

$q^5$

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Augmenting the  $q$ -factor model with an expected growth factor to form **the  $q^5$  model**:

$$E[R_i - R_f] = \beta_{\text{MKT}}^i E[\text{MKT}] + \beta_{\text{Me}}^i E[R_{\text{Me}}] \\ + \beta_{\text{I/A}}^i E[R_{\text{I/A}}] + \beta_{\text{Roe}}^i E[R_{\text{Roe}}] + \beta_{\text{Eg}}^i E[R_{\text{Eg}}]$$

The  $q^5$  model outperforms the Fama-French 6-factor model in explaining a large set of 150 significant anomalies

- 1 The Expected Growth Factor
- 2 Stress-testing Factor Models
- 3 Examples of Individual Factor Regressions

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# The Expected Growth Factor

Economic foundation, Cochrane (1991)

In the multiperiod investment framework:

$$r_{t+1} \approx \frac{X_{t+1} + (1 - \delta) [1 + a (I_{t+1}/A_{t+1})]}{1 + a (I_t/A_t)}$$

The “dividend yield” component,  $X_{t+1}/[1 + a (I_t/A_t)]$ , motivates the  $q$ -factor model

The “capital gain” component roughly proportional to investment-to-assets growth,  $(I_{t+1}/A_{t+1}) / (I_t/A_t)$

# The Expected Growth Factor

Monthly cross-sectional forecasting regressions

Forecast  $\tau$ -year-ahead investment-to-assets changes, with 3 predictors

Tobin's  $q$ : Erickson and Whited (2000)

Cash flows: Internal funds available for investments, Fazzari, Hubbard, and Petersen (1988); better than earnings in capturing the expected growth, likely due to intangibles, Ball et al. (2016)

dRoe: Capturing short-term dynamics of investment growth, Liu, Whited, and Zhang (2009)

# The Expected Growth Factor

Key results

$\tau$	$\log(q)$	Cop	dRoe	$R^2$	Pearson	Rank
1	-0.03 (-5.63)	0.52 (12.75)	0.77 (7.62)	6.42	0.14 [0.00]	0.21 [0.00]

$E_t[d^1|/A]$  and  $d^1|/A$  aligned at the portfolio level (Corr = 0.64):

	Low	2	3	4	5	6	7	8	9	High	H-L
$E_t[d^1 /A]$	-15.2	-7.7	-5.6	-4.2	-3.0	-2.0	-0.9	0.5	2.5	7.7	22.9
$d^1 /A$	-16.7	-12.3	-4.1	-3.6	-1.1	-0.4	-0.3	0.6	1.6	6.0	22.7

$R_{Eg}$ , independent  $2 \times 3$  monthly sorts on size and  $E_t[d^1|/A]$ :

$\bar{R}_{Eg}$	$\alpha$	$\beta_{Mkt}$	$\beta_{Me}$	$\beta_{ /A}$	$\beta_{Roe}$	$R^2$
0.84 (10.27)	0.67 (9.75)	-0.11 (-6.38)	-0.09 (-3.56)	0.21 (4.86)	0.30 (9.13)	0.44

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The playing field, the right-hand side, 8 competing factor models

- The  $q$ -factor model, the  $q^5$  model
- The Fama-French 5-factor model, the 6-factor model, the alternative 6-factor model with RMWc
- The **replicated** Stambaugh-Yuan 4-factor model
- The Barillas-Shanken 6-factor model, including MKT, SMB,  $R_{I/A}$ ,  $R_{Roe}$ , the Asness-Frazzini monthly formed HML, UMD
- The **replicated** Daniel-Hirