Aggregation, Capital Heterogeneity, and the Investment CAPM

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A detailed treatment of aggregation and capital heterogeneity substantially improves the performance of the investment CAPM

Markowitz (1952)

Treynor (1962), Sharpe (1964), Lintner (1965), Mossin (1966)

Merton (1973), Ross (1976)

Rubinstein (1976), Lucas (1978), Breeden (1979)

Hansen and Singleton (1982, 1983), Breeden, Gibbons, and Litzenberger (1989)

Cochrane (2005), Back (2010), Campbell (2017)

Berk and DeMarzo (2013), Bodie, Kane, and Marcus (2014)

Böhm-Bawert (1891)

Fisher (1930), Hirshleifer (1958, 1965, 1970)

Modigliani and Miller (1958)

Cochrane (1991)

Zhang (2005, 2017)

Liu, Whited, and Zhang (2009), Liu and Zhang (2014)

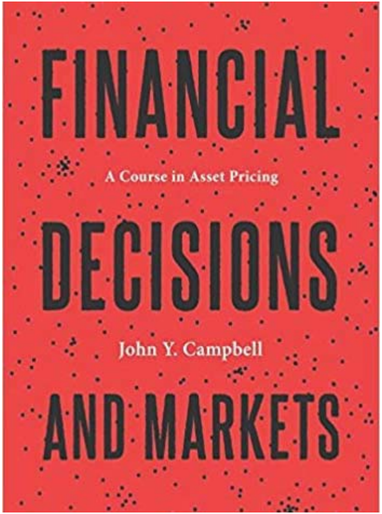
Hou, Xue, and Zhang (2015), Hou, Mo, Xue, and Zhang (2018)

Ricardo and Mill: Costs of production determine value
Jevons, Menger, and Walras: Marginal utility determines value

- The water versus diamond debate

"We might as reasonably dispute whether it is the upper or under blade of a pair of scissors that cuts a piece of paper, as whether value is governed by utility or costs of production. It is true that when one blade is held still, and the cutting is affected by moving the other, we may say with careless brevity that the cutting is done by the second; but the statement is not strictly accurate, and is to be excused only so long as it claims to be merely a popular and not a strictly scientific account of what happens (our emphasis)."

Campbell (2017): An entire chapter on investment-based asset pricing



An empirical challenge facing the structural investment model:

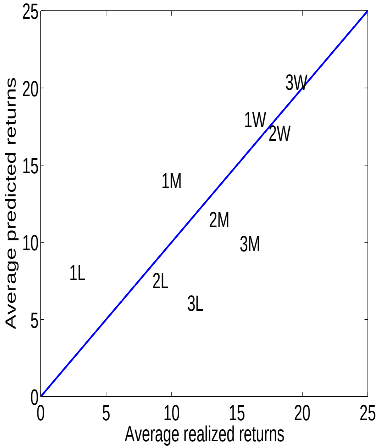
"This problem, that different parameters are needed to fit each anomaly, is a pervasive one in the *q*-theoretic asset pricing literature (p. 275)."

Liu, Whited, and Zhang (2009):

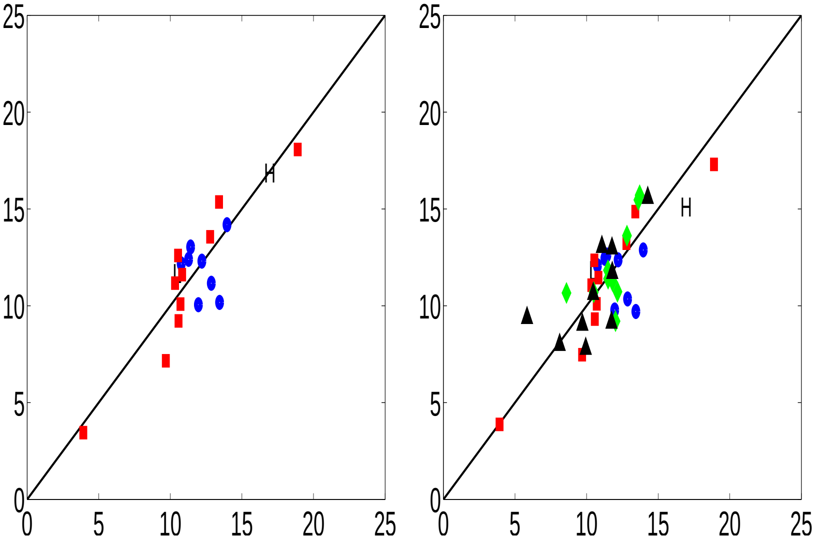
TABLE 2
PARAMETER ESTIMATES AND TESTS OF OVERIDENTIFICATION

	SUE	B/M	CI
	A. Matching Expected Returns		
a	7.7	22.3	1.0
	[1.7]	[25.5]	[.3]
α	.3	.5	.2
	[.0]	[.3]	[.0]
χ^2	4.4	6.0	6.5
d.f.	8	8	8
p	.8	.7	.6
m.a.e.	.7	2.3	1.5

Liu and Zhang (2014):



Average predicted versus realized stock returns, value, momentum, I/A, and Roe



Outline

1. The Model of the Firms
2. Econometric Methods
3. Data
4. GMM Estimation and Tests
5. Diagnostics: Dynamics of Factor Premiums
6. Out-of-sample Tests

Operating profits: $\Pi(K_{it}, W_{it}, X_{it})$

- K_{it}: Physical capital; W_{it}: Working capital
- X_{it}: A vector of exogenous shocks
- Constant returns to scale, Cobb-Douglas

Capital accumulation:

$$\begin{aligned} K_{it+1} &= I_{it} + (1 - \delta_{it})K_{it} \\ W_{it+1} &= \triangle W_{it} + W_{it} \end{aligned}$$

Adjustment costs on physical (not working) capital:

$$\Phi(I_{it}, K_{it}) = \frac{a}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it}$$

Optimal physical capital investment: $E_t[M_{t+1} r^K_{it+1}] = 1$, in which the physical capital investment return:

$$r^K_{it+1} = \frac{(1-\tau_{t+1})\left[\gamma_K \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1})\left[1 + (1-\tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]}{1 + (1-\tau_t)a\left(\frac{I_{it}}{K_{it}}\right)}$$

Optimal working capital investment: $E_t[M_{t+1} r^W_{it+1}] = 1$, in which the working capital investment return:

$$r^W_{it+1} = 1 + (1-\tau_{t+1})\gamma_W \frac{Y_{it+1}}{W_{it+1}}$$

The weighted average of the investment returns equals the weighted average of the cost of equity and after-tax cost of debt:

$$w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W = w_{it}^B \, r_{it+1}^{Ba} + (1 - w_{it}^B) \, r_{it+1}^S$$

