### Lecture Notes

Li and Zhang (2010, J. of Financial Economics): Does *Q*-Theory with Investment Frictions Explain Anomalies in the Cross-Section of Returns?

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### News

Theory: demonstrate that the expected return-investment relation should be steeper in firms with high investment frictions

# Empirics:

- ► Some evidence that the investment-to-assets and asset growth anomalies are stronger in financially more constrained firms
- ▶ No evidence that investment frictions affect the investment growth, net stock issues, abnormal corporate investment, and net operating assets anomalies
- ▶ Investment frictions dominated by limits-to-arbitrage

# Outline

Model

Tests

Summary and Interpretation

Why should investment frictions affect investment-related anomalies?

Two periods, 0 and 1

Firm i's capital:  $K_{i0}$  and  $K_{i1}$ ,  $K_{i1} = I_{i0} + (1 - \delta)K_{i0}$ 

Firm i's return on assets, ROA:  $\Pi$ , constant over two periods

Firm i's operating profits:  $\Pi K_{i0}$  and  $\Pi K_{i1}$ 

Firm i's investment costs:

$$C(I_{i0}, K_{i0}) = \frac{\lambda_i}{2} \left(\frac{I_{i0}}{K_{i0}}\right)^2 K_{i0}, \quad \lambda_i > 0$$

The first-order condition

Firm i's discount rate:  $R_i$ 

Firm i's value-maximization problem:

$$\max_{\{I_{i0}\}} \; \Pi K_{i0} - I_{i0} - \frac{\lambda_i}{2} \left(\frac{I_{i0}}{K_{i0}}\right)^2 K_{i0} + \frac{1}{R_i} \left[\Pi K_{i1} + (1-\delta)K_{i1}\right]$$

Firm i's first-order condition:

$$R_i = \frac{\Pi + 1 - \delta}{1 + \lambda_i (I_{i0}^* / K_{i0})}$$

The investment-discount rate relation and its interaction with investment frictions

Totally differentiating the first-order condition w.r.t.  $R_i$ :

$$\frac{d(I_{i0}^*/K_{i0})}{dR_i} = -\frac{[1 + \lambda_i(I_{i0}^*/K_{i0})]^2}{\lambda_i(\Pi + 1 - \delta)} < 0$$

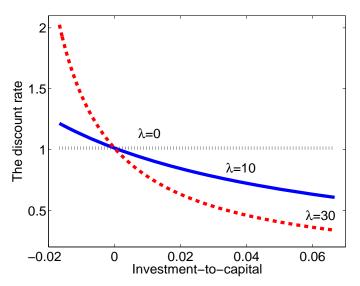
as in Cochrane (1991) and Liu, Whited, and Zhang (2009)

The investment-discount rate relation varies with investment costs:

$$d\left|\frac{d(I_{i0}^*/K_{i0})}{dR_i}\right|/d\lambda_i = -\frac{[1+\lambda_i(I_{i0}^*/K_{i0})]^2}{\lambda_i^2(\Pi+1-\delta)} < 0$$

Model

Plot  $R_i = (\Pi + 1 - \delta)/(1 + \lambda_i(I_{i0}^*/K_{i0}))$  with  $\Pi = .15/12$  per month and  $\delta = 0$ 



How investment frictions affect the expected return-investment relation? Intuition

$$R_i = \frac{\Pi + 1 - \delta}{1 + \lambda_i (I_{i0}^* / K_{i0})}$$

When investment is frictionless,  $\lambda_i = 0$ , investment is infinitely elastic to the discount rate, or  $R_i$  is flat in  $I_{i0}^*/K_{i0}$ 

With frictions,  $\lambda_i > 0$ , investment now predicts future returns

The greater is  $\lambda_i$ , the less elastic investment is, a given change in  $I_{i0}^*/K_{i0}$  corresponds to a higher magnitude change in  $R_i$ 



The negative expected return-investment relation is steeper in firms with high investment costs than in firms with low investment costs

