

$q^5$

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2019 JHU Carey Finance Conference

June 28, 2019

Augmenting the Hou-Xue-Zhang (2015)  $q$ -factor model with an expected growth factor to form **the  $q^5$  model**:

$$E[R_i - R_f] = \beta_{\text{MKT}}^i E[\text{MKT}] + \beta_{\text{Me}}^i E[R_{\text{Me}}] \\ + \beta_{\text{I/A}}^i E[R_{\text{I/A}}] + \beta_{\text{Roe}}^i E[R_{\text{Roe}}] + \beta_{\text{Eg}}^i E[R_{\text{Eg}}]$$

Stress-testing factor models with a large set of 150 anomalies:

- The  $q^5$  model improves on the  $q$ -factor model substantially
- The  $q$ -factor model already compares favorably with the Fama-French (2018) 6-factor model

- 1 The Expected Growth Factor
- 2 Stress-testing Factor Models
- 3 Examples of Individual Factor Regressions

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# The Expected Growth Factor

Economic foundation, Cochrane (1991)

In the multiperiod investment framework:

$$r_{t+1} \approx \frac{X_{t+1} + (1 - \delta)[1 + a(I_{t+1}/A_{t+1})]}{1 + a(I_t/A_t)}$$

The “dividend yield” component,  $X_{t+1}/[1 + a(I_t/A_t)]$ , motivates the  $q$ -factor model

The “capital gain” component roughly proportional to investment-to-assets growth,  $(I_{t+1}/A_{t+1}) / (I_t/A_t)$

# The Expected Growth Factor

Cross-sectional forecasting framework

Forecast  $d^{\tau}I/A$ ,  $\tau$ -year ahead investment-to-assets changes, with monthly cross-sectional regressions

Motivating predictors based on a priori conceptual arguments:

- Tobin's  $q$ : Erickson and Whited (2000)
- Cash flows: Fazzari, Hubbard, and Petersen (1988)
- Change in return on equity: Liu, Whited, and Zhang (2009)

# The Expected Growth Factor

A priori conceptual arguments

Cash flows: We use the cash-based operating profitability from Ball, Gerakos, Linnainmaa, and Nikolaev (2016)

- Total revenue minus cost of goods sold, minus SG&A, plus R&D, minus change in accounts receivable, minus change in inventory, minus change in prepaid expenses, plus change in deferred revenue, plus change in trade accounts payable, and plus change in accrued expenses, all scaled by book assets

dRoe: Capturing short-term dynamics of investment growth

# The Expected Growth Factor

Monthly cross-sectional regressions of  
future investment-to-assets changes, 7/1963–12/2018

$\tau$	$\log(q)$	Cop	dRoe	$R^2$	Pearson	Rank
1	-0.03 (-5.63)	0.52 (12.75)	0.77 (7.62)	6.42	0.14 [0.00]	0.21 [0.00]
2	-0.07 (-9.76)	0.70 (12.34)	0.91 (10.07)	8.61	0.15 [0.00]	0.22 [0.00]
3	-0.09 (-12.39)	0.75 (12.17)	0.72 (8.60)	8.98	0.15 [0.00]	0.22 [0.00]

Relatively reliable out-of-sample correlations with subsequent,  
realized investment-to-assets changes



# The Expected Growth Factor

Properties of the expected growth deciles, 1/1967–12/2018

Low	2	3	4	5	6	7	8	9	High	H–L
Average excess returns, $\bar{R}$										
-0.12	0.20	0.28	0.42	0.45	0.49	0.56	0.64	0.77	0.95	1.07
-0.40	0.84	1.21	2.00	2.36	2.61	3.00	3.54	4.17	4.69	6.48
The expected 1-year-ahead growth, $E_t[d^1/A]$										
-15.21	-7.67	-5.61	-4.20	-3.03	-1.97	-0.86	0.47	2.52	7.65	22.87
-36.75	-31.37	-25.19	-20.56	-15.96	-11.01	-5.08	3.01	16.53	37.98	45.21
Average future 1-year-ahead realized growth, $d^1/A$										
-16.69	-12.30	-4.11	-3.56	-1.10	-0.43	-0.32	0.64	1.57	5.96	22.65
-11.71	-8.36	-7.15	-5.22	-2.24	-0.90	-0.71	1.18	3.59	9.07	14.72

$E_t[d^1/A]$  and  $d^1/A$  aligned at the portfolio level (Corr = 0.64)

# The Expected Growth Factor

$R_{Eg}$ , independent  $2 \times 3$  monthly sorts on size and  $E_t[d^1 I/A]$ , 1/1967–12/2018

$\bar{R}_{Eg}$	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{I/A}$	$\beta_{Roe}$	$R^2$	
0.84 (10.27)	0.67 (9.75)	-0.11 (-6.38)	-0.09 (-3.56)	0.21 (4.86)	0.30 (9.13)	0.44	
	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{I/A}$	$\beta_{Roe}$	$\beta_{\log(q)}$	$R^2$
	0.67 (9.80)	-0.11 (-6.40)	-0.09 (-3.61)	0.23 (4.72)	0.30 (8.83)	-0.02 (-0.48)	0.44
	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{I/A}$	$\beta_{Roe}$	$\beta_{Cop}$	$R^2$
	0.37 (6.35)	-0.02 (-1.66)	-0.02 (-0.54)	0.31 (9.51)	0.14 (4.37)	0.60 (10.63)	0.65
	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{I/A}$	$\beta_{Roe}$	$\beta_{dRoe}$	$R^2$
	0.63 (8.56)	-0.11 (-6.62)	-0.10 (-3.93)	0.18 (3.57)	0.23 (5.00)	0.16 (2.41)	0.46

Results robust to the use of percentile rankings and composite score across  $\log(q)$ , Cop, and dRoe

- 1 The Expected Growth Factor
- 2 Stress-testing Factor Models**
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8 competing factor models:

- The  $q$ -factor model, the  $q^5$  model
- The Fama-French 5-factor model, the 6-factor model, the alternative 6-factor model with RMWc
- The Stambaugh-Yuan 4-factor model
- The Barillas-Shanken 6-factor model, including MKT, SMB,  $R_{I/A}$ ,  $R_{Roe}$ , the Asness-Frazzini monthly formed HML, UMD
- The Daniel-Hirshleifer-Sun 3-factor model

Use the replicated Stambaugh-Yuan and Daniel-Hirshleifer-Sun models via the common construction (Hou et al. 2019)

# Stress Tests

The playing field, monthly Sharpe ratios for factors and factor models

Individual factors							
$R_{Mkt}$	$R_{Me}$	$R_{I/A}$	$R_{Roe}$	$R_{Eg}$	SMB	HML	CMA
0.11	0.09	0.20	0.22	0.44	0.07	0.11	0.15
RMW	RMW <sub>c</sub>	UMD	HML <sup>m</sup>	MGMT	PERF	FIN	PEAD
0.12	0.19	0.15	0.08	0.20	0.16	0.10	0.32
Factor models							
$q$	$q^5$	FF5	FF6	FF6c	BS6	SY4	DHS
0.42	0.63	0.32	0.37	0.43	0.48	0.41	0.42

150 anomalies with NYSE breakpoints and value-weighted returns significant at the 5% level (Hou, Xue, and Zhang 2019)

- Momentum: 39
- Value-versus-growth: 15
- Investment: 26
- Profitability: 40
- Intangibles: 27
- Trading frictions: 3

Sue1 Earnings surprise  
(1-month holding period),  
Foster, Olsen, and Shevlin (1984)

Abr6 Cumulative abnormal returns  
around earnings announcements  
(6-month holding period), Chan,  
Jegadeesh, and Lakonishok (1996)

Re1 **Revisions in analysts' forecasts**  
(1-month holding period), Chan,  
Jegadeesh, and Lakonishok (1996)

R<sup>6</sup>1 Price momentum (6-month prior  
returns, 1-month holding period),  
Jegadeesh and Titman (1993)

Abr1 **Cumulative abnormal returns  
around earnings announcements**  
(1-month holding period), Chan,  
Jegadeesh, and Lakonishok (1996)

Abr12 Cumulative abnormal returns  
around earnings announcements  
(12-month holding period), Chan,  
Jegadeesh, and Lakonishok (1996)

Re6 Revisions in analysts' forecasts  
(6-month holding period), Chan,  
Jegadeesh, and Lakonishok (1996)

R<sup>6</sup>6 Price momentum (6-month prior  
returns, 6-month holding period),  
Jegadeesh and Titman (1993)

# Stress Tests

## Testing deciles, momentum (39)

$R^{612}$	Price momentum (6-month prior returns, 12-month holding period), Jegadeesh and Titman (1993)	$R^{111}$	Price momentum (11-month prior returns, 1-month holding period), Fama and French (1996)
$R^{116}$	Price momentum, (11-month prior returns, 6-month holding period), Fama and French (1996)	$R^{1112}$	Price momentum, (11-month prior returns, 12-month holding period), Fama and French (1996)
Im1	Industry momentum, (1-month holding period), Moskowitz and Grinblatt (1999)	Im6	Industry momentum, (6-month holding period), Moskowitz and Grinblatt (1999)
Im12	Industry momentum (12-month holding period), Moskowitz and Grinblatt (1999)	Rs1	Revenue surprise (1-month holding period), Jegadeesh and Livnat (2006)
dEf1	Analysts' forecast change (1-month hold period), Hawkins, Chamberlin, and Daniel (1984)	dEf6	Analysts' forecast change (6-month hold period), Hawkins, Chamberlin, and Daniel (1984)



# Stress Tests

## Testing deciles, momentum (39)

dEf12	Analysts' forecast change (12-month hold period), Hawkins, Chamberlin, and Daniel (1984)	Nei1	# consecutive quarters with earnings increases (1-month holding period), Barth, Elliott, and Finn (1999)
52w6	52-week high (6-month holding period), George and Hwang (2004)	52w12	52-week high (12-month holding period), George and Hwang (2004)
$\epsilon^6_6$	Six-month residual momentum (6-month holding period), Blitz, Huij, and Martens (2011)	$\epsilon^6_{12}$	Six-month residual momentum (12-month holding period), Blitz, Huij, and Martens (2011)
$\epsilon^{11}_1$	11-month residual momentum, 1-month, Blitz, Huij, and Martens (2011)	$\epsilon^{11}_6$	11-month residual momentum, 6-month, Blitz, Huij, and Martens (2011)
$\epsilon^{11}_{12}$	11-month residual momentum, 12-month, Blitz, Huij, and Martens (2011)	Sm1	Segment momentum 1-month, Cohen and Lou (2012)

# Stress Tests

## Testing deciles, momentum (39)

- |       |   |       |   |
|-------|---|-------|---|
| Sm12  | Segment momentum, 12-month, Cohen and Lou (2012)                | llr1  | Industry lead-lag effect in prior returns, 1-month, Hou (2007)  |
| llr6  | Industry lead-lag effect in prior returns, 6-month, Hou (2007)  | llr12 | Industry lead-lag effect in prior returns, 12-month, Hou (2007) |
| lle1  | Industry lead-lag effect in earnings news, 1-month, Hou (2007)  | Cm1   | Customer momentum, 1-month Cohen and Frazzini (2008)            |
| Cm12  | Customer momentum, 12-month Cohen and Frazzini (2008)           | Sim1  | Supplier industries momentum, 1-month, Menzly and Ozbas (2010)  |
| Cim1  | Customer industries momentum, 1-month, Menzly and Ozbas (2010)  | Cim6  | Customer industries momentum, 6-month, Menzly and Ozbas (2010)  |
| Cim12 | Customer industries momentum, 12-month, Menzly and Ozbas (2010) |       |   |

# Stress Tests

## Testing deciles, value-versus-growth (15)

Bm	Book-to-market equity, Rosenberg, Reid, and Lanstein (1985)	Ep <sup>q1</sup>	Quarterly earnings-to-price (1-month holding period)
Ep <sup>q6</sup>	Quarterly earnings-to-price (6-month holding period)	Ep <sup>q12</sup>	Quarterly earnings-to-price (12-month holding period)
Cp <sup>q1</sup>	Quarterly cash flow-to-price (1-month holding period)	Cp <sup>q6</sup>	Quarterly cash flow-to-price (6-month holding period)
Nop	Net payout yield, Boudoukh, Michaely Richardson, and Roberts (2007)	Em	Enterprise multiple, Loughran and Wellman (2011)
Em <sup>q1</sup>	Quarterly enterprise multiple (1-month holding period)	Sp	Sales-to-price Barbee, Mukherji, and Raines (1996))
Sp <sup>q1</sup>	Quarterly sales-to-price (1-month holding period)	Sp <sup>q6</sup>	Quarterly sales-to-price (6-month holding period)
Sp <sup>q12</sup>	Quarterly sales-to-price (12-month holding period)	Ocp	Operating cash flow-to-price, Desai, Rajgopal, and Venkatachalam (2004)
Ocp <sup>q1</sup>	Operating cash flow-to-price (1-month holding period)		

# Stress Tests

## Testing deciles, investment (26)

I/A	Investment-to-assets, Cooper, Gulen, and Schill (2008)	Ia <sup>q6</sup>	Quarterly investment-to-assets (6-month holding period)
Ia <sup>q12</sup>	Quarterly investment-to-assets (12-month holding period)	dPia	(Changes in PPE and inventory)/assets Lyandres, Sun, and Zhang (2008)
Noa	<b>Net operating assets</b> , Hirshleifer Hou, Teoh, and Zhang (2004)	dNoa	Changes in net operating assets, Hirshleifer, Hou, Teoh, and Zhang (2004)
Ig	Investment growth Xing (2008)	2Ig	Two-year investment growth Anderson and Garcia-Feijoo (2006)
Nsi	<b>Net stock issues</b> Pontiff and Woodgate (2008)	Cei	Composite equity issuance Daniel and Titman (2006)
dli	% change in investment— % change in industry investment, Abarbanell and Bushee (1998)	dLno	Change in long-term net operating assets, Fairfield, Whisenant, and Yohn (2003)
Ivg	Inventory growth Belo and Lin (2011)	Ivc	Inventory changes Thomas and Zhang (2002)

Oa	<b>Operating accruals</b> Sloan (1996)	dWc	Change in net non-cash working capital, Richardson, Sloan, Soliman, and Tuna (2005)
dCoa	Change in current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)	dNco	Change in net non-current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)
dNca	Change in non-current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)	dFin	<b>Change in net financial assets</b> Richardson, Sloan, Soliman, and Tuna (2005)
dFnl	Change in financial liabilities Richardson, Sloan, Soliman, and Tuna (2005)	dBe	Change in common equity, Richardson Sloan, Soliman, and Tuna (2005)
Dac	<b>Discretionary accruals</b> Xie (2001)	Poa	Percent operating accruals, Hafzalla, Lundholm, and Van Winkle (2011)
Pta	Percent total accruals, Hafzalla, Lundholm, and Van Winkle (2011)	Pda	Percent discretionary accruals

# Stress Tests

Testing deciles, profitability (40)

Roe1 Return on equity, 1-month,  
Hou, Xue, and Zhang (2015)

Roe6 Return on equity, 6-month,  
Hou, Xue, and Zhang (2015)

dRoe1 Change in Roe, 1-month horizon

dRoe6 Change in Roe, 6-month horizon

dRoe12 Change in Roe, 12-month horizon

Roa1 Return on assets, 1-month horizon,  
Balakrishnan, Bartov, and Faurel (2010)

dRoa1 Change in Roa, 1-month horizon

dRoa6 Change in Roa, 6-month horizon

Ato Asset turnover  
Soliman (2008)

Cto Capital turnover  
Haugen and Baker (1996)

Rna<sup>q1</sup> Return on net operating assets,  
1-month horizon

Rna<sup>q6</sup> Return on net operating assets,  
6-month horizon

Ato<sup>q1</sup> Quarterly asset turnover,  
1-month horizon

Ato<sup>q6</sup> Quarterly asset turnover,  
6-month horizon

# Stress Tests

## Testing deciles, profitability (40)

Ato <sup>q12</sup>	Quarterly asset turnover, 12-month horizon	Cto <sup>q1</sup>	Quarterly capital turnover, 1-month horizon
Cto <sup>q6</sup>	Quarterly capital turnover, 6-month horizon	Cto <sup>q12</sup>	Quarterly capital turnover, 12-month horizon
Gpa	Gross profits-to-assets, Novy-Marx (2013)	Gla <sup>q1</sup>	Gross profits-to-lagged assets, 1-month horizon
Gla <sup>q6</sup>	Gross profits-to-lagged assets, 6-month horizon	Gla <sup>q12</sup>	Gross profits-to-lagged assets, 12-month horizon
Ole <sup>q1</sup>	Operating profits-to-lagged equity, 1-month horizon	Ole <sup>q6</sup>	Operating profits-to-lagged equity , 6-month horizon
Opa	Operating profits-to-assets, Ball, Gerakos, Linnainmaa, and Nikolaev (2015)	Ola <sup>q1</sup>	Operating profits-to- lagged assets, 1-month horizon
Ola <sup>q6</sup>	Operating profits-to-lagged assets, 6-month horizon	Ola <sup>q12</sup>	Operating profits-to- lagged assets, 12-month horizon

