Lecture Notes

Petrosky-Nadeau, Zhang, and Kuehn (2018, American Economic Review, "Endogenous Disasters")

Lu Zhang[1]

[1]Ohio State and NBER

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The textbook Diamond-Mortensen-Pissarides model of equilibrium unemployment gives rise endogenously to rare disasters

Outline

Outline

The representative firm posts job vacancies, V_t, to attract unemployed workers, U_t, via a CRS matching function:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{1/\iota}}$$

The job filling rate:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^{\iota})^{1/\iota}}$$

in which $\theta_t = V_t / U_t$ is labor market tightness: $q'(\theta_t) < 0$

The representative firm incurs costs of vacancy posting, $\kappa_t V_t$, with the unit costs, κ_t, given by:

$$\kappa_t \equiv \kappa_0 + \kappa_1 q(\theta_t)$$

in which

- κ_0: The proportional costs
- κ_1: The fixed costs

Fixed matching costs (paid after a hired worker arrives): Training, interviewing, negotiation, and administrative setup costs of adding a worker to the payroll, etc., as in Pissarides (2009)

Once matched, jobs are destroyed at a constant rate s:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t$$

in which

$$V_t \geq 0$$

The firm uses labor to produce via a CRS production function:

$$Y_t = X_t N_t \quad \text{in which} \quad \log(X_{t+1}) = \rho \log(X_t) + \sigma \epsilon_{t+1}$$

Dividends to shareholders:

$$D_t = X_t N_t - W_t N_t - \kappa_t V_t$$

in which W_t is the wage rate

Taking the stochastic discount factor, $M_{t+1} = \beta(C_t / C_{t+1})$, as given, the firm maximizes the market value of equity, S_t:

$$S_t \equiv \max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} E_t \left[\sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} \right]$$

subject to $N_{t+1} = (1 - s) N_t + q(\theta_t) V_t$ and $V_t \geq 0$

Let λ_t be the multiplier on the $q(\theta_t)V_t \geq 0$ constraint:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1}\right]\right]\right]$$

The Kuhn-Tucker conditions: $V_t \geq 0, \lambda_t \geq 0, \lambda_t V_t = 0$

Wages as the endogenous outcome of a generalized Nash bargaining process between a worker and the firm:

$$W_t = \eta \left(X_t + \kappa_t \theta_t \right) + (1 - \eta) b$$

- η: Relative bargaining weight of the worker
- X_t: Marginal product of labor
- $\kappa_t \theta_t = \kappa_t V_t / U_t$: Vacancy costs per unemployed worker
- b: The flow value of unemployment activities

η and b govern wage elasticity to labor productivity

The goods market clearing condition:

$$C_t + \kappa_t V_t = X_t N_t$$

The recursive competitive equilibrium consists of vacancies, $V_t^\star \geq 0$; multiplier, $\lambda_t^\star \geq 0$; consumption, $C_t^\star$; and indirect utility, $J_t^\star$:

- $V_t^\star$ and $\lambda_t^\star$ satisfy the intertemporal job creation condition and the Kuhn-Tucker conditions, while taking the wage equation and the representative household's SDF as given;
- $C_t^\star$ satisfies the optimality condition $1 = E_t[M_{t+1} R_{t+1}]$;
- the goods market and the financial market clear

Solve for $\lambda_t \equiv \lambda(N_t, X_t)$ from

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t \left[M_{t+1} \left[X_{t+1} - W_{t+1} + (1-s) \left(\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} \right) \right] \right]$$

while obeying the Kuhn-Tucker conditions

$\log(X_t)$ discretized with 17 grid points; cubic splines (75 basis functions) in N for each $\log(X)$-level

Parameterizing the conditional expectation, $\mathcal{E}_t \equiv \mathcal{E}(N_t, X_t)$ eliminates the need to parameterizing λ_t separately:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = \mathcal{E}_t$$

After obtaining $\mathcal{E}_t$, calculate $\tilde{q}(\theta_t) = \kappa_0 / (\mathcal{E}_t - \kappa_1)$:

