of Financial Constraints

Using simulated panels, we report the Fama-MacBeth (1973) monthly cross-sectional regressions of the shadow price of new debt, ν_{jt}, on the Kaplan and Zingles (1997, *KZ*) index and the Whited and Wu (2006, *WW*) index of financial constraints, both separately and jointly. To make the magnitudes of their slopes comparable, we standardize both indexes by dividing their demeaned values by their respective standard deviations before using them in the cross-sectional regressions. We simulate 100 artificial panels, each of which has 3,000 firms and 480 monthly observations, and then report across-simulation average Fama-MacBeth slopes, *t*-statistics (in parentheses), and average cross-sectional R^2s. We also report the average cross-sectional correlation between the *KZ* index and the *WW* index. Panel A reports the quantitative results from the benchmark parametrization with the parameter values in Table I. The remaining panels report four comparative static experiments: the high liquidation costs case with $s_0 = 0.70$ (Panel B), the low fixed flotation costs case with $\lambda_0 = 0.02$, the countercyclical liquidation costs case with $s_1 > 0$, and the countercyclical flotation costs case with $\lambda_2 > 0$.

KZ	R^2	*WW*	R^2	*KZ*	*WW*	R^2	Corr(*KZ*, *WW*)
Panel A: Benchmark Parametrization							
0.017	0.05	0.108	0.10	0.014	0.108	0.12	0.50
(5.28)		(11.47)		(2.97)	(11.53)		
Panel B: High Liquidation Costs, $s_0 = 0.70$							
0.010	0.04	0.080	0.10	0.010	0.063	0.10	0.56
(7.22)		(13.30)		(7.28)	(4.51)		
Panel C: Low Fixed Flotation Costs, $\lambda_2 = 0.02$							
0.021	0.07	0.151	0.09	0.011	0.120	0.09	0.47
(6.02)		(6.82)		(3.65)	(3.81)		
Panel D: Countercyclical Liquidation Costs, $s_1 > 0$							
0.002	0.08	0.080	0.09	0.002	0.011	0.08	0.46
(6.58)		(4.47)		(6.58)	(1.04)		
Panel E: Countercyclical Flotation Costs, $\lambda_2 > 0$							
0.001	0.05	0.021	0.06	0.001	0.016	0.07	0.52
(2.41)		(2.83)		(0.92)	(2.49)		

proxy for financial constraints. For example, in the benchmark parametrization (Panel A), the slope of the *WW* index in the joint regression with both indexes is 0.108, more than seven times the magnitude for the *KZ* slope, 0.014. The results from the four alternative parameterizations are largely similar.

C. Cyclical Properties of the Shadow Price of New Debt and Leverage Ratios

The population correlation between the shadow price of new debt averaged across firms, $\sum_{j=1}^{N} \nu_{jt}$, and the aggregate productivity, x_t, is 0.20 in the benchmark parametrization. Allowing for low fixed equity flotation costs ($\lambda_0 = 0.02$) and countercyclical equity flotation costs ($\lambda_2 > 0$) reduces this correlation to 0.02 and 0.06, respectively. Intuitively, both perturbations make new equity a more affordable source of external finance in good times. This effect reduces the importance of new debt when aggregate productivity is high, and subsequently lowers the correlation between the shadow price of new debt and aggregate productivity.

Although these results go in the same direction as the evidence of Gomes et al. (2006), the shadow price in our simulations seems less procyclical than what Gomes et al. document in the data. (In fact, the shadow price is largely acyclical when $\lambda_0 = 0.02$ or $\lambda_2 > 0$.) However, the evidence of Gomes et al. hinges on their modeling of financial constraints as dividend nonnegativity constraints, which rule out all sources of external financing.[7] It is conceivable that more realistic financial constraints can overturn their conclusion on the cyclical properties of the shadow price.

The aggregate market leverage, defined as $\sum_{j=1}^{N} b_{jt} / \sum_{j=1}^{N} v_{jt}$, is strongly countercyclical in our model simulations. The population correlation between aggregate market leverage and aggregate productivity is -0.70 in the benchmark parametrization. If we allow low fixed equity flotation costs, the correlation is strengthened to -0.90. Allowing countercyclical equity flotation costs also strengthens the correlation to -0.88. Intuitively, by making new equity more affordable, especially in good times, both perturbations make the new equity more procyclical. The strong countercyclical property of the market leverage is consistent with the evidence documented by, for example, Korajczyk and Levy (2003, Figure 1).

