Lecture Notes

Gonçalves, Xue, and Zhang (2020, Review of Financial Studies, "Aggregation, Capital Heterogeneity, and the Investment CAPM")

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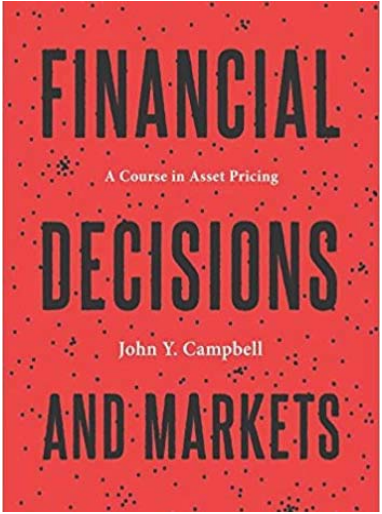
A detailed treatment of aggregation and capital heterogeneity substantially improves the performance of the investment CAPM



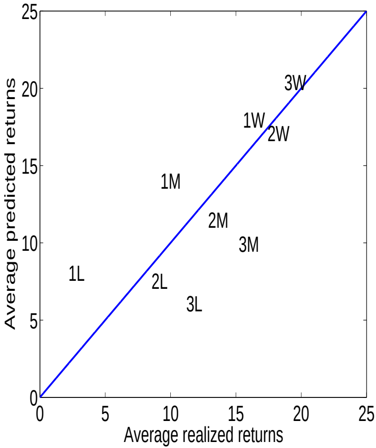
"This problem, that different parameters are needed to fit each anomaly, is a pervasive one in the *q*-theoretic asset pricing literature (p. 275)."

Liu, Whited, and Zhang (2009):

TABLE 2
PARAMETER ESTIMATES AND TESTS OF OVERIDENTIFICATION

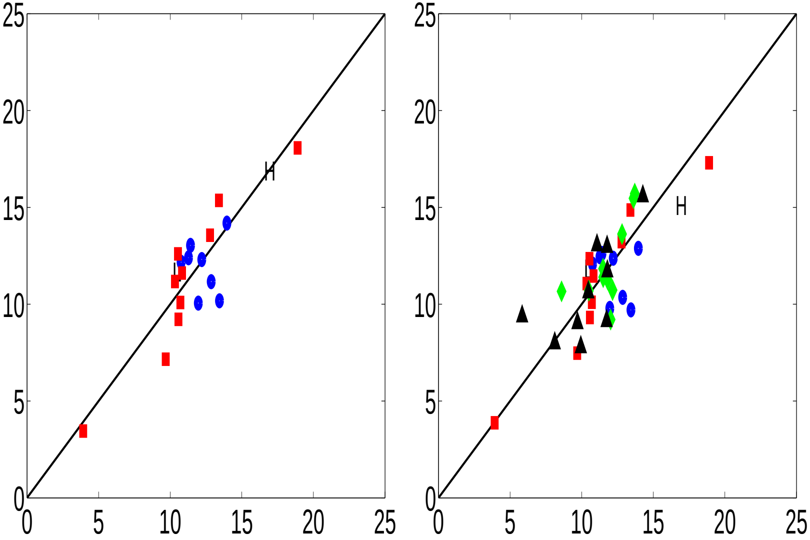
	SUE	B/M	CI
	A. Matching Expected Returns		
a	7.7	22.3	1.0
	[1.7]	[25.5]	[.3]
α	.3	.5	.2
	[.0]	[.3]	[.0]
χ^2	4.4	6.0	6.5
d.f.	8	8	8
p	.8	.7	.6
m.a.e.	.7	2.3	1.5

Liu and Zhang (2014):



Introduction

Average predicted versus realized stock returns, value, momentum, I/A, and Roe



Outline

Outline

The Model

A 2-capital setup

Operating profits: $\Pi(K_{it}, W_{it}, X_{it})$, with K_{it} physical capital; W_{it} working capital; X_{it} a vector of exogenous shocks

Constant returns to scale, Cobb-Douglas

Capital accumulation:

$$\begin{aligned} K_{it+1} &= I_{it} + (1 - \delta_{it})K_{it} \\ W_{it+1} &= \triangle W_{it} + W_{it} \end{aligned}$$

Adjustment costs on physical (not working) capital:

$$\Phi(I_{it}, K_{it}) = \frac{a}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}$$

The fraction of PPENT in the sum of PPENT and working capital (Cash, account receivables, inventory) on average only 38%

Working capital ordinary but essential to daily operations

Why no labor? 80.1% missing wages data; unlike working capital, without adjustment costs, labor does not affect expected returns

Why no intangibles? Measurement errors in investment flows and amortization rates; expected return moments and valuation moments contain different information

The Model

Optimal investment

Optimal physical capital investment: $E_t[M_{t+1} r^K_{it+1}] = 1$, in which the physical capital investment return:

$$r^K_{it+1} = \frac{\begin{array}{c}(1-\tau_{t+1})\left[\gamma_K \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} \\ +(1-\delta_{it+1})\left[1+(1-\tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]\end{array}}{1+(1-\tau_t)a\left(\frac{I_{it}}{K_{it}}\right)}$$

Optimal working capital investment: $E_t[M_{t+1} r^W_{it+1}] = 1$, in which the working capital investment return:

$$r^W_{it+1} = 1 + (1-\tau_{t+1})\gamma_W \frac{Y_{it+1}}{W_{it+1}}$$

The weighted average of the investment returns equals the weighted average of the cost of equity and after-tax cost of debt:

$$w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W = w_{it}^B r_{it+1}^{Ba} + (1 - w_{it}^B) r_{it+1}^S$$