$w_{it}^K = q_{it} K_{it+1} / (q_{it} K_{it+1} + W_{it+1})$ and $w_{it}^B = B_{it+1} / (P_{it} + B_{it+1})$

- Modigliani and Miller (1958, Proposition III)

The investment CAPM:

$$r_{it+1}^S = \underbrace{\frac{w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W - w_{it}^B r_{it+1}^{Ba}}{1 - w_{it}^B}}_{\textbf{The fundamental return}, \; r_{it+1}^F}$$

Test the expected return implications of the investment CAPM:

$$E[r^S_{pt+1} - r^F_{pt+1}] = 0,$$

r^S_{pt+1}: Portfolio p's stock return, r^F_{pt+1}: The fundamental return

- The investment CAPM alpha: $\alpha_p = E_T[r^S_{pt+1} - r^F_{pt+1}]$, with $E_T[\cdot]$ the sample mean

The supply counterpart of Hansen and Singleton (1982)

γ_K and γ_W enter the moment condition only in the form of $\gamma = \gamma_K + \gamma_W$:

$$w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W = \frac{(1-\tau_{t+1})(\gamma_K + \gamma_W) Y_{it+1}/(K_{it+1} + W_{it+1})}{q_{it} K_{it+1}/(K_{it+1} + W_{it+1}) + W_{it+1}/(K_{it+1} + W_{it+1})} + w_{it}^K \frac{(1-\tau_{t+1})(a/2)\left(I_{it+1}/K_{it+1}\right)^2 + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1}) q_{it+1}}{q_{it}} + (1 - w_{it}^K)$$

The 2-capital model as parsimonious as the physical capital model

Let $\mathbf{c} \equiv (\gamma, a)$, $\mathbf{g}_T$ the sample moments, $\mathbf{D} = \partial \mathbf{g}_T / \partial \mathbf{c}$

The GMM objective function: $\mathbf{g}_T' \mathbf{W} \mathbf{g}_T$, in which $\mathbf{W} = \mathbf{I}$

$\text{Var}(\hat{\mathbf{c}}) = (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}/T$

$\text{Var}(\mathbf{g}_T) = \left[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\right] \mathbf{S} \left[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\right]' / T$

The overidentification test:

$$\mathbf{g}_T' \left[\text{var}(\mathbf{g}_T)\right]^{+} \mathbf{g}_T \sim \chi^2(\#\,\text{moments} - \#\,\text{parameters})$$

Portfolio-level fundamental returns are constructed from portfolio-level accounting variables aggregated from the firm level:

$$E\left[\begin{array}{c} \sum_{i=1}^{N_{pt}} w_{ipt} r^S_{ipt+1} - \\ r^F_{pt+1}\left(\gamma_K, a; Y_{pt+1}, K_{pt+1}, I_{pt+1}, \delta_{pt+1}, I_{pt}, K_{pt}, r^{Ba}_{pt+1}, w^B_{pt}\right) \end{array}\right] = 0$$

- N_{pt}: The number of firms in portfolio p at the start of t, w_{ipt}: Stock i's weight in portfolio p, r^S_{ipt+1}: The return of stock i in p over time t, r^F_{pt+1}: The fundamental return of p

Aggregating firm-level characteristics to the portfolio level:
$I_{pt+1} = \sum_{i=1}^{N_{pt}} I_{ipt+1}$, $w^B_{pt} = \sum_{i=1}^{N_{pt}} B_{ipt+1} / \sum_{i=1}^{N_{pt}} (P_{ipt} + B_{ipt+1})$, etc

Construct firm-level fundamental returns from firm-level accounting variables, then aggregate to portfolio-level fundamental returns:

$$E\left[\begin{array}{c}\sum_{i=1}^{N_{pt}} w_{ipt} r_{ipt+1}^{S} - \sum_{i=1}^{N_{pt}} w_{ipt} \times \\ r_{ipt+1}^{F}\left(\gamma, a; Y_{ipt+1}, K_{ipt+1}, I_{ipt+1}, \delta_{ipt+1}, I_{ipt}, K_{ipt}, r_{ipt+1}^{Ba}, w_{ipt}^{B}\right)\end{array}\right] = 0$$

- r_{ipt+1}^{F}: Firm i's fundamental return, r_{pt+1}^{F} varies with w_{ipt}

Why?

- Economics: Firms can make different investment choices
- Econometrics: The substantial firm-level heterogeneity helps identify structural parameters

Outline

40 testing deciles formed on:

- Book-to-market: Bm
- Momentum (prior 11-month returns, 1-month horizon): R^{11}
- Asset growth: I/A
- Return on equity: Roe

NYSE breakpoints and value-weighted returns

Average returns of the 40 testing deciles, January 1967–June 2017

	L	2	3	4	5	6	7	8	9	H	H−L
					The Bm deciles						
$\overline{R}$	0.43	0.53	0.60	0.46	0.53	0.56	0.67	0.63	0.73	0.90	0.47
$t_{\overline{R}}$	1.85	2.74	3.16	2.26	2.89	3.19	3.65	3.40	4.07	3.93	2.15
					The R^{11} deciles						
$\overline{R}$	−0.03	0.40	0.47	0.48	0.45	0.48	0.46	0.63	0.68	1.08	1.12
$t_{\overline{R}}$	−0.10	1.53	2.16	2.47	2.43	2.54	2.63	3.25	3.25	3.98	3.88
					The I/A deciles						
$\overline{R}$	0.69	0.68	0.63	0.52	0.53	0.56	0.59	0.48	0.58	0.33	−0.36
$t_{\overline{R}}$	2.98	3.42	3.84	3.19	3.09	3.13	3.24	2.49	2.42	1.27	−2.20
					The Roe deciles						
$\overline{R}$	0.06	0.25	0.42	0.40	0.54	0.44	0.57	0.53	0.57	0.74	0.68
$t_{\overline{R}}$	0.18	1.03	2.03	2.20	2.98	2.24	3.14	2.90	2.97	3.42	3.01

Y_{it}: Sales

K_{it}: Net property, plant, and equipment

W_{it}: Current assets

B_{it+1}: Long-term debt plus short-term debt (zero if missing)

P_{it}: Market equity, from CRSP

τ_t: The statutory corporate income tax rate from the Commerce Cleaning House

δ_{it}: The amount of depreciation and amortization minus amortization, scaled by net PPE

I_{it}: $K_{it+1} - (1 - \delta_{it})K_{it}$

r^B_{it}: Total interest and related expenses, scaled by total debt

Construct monthly fundamental returns from annual accounting variables to match with monthly stock returns

For each month t, take firm-level accounting variables from the fiscal year end closest to month t to measure time-t variables in the model, and to take accounting variables from the subsequent fiscal year end to measure time-$t+1$ variables

Compound the portfolio stock returns within a 12-month rolling window with month t in the middle of the window to match with the portfolio fundamental return for month t