Intriguingly, the aggregate book leverage, defined as $\sum_{j=1}^{N} b_{jt} / \sum_{j=1}^{N} k_{jt}$, is weakly procyclical in our simulations. The population correlation between aggregate book leverage and aggregate productivity is 0.25 in the benchmark parametrization. Allowing low or countercyclical equity flotation costs lowers this correlation to be around 0.12. Intuitively, because the forward-looking market value of equity incorporates the effect of countercyclical aggregate discount rates embedded in the stochastic discount factor (equations (5) and (6)), the market value of equity is more procyclical than the book value of equity (defined

[7] A previous version of our paper verifies that the shadow price of new funds is strongly procyclical in a model with dividend nonnegativity constraints and no debt (see Livdan et al. (2006)).

as $k_{jt} - b_{jt}$) in the model economies. Accordingly, market leverage is strongly countercyclical, even though book leverage is weakly procyclical.

D. The Leverage–Return Relation

The inflexibility mechanism underlying the positive relation between financial constraints and average returns also provides a new channel through which leverage affects risk and expected returns.

The standard leverage hypothesis discussed in corporate finance textbooks says that, all else equal, higher leverage means higher equity risk, which in turn means higher average equity returns. For example, Grinblatt and Titman (2001, pp. 381–384) argue that, when the debt is default free (as in our model), a firm's equity holders bear all the risk from movements in asset values. Specifically, the equity beta is a linear function of market leverage: Grinblatt and Titman's equation (11.2b) says that $\beta^E = (1 + D/E)\beta^A$, where β^E is equity beta, β^A is asset beta, and D/E is market leverage. Crucially, the textbook discussion assumes that the asset beta is fixed. "If a firm's total cash flows are independent of its capital structure—that is, its mix of debt of equity financing—the total risk borne by the aggregation of the investors of a firm, debt holders plus equity holders, does not change when the firm changes its capital structure (Grinblatt and Titman, p. 382)."

Our framework encompasses but goes beyond the standard leverage hypothesis. The inflexibility mechanism allows the asset beta to covary positively with market leverage. Intuitively, more levered firms are burdened with more debt, and must repay existing debt before they can finance new investments. More levered firms are more likely to face binding collateral constraints, are less flexible in adjusting capital to smooth dividends, and are therefore riskier and earn higher expected returns than less levered firms. With the notation of Grinblatt and Titman (2001), the inflexibility mechanism says $\beta^E = (1 + D/E)\beta^A(D/E)$, where the asset beta is an increasing function of D/E. Accordingly, our model predicts a convex relation between market leverage and expected returns, whereas the standard leverage hypothesis predicts a linear relation.

Figure 4 plots the equity beta, β_{jt}, against market leverage, b_{jt}/v_{jt}, under the benchmark parametrization of our model. We let both variables vary with firm-specific productivity, while fixing the aggregate productivity and capital stock at their long-run average levels. The figure shows that the leverage–risk relation is largely convex, especially for firms with low firm-specific productivity. Intuitively, the inflexibility from high financial leverage is more severe for less productive firms because it interacts with other sources of inflexibility such as asymmetric adjustment costs and operating leverage, which also are more important for less productive firms.

The inflexibility mechanism is quantitatively important in driving the leverage–return relation in our model economies. Using artificial panels we run the Fama-MacBeth (1973) monthly cross-sectional regressions of stock returns on market leverage, with and without controlling for the asset beta. We measure the asset beta as the levered equity beta, namely, $\beta^A_{jt} = (v_{jt}/(b_{jt} + v_{jt}))\beta_{jt}$,

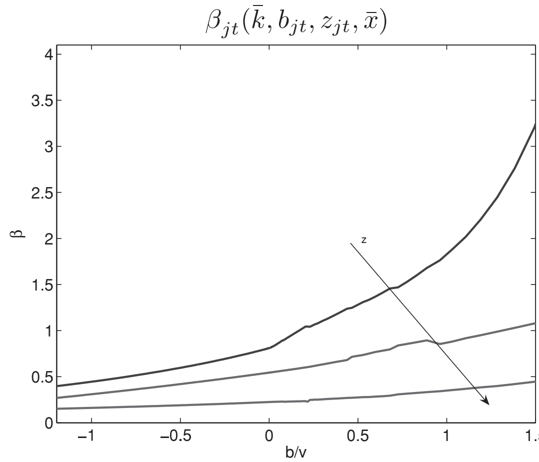

Figure 4. The relation between equity risk and market leverage, the benchmark parametrization. We plot risk (β_{jt}) against market leverage (b_{jt}/v_{jt}), while fixing aggregate productivity x_t and capital stock k_{jt} at their long-run average levels ($\bar{x}$ and $\bar{k}$, respectively). $\bar{k}$, determined in simulations, is around one. The panel has a class of curves that correspond to different values of z_{jt}. The arrow indicates the direction along which z_{jt} increases.

where β_{jt} is the equity beta as defined in equation (21). Table VII shows that, when used alone, market leverage significantly covaries with future stock returns. If this effect is purely due to the standard leverage hypothesis (which assumes the asset beta is fixed), then adding the asset beta into the regressions should not affect the leverage slopes and their significance. We observe the exact opposite. When b_{jt}/v_{jt} and β_{jt}^A are jointly used, the leverage slopes become smaller and insignificant across different parameterizations, but the slopes of β_{jt}^A are all significantly positive.

IV. Conclusion

Guided by neoclassical economic principles, we extend the investment-based asset pricing framework à la Zhang (2005) to incorporate debt dynamics à la Hennessy and Whited (2005). In our setting, facing aggregate and firm-specific shocks and a stochastic discount factor, firms choose optimal investment and next-period debt to maximize their equity value. Firms can retain earnings, borrow, and raise equity with flotation costs. When borrowing, firms face collateral constraints on debt capacity. Quantitative results show that firms with smaller capital stocks, lower firm-specific productivity, and higher current-period debt are more financially constrained. More important, more constrained firms are riskier and earn higher expected returns than less constrained firms. Intuitively, collateral constraints prevent firms from funding all desired investments, thereby reducing their flexibility in using the investment channel to smooth dividend streams in the face of aggregate shocks.

Table VII

Fama-MacBeth (1973) Monthly Cross-sectional Regressions of Stock Returns on Market Leverage, with and without Controlling for Asset Beta

Using simulated panels, we report the Fama-MacBeth (1973) monthly cross-sectional regressions of the stock returns, r_{jt+1}, on market leverage, b_{jt}/v_{jt}, with and without controlling for the asset beta, β_{jt}^A. β_{jt}^A is defined as $\beta_{jt} v_{jt}/(b_{jt}+v_{jt})$, where β_{jt} is the equity beta. Appendix B details the numerical calculations of β_{jt}. We simulate 100 artificial panels, each of which has 3,000 firms and 480 monthly observations, and then report across-simulation average Fama-MacBeth slopes, t-statistics (in parentheses), and average cross-sectional R^2s. Panel A reports the quantitative results from the benchmark parametrization with the parameter values in Table I. The remaining panels report four comparative static experiments: the high liquidation costs case with $s_0 = 0.70$ (Panel B), the low fixed flotation costs case with $\lambda_0 = 0.02$ (Panel C), the countercyclical liquidation costs case with $s_1 > 0$ (Panel D), and the countercyclical flotation costs case with $\lambda_2 > 0$ (Panel E).

b_{jt}/v_{jt}	R^2	b_{jt}/v_{jt}	β_{jt}^A	R^2
Panel A: Benchmark Parametrization				
1.093	0.28	1.051	0.0049	0.39
(5.15)		(1.26)	(3.31)	
Panel B: High Liquidation Costs, $s_0 = 0.70$				
3.617	0.14	2.456	0.0048	0.15
(7.22)		(0.46)	(10.29)	
Panel C: Low Fixed Flotation Costs, $\lambda_2 = 0.02$				
3.490	0.09	3.083	0.0062	0.12
(4.37)		(0.82)	(10.40)	
Panel D: Countercyclical Liquidation Costs, $s_1 > 0$				
3.827	0.14	2.079	0.0031	0.15
(6.58)		(0.78)	(9.60)	
Panel E: Countercyclical Flotation Costs, $\lambda_2 > 0$				
3.062	0.08	1.032	0.0046	0.09
(8.55)		(1.74)	(9.63)	

The interplay between asset pricing and corporate finance is likely to remain a fertile ground for future research. An exciting direction is to incorporate defaultable bonds into the neoclassical investment framework. Doing so would allow one to study the economic mechanism underlying the distress anomaly, which is the anomalous negative relation between financial distress and average stock returns (e.g., Dichev (1998) and Campbell et al. (2008)). Moreover, it seems important to address the distress anomaly together with the credit spread puzzle (e.g., Huang and Huang (2003)). Bhamra, Kuehn, and Strebulaev (2007), Chen (2007), and Chen, Collin-Dufresne, and Goldstein (2007) make important progress on the credit spread puzzle. But a deeper puzzle arises. Given that equity and bond are different contingent claims written on the same productive assets, why do more distressed firms earn lower average stock returns but higher credit spreads than less distressed firms? The Lucas-Prescott neoclassical paradigm has provided penetrating insights into important economic questions before. This same paradigm is likely to offer a promising framework for understanding the risk and returns of financially distressed firms.

Appendix A: The Shadow Price of New Debt

To characterize the shadow price of new debt given in equation (18), we first specify the infinite-horizon Lagrangian formulation of the value function, denoted $\mathcal{L}$, as follows:

$$\begin{aligned}\mathcal{L}_{jt} = \cdots + \pi_{jt} + \frac{b_{jt+1}}{\iota_{jt}} - \phi(i_{jt}, k_{jt}) - b_{jt} - \lambda(e_{jt}, k_{jt}) - \nu_{jt}(b_{jt+1} - s(1-\delta)k_{jt+1}) \\ + E_t\left[m_{t+1}\left(\pi_{jt+1} + \frac{b_{jt+2}}{\iota_{jt+1}} - \phi(i_{jt+1}, k_{jt+1}) - b_{jt+1} - \lambda(e_{jt+1}, k_{jt+1})\right.\right. \\ \left.\left. - \nu_{jt+1}(b_{jt+2} - s(1-\delta)k_{jt+2}) + \cdots\right)\right]. \end{aligned} \tag{A1}$$

The Lagrangian $\mathcal{L}_{jt}$ is differentiable almost everywhere except when $\pi(k_{jt}, z_{jt}, x_t) + b_{jt+1}/\iota_{jt} - \phi(i_{jt}, k_{jt}) - b_{jt} = 0$. Thus, we can characterize ν_{jt} analytically only in the case when e_{jt} is strictly positive.

When $e_{jt} > 0$, differentiating $\mathcal{L}_{jt}$ with respect to b_{jt+1} and recognizing

$$e_{jt} = \phi(i_{jt}, k_{jt}) + b_{jt} - \pi(k_{jt}, z_{jt}, x_t) - \frac{b_{jt+1}}{\iota_{jt}}, \tag{A2}$$

we obtain

$$\frac{\partial \mathcal{L}_{jt}}{\partial b_{jt+1}} = \frac{1}{\iota_{jt}} + \frac{1}{\iota_{jt}}\lambda_e(e_{jt}, k_{jt})\mathbf{1}^e_{jt} - \nu_{jt} - E_t\left[m_{t+1}\left(1 + \lambda_e(e_{jt+1}, k_{jt+1})\mathbf{1}^e_{jt+1}\right)\right] = 0. \tag{A3}$$

Solving for ν_{jt} gives

$$\nu_{jt} = \frac{1}{\iota_{jt}}\left[1 + \lambda_e(e_{jt}, k_{jt})\mathbf{1}^e_{jt}\right] - E_t\left[m_{t+1}\left(1 + \lambda_e(e_{jt+1}, k_{jt+1})\mathbf{1}^e_{jt+1}\right)\right]. \tag{A4}$$

We can simplify equation (A4) further by noting that the collateral constraints bind ($\nu_{jt} > 0$) when $b_{jt+1} > 0$ and $\iota_{jt} = r_{ft} = 1/E_t[m_{t+1}]$. Equation (A4) then becomes equation (18) in the main text.

Appendix B: Solution Algorithm

We use the discrete-state-space value function iteration technique to solve the dynamic equity value maximization problems of firms given by equation (17). The state variables x and z are defined on continuous state spaces that can be transformed into discrete state spaces using Rouwenhorst's (1995) methods. We use five grid points for the x process and nine points for the z process. In all cases our results are robust to finer grids. Once the discrete space is available, the conditional expectation operator can be carried out as a matrix multiplication.

The capital stock in each period is constrained to be an element of the linear finite time-invariant set $\mathcal{K} = \{k_1, \ldots, k_{N_K}\}$, with a total of $N_K = 50$ elements (grid points). For any optimal capital stock on the grid, the finer grid used for the interpolation consists of 1,000 evenly spaced points. The face value of one-period debt, b, in each period is constrained to be an element of the linear finite time-invariant set $\mathcal{B} = \{b_1, \ldots, b_{N_B}\}$, centered around zero with a total of $N_B = 2N_K + 1 = 101$ elements (grid points). The boundaries of the set, $\{b_1, b_{N_B}\}$, are the same for any $k_i \in \mathcal{K}$, and are chosen to satisfy $\{b_1, b_{N_B}\} = \pm s_0(1-\delta)k_{N_K}$. For any optimal debt on the grid chosen from a first-pass optimization, the finer grid used for the interpolation consists of 1,001 evenly spaced points.