$w_{it}^K = q_{it} K_{it+1} / (q_{it} K_{it+1} + W_{it+1})$ and $w_{it}^B = B_{it+1} / (P_{it} + B_{it+1})$

The investment CAPM:

$$r_{it+1}^S = \underbrace{\frac{w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W - w_{it}^B r_{it+1}^{Ba}}{1 - w_{it}^B}}_{\textbf{The fundamental return},\ r_{it+1}^F}$$

Outline

Test the expected return implications of the investment CAPM:

$$E[r^S_{pt+1} - r^F_{pt+1}] = 0,$$

r^S_{pt+1}: Portfolio p's stock return, r^F_{pt+1}: The fundamental return

The investment CAPM alpha: $\alpha_p = E_T[r^S_{pt+1} - r^F_{pt+1}]$

Econometric Methods

A technical aside: γ_K and γ_W enter the estimation as $\gamma = \gamma_K + \gamma_W$

The 2-capital model as parsimonious as the physical capital model:

$$\begin{aligned} w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W \quad = & \\ & \frac{(1-\tau_{t+1})(\gamma_K + \gamma_W) Y_{it+1}/(K_{it+1} + W_{it+1})}{q_{it} K_{it+1}/(K_{it+1} + W_{it+1}) + W_{it+1}/(K_{it+1} + W_{it+1})} + \\ & w_{it}^K \frac{(1-\tau_{t+1})(a/2)\left(I_{it+1}/K_{it+1}\right)^2 + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1}) q_{it+1}}{q_{it}} \\ & +(1 - w_{it}^K) \end{aligned}$$

$(\gamma_K + \gamma_W) Y_{it+1}/(K_{it+1} + W_{it+1})$ as the correct measure of marginal product of capital

Let $\mathbf{c} \equiv (\gamma, a)$, $\mathbf{g}_T$ the sample moments, $\mathbf{D} = \partial \mathbf{g}_T / \partial \mathbf{c}$

The GMM objective function: $\mathbf{g}_T' \mathbf{W} \mathbf{g}_T$, in which $\mathbf{W} = \mathbf{I}$

$\mathrm{Var}(\hat{\mathbf{c}}) = (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}/T$

$\mathrm{Var}(\mathbf{g}_T) = \left[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\right] \mathbf{S} \left[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\right]' / T$

The overidentification test:

$$\mathbf{g}_T' \left[\mathrm{var}(\mathbf{g}_T)\right]^{+} \mathbf{g}_T \sim \chi^2(\#\,\text{moments} - \#\,\text{parameters})$$

Econometric Methods

Portfolio-level aggregation (Liu, Whited, and Zhang 2009; Liu and Zhang 2014)

Portfolio-level fundamental returns constructed from portfolio-level accounting variables aggregated from the firm level:

$$E\left[\begin{array}{c}\sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^{S} - \\ r_{pt+1}^{F}\left(\gamma_K, a; Y_{pt+1}, K_{pt+1}, I_{pt+1}, \delta_{pt+1}, I_{pt}, K_{pt}, r_{pt+1}^{Ba}, w_{pt}^{B}\right)\end{array}\right] = 0$$

- N_{pt}: The number of firms in portfolio p at the start of t, w_{ipt}: Stock i's weight in portfolio p, r_{ipt+1}^{S}: The return of stock i in p over time t, r_{pt+1}^{F}: The fundamental return of p

Aggregating firm-level characteristics to the portfolio level: $I_{pt+1} = \sum_{i=1}^{N_{pt}} I_{ipt+1}$, $w_{pt}^{B} = \sum_{i=1}^{N_{pt}} B_{ipt+1} / \sum_{i=1}^{N_{pt}} (P_{ipt} + B_{ipt+1})$, etc

Construct firm-level fundamental returns from firm-level accounting variables, then aggregate to portfolio-level fundamental returns:

$$E\left[\begin{array}{c}\sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^{S} - \sum_{i=1}^{N_{pt}} w_{ipt} \times \\ r_{ipt+1}^{F}\left(\gamma, a; Y_{ipt+1}, K_{ipt+1}, I_{ipt+1}, \delta_{ipt+1}, I_{ipt}, K_{ipt}, r_{ipt+1}^{Ba}, w_{ipt}^{B}\right)\end{array}\right] = 0$$

- r_{ipt+1}^{F}: Firm i's fundamental return, r_{pt+1}^{F} varies with w_{ipt}

Why?

- Firms can make different investment choices
- Substantial firm-level heterogeneity helps identify parameters

Outline

Data

Testing deciles

40 testing deciles formed on:

- Book-to-market: Bm
- Momentum (prior 11-month returns, 1-month horizon): R^{11}
- Asset growth: I/A
- Return on equity: Roe

NYSE breakpoints and value-weighted returns

All-but-micro breakpoints and equal-weighted returns (the Online Appendix, with stronger results)

Data

Average returns of the 40 testing deciles, January 1967–June 2017

	L	2	3	4	5	6	7	8	9	H	H−L
	The Bm deciles										
$\overline{R}$	0.43	0.53	0.60	0.46	0.53	0.56	0.67	0.63	0.73	0.90	0.47
$t_{\overline{R}}$	1.85	2.74	3.16	2.26	2.89	3.19	3.65	3.40	4.07	3.93	2.15
	The R^{11} deciles										
$\overline{R}$	−0.03	0.40	0.47	0.48	0.45	0.48	0.46	0.63	0.68	1.08	1.12
$t_{\overline{R}}$	−0.10	1.53	2.16	2.47	2.43	2.54	2.63	3.25	3.25	3.98	3.88
	The I/A deciles										
$\overline{R}$	0.69	0.68	0.63	0.52	0.53	0.56	0.59	0.48	0.58	0.33	−0.36
$t_{\overline{R}}$	2.98	3.42	3.84	3.19	3.09	3.13	3.24	2.49	2.42	1.27	−2.20
	The Roe deciles										
$\overline{R}$	0.06	0.25	0.42	0.40	0.54	0.44	0.57	0.53	0.57	0.74	0.68
$t_{\overline{R}}$	0.18	1.03	2.03	2.20	2.98	2.24	3.14	2.90	2.97	3.42	3.01