Tests
Design

Fama-MacBeth cross-sectional regressions of monthly percent returns on a given investment-related anomaly variable in subsamples with high, medium, and low investment frictions

Null Hypothesis: The magnitude of the slope is higher in the high-frictions subsample than in the low-frictions subsample

Alternative: Mispricing can persist when arbitrage costs outweigh arbitrage benefits, Shleifer and Vishny (1997). Horse races between investment frictions and limits-to-arbitrage proxies

# **Tests**

Identify investment frictions with firm-level proxies of financing constraints

Asset size: Total assets, annual sorts, the small-assets tercile = more constrained, the big-assets tercile = less constrained

Payout ratio: (Dividends for preferred stocks + Dividends for common stocks + Share repurchases)/Operating income before depreciation, annual sorts, the low-payout tercile = more constrained, the big-payout tercile = less constrained

 For firms with negative earnings (zero dividends = more constrained, positive dividends = less constrained)

Bond ratings: Unrated = more constrained, rated = less constrained

Tests
Proxies for limits-to-arbitrage

Idiosyncratic volatility: Residual volatility from daily market regressions over 250 days ending on June 30 of year t, annual sorts, the low-lvol tercile = low arbitrage costs, the high-lvol tercile = high arbitrage costs

Dollar trading volume: Share volume times daily closing price over the past 12 months, annual sorts, the low-volume tercile = high arbitrage costs, the high-volume tercile = low arbitrage costs

### Tests

Investment-related anomaly variables

Investment-to-assets, I/A: (Change in PPE + Change in inventories)/Lagged total assets, Chen and Zhang (2009)

Asset growth,  $\triangle A/A$ : Change in total assets/Lagged total assets, Cooper, Gulen, and Schill (2008)

Investment growth,  $\triangle I/I$ : Change in CAPX/Lagged CAPX, Xing (2008)

#### **Tests**

Investment-related anomaly variables

Net stock issues, *NSI*: log growth rate of the split-adjusted shares outstanding, Fama and French (2008)

Abnormal corporate investment, ACI:

 $3CE_t/(CE_{t-1}+CE_{t-2}+CE_{t-3})-1$  with CE=CAPX/Sales, Titman, Wei, and Xie (2004)

Net operating assets, NOA: (Operating assets — Operating liabilities)/Lagged total assets, Hirshleifer, Hou, Teoh, and Zhang (2004)

# Tests Cross-correlations

	Asset size	Payout ratio	Bond rating	Ivol	Volume
Asset size	1				
Payout ratio	0.45	1			
Bond rating	-0.37	-0.21	1		
Ivol	-0.64	-0.55	0.29	1	
Volume	0.73	0.27	-0.35	-0.39	1

Tests
Testing the investment frictions hypothesis

	I/A	$\triangle A/A$	$\triangle I/I$	NSI	ACI	NOA
Full Sample	-0.69 $(-4.9)$	-0.74 (-8.3)	$-0.08 \ (-5.5)$	-1.87 $(-7.0)$	$-0.05 \ (-1.6)$	$-0.51 \ (-5.1)$
Small asset size Big asset size Small-minus-big	-0.85 -0.33 [-2.1]	-0.83 -0.47 [-2.4]	-0.09 -0.05 [-0.9]	-1.27 $-1.50$ [0.6]	-0.04 $0.02$ $[-1.0]$	-0.47 $-0.45$ $[-0.1]$
Low payout ratio High payout ratio Low-minus-high	-0.93 -0.39 [-2.5]	-0.81 $-0.66$ $[-1.2]$	-0.10 $-0.06$ $[-1.4]$	-1.39 $-2.20$ [1.9]	-0.08 $-0.03$ $[-1.2]$	-0.50 -0.56 [0.5]
With bond rating Without bond rating Without-minus-with	-0.47 -0.86 [-2.5]	-0.50 -0.90 [-3.8]	-0.05 -0.10 [-2.4]	-1.82 $-1.86$ [-0.1]	-0.09 -0.03 [1.6]	-0.51 -0.50 [0.2]