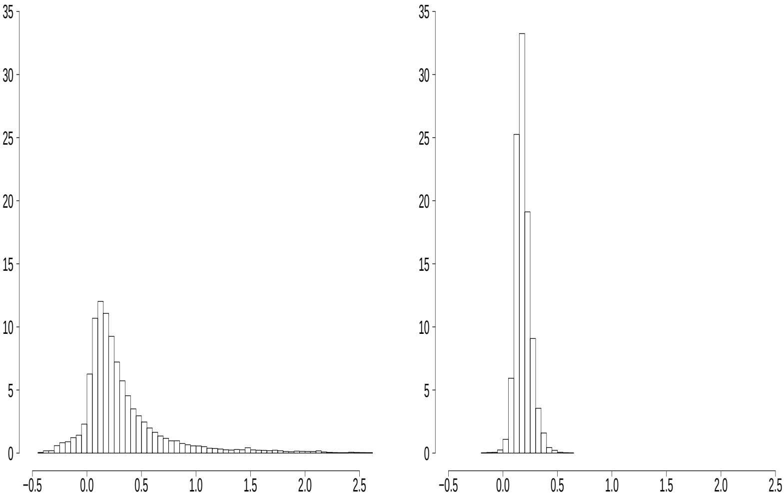
Descriptive statistics, firm-level variables in the fundamental returns, June 1967–December 2016

	Mean	σ	5%	25%	50%	75%	95%
I_{it}/K_{it}	0.36	0.44	−0.03	0.11	0.23	0.44	1.32
$\triangle W_{it}/W_{it}$	0.13	0.32	−0.30	−0.05	0.07	0.22	0.82
Y_{it+1}/K_{it+1}	9.05	11.59	0.45	2.38	5.24	10.17	35.52
Y_{it+1}/W_{it+1}	3.09	2.00	0.76	1.77	2.61	3.83	7.46
$Y_{it+1}/(K_{it+1} + W_{it+1})$	1.62	0.93	0.30	0.97	1.50	2.11	3.80
$K_{it+1}/(K_{it+1} + W_{it+1})$	0.38	0.25	0.07	0.18	0.32	0.55	0.88
w_{it}^B	0.26	0.22	0.00	0.07	0.22	0.42	0.68
δ_{it+1}	0.19	0.12	0.05	0.11	0.16	0.25	0.49
r_{it+1}^B	8.74	5.77	0.02	5.65	7.98	10.54	24.89

Data

Correlation matrix, firm-level variables in the fundamental returns, June 1967–December 2016

	$\frac{I_{it+1}}{K_{it+1}}$	$\frac{\triangle W_{it}}{W_{it}}$	$\frac{\triangle W_{it+1}}{W_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}}$	$\frac{Y_{it+1}}{W_{it+1}}$	$\frac{Y_{it+1}}{K_{it+1}+W_{it+1}}$	$\frac{K_{it+1}}{K_{it+1}+W_{it+1}}$	w^B_{it}	δ_{it+1}	r^B_{it+1}
I_{it}/K_{it}	0.32	0.30	0.10	0.15	−0.06	0.06	−0.18	−0.18	0.28	0.06
I_{it+1}/K_{it+1}		0.23	0.30	0.36	0.00	0.20	−0.28	−0.29	0.53	0.16
$\triangle W_{it}/W_{it}$			0.04	0.07	−0.04	0.01	−0.06	−0.08	0.05	0.03
$\triangle W_{it+1}/W_{it+1}$				0.09	0.25	0.20	0.08	−0.13	0.07	0.15
Y_{it+1}/K_{it+1}					0.07	0.56	−0.60	−0.18	0.52	0.03
Y_{it+1}/W_{it+1}						0.55	0.46	0.19	−0.19	0.09
$Y_{it+1}/(K_{it+1}+W_{it+1})$							−0.33	−0.08	0.24	0.13
$K_{it+1}/(K_{it+1}+W_{it+1})$								0.37	−0.59	0.00
w^B_{it}									−0.33	0.04
δ_{it+1}										0.06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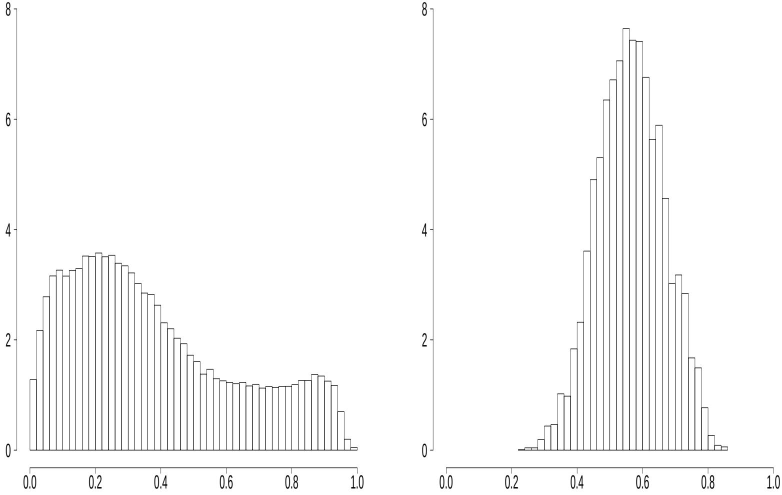
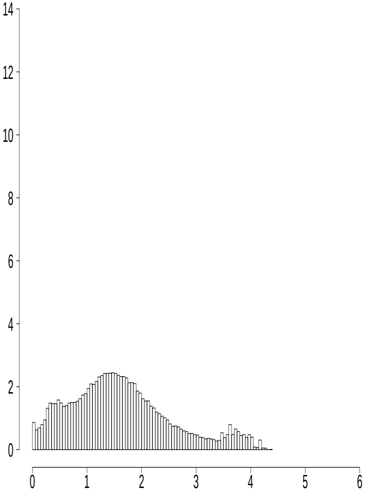
Data

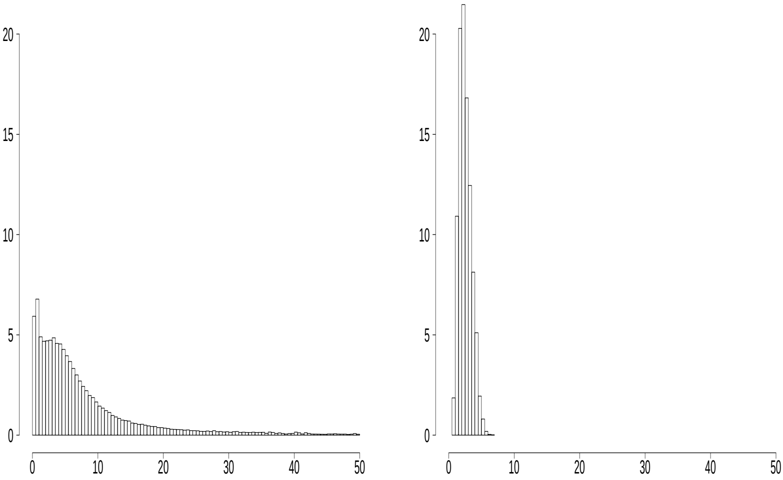
Histograms, firm versus portfolio $K_{it+1}/(K_{it+1}+W_{it+1})$



Data

Histograms, firm versus portfolio $Y_{it+1}/(K_{it+1}+W_{it+1})$

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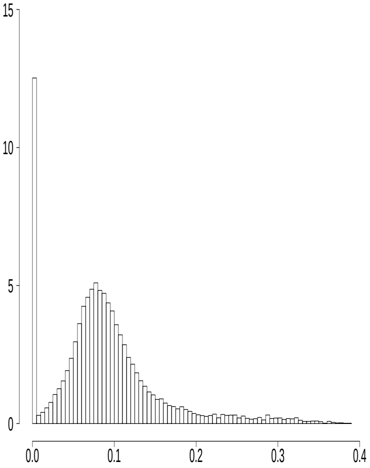
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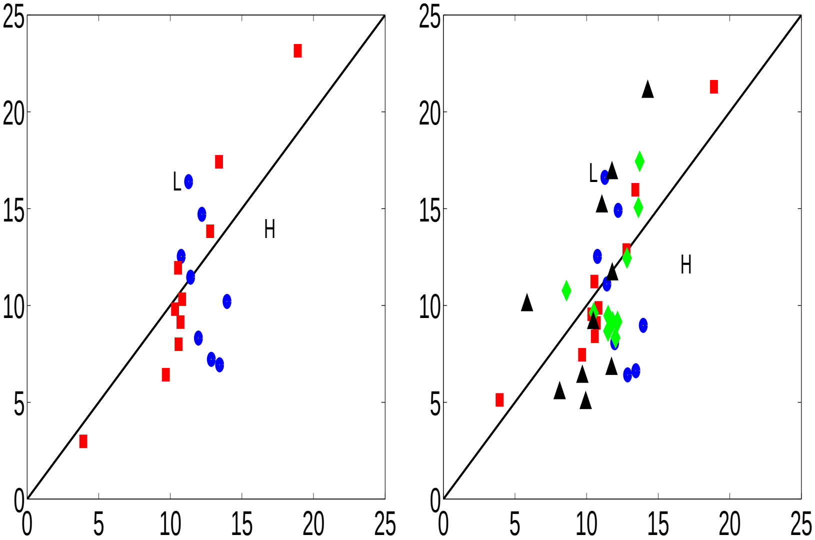
	d.f.	γ_K	$[\gamma_K]$	a	$[a]$	$\overline{\vert\alpha\vert}$	$\overline{\vert\alpha_{\text{H--L}}\vert}$	p
Bm	8	16.56	2.40	6.27	1.94	2.52	0.32	0.01
R^{11}	8	12.00	1.14	1.28	0.56	1.34	1.46	8.37
I/A	8	12.20	1.06	1.06	0.40	2.04	0.54	0.00
Roe	8	10.32	0.97	0.00	0.07	3.35	0.21	0.00
Bm-R^{11}	18	13.44	1.21	2.54	0.52	2.90	7.02	0.00
I/A-Roe	18	11.43	0.99	0.71	0.34	2.86	1.64	0.00
Bm-R^{11}-I/A-Roe	38	12.51	1.08	1.74	0.34	2.96	4.12	0.00

GMM Estimation and Tests

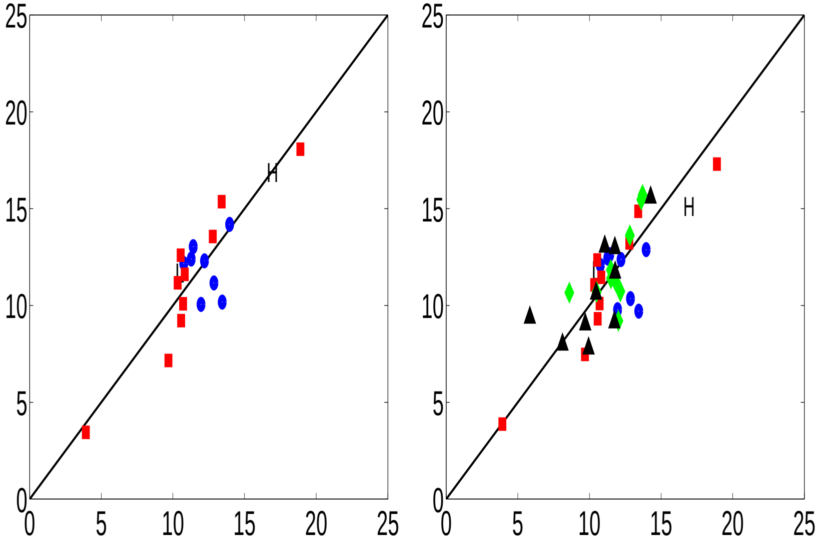
Average predicted versus realized stock returns, Bm-R^{11}, Bm-R^{11}-I/A-Roe, the physical capital model at the portfolio level



	d.f.	γ	$[\gamma]$	a	$[a]$	$\overline{\|\alpha\|}$	$\overline{\|\alpha_{H-L}\|}$	p
Bm	8	17.62	2.07	3.75	0.68	1.34	0.16	0.07
R^{11}	8	13.37	2.84	8.11	0.00	0.82	0.74	85.28
I/A	8	17.44	1.77	1.63	0.70	0.89	2.31	0.31
Roe	8	14.90	3.20	7.63	0.00	0.79	1.16	92.46
Bm-R^{11}	18	17.89	2.03	3.44	0.55	1.27	0.77	0.00
I/A-Roe	18	17.35	1.79	1.65	0.67	1.14	2.15	0.00
Bm-R^{11}-I/A-Roe	38	17.77	1.94	2.84	0.47	1.33	1.73	0.00

GMM Estimation and Tests

Average predicted versus realized stock returns, Bm-R^{11}, Bm-R^{11}-I/A-Roe, the 2-capital model at the firm level

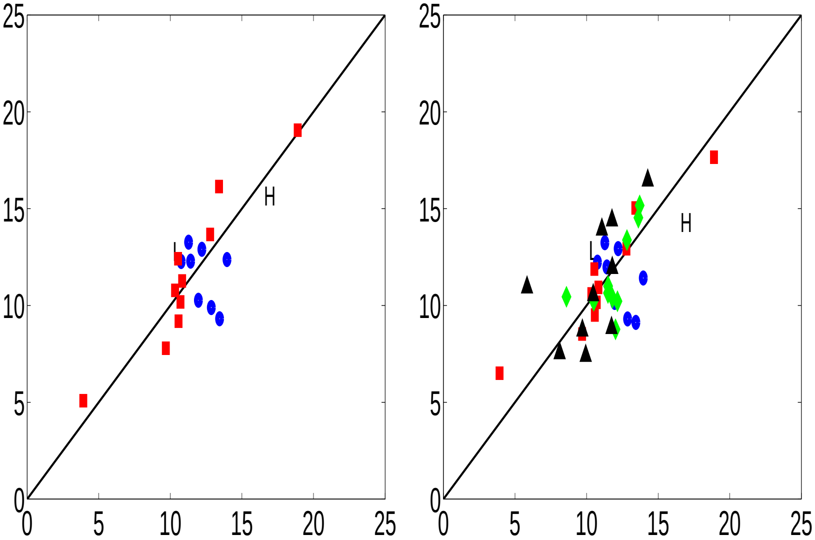


The physical capital investment return:

$$r^I_{it+1} = \frac{(1-\tau_{t+1})\left[\gamma_K \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} + (1-\delta_{it+1})\left[1+(1-\tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]}{1+(1-\tau_t)a\left(\frac{I_{it}}{K_{it}}\right)}$$

The “tug of war” between current investment-to-physical capital and expected investment-to-physical capital

	Bm	R^{11}	I/A	Roe
Benchmark	3.09	1.55	−0.06	2.23
$\overline{I_{it}/K_{it}}$	36.28	−7.65	−21.75	−6.34
$\overline{I_{it+1}/K_{it+1}}$	−27.79	20.71	13.36	14.83
$\overline{Y_{it+1}/(K_{it+1}+W_{it+1})}$	−7.04	6.82	2.40	8.72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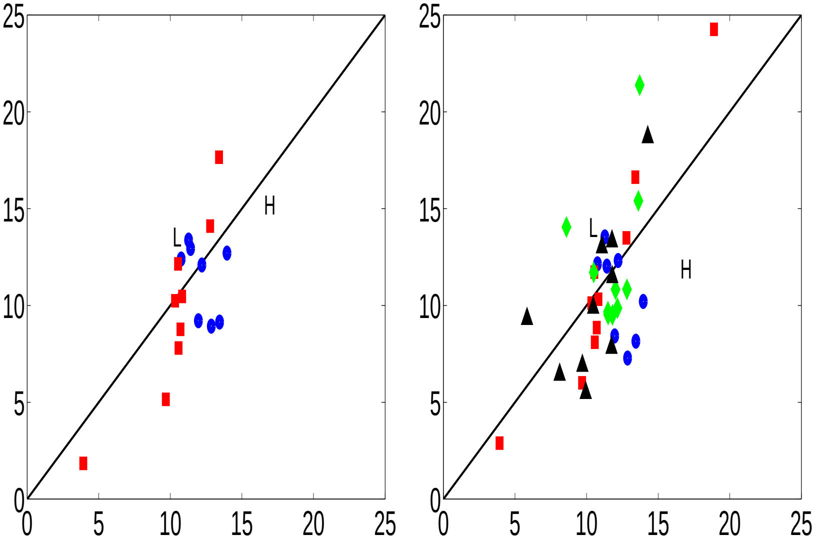

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Outline

Correlations between the stock returns of various leads and lags and the contemporaneous fundamental return, r_{it}^F

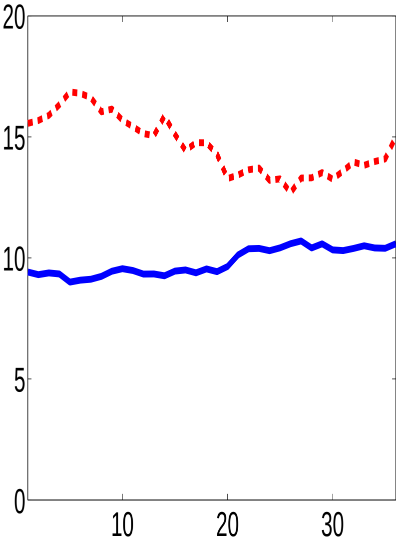
	Correlations of the stock returns with the fundamental returns, r_{it}^F										
	r_{it-60}^S	r_{it-36}^S	r_{it-24}^S	r_{it-12}^S	r_{it-3}^S	r_{it}^S	r_{it+3}^S	r_{it+12}^S	r_{it+24}^S	r_{it+36}^S	r_{it+60}^S
Firms	-0.02^c	-0.03^c	-0.03^c	0.02^c	0.10^c	0.11^c	0.12^c	0.05^c	0.00	0.01	-0.01^a
Port	0.05^a	0.09^c	0.05^a	0.09^c	0.17^c	0.19^c	0.20^c	0.12^c	0.08^c	0.12^c	0.11^c

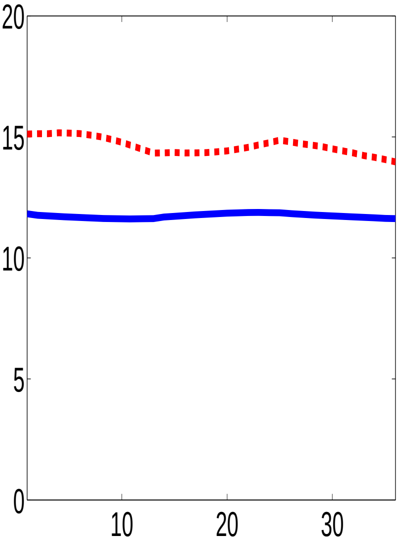
	Correlations between the stock and fundamental returns across the testing deciles										
	L	2	3	4	5	6	7	8	9	H	H−L
Bm	0.13	0.19	0.12	0.04	0.13^b	0.20^a	0.00	0.00	0.05	0.15	0.26^c
R^{11}	0.20^b	0.09	0.06	−0.05	−0.03	0.04	0.01	0.08	0.10	0.22^c	0.14^a
I/A	0.19^b	0.11	0.10	−0.03	0.12	−0.02	0.02	−0.02	0.11	0.30^c	0.42^c
Roe	0.19	0.18	0.11	0.14^a	−0.02	0.01	0.09	0.01	−0.02	0.09	0.16

^{a}, ^{b}, ^{c}: Significant at the 10%, 5%, and 1% levels, respectively

Diagnostics

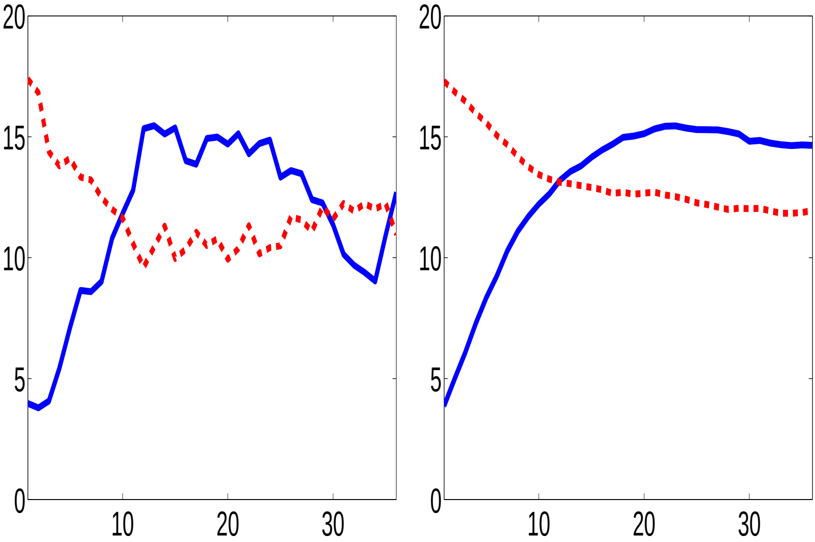
The long-term dynamics of the value premium, the stock and fundamental returns in event-time





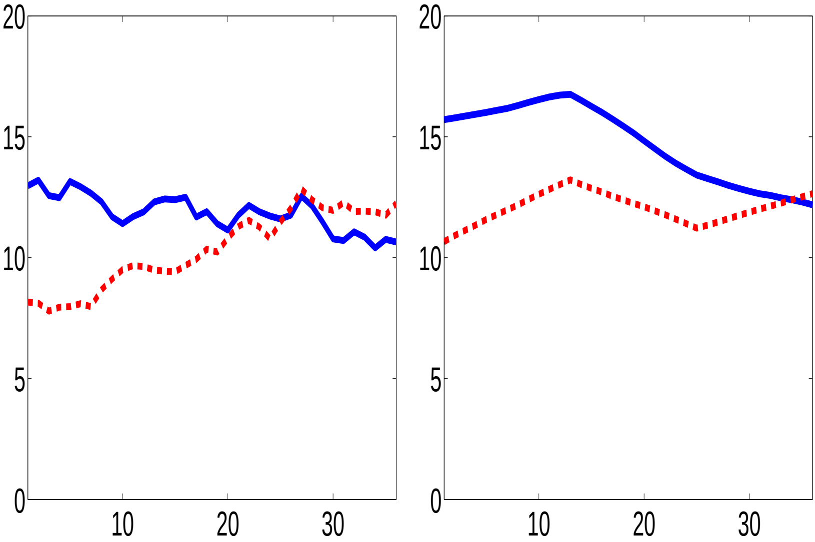
Diagnostics

The short-term dynamics of the momentum premium, the stock and fundamental returns in event-time



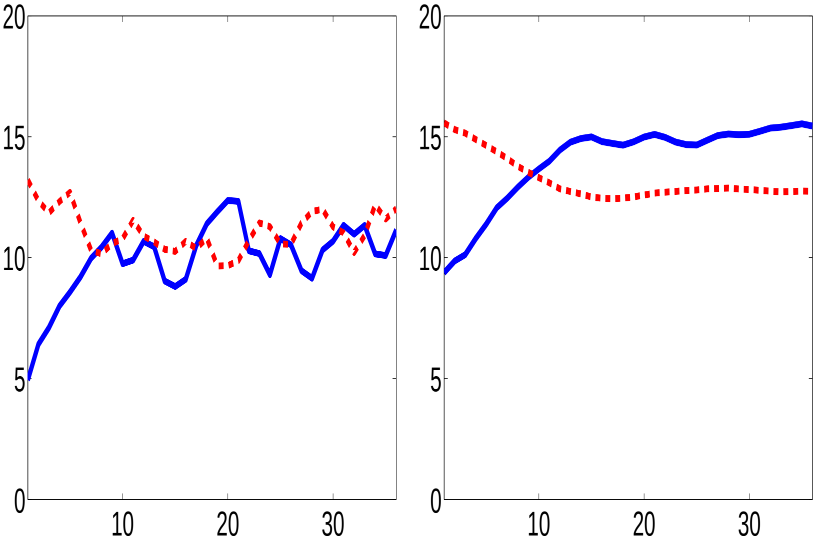
Diagnostics

The long-term dynamics of the investment premium, the stock and fundamental returns in event-time



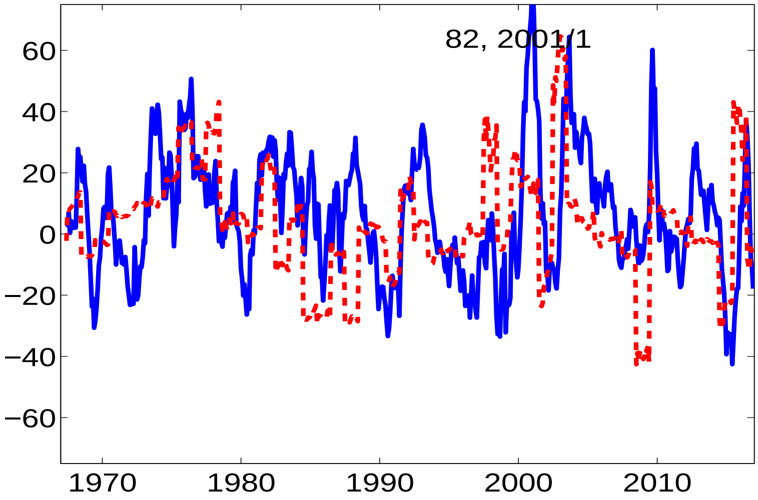
Diagnostics

The short-term dynamics of the Roe premium, the stock and fundamental returns in event-time



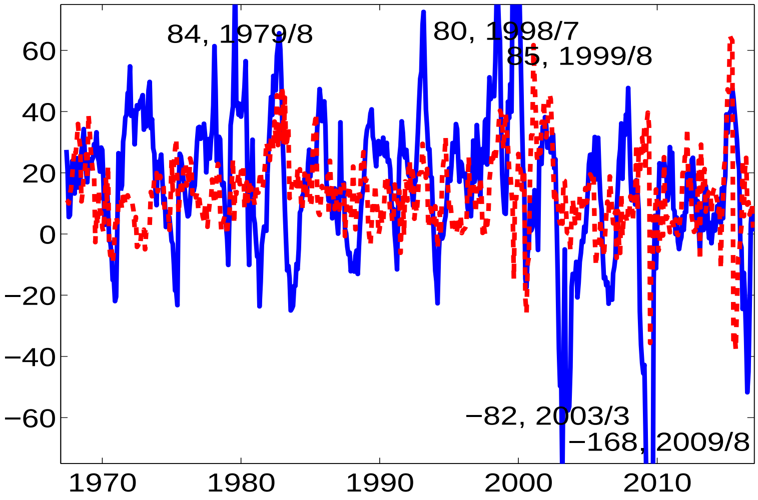
Diagnostics

Time series, the stock and fundamental value premium, corr = 0.26



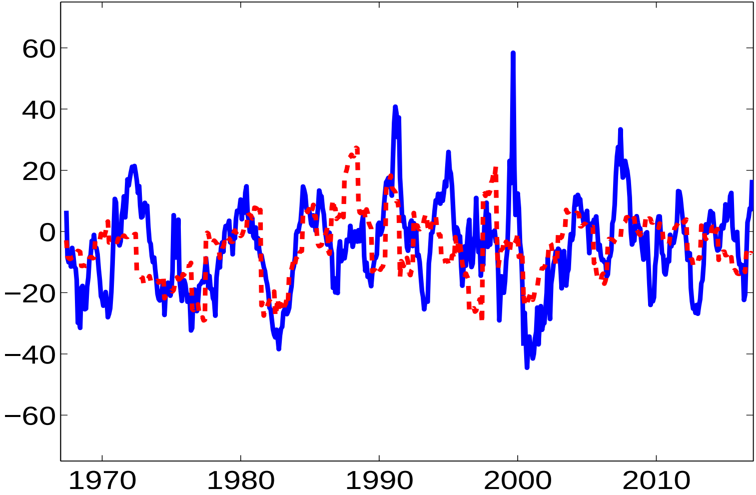
Diagnostics

Time series, the stock and fundamental momentum premium, corr = 0.14



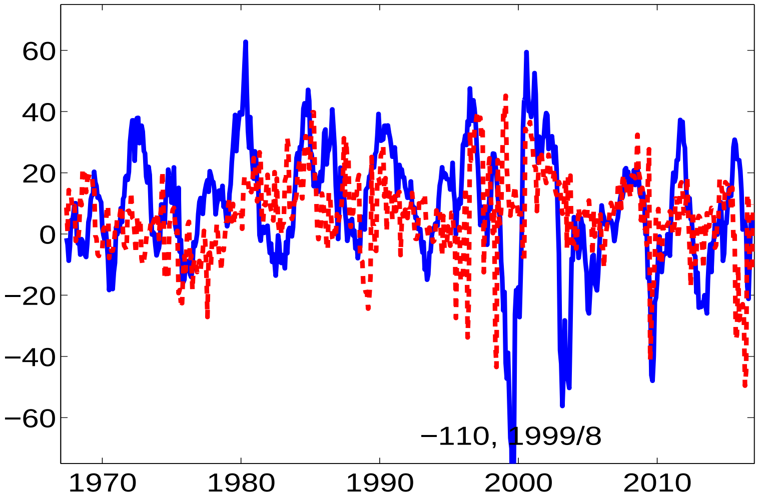
Diagnostics

Time series, the stock and fundamental investment premium, corr = 0.42



Diagnostics

Time series, the stock and fundamental Roe premium, corr = 0.16

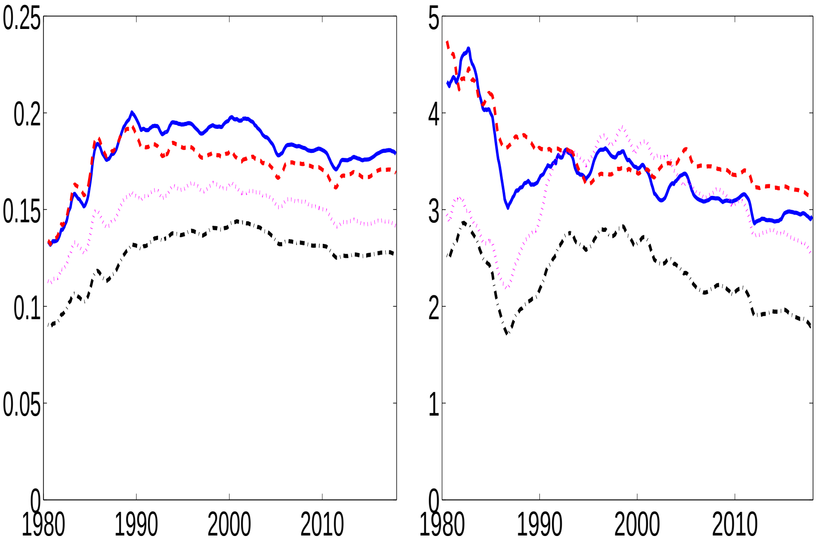


Bm		L	2	3	4	5	6	7	8	9	H	H−L
σ	r^S	0.20	0.18	0.18	0.19	0.17	0.16	0.17	0.17	0.17	0.22	0.20[c]
	r^F	0.05	0.06	0.06	0.07	0.08	0.10	0.07	0.11	0.13	0.18	0.18[c]
S_k	r^S	−0.24	0.03	−0.08	−0.04	−0.16	−0.07	−0.20	−0.48	−0.14	0.12	0.42
	r^F	−0.96	−1.26	1.05	0.57	0.81	−1.57	0.67	1.27	0.73	0.63	0.36
K_u	r^S	3.04	3.12	2.75	3.43	3.20	3.57	3.52	4.36	3.94	4.47	3.28
	r^F	3.97	6.24	8.33	5.36	4.81	8.13	2.95	6.62	4.29	4.64	4.03
R^{11}		L	2	3	4	5	6	7	8	9	H	H−L
σ	r^S	0.30	0.24	0.20	0.18	0.16	0.17	0.16	0.18	0.19	0.26	0.28[c]
	r^F	0.12	0.08	0.08	0.07	0.07	0.06	0.07	0.07	0.07	0.07	0.13[c]
S_k	r^S	1.47	0.94	0.19	0.42	−0.10	−0.14	−0.23	−0.16	−0.11	−0.03	−1.78[a]
	r^F	−0.56	−0.03	0.33	0.38	0.57	0.69	1.01	0.62	0.13	−0.41	0.30[a]
K_u	r^S	9.92	8.05	3.91	4.07	3.70	3.58	3.02	3.07	3.57	3.19	11.59[c]
	r^F	6.58	4.10	6.00	4.81	5.51	5.24	6.73	5.06	4.07	3.91	5.29[b]

I/A		L	2	3	4	5	6	7	8	9	H	H−L
σ	r^S	0.22	0.18	0.16	0.15	0.16	0.16	0.17	0.17	0.21	0.23	0.14^c
	r^F	0.09	0.07	0.08	0.07	0.06	0.07	0.06	0.05	0.07	0.08	0.11^c
S_k	r^S	0.36	−0.01	−0.01	−0.16	−0.25	−0.18	−0.20	−0.15	−0.30	−0.22	0.13
	r^F	0.22	0.88	0.41	1.00	0.40	0.03	−0.27	0.43	−0.29	−0.60	0.08
K_u	r^S	4.13	3.67	3.18	3.48	3.55	3.19	3.22	3.07	3.33	3.15	3.44
	r^F	2.71	4.60	2.95	5.17	3.01	3.43	4.48	4.15	3.59	5.03	3.18
Roe		L	2	3	4	5	6	7	8	9	H	H−L
σ	r^S	0.27	0.22	0.19	0.16	0.17	0.18	0.16	0.17	0.17	0.20	0.20^c
	r^F	0.14	0.12	0.09	0.08	0.08	0.07	0.07	0.05	0.05	0.05	0.14^c
S_k	r^S	0.20	0.23	−0.03	−0.02	−0.25	−0.38	−0.39	−0.14	−0.20	−0.06	-0.84^a
	r^F	0.46	0.38	0.58	0.38	0.50	1.31	0.07	−0.38	−0.15	−0.09	−0.38
K_u	r^S	3.69	3.94	4.13	3.36	3.12	3.66	3.14	2.90	3.35	2.70	5.75^c
	r^F	4.99	5.45	6.73	4.53	4.85	6.56	4.19	3.88	2.98	3.08	4.45^c