# Stress Tests

## Testing deciles, profitability (40)

Cop	Cash-based operating profitability, Ball, Gerakos, Linnainmaa, and Nikolaev (2016)	Cl	Cash-based operating profits-to-lagged assets
Cl <sup>q1</sup>	Cash-based operating profits-to-lagged assets, 1-month horizon	Cl <sup>q6</sup>	Cash-based operating profits-to-lagged assets, 6-month horizon
Cl <sup>q12</sup>	Cash-based operating profits-to-lagged assets, 12-month horizon	F <sup>q1</sup>	Quarterly F-score, 1-month horizon
F <sup>q6</sup>	Quarterly F-score, 6-month horizon	F <sup>q12</sup>	Quarterly F-score, 12-month horizon
Fp <sup>q6</sup>	Failure probability, 6-month horizon Campbell, Hilscher, and Szilagyi (2008)	O <sup>q1</sup>	Quarterly O-score 1-month horizon
Tbi <sup>q12</sup>	Quarterly taxable income-to-book income 12-month horizon	Sg <sup>q1</sup>	Quarterly sales growth 1-month horizon



# Stress Tests

## Testing deciles, intangibles (27)

Oca	Organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)	loca	Industry-adjusted organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)
Adm	Advertising expense-to-market, Chan, Lakonishok, and Sougiannis (2001)	Rdm	<b>R&amp;D-to-market</b> , Chan, Lakonishok, and Sougiannis (2001)
Rdm <sup>q1</sup>	<b>Quarterly R&amp;D-to-market</b> , 1-month horizon	Rdm <sup>q6</sup>	Quarterly R&D-to-market, 6-month horizon
Rdm <sup>q12</sup>	Quarterly R&D-to-market, 12-month horizon	Rds <sup>q6</sup>	Quarterly R&D-to-sales 6-month horizon
Rds <sup>q12</sup>	Quarterly R&D-to-sales, 12-month horizon	OI	Operating leverage, Novy-Marx (2011)
OI <sup>q1</sup>	Quarterly operating leverage, 1-month horizon	OI <sup>q6</sup>	Quarterly operating leverage, 6-month horizon
OI <sup>q12</sup>	Quarterly operating leverage, 12-month horizon	Hs	Industry concentration (sales), Hou and Robinson (2006)
Rer	Real estate ratio, Tuzel (2010)	Eprd	Earnings predictability, Francis, Lafond, Olsson, and Schipper (2004)

# Stress Tests

## Testing deciles, intangibles and trading frictions (3)

Etl	Earnings timeliness, Francis, Lafond, Lafond, Olsson, and Schipper (2004)	Alm <sup>q</sup> 1	Asset liquidity (market assets) 1-month horizon
Alm <sup>q</sup> 6	Asset liquidity (market assets) 6-month horizon	Alm <sup>q</sup> 12	Asset liquidity (market assets) 12-month horizon
$R_a^1$	12-month-lagged return Heston and Sadka (2008)	$R_n^1$	Year 1-lagged return, nonannual Heston and Sadka (2008)
$R_a^{[2,5]}$	Years 2–5 lagged returns, annual Heston and Sadka (2008)	$R_a^{[6,10]}$	Years 6–10 lagged returns, annual Heston and Sadka (2008)
$R_n^{[6,10]}$	Years 6–10 lagged returns, nonannual Heston and Sadka (2008)	$R_a^{[11,15]}$	Years 11–15 lagged returns, annual Heston and Sadka (2008)
$R_a^{[16,20]}$	Years 16–20 lagged returns, annual Heston and Sadka (2008)		

### Trading frictions (3)

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Dtv12	Dollar trading volume, 12-month horizon, Brennan, Chordia, and Subrahmanyam (1998))	lsff1	Idiosyncratic skewness per the FF 3-factor model, 1-month horizon
lsq1	Idiosyncratic skewness, per the $q$ -factor model, 1-month horizon		

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	All (150)				
$q$	0.28	52	25	0.11	101
$q^5$	0.19	23	6	0.10	57
FF5	0.43	100	69	0.13	112
FF6	0.30	74	37	0.11	91
FF6 <sub>c</sub>	0.27	59	25	0.11	71
BS6	0.29	63	37	0.13	132
SY4	0.29	64	25	0.11	87
DHS	0.37	70	33	0.14	97