Data

Measuring capital, K_{it}, as net property, plant, and equipment; PPENT/PPEGT on average 0.56, with a standard deviation of 0.16, 5th percentile 0.3, 95th 0.83

Recursively substituting K_{is}:

$$
\begin{aligned}
K_{i1} &= (1-\delta_{i0})K_{i0} + I_{i0} \\
K_{i2} &= (1-\delta_{i1})K_{i1} + I_{i1} \\
\vdots & \qquad\qquad \vdots \\
K_{it} &= (1-\delta_{it-1})K_{it-1} + I_{it-1}
\end{aligned}
$$

yields:

$$
\overbrace{K_{it}}^{\textbf{Net PPE}} = \overbrace{\left(K_{i0} + \sum_{s=0}^{t-1} I_{is}\right)}^{\textbf{Gross PPE}} - \overbrace{\sum_{s=0}^{t-1} \delta_{is} K_{is}}^{\textbf{Accumulated depreciation}}
$$

Most prior studies measure I_{it} as CAPX minus sales of PPE

The differences between CAPX minus SPPE and $K_{it+1} - (1 - \delta_{it})K_{it}$ more than 10.28%, 31.5%, and 57.45% of K_{it} in magnitude for 25%, 10%, and 5% of firm-level observations

- δ_{it}: The amount of depreciation and amortization minus amortization, scaled by net PPE

Mergers and acquisitions, asset exchanges, restructuring charges, impairment loss, capital lease

Measure I_{it} as $K_{it+1} - (1 - \delta_{it})K_{it}$

Hayashi and Inoue (1991); Lewellen and Badrinath (1997)

Data

Measurement of other variables

Y_{it}: Sales

W_{it}: Current assets

B_{it+1}: Long-term debt plus short-term debt (zero if missing)

P_{it}: The market equity, from CRSP

τ_t: The statutory corporate income tax rate from the Commerce Cleaning House

r^B_{it}: Total interest and related expenses, scaled by total debt

Data

Timing alignment built on Liu and Zhang (2014)

Construct monthly fundamental returns from annual accounting variables to match with monthly stock returns

For each month t, take firm-level accounting variables from the fiscal year end closest to month t to measure time-t variables in the model, and to take accounting variables from the subsequent fiscal year end to measure time-$t+1$ variables

Compound the portfolio stock returns within a 12-month rolling window with month t in the middle of the window to match with the portfolio fundamental return for month t

Data

Descriptive statistics, firm-level variables in the fundamental returns, June 1967–December 2016

	Mean	σ	5%	25%	50%	75%	95%
I_{it}/K_{it}	0.36	0.44	−0.03	0.11	0.23	0.44	1.32
$\triangle W_{it}/W_{it}$	0.13	0.32	−0.30	−0.05	0.07	0.22	0.82
Y_{it+1}/K_{it+1}	9.05	11.59	0.45	2.38	5.24	10.17	35.52
Y_{it+1}/W_{it+1}	3.09	2.00	0.76	1.77	2.61	3.83	7.46
$Y_{it+1}/(K_{it+1}+W_{it+1})$	1.62	0.93	0.30	0.97	1.50	2.11	3.80
$K_{it+1}/(K_{it+1}+W_{it+1})$	0.38	0.25	0.07	0.18	0.32	0.55	0.88
w_{it}^B	0.26	0.22	0.00	0.07	0.22	0.42	0.68
δ_{it+1}	0.19	0.12	0.05	0.11	0.16	0.25	0.49
r_{it+1}^B	8.74	5.77	0.02	5.65	7.98	10.54	24.89

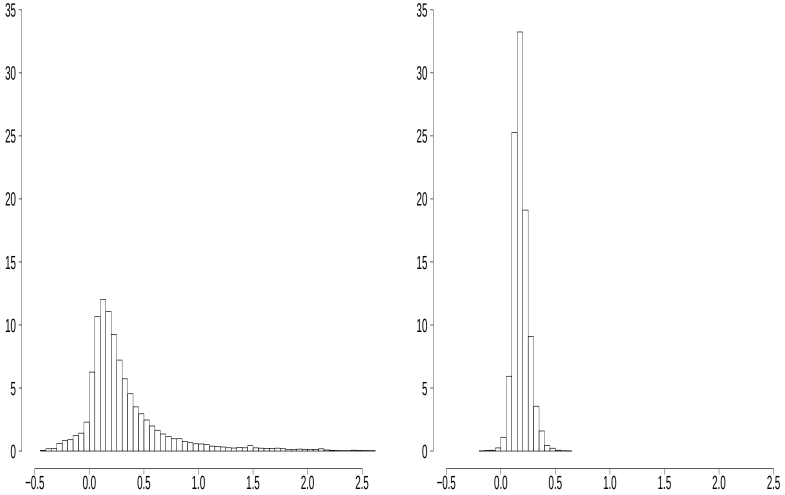
Data

Correlation matrix, firm-level variables in the fundamental returns, June 1967–December 2016