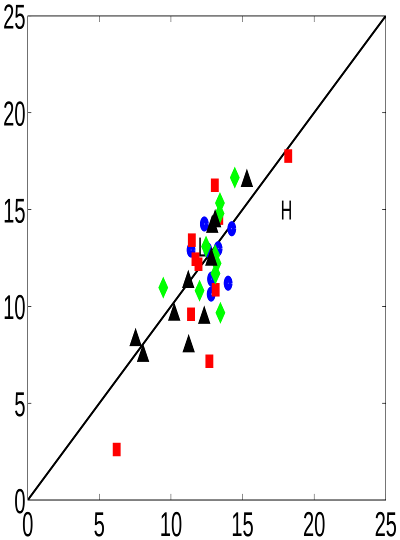
Out-of-sample Tests

Recursive estimation: The 1st window from 1967/6 to 1980/7, add 1 month at a time until 2017/12, time series of γ and a estimates



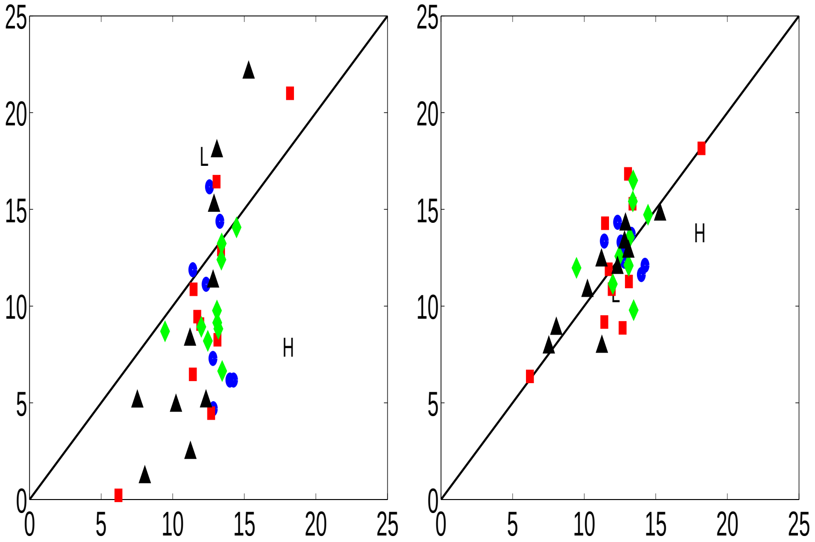
Out-of-sample Tests

1-month-ahead fits, stock versus fundamental returns, the 2-capital model at the firm level, July 1980–December 2016



Out-of-sample Tests

1-month-ahead fits, stock versus fundamental returns, the physical capital model at the portfolio level and the q-factor model, July 1980–December 2016



Investment-to-capital, I_{t+1}/K_{t+1}				Annual sales growth, Y_{t+1}/Y_t			
	Slope	$[t]$	R^2		Slope	$[t]$	R^2
$\log(Q_t)$	0.11	30.08	28.34	g^Y_{q-1}	0.43	54.09	67.45
$Y_t/(K_t + W_t)$	0.02	9.29		g^Y_{q-2}	0.14	19.96	
I_t/K_t	0.34	34.07		g^Y_{q-3}	0.08	10.77	
				g^Y_{q-4}	0.10	14.54	

Out-of-sample Tests

Deciles formed on the expected return estimates, July 1980–December 2017

h	L	2	3	4	5	6	7	8	9	H	H−L
	The two-capital model estimated at the firm level										
1	0.39	0.80	0.61	0.75	0.80	0.57	0.80	0.82	0.85	0.86	0.48
	1.34	3.08	2.50	3.13	3.88	2.75	4.13	4.26	4.21	4.06	2.52
6	0.44	0.76	0.69	0.76	0.75	0.60	0.79	0.78	0.81	0.83	0.39
	1.55	2.87	2.86	3.21	3.64	2.93	4.07	4.09	4.04	3.98	2.21
12	0.49	0.77	0.65	0.78	0.75	0.64	0.76	0.74	0.83	0.77	0.28
	1.78	2.99	2.70	3.29	3.62	3.22	3.91	3.88	4.14	3.68	1.66

h	L	2	3	4	5	6	7	8	9	H	H−L
	The physical capital model estimated at the portfolio level										
1	0.42	0.75	0.59	0.66	0.72	0.79	0.83	0.82	0.95	0.83	0.41
	1.39	2.82	2.76	2.98	3.39	4.12	3.93	3.60	3.98	3.30	2.43
6	0.46	0.77	0.61	0.63	0.74	0.71	0.87	0.80	0.88	0.79	0.33
	1.55	3.03	2.89	2.80	3.43	3.74	4.20	3.51	3.80	3.11	2.07
12	0.54	0.76	0.61	0.64	0.75	0.68	0.81	0.81	0.87	0.79	0.26
	1.88	3.05	2.90	2.83	3.53	3.55	3.93	3.56	3.83	3.15	1.77

Out-of-sample Tests

Deciles formed on the expected return estimates, July 1980–December 2017, evidence consistent with Fama and French (1997)

h	L	2	3	4	5	6	7	8	9	H	H−L
	The q-factor model										
1	0.64	0.70	0.78	0.65	0.79	0.76	0.74	0.80	0.76	0.83	0.20
	2.18	2.89	3.79	3.23	4.02	4.00	3.50	3.69	3.24	2.79	0.94
6	0.63	0.83	0.74	0.69	0.78	0.75	0.70	0.74	0.75	0.82	0.19
	2.13	3.56	3.67	3.53	4.09	3.95	3.49	3.48	3.19	2.83	0.93
12	0.61	0.75	0.79	0.72	0.75	0.76	0.71	0.73	0.77	0.86	0.25
	2.07	3.33	3.93	3.72	3.99	3.95	3.59	3.49	3.30	2.96	1.22

A detailed treatment of aggregation and capital heterogeneity substantially improves the performance of the investment CAPM

Future work: The fundamental cost of capital