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	Momentum (39)				
$q$	0.25	11	3	0.10	24
$q^5$	0.17	4	1	0.09	15
FF5	0.62	37	29	0.15	36
FF6	0.27	19	6	0.10	21
FF6 <sub>c</sub>	0.24	14	5	0.09	18
BS6	0.23	12	4	0.12	33
SY4	0.32	19	6	0.10	23
DHS	0.25	10	3	0.14	26

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	Value-versus-growth (15)				
$q$	0.21	1	0	0.11	8
$q^5$	0.22	3	0	0.13	7
FF5	0.15	2	0	0.10	7
FF6	0.19	4	0	0.10	9
FF6 <sub>c</sub>	0.17	3	0	0.10	6
BS6	0.23	6	2	0.13	14
SY4	0.24	4	1	0.12	9
DHS	0.78	15	13	0.23	15

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$ \overline{\alpha_{H-L}} $	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$ \overline{\alpha} $	$\#_{p < 5\%}^{\text{GRS}}$
	Investment (26)				
$q$	0.22	9	4	0.10	19
$q^5$	0.10	1	0	0.08	6
FF5	0.24	10	7	0.09	17
FF6	0.22	10	6	0.09	16
FF6 <sub>c</sub>	0.18	8	2	0.08	7
BS6	0.22	8	6	0.11	24
SY4	0.19	8	3	0.09	17
DHS	0.34	20	4	0.10	22

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	Profitability (40)				
$q$	0.25	16	6	0.10	28
$q^5$	0.14	5	1	0.09	14
FF5	0.43	32	23	0.12	32
FF6	0.31	26	13	0.10	25
FF6 <sub>c</sub>	0.26	18	7	0.10	21
BS6	0.31	20	12	0.12	37
SY4	0.29	20	9	0.10	24
DHS	0.18	6	1	0.09	13

# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	Intangibles (27)				
$q$	0.47	13	11	0.18	19
$q^5$	0.36	8	4	0.15	13
FF5	0.50	17	9	0.16	18
FF6	0.48	13	11	0.17	18
FF6 <sub>c</sub>	0.50	14	11	0.17	18
BS6	0.49	15	11	0.20	21
SY4	0.38	11	6	0.15	12
DHS	0.60	16	10	0.19	18



# Stress Tests

Relative performance of factor models

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
Trading frictions (3)					
$q$	0.24	2	1	0.10	3
$q^5$	0.19	2	0	0.08	2
FF5	0.22	2	1	0.07	2
FF6	0.20	2	1	0.07	2
FF6 <sub>c</sub>	0.20	2	0	0.07	1
BS6	0.23	2	2	0.09	3
SY4	0.18	2	0	0.09	2
DHS	0.50	3	2	0.18	3

All (150):  $\bar{R} = 1.69$  ( $t = 9.62$ )

	$\alpha_{H-L}$	$t_{H-L}$	$ \bar{\alpha} $	$p_{GRS}$
$q$	0.86	5.64	0.16	0.00
$q^5$	0.37	2.62	0.10	0.01
FF5	1.33	7.94	0.25	0.00
FF6	0.94	7.46	0.16	0.00
FF6c	0.82	6.77	0.14	0.00
BS6	0.68	4.85	0.13	0.00
SY4	0.90	7.61	0.16	0.00
DHS	0.74	4.98	0.14	0.00

Momentum (39):  $\bar{R} = 1.09$  ( $t = 4.21$ )

	$\alpha_{H-L}$	$t_{H-L}$	$ \bar{\alpha} $	$\rho_{GRS}$
$q$	0.35	1.04	0.10	0.08
$q^5$	-0.25	-0.85	0.10	0.35
FF5	1.21	3.74	0.27	0.00
FF6	0.33	2.08	0.09	0.06
FF6c	0.29	1.82	0.10	0.04
BS6	0.21	1.26	0.09	0.07
SY4	0.43	1.93	0.10	0.01
DHS	-0.36	-1.49	0.16	0.00

Value-versus-growth (15):  $\bar{R} = 0.70$  ( $t = 3.47$ )

	$\alpha_{H-L}$	$t_{H-L}$	$ \bar{\alpha} $	$p_{GRS}$
$q$	0.28	1.48	0.13	0.00
$q^5$	0.38	2.14	0.16	0.00
FF5	0.04	0.30	0.11	0.00
FF6	0.19	1.58	0.10	0.00
FF6c	0.12	1.05	0.10	0.00
BS6	-0.16	-1.17	0.12	0.00
SY4	0.34	2.20	0.14	0.00
DHS	0.98	5.34	0.31	0.00

Investment (26):  $\bar{R} = 0.66$  ( $t = 4.44$ )

	$\alpha_{H-L}$	$t_{H-L}$	$ \bar{\alpha} $	$\rho_{GRS}$
$q$	0.25	2.61	0.10	0.00
$q^5$	0.06	0.54	0.06	0.15
FF5	0.29	3.11	0.08	0.00
FF6	0.27	2.84	0.07	0.01
FF6c	0.27	2.62	0.06	0.06
BS6	0.18	1.73	0.09	0.00
SY4	0.10	1.00	0.07	0.01
DHS	0.55	3.83	0.12	0.00

Profitability (40):  $\bar{R} = 0.80$  ( $t = 4.64$ )

	$\alpha_{H-L}$	$t_{H-L}$	$ \bar{\alpha} $	$p_{GRS}$
$q$	0.28	2.31	0.07	0.01
$q^5$	-0.14	-1.21	0.08	0.09
FF5	0.60	5.35	0.12	0.00
FF6	0.43	3.94	0.09	0.00
FF6c	0.30	2.30	0.07	0.09
BS6	0.34	2.61	0.09	0.00
SY4	0.37	2.86	0.09	0.00
DHS	-0.09	-0.56	0.07	0.35

Intangibles (27):  $\bar{R} = 0.94$  ( $t = 5.27$ )

	$\alpha_{H-L}$	$t_{H-L}$	$ \bar{\alpha} $	$\rho_{GRS}$
$q$	0.42	2.62	0.18	0.00
$q^5$	0.50	3.19	0.19	0.00
FF5	0.43	3.24	0.18	0.00
FF6	0.54	4.25	0.20	0.00
FF6c	0.57	4.17	0.21	0.00
BS6	0.26	1.85	0.15	0.00
SY4	0.46	3.16	0.18	0.00
DHS	0.89	5.24	0.28	0.00

Trading frictions (3):  $\bar{R} = 0.23$  ( $t = 1.77$ )

	$\alpha_{H-L}$	$t_{H-L}$	$ \bar{\alpha} $	$\rho_{GRS}$
$q$	0.16	1.80	0.10	0.00
$q^5$	0.15	1.60	0.08	0.06
FF5	0.14	1.80	0.08	0.05
FF6	0.12	1.53	0.07	0.07
FF6c	0.12	1.34	0.06	0.28
BS6	0.14	1.60	0.11	0.00
SY4	0.13	1.50	0.09	0.01
DHS	0.57	4.29	0.13	0.00



- 1 The Expected Growth Factor
- 2 Stress-testing Factor Models
- 3 Examples of Individual Factor Regressions**

# Individual Factor Regressions

Examples, 1/1967–12/2018

	Sue1	R <sup>6</sup>	Bm	Oa	dFin	Dac	Cop	Rdm
$\bar{R}$	0.45	0.83	0.43	-0.29	0.27	-0.45	0.68	0.73
$t_{\bar{R}}$	3.50	3.66	2.14	-2.36	2.43	-3.47	3.94	2.96
$\alpha_q$	0.05	0.30	0.11	-0.57	0.41	-0.74	0.75	0.81
$\alpha_{q^5}$	-0.07	-0.16	0.05	-0.20	0.14	-0.31	0.11	0.27
$t_q$	0.39	1.04	0.71	-4.25	2.97	-5.33	5.57	3.64
$t_{q^5}$	-0.52	-0.64	0.32	-1.30	0.97	-2.16	0.96	1.24
$\alpha_{FF6}$	0.26	0.19	-0.09	-0.48	0.46	-0.69	0.75	0.68
$\alpha_{FF6c}$	0.22	0.16	-0.09	-0.32	0.34	-0.59	0.55	0.79
$t_{FF6}$	2.23	1.92	-0.82	-3.49	3.81	-5.08	6.44	3.24
$t_{FF6c}$	1.84	1.57	-0.74	-2.13	2.63	-4.12	4.75	3.64

The  $q^5$  model outperforms the Fama-French 6-factor model in a large-scale empirical horse race