	$\frac{I_{it+1}}{K_{it+1}}$	$\frac{\triangle W_{it}}{W_{it}}$	$\frac{\triangle W_{it+1}}{W_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}}$	$\frac{Y_{it+1}}{W_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}+W_{it+1}}$	$\frac{K_{it+1}}{K_{it+1}+W_{it+1}}$	w_{it}^{B}	δ_{it+1}	r_{it+1}^{B}
I_{it}/K_{it}	0.32	0.30	0.10	0.15	−0.06	0.06	−0.18	−0.18	0.28	0.06
I_{it+1}/K_{it+1}		0.23	0.30	0.36	0.00	0.20	−0.28	−0.29	0.53	0.16
$\triangle W_{it}/W_{it}$			0.04	0.07	−0.04	0.01	−0.06	−0.08	0.05	0.03
$\triangle W_{it+1}/W_{it+1}$				0.09	0.25	0.20	0.08	−0.13	0.07	0.15
Y_{it+1}/K_{it+1}					0.07	0.56	−0.60	−0.18	0.52	0.03
Y_{it+1}/W_{it+1}						0.55	0.46	0.19	−0.19	0.09
$Y_{it+1}/(K_{it+1}+W_{it+1})$							−0.33	−0.08	0.24	0.13
$K_{it+1}/(K_{it+1}+W_{it+1})$								0.37	−0.59	0.00
w_{it}^{B}									−0.33	0.04
δ_{it+1}										0.06

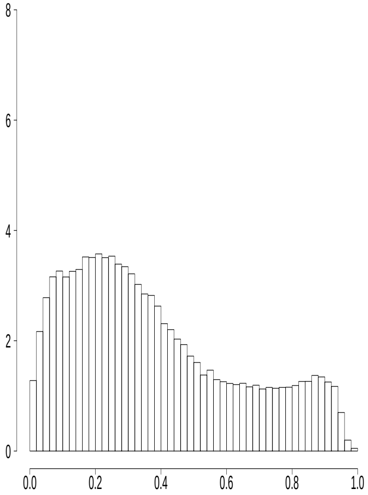
Data

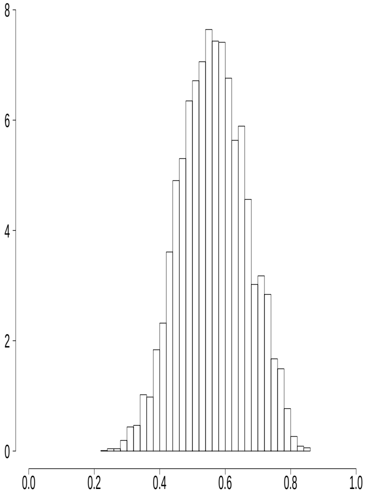
Histograms, firm versus portfolio I_{it}/K_{it}, 2.5%–97.5% winsorization



Data

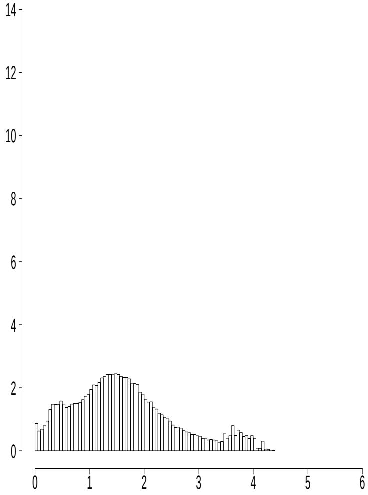
Histograms, firm versus portfolio $K_{it+1}/(K_{it+1} + W_{it+1})$, $[0, 1]$, no winsorization

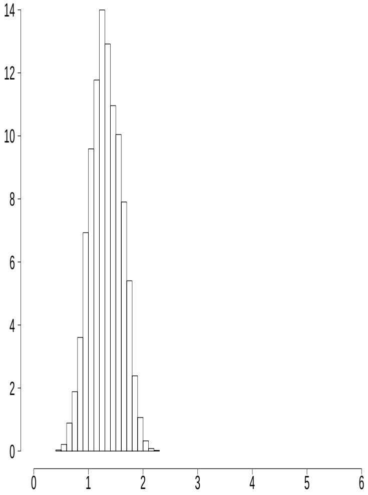




Data

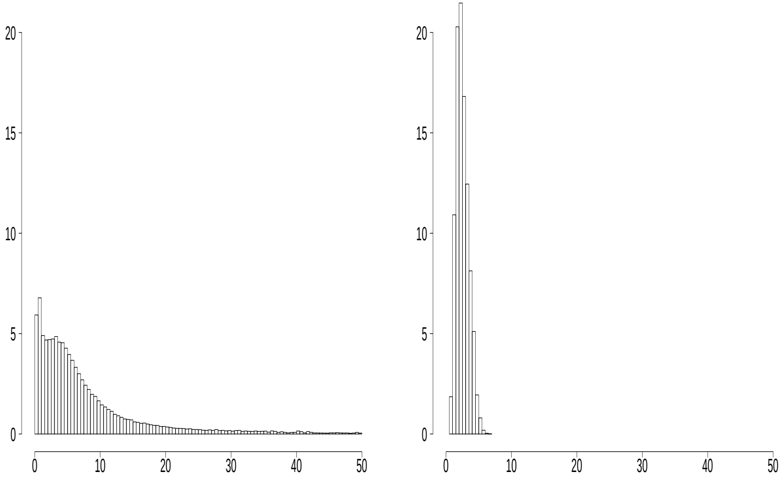
Histograms, firm versus portfolio $Y_{it+1}/(K_{it+1} + W_{it+1})$, 0–95% winsorization