Tests Testing the investment frictions hypothesis, controlling for size, B/M, and momentum

	I/A	$\triangle A/A$	$\triangle I/I$	NSI	ACI	NOA
Full Sample	-0.49	-0.52	-0.07	-1.28	-0.02	-0.56
	(-3.8)	(-6.4)	(-5.2)	(-5.7)	(-1.0)	(-6.8)
Small asset size	-0.68	-0.57	-0.07	-0.88	-0.07	-0.67
Big asset size	-0.20	-0.38	-0.04	-1.38	0.02	-0.43
Small-minus-big	[-2.1]	[-1.3]	[-0.6]	[1.4]	[-1.7]	[-1.7]
Low payout ratio	-0.62	-0.51	-0.06	-0.89	-0.05	-0.51
High payout ratio	-0.27	-0.45	-0.06	-1.73	-0.01	-0.63
Low-minus-high	[-1.8]	[-0.5]	[-0.2]	[2.4]	[-1.0]	[1.1]
With bond rating	-0.23	-0.29	-0.05	-1.28	-0.05	-0.44
Without bond rating	-0.65	-0.65	-0.08	-1.28	-0.01	-0.59
Without-minus-with	[-2.8]	[-3.6]	[-1.3]	[-0.0]	[1.1]	[-1.8]

Tests
Do limits-to-arbitrage affect anomalies?

	I/A	$\triangle A/A$	$\triangle I/I$	NSI	ACI	NOA
Low Ivol	-0.10	-0.16	-0.02	-1.49	-0.01	-0.29
High Ivol	-1.01	-0.99	-0.10	-1.54	-0.05	-0.61
High-minus-low Ivol	[-4.2]	[-5.7]	[-2.7]	[-0.1]	[-0.8]	[-2.4]
Low Dvol	-1.18	-0.94	-0.09	-1.82	-0.12	-0.80
High Dvol	-0.45	-0.50	-0.09	-1.54	-0.02	-0.47
Low-minus-high Dvol	[-2.8]	[-2.2]	[-0.0]	[-0.6]	[-1.8]	[-2.2]

 $Tests \\ Do \ limits-to-arbitrage \ affect \ anomalies? \ controlling \ for \ size, \ B/M, \ and \ momentum$ 

	I/A	$\triangle A/A$	$\triangle I/I$	NSI	ACI	NOA
Low Ivol	0.01	-0.11	-0.03	-1.15	0.00	-0.33
High Ivol	-0.83	-0.70	-0.08	-0.98	-0.04	-0.71
High-minus-low Ivol	[-4.1]	[-4.4]	[-1.5]	[0.5]	[-0.9]	[-2.9]
Low Dvol	-0.90	-0.73	-0.07	-1.50	-0.07	-0.71
High Dvol	-0.25	-0.36	-0.07	-1.38	-0.02	-0.50
Low-minus-high Dvol	[-2.8]	[-2.3]	[-0.0]	[-0.3]	[-1.1]	[-1.4]

Tests

Horse races with two-by-two splits: the effect of financing constraints after controlling for idiosyncratic volatility

	I/A	$\triangle A/A$	$\triangle I/I$	NSI	ACI	NOA
Low Ivol,	0.06	0.04	-0.06	-0.58	-0.04	0.10
small-minus-big asset	[0.3]	[0.3]	[-1.7]	[-1.3]	[-0.9]	[0.9]
High Ivol,	-0.14	-0.16	0.01	-0.07	-0.01	0.05
small-minus-big asset	[-0.6]	[-1.1]	[0.4]	[-0.2]	[-0.3]	[0.4]
Low Ivol,	-0.40	-0.18	-0.05	-0.31	-0.12	-0.06
low-minus-high payout	[-2.1]	[-1.4]	[-1.6]	[-0.8]	[-2.6]	[-0.6]
High Ivol,	-0.16	-0.15	-0.01	0.47	0.00	-0.02
low-minus-high payout	[-0.7]	[-1.0]	[-0.3]	[1.0]	[0.1]	[-0.1]
Low Ivol,	-0.19	-0.15	-0.04	-0.29	-0.02	0.16
without-minus-with rating	[-1.1]	[-1.1]	[-1.5]	[-0.8]	[-0.4]	[1.7]
High Ivol,	-0.21	-0.33	-0.03	-0.04	0.08	-0.06
without-minus-with rating	[-1.0]	[-2.5]	[-1.1]	[-0.1]	[1.5]	[-0.5]