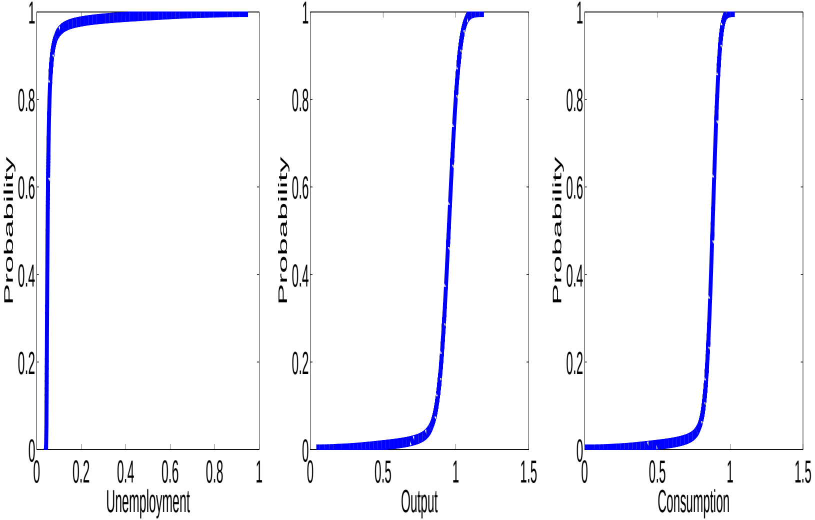
- If $\tilde{q}(\theta_t) \geq 1$ (binding constraint): set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$
- If $\tilde{q}(\theta_t) < 1$ (nonbinding constraint): set $\lambda_t = 0$, $q(\theta_t) = \tilde{q}(\theta_t) \Rightarrow \theta_t = q^{-1}(\tilde{q}(\theta_t))$, $V_t = \theta_t(1 - N_t)$

Rate of time preference, β	0.9954	
Aggregate productivity persistence, ρ	0.983	Gertler and Trigari (2009)
Conditional volatility of shocks, σ	0.01	Gertler and Trigari (2009)
Workers' bargaining weight, η	0.04	Hagedorn and Manovskii (2008)
Job destruction rate, s	0.04	Davis et al. (2006)
Elasticity of matching function, ι	1.25	Den Haan et al. (2000)
Value of unemployment activities, b	0.85	Hagedorn and Manovskii (2008)
The proportional costs, κ_0	0.5	
The fixed costs, κ_1	0.5	

Panel A: Log output growth						Panel B: Log consumption growth					
	data	mean	5%	95%	p-value		data	mean	5%	95%	p-value
σ_Y	5.63	5.31	2.90	11.56	0.31	σ_C	6.37	4.65	2.12	11.26	0.21
S_Y	−1.02	0.85	−0.39	3.35	0.99	S_C	−0.55	0.91	−0.50	3.57	0.96
K_Y	11.87	12.8	3.08	34.99	0.41	K_C	9.19	14.4	3.16	38.59	0.54
ρ_1^Y	0.16	0.24	0.01	0.63	0.58	ρ_1^C	0.07	0.23	−0.02	0.65	0.79
ρ_2^Y	0	−0.11	−0.31	0.23	0.16	ρ_2^C	0.03	−0.12	−0.33	0.24	0.14
ρ_3^Y	0.02	−0.12	−0.32	0.08	0.1	ρ_3^C	0	−0.12	−0.34	0.09	0.15
ρ_4^Y	−0.02	−0.11	−0.31	0.08	0.23	ρ_4^C	−0.02	−0.11	−0.32	0.09	0.23

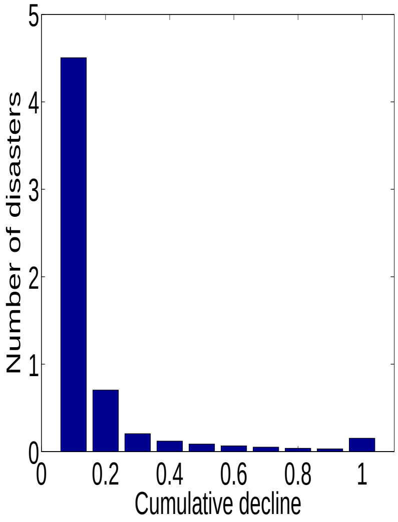
Panel C: Unemployment

	data	mean	5%	95%	p-value		data	mean	5%	95%	p-value
$E[U]$	6.98	6.28	4.83	10.54	0.19	S_U	2.02	3.52	1.49	5.82	0.85
K_U	7.26	19.18	5.24	41.78	0.87	σ_U	21.76	23.41	5.45	53.49	0.43