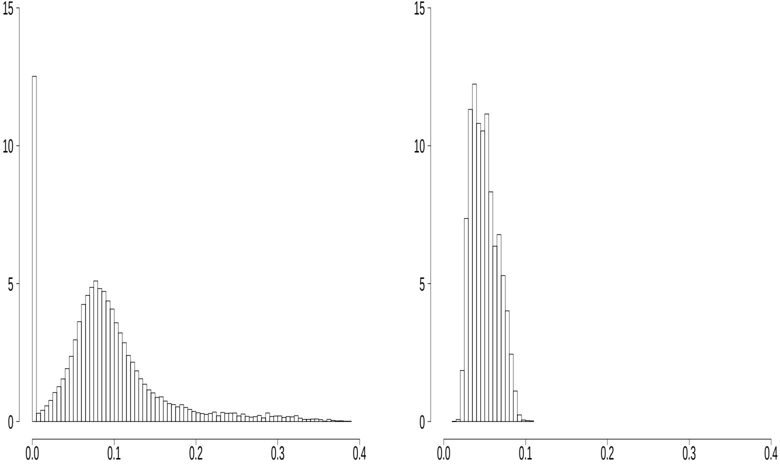
Data

Histograms, firm versus portfolio Y_{it+1}/K_{it+1}, 0–95% winsorization



Data

Histograms, firm versus portfolio r^B_{it+1}, 0–95% winsorization



Outline

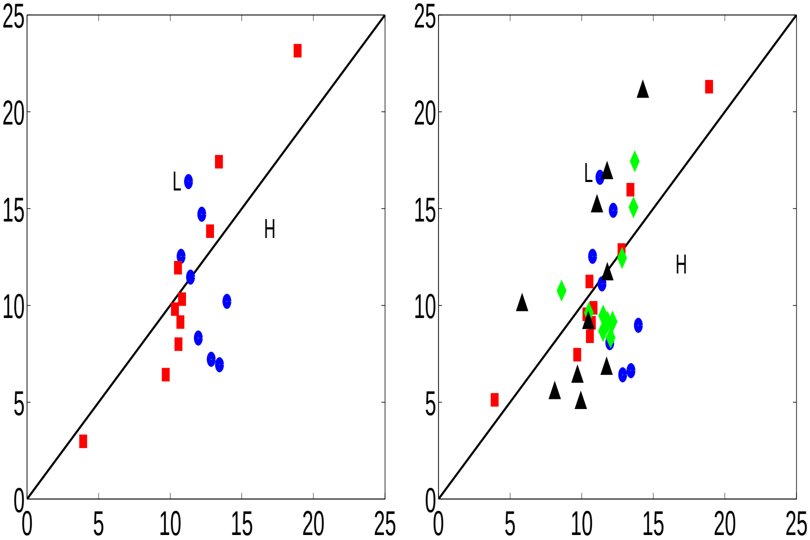
GMM

Replication: The 1-capital model estimated at the portfolio level

	d.f.	γ_K	$[\gamma_K]$	a	$[a]$	$\overline{\lvert\alpha\rvert}$	$\overline{\lvert\alpha_{\mathrm{H-L}}\rvert}$	p
Bm	8	16.56	2.40	6.27	1.94	2.52	0.32	0.01
R^{11}	8	12.00	1.14	1.28	0.56	1.34	1.46	8.37
I/A	8	12.20	1.06	1.06	0.40	2.04	0.54	0.00
Roe	8	10.32	0.97	0.00	0.07	3.35	0.21	0.00
Bm-R^{11}	18	13.44	1.21	2.54	0.52	2.90	7.02	0.00
I/A-Roe	18	11.43	0.99	0.71	0.34	2.86	1.64	0.00
Bm-R^{11}-I/A-Roe	38	12.51	1.08	1.74	0.34	2.96	4.12	0.00

GMM

Average predicted versus realized returns, the 1-capital model at the portfolio level; the value premium alpha, Bm-R^{11}, 8.85% ($t = 2.76$); Bm-R^{11}-I/A-Roe, 11.11% ($t = 3.89$)



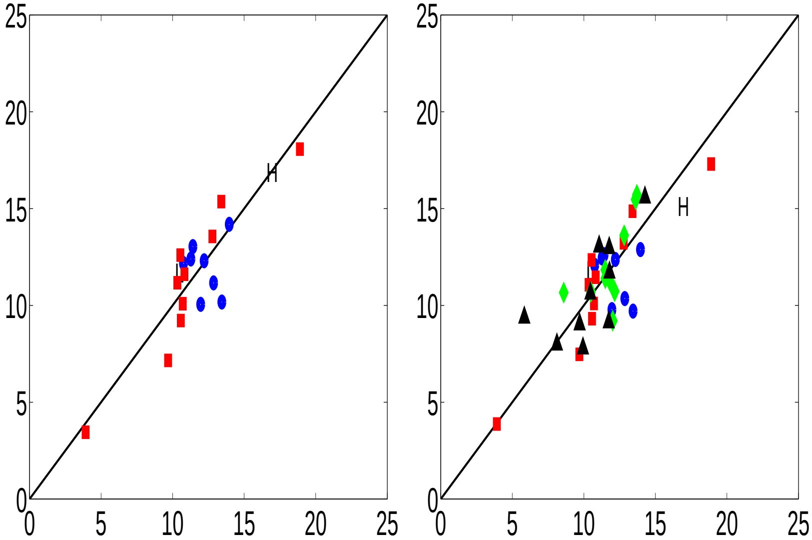
GMM

The 2-capital model estimated at the firm level

	d.f.	γ	$[\gamma]$	a	$[a]$	$\overline{\|\alpha\|}$	$\overline{\|\alpha_{\text{H–L}}\|}$	p
Bm	8	17.62	2.07	3.75	0.68	1.34	0.16	0.07
R^{11}	8	13.37	2.84	8.11	0.00	0.82	0.74	85.28
I/A	8	17.44	1.77	1.63	0.70	0.89	2.31	0.31
Roe	8	14.90	3.20	7.63	0.00	0.79	1.16	92.46
Bm-R^{11}	18	17.89	2.03	3.44	0.55	1.27	0.77	0.00
I/A-Roe	18	17.35	1.79	1.65	0.67	1.14	2.15	0.00
Bm-R^{11}-I/A-Roe	38	17.77	1.94	2.84	0.47	1.33	1.73	0.00