We can formulate the dynamic value-maximization problem on the grid as follows:

$$\begin{aligned}
&v^n(k_i, b_j, z_l, x_m)\\
&= \max_{\{k',b'\}\in\mathcal{K}'\times\mathcal{B}'} \Bigg\{ \pi(k_i, z_l, x_m) + \frac{b'}{\iota(x_m)} - b_j - \phi(k', k_i) - \lambda(e(k_i, b_j, z_l, x_m, k', b'), k_j)\\
&+ \sum_{l=1}^{9}\sum_{m=1}^{5} \eta e^{(\gamma_0+\gamma_1(x_m-\bar{x}))(x_m-x')} \tilde{v}^{n-1}(k', b', z_{l'}, x_{m'}) Q_z(z_{l'} \mid z_l) Q_x(x_{m'} \mid x_m), \qquad \text{(B1)}
\end{aligned}$$

where $\tilde{v}$ incorporates the collateral constraint as $\tilde{v}^n = v^n \mathbf{1}_{\{b' \le s(1-\delta)k'\}} - e^{10}(1 - \mathbf{1}_{\{b' \le s(1-\delta)k'\}})$ and n indicates the number of value iterations. The four-dimensional matrix $v^n : \mathcal{K} \times \mathcal{B} \times \mathcal{X} \times \mathcal{Z}$ has 227,250 values as compared to 8,250 values used for Model 1. The algorithm can be described as follows:

1. Make a guess for $v^{n-1}(k, b, z, x)$ on the right-hand side of (B1). It is a $25 \times 51 \times 9 \times 5$ object.
2. For each $\{k, b, z, x\} \in \mathcal{K} \times \mathcal{B} \times \mathcal{X} \times \mathcal{Z}$, use local linear interpolation to construct $v^{n-1}(k', b', z, x)$. Reshape the v^{n-1} into a traditional $\mathcal{K} \times \mathcal{B}$ matrix and again reshape after the interpolation to form a three-dimensional object $v^{n-1}(k', b', z, x)$. Because $\mathcal{K} \times \mathcal{B}$ is not a square matrix, we must perform interpolation along each dimension separately.
3. On the $\mathcal{K}' \times \mathcal{B}'$ grid construct $\mathbf{1}_{\{b' \le s(1-\delta)k'\}}$, which is a three-dimensional object of zeros and ones, and use it to construct $\tilde{v}^{n-1}(k', b', z, x)$.
4. For each $\{k, b, z, x\} \in \mathcal{K} \times \mathcal{B} \times \mathcal{X} \times \mathcal{Z}$, solve for the optimal $\{k^*(k, b, z, x), b^*(k, b, z, x)\} \in \mathcal{K}' \times \mathcal{B}'$ from

$$\begin{aligned}
&\{k'(k, b, z, x), b'(k, b, z, x)\}\\
&= \arg\max_{\{k',b'\}\in\mathcal{K}'\times\mathcal{B}'} \Bigg\{ \frac{b'}{\iota(x_m)} - \phi(k', k_i) - \lambda(e(k_i, b_j, z_l, x_m, k', b'), k_j)\\
&+ \sum_{l=1}^{9}\sum_{m=1}^{5} \eta e^{(\gamma_0+\gamma_1(x_m-\bar{x}))(x_m-x')} Q_z(z_{l'} \mid z_l) Q_x(x_{m'} \mid x_m) \tilde{v}^{n-1}(k', b', z_{l'}, x_{m'}) \Bigg\}
\end{aligned}$$

(B2)

by doing a simple grid search along k' holding b' fixed and then along b' for each k^*.

5. Construct $v^n(k, b, z, x)$ from equation (B1).
   Check for the conversion using maximum error criterion

$$\max |v^n(k, b, z, x) - v^{n-1}(k, b, z, x)| < \epsilon = 10^{-5}. \tag{B3}$$

6. If the conversion criterion is not satisfied, set $v^n(k, b, z, x)$ as a new guess and repeat all of the steps above.

Once the algorithm converges, the expected return $E_t[r_{jt+1}] = E_t[v_{jt+1}]/(v_{jt} - d_{jt})$ can be calculated in a similar way. The equity beta β_{jt} can be backed out from equation (20) because the risk-free rate r_{ft} and the price of risk ζ_{mt} are known functions of the pricing kernel. Piecewise linear interpolation is used extensively to obtain firm value, optimal investment, and expected return, which do not lie directly on the grid points.

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