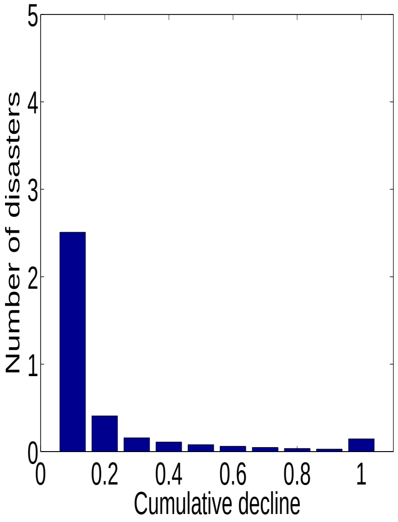


Probability
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
Unemployment
Probability
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
Output
Probability
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
Consumption

	Data	Model			
		Mean	5%	95%	p-value
Panel A: Output					
Probability	7.83	5.04	2.24	8.57	0.09
Size	21.99	22.22	12.7	46.24	0.33
Duration	3.72	4.44	3.2	6	0.79
Panel B: Consumption					
Probability	8.57	2.86	0.71	5.83	0.00
Size	23.16	25.64	11.26	62.13	0.36
Duration	3.75	4.91	3	7	0.81



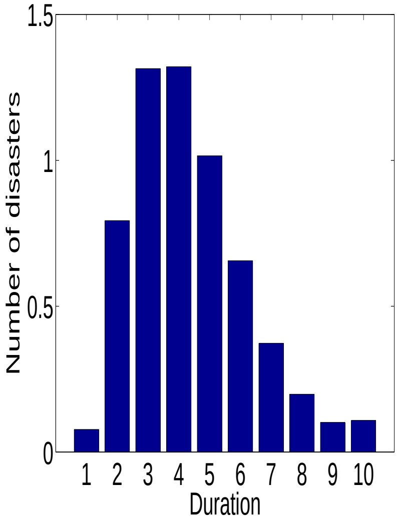
Number of disasters
Cumulative decline
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5

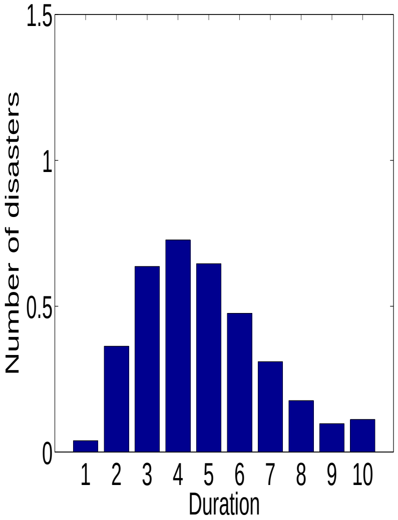


Number of disasters
Cumulative decline
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5

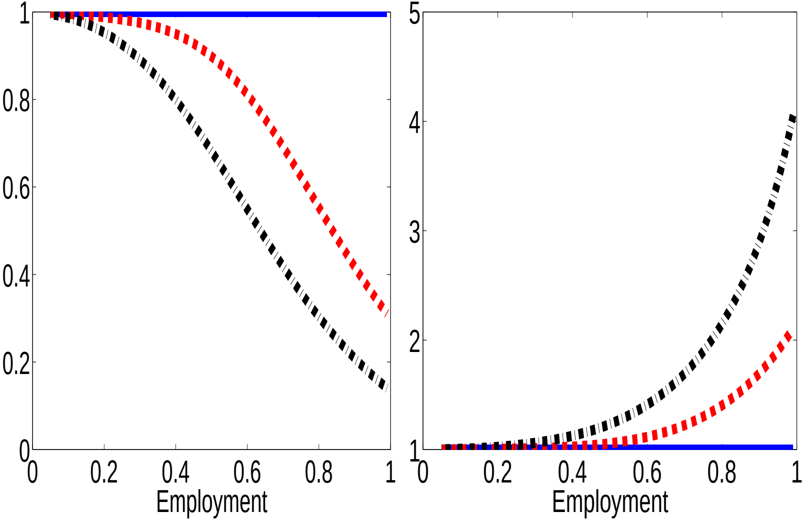
Quantitative Results

Distributions of disasters by duration





	Baseline	$b = 0.825$	$b = 0.4$	$s = 0.035$	$\kappa_t = 0.7$	$\iota = 1.1$	$\eta = 0.05$
	Panel A: Output						
Probability	5.04	3.61	2.53	4.42	4.05	5.29	5.57
Size	22.22	16.07	13.41	19.87	18.2	21.97	22.69
Duration	4.44	4.57	4.7	4.5	4.51	4.41	4.4
	Panel B: Consumption						
Probability	2.86	1.62	1.32	2.43	1.85	3.04	3.59
Size	25.64	16.31	12.35	22.25	20.19	25.05	25.21
Duration	4.91	5.19	5.2	4.97	5.1	4.88	4.78


1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
Employment
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
Employment

Outline

Let C_{mt} be market consumption, C_{ht} home consumption, and the composite consumption bundle:

$$C_t \equiv \left[aC_{mt}^e + (1-a)C_{ht}^e\right]^{1/e},$$

in which $e \in (0,1]$ and $a \in [0,1]$

The stochastic discount factor:

$$M_{t+1} = \beta \left(\frac{C_{mt+1}}{C_{mt}}\right)^{e-1} \left(\frac{C_t}{C_{t+1}}\right)^{e}$$

Home production technology:

$$C_{ht} = X_h U_t, \qquad \text{with} \qquad X_h > 0$$

The equilibrium Nash-wage becomes:

$$W_t = \eta(X_t + \kappa_t \theta_t) + (1 - \eta) z_t,$$

with

$$z_t \equiv X_h \left(\frac{1 - a}{a} \right) \left(\frac{C_{mt}}{C_{ht}} \right)^{1-e} + b$$

The market clearing condition: $C_{mt} + \kappa_t V_t = X_t N_t$

	Data	Model					Data	Model			
σ		0.01	0.014	0.014	0.014	σ		0.01	0.014	0.014	0.014
a		0.8	0.8	0.8	0.85	a		0.8	0.8	0.8	0.85
e		0.85	0.85	0.9	0.85	e		0.85	0.85	0.9	0.85
σ_Y	5.63	3.41	5.29	4.62	3.9	σ_C	6.37	2.91	4.67	3.74	2.9
S_Y	−1.02	0.06	0.15	0.13	0.01	S_C	−0.55	0.09	0.2	0.2	0.03
K_Y	11.87	3.83	4.92	4.95	3.42	K_C	9.19	4.22	5.73	5.97	3.48
ρ_1^Y	0.16	0.15	0.16	0.15	0.14	ρ_1^C	0.07	0.15	0.16	0.15	0.14
ρ_2^Y	0	−0.13	−0.13	−0.13	−0.12	ρ_2^C	0.03	−0.13	−0.14	−0.14	−0.12
ρ_3^Y	0.02	−0.1	−0.11	−0.1	−0.1	ρ_3^C	0	−0.1	−0.11	−0.11	−0.1
ρ_4^Y	−0.02	−0.08	−0.08	−0.08	−0.08	ρ_4^C	−0.02	−0.08	−0.09	−0.08	−0.08
Prob_Y	7.83	5	9.95	8.2	6.88	Prob_C	8.57	3.35	7.52	4.95	3.43
Size_Y	21.99	15	18.58	17.21	15.43	Size_C	23.16	14.42	18.06	16.34	13.81
Dur_Y	3.74	4.32	3.74	3.88	3.98	Dur_C	3.75	4.65	3.99	4.31	4.59
$E[U]$	6.98	5.97	6.58	5.33	4.5	S_U	2.02	1.86	2.44	3.06	2.1
K_U	7.26	7.48	10.42	15.73	9.87	σ_U	21.76	10.75	19.53	15.3	4.07

Production $Y_t = X_t K_t^{\alpha} N_t^{1-\alpha}$, $\alpha \in (0, 1)$, and $x_t = \log(X_t)$:

$$x_{t+1} = (1-\rho)\bar{x} + \rho x_t + \sigma \epsilon_{t+1}$$

Capital accumulation:

$$K_{t+1} = (1-\delta)K_t + \Phi(I_t, K_t),$$

in which δ is the capital depreciation rate, I_t is investment, and

$$\Phi(I_t, K_t) \equiv \left[a_1 + \frac{a_2}{1-1/\nu}\left(\frac{I_t}{K_t}\right)^{1-1/\nu}\right] K_t$$

The investment Euler equation:

$$\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{1/\nu} = E_t\left[M_{t+1}\left(\alpha\frac{Y_{t+1}}{K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}}\right)\right]$$

The intertemporal job creation condition:

$$\frac{\kappa_0}{q(\theta_t)}+\kappa_1-\lambda_t = E_t\left[M_{t+1}\left((1-\alpha)\frac{Y_{t+1}}{N_{t+1}} - W_{t+1} + (1-s)\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1}\right]\right)\right]$$

The equilibrium wage:

$$W_t = \eta\left[(1-\alpha)\frac{Y_t}{N_t} + \kappa_t\theta_t\right] + (1-\eta)b$$