GMM

Average predicted versus realized returns, the 2-capital model at the firm level; the value premium alpha, Bm-R^{11}, 1.18% ($t = 0.51$); Bm-R^{11}-I/A-Roe, 3.09% ($t = 1.37$)



The physical capital investment return:

$$r^I_{it+1} = \frac{\begin{array}{c}(1-\tau_{t+1})\left[\gamma_K \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} \\ +(1-\delta_{it+1})\left[1+(1-\tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]\end{array}}{1+(1-\tau_t)a\left(\frac{I_{it}}{K_{it}}\right)}$$

A "tug of war" between current investment and expected investment

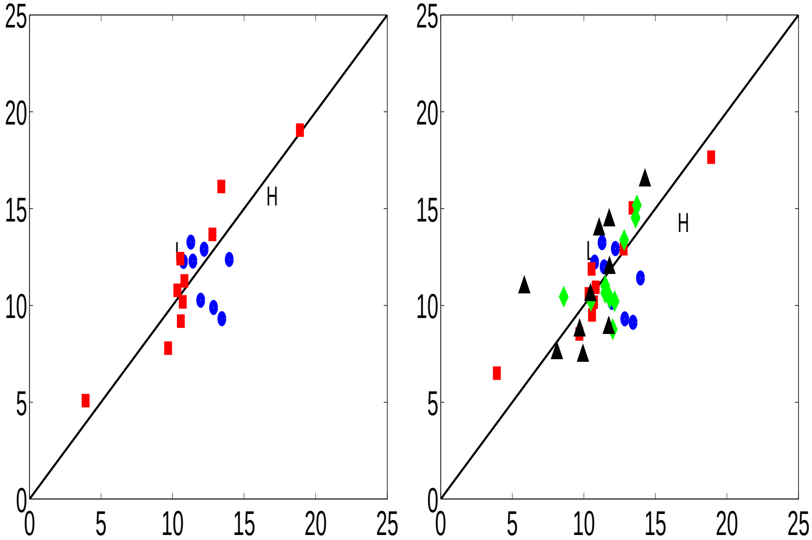
GMM

Comparative statics on the high-minus-low investment CAPM alphas

	Bm	R^{11}	I/A	Roe
Benchmark	3.09	1.55	−0.06	2.23
$\overline{I_{it}/K_{it}}$	36.28	−7.65	−21.75	−6.34
$\overline{I_{it+1}/K_{it+1}}$	−27.79	20.71	13.36	14.83
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−7.04	6.82	2.40	8.72

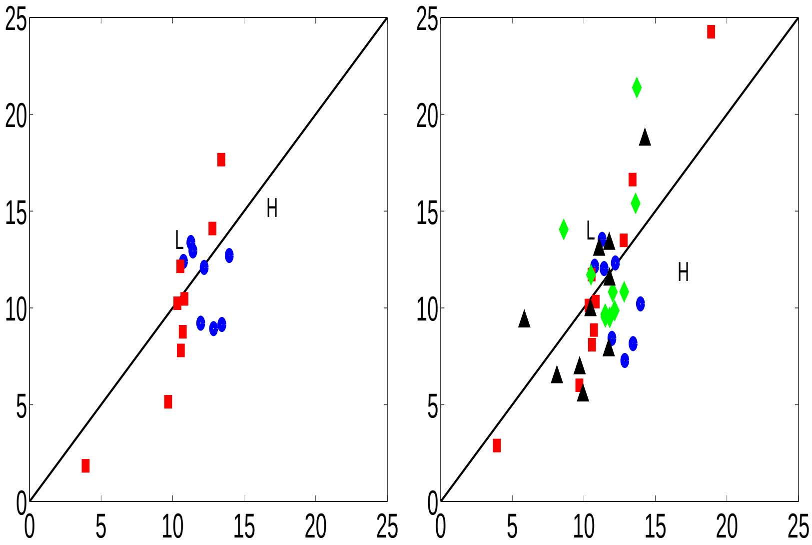
GMM

Aggregation, the 2-capital model at the portfolio level; the value premium alpha: Bm-R^{11}, 3.51% ($t = 1.23$); Bm-R^{11}-I/A-Roe, 4.94% ($t = 1.93$)



GMM

Capital heterogeneity, the physical capital model at the firm level; the value premium alpha: Bm-R^{11}, 4.75% ($t = 1.86$); Bm-R^{11}-I/A-Roe, 8.52% ($t = 3.41$)



Outline

Diagnostics

The stock-fundamental return correlations: The 12-month rolling window design helps resolve the Liu-Whited-Zhang (2009) correlation puzzle