Tests

Horse races with two-by-two splits: the effect of financing constraints after controlling for dollar trading volume

	I/A	$\triangle A/A$	$\triangle I/I$	NSI	ACI	NOA
Low Dvol,	-0.96	-0.34	-0.06	-0.21	-0.10	-0.18
small-minus-big asset	[-3.1]	[-1.6]	[-1.3]	[-0.4]	[-1.6]	[-0.9]
High Dvol,	0.10	-0.10	-0.01	0.31	-0.10	0.17
small-minus-big asset	[0.3]	[-0.4]	[-0.2]	[0.4]	[-1.3]	[0.9]
Low Dvol,	-0.41	-0.21	-0.04	1.16	-0.03	0.06
low-minus-high payout	[-1.6]	[-1.2]	[-1.4]	[2.0]	[-0.6]	[0.4]
High Dvol,	-0.33	-0.13	-0.02	0.35	-0.05	0.09
low-minus-high payout	[-1.4]	[-0.9]	[-0.6]	[0.7]	[-0.8]	[0.6]
Low Dvol,	-0.57	-0.71	-0.03	-0.62	0.04	-0.18
without-minus-with rating	[-2.0]	[-3.7]	[-0.8]	[-1.1]	[0.8]	[-1.1]
High Dvol,	-0.37	-0.25	-0.06	-0.25	0.08	-0.04
without-minus-with rating	[-1.7]	[-1.6]	[-1.7]	[-0.6]	[1.5]	[-0.3]

Tests Horse races with two-by-two splits: the effect of idiosyncratic volatility after controlling for financing constraints

	I/A	$\triangle A/A$	$\triangle I/I$	NSI	ACI	NOA
Small asset,	-0.63	-0.57	-0.01	0.83	0.03	-0.25
high-minus-low Ivol	[-2.9]	[-3.8]	[-0.6]	[1.8]	[0.7]	[-1.9]
Big asset,	-0.43	-0.37	-0.09	0.32	0.01	-0.20
high-minus-low Ivol	[-1.8]	[-2.4]	[-2.2]	[0.7]	[0.1]	[-1.6]
Low payout,	-0.38	-0.43	-0.02	0.54	0.09	-0.18
high-minus-low Ivol	[-1.9]	[-3.1]	[-0.8]	[1.3]	[1.9]	[-1.5]
High payout,	-0.61	-0.46	-0.06	-0.24	-0.03	-0.22
high-minus-low Ivol	[-2.4]	[-2.7]	[-1.8]	[-0.5]	[-0.5]	[-1.6]
With rating,	-0.57	-0.43	-0.06	0.16	-0.06	-0.09
high-minus-low Ivol	[-2.4]	[-2.7]	[-1.6]	[0.4]	[-1.0]	[-0.7]
Without rating,	-0.59	-0.61	-0.05	0.40	0.03	-0.32
high-minus-low Ivol	[-2.8]	[-4.2]	[-1.6]	[1.0]	[0.7]	[-2.7]

Tests Horse races with two-by-two splits: the effect of dollar trading volume after controlling for financing constraints