The goods market clearing condition: $C_t + I_t + \kappa_t V_t = Y_t$

	Data	Model					Data	Model			
σ		0.01	0.014	0.014	0.014	σ		0.01	0.014	0.014	0.014
ν		2	2	1.5	0.5	ν		2	2	1.5	0.5
σ_Y	5.63	3.35	5.11	5.1	4.93	σ_C	6.37	2.38	3.74	4	4.75
S_Y	−1.02	0.1	0.12	0.11	0.1	S_C	−0.55	0.08	0.12	0.14	0.17
K_Y	11.87	4.11	4.5	4.49	4.34	K_C	9.19	4.67	5.18	5.1	4.79
ρ_1^Y	0.16	0.18	0.19	0.19	0.17	ρ_1^C	0.07	0.21	0.22	0.2	0.17
ρ_2^Y	0	−0.1	−0.09	−0.1	−0.12	ρ_2^C	0.03	−0.08	−0.07	−0.09	−0.12
Prob_Y	7.83	4.55	9.45	9.4	9.07	Prob_C	8.57	2.08	5.31	5.95	8.18
Size_Y	21.99	15.76	18.97	18.81	18.08	Size_C	23.16	14.9	17.69	17.68	17.98
Dur_Y	3.72	4.58	3.89	3.87	3.8	Dur_C	3.75	5.39	4.51	4.33	3.9
σ_I	23.33	4.52	6.98	6.06	2.88	$E[U]$	6.98	5.98	7.46	7.45	6.92
S_I	−0.79	0.2	0.2	0.17	0	S_U	2.02	2.51	2.55	2.55	2.64
K_I	8.72	4.51	4.94	4.92	4.66	K_U	7.26	11	11.09	11.12	11.65
ρ_1^I	0.22	0.17	0.17	0.18	0.19	σ_U	21.76	14	22.51	22.57	22.27
ρ_2^I	−0.04	−0.12	−0.12	−0.11	−0.1						

Outline

Preferences:

$$J_t = \max_{\{C_t\}} \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left(E_t \left[J_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}$$

Optimality condition: $1 = E_t[M_{t+1} R_{t+1}]$:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{J_{t+1}}{E_t[J_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}$$

$$R_{t+1} = \frac{X_{t+1} - W_{t+1} + (1-s)\left[\kappa_0/q(\theta_{t+1}) + \kappa_1 - \lambda_{t+1}\right]}{\kappa_0/q(\theta_t) + \kappa_1 - \lambda_t}$$

	Data	Model					Data	Model			
γ		10	7.5	10	1	γ		10	7.5	10	1
ψ		1.5	1.5	1	1	ψ		1.5	1.5	1	1
σ_Y	5.63	5.67	4.97	4.99	4.11	σ_C	6.37	5.05	4.35	4.35	3.44
S_Y	−1.02	0.87	0.81	0.76	0.61	S_C	−0.55	0.88	0.81	0.81	0.67
K_Y	11.87	15.47	14.18	12.36	10.36	K_C	9.19	17.09	15.69	14.2	11.9
ρ_1^Y	0.16	0.21	0.19	0.23	0.20	ρ_1^C	0.07	0.19	0.18	0.22	0.19
ρ_2^Y	0	−0.14	−0.14	−0.12	−0.12	ρ_2^C	0.03	−0.15	−0.15	−0.13	−0.14
ρ_3^Y	0.02	−0.13	−0.12	−0.12	−0.12	ρ_3^C	0	−0.13	−0.12	−0.13	−0.12
ρ_4^Y	−0.02	−0.1	−0.1	−0.11	−0.1	ρ_4^C	−0.02	−0.1	−0.09	−0.11	−0.1
Prob_Y	7.83	4.49	4.03	4.53	5.03	Prob_C	8.57	2.51	2.12	2.51	2.84
Size_Y	21.99	23.92	22.17	21.92	22.25	Size_C	23.16	28.86	26.51	25.6	25.7
Dur_Y	3.72	4.46	4.56	4.5	4.45	Dur_C	3.75	4.84	5.01	4.9	4.93
$E[U]$	6.98	6.26	5.88	6.23	5.7	$E[R - R^f]$	4.69	4.45	1.1	4.97	0.22
S_U	2.02	3.66	3.57	3.46	3.29	$E[R^f]$	1.04	2.58	2.87	2.6	2.93
K_U	7.26	20.71	20.75	18.48	18	σ_R	20	15.79	15.15	15.73	14.5
σ_U	21.76	25.67	21.99	22.93	17.88	σ_{R^f}	12.32	1.64	1.39	1.98	1.54

The textbook Diamond-Mortensen-Pissarides model of equilibrium unemployment gives rise endogenously to rare disasters