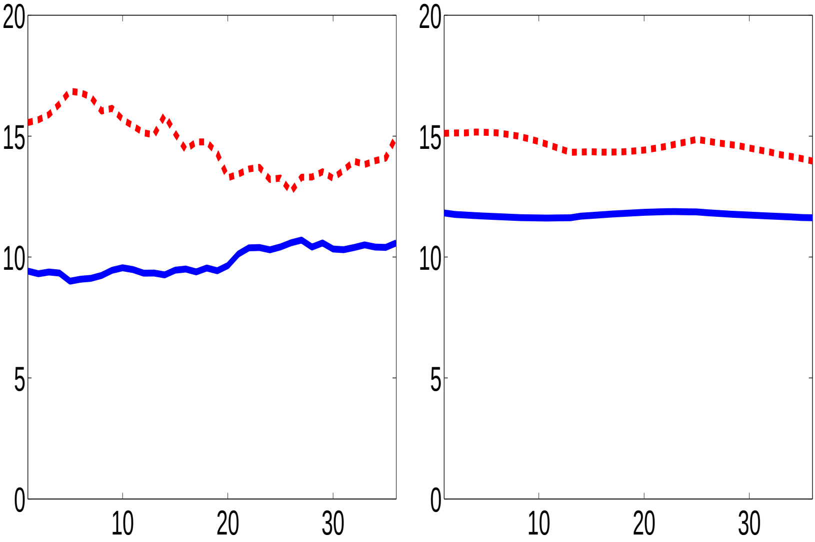
	Correlations of the stock returns with the fundamental returns, r_{it}^F										
	r_{it-60}^S	r_{it-36}^S	r_{it-24}^S	r_{it-12}^S	r_{it-3}^S	r_{it}^S	r_{it+3}^S	r_{it+12}^S	r_{it+24}^S	r_{it+36}^S	r_{it+60}^S
Firms	-0.02^c	-0.03^c	-0.03^c	0.02^c	0.10^c	0.11^c	0.12^c	0.05^c	0.00	0.01	-0.01^a
Port	0.05^a	0.09^c	0.05^a	0.09^c	0.17^c	0.19^c	0.20^c	0.12^c	0.08^c	0.12^c	0.11^c

	Correlations between the stock and fundamental returns across the testing deciles										
	L	2	3	4	5	6	7	8	9	H	H−L
Bm	0.13	0.19	0.12	0.04	0.13^b	0.20^a	0.00	0.00	0.05	0.15	0.26^c
R^{11}	0.20^b	0.09	0.06	−0.05	−0.03	0.04	0.01	0.08	0.10	0.22^c	0.14^a
I/A	0.19^b	0.11	0.10	−0.03	0.12	−0.02	0.02	−0.02	0.11	0.30^c	0.42^c
Roe	0.19	0.18	0.11	0.14^a	−0.02	0.01	0.09	0.01	−0.02	0.09	0.16

^{a}, ^{b}, ^{c}: Significant at the 10%, 5%, and 1% levels, respectively

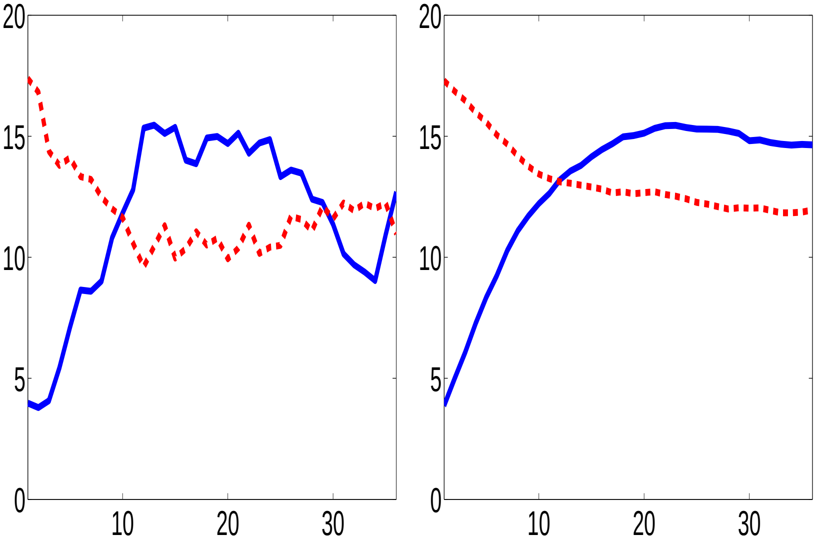
Diagnostics

The value premium, stock and fundamental returns in event-time



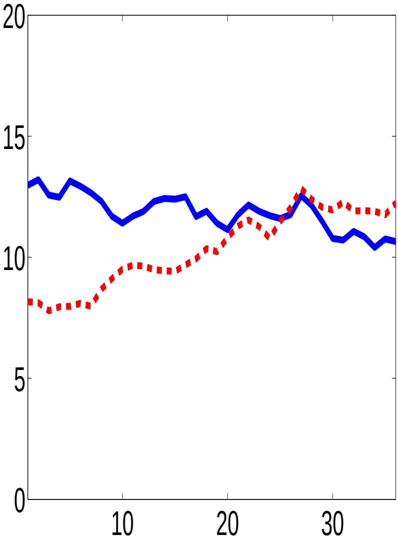
Diagnostics

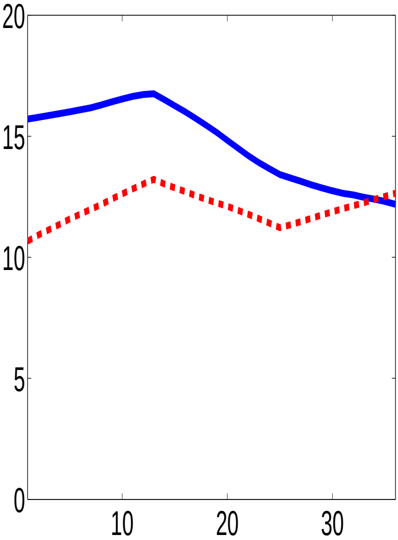
The momentum premium, stock and fundamental returns in event-time



Diagnostics

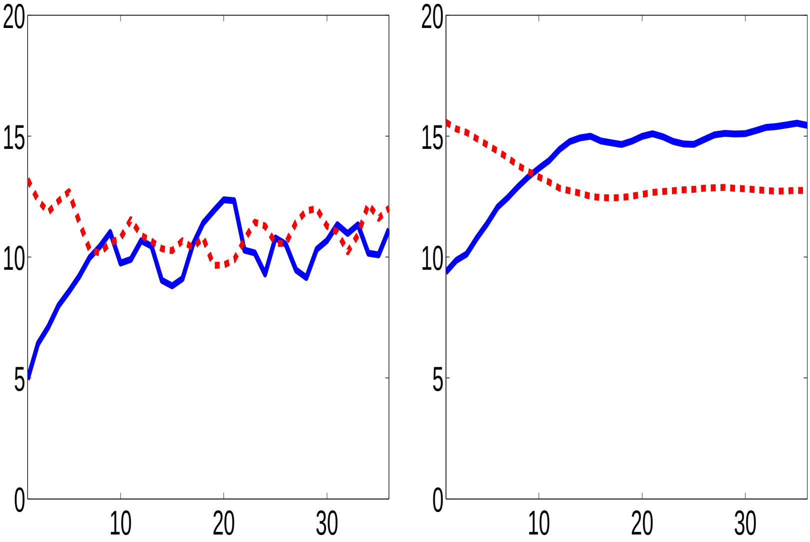
The investment premium, stock and fundamental returns in event-time





Diagnostics

The Roe premium, stock and fundamental returns in event-time



Diagnostics

Higher moments: Lower volatility, weakly positive (not negative) skewness for momentum

Bm		L	2	3	4	5	6	7	8	9	H	H−L
σ	r^S	0.20	0.18	0.18	0.19	0.17	0.16	0.17	0.17	0.17	0.22	0.20^c
	r^F	0.05	0.06	0.06	0.07	0.08	0.10	0.07	0.11	0.13	0.18	0.18^c
S_k	r^S	−0.24	0.03	−0.08	−0.04	−0.16	−0.07	−0.20	−0.48	−0.14	0.12	0.42
	r^F	−0.96	−1.26	1.05	0.57	0.81	−1.57	0.67	1.27	0.73	0.63	0.36
K_u	r^S	3.04	3.12	2.75	3.43	3.20	3.57	3.52	4.36	3.94	4.47	3.28
	r^F	3.97	6.24	8.33	5.36	4.81	8.13	2.95	6.62	4.29	4.64	4.03
R^{11}		L	2	3	4	5	6	7	8	9	H	H−L
σ	r^S	0.30	0.24	0.20	0.18	0.16	0.17	0.16	0.18	0.19	0.26	0.28^c
	r^F	0.12	0.08	0.08	0.07	0.07	0.06	0.07	0.07	0.07	0.07	0.13^c
S_k	r^S	1.47	0.94	0.19	0.42	−0.10	−0.14	−0.23	−0.16	−0.11	−0.03	-1.78^a
	r^F	−0.56	−0.03	0.33	0.38	0.57	0.69	1.01	0.62	0.13	−0.41	0.30^a
K_u	r^S	9.92	8.05	3.91	4.07	3.70	3.58	3.02	3.07	3.57	3.19	11.59^c
	r^F	6.58	4.10	6.00	4.81	5.51	5.24	6.73	5.06	4.07	3.91	5.29^b

Diagnostics

Higher moments

I/A		L	2	3	4	5	6	7	8	9	H	H−L
σ	r^S	0.22	0.18	0.16	0.15	0.16	0.16	0.17	0.17	0.21	0.23	0.14[c]
	r^F	0.09	0.07	0.08	0.07	0.06	0.07	0.06	0.05	0.07	0.08	0.11[c]
S_k	r^S	0.36	−0.01	−0.01	−0.16	−0.25	−0.18	−0.20	−0.15	−0.30	−0.22	0.13
	r^F	0.22	0.88	0.41	1.00	0.40	0.03	−0.27	0.43	−0.29	−0.60	0.08
K_u	r^S	4.13	3.67	3.18	3.48	3.55	3.19	3.22	3.07	3.33	3.15	3.44
	r^F	2.71	4.60	2.95	5.17	3.01	3.43	4.48	4.15	3.59	5.03	3.18
Roe		L	2	3	4	5	6	7	8	9	H	H−L
σ	r^S	0.27	0.22	0.19	0.16	0.17	0.18	0.16	0.17	0.17	0.20	0.20[c]
	r^F	0.14	0.12	0.09	0.08	0.08	0.07	0.07	0.05	0.05	0.05	0.14[c]
S_k	r^S	0.20	0.23	−0.03	−0.02	−0.25	−0.38	−0.39	−0.14	−0.20	−0.06	−0.84[a]
	r^F	0.46	0.38	0.58	0.38	0.50	1.31	0.07	−0.38	−0.15	−0.09	−0.38
K_u	r^S	3.69	3.94	4.13	3.36	3.12	3.66	3.14	2.90	3.35	2.70	5.75[c]
	r^F	4.99	5.45	6.73	4.53	4.85	6.56	4.19	3.88	2.98	3.08	4.45[c]

Conclusion

A detailed treatment of aggregation and capital heterogeneity substantially improves the performance of the investment CAPM