I/A	$\triangle A/A$	$\triangle I/I$	NSI	ACI	NOA
-0.80	-0.37	-0.04	-0.51	0.00	-0.28
[-2.3]	[-1.6]	[-0.8]	[-0.7]	[0.1]	[-1.4]
0.26	-0.13	0.01	0.01	0.01	0.07
[1.0]	[-0.6]	[0.1]	[0.0]	[0.1]	[0.4]
-0.57	-0.38	-0.01	-0.15	-0.03	-0.26
[-2.4]	[-2.2]	[-0.4]	[-0.3]	[-0.6]	[-1.7]
-0.49	-0.30	0.01	-0.96	-0.05	-0.23
[-2.1]	[-1.6]	[0.2]	[-1.9]	[-1.0]	[-1.5]
-0.30	0.03	-0.03	0.11	-0.07	-0.08
[-1.0]	[0.2]	[-0.7]	[0.2]	[-1.2]	[-0.4]
-0.50	-0.44	0.00	-0.26	-0.10	-0.22
[-2.0]	[-2.5]	[0.2]	[-0.5]	[-1.9]	[-1.5]
	-0.80 [-2.3] 0.26 [1.0] -0.57 [-2.4] -0.49 [-2.1] -0.30 [-1.0] -0.50	-0.80 -0.37 [-2.3] [-1.6] 0.26 -0.13 [1.0] [-0.6] -0.57 -0.38 [-2.4] [-2.2] -0.49 -0.30 [-2.1] [-1.6] -0.30 0.03 [-1.0] [0.2] -0.50 -0.44	-0.80         -0.37         -0.04           [-2.3]         [-1.6]         [-0.8]           0.26         -0.13         0.01           [1.0]         [-0.6]         [0.1]           -0.57         -0.38         -0.01           [-2.4]         [-2.2]         [-0.4]           -0.49         -0.30         0.01           [-2.1]         [-1.6]         [0.2]           -0.30         0.03         -0.03           [-1.0]         [0.2]         [-0.7]           -0.50         -0.44         0.00	-0.80         -0.37         -0.04         -0.51           [-2.3]         [-1.6]         [-0.8]         [-0.7]           0.26         -0.13         0.01         0.01           [1.0]         [-0.6]         [0.1]         [0.0]           -0.57         -0.38         -0.01         -0.15           [-2.4]         [-2.2]         [-0.4]         [-0.3]           -0.49         -0.30         0.01         -0.96           [-2.1]         [-1.6]         [0.2]         [-1.9]           -0.30         0.03         -0.03         0.11           [-1.0]         [0.2]         [-0.7]         [0.2]           -0.50         -0.44         0.00         -0.26	-0.80         -0.37         -0.04         -0.51         0.00           [-2.3]         [-1.6]         [-0.8]         [-0.7]         [0.1]           0.26         -0.13         0.01         0.01         0.01           [1.0]         [-0.6]         [0.1]         [0.0]         [0.1]           -0.57         -0.38         -0.01         -0.15         -0.03           [-2.4]         [-2.2]         [-0.4]         [-0.3]         [-0.6]           -0.49         -0.30         0.01         -0.96         -0.05           [-2.1]         [-1.6]         [0.2]         [-1.9]         [-1.0]           -0.30         0.03         -0.03         0.11         -0.07           [-1.0]         [0.2]         [-0.7]         [0.2]         [-1.2]           -0.50         -0.44         0.00         -0.26         -0.10

# Conclusion Summary and interpretation

The expected return-investment relation should be steeper in firms with high investment frictions as predicted by q-theory

Some evidence that investment frictions affect the investment-to-assets and asset growth anomalies, but not the investment growth, net stock issues, abnormal corporate investment, and net operating assets anomalies

Investment frictions dominated by limits-to-arbitrage in direct horse races: Mispricing seems to better explain the anomalies in question

Conclusion
Update

Lam and Wei (2011) conduct cross-sectional regressions of returns on asset growth on subsamples split by a given measure of limits-to-arbitrage or investment frictions

#### Main findings:

- Proxies for limits-to-arbitrage and proxies for investment frictions are often highly correlated;
- the evidence based on equal-weighted returns shows significant support for both hypotheses, while the evidence from value-weighted returns is weaker;
- in direct comparisons, each hypothesis is supported by a fair and similar amount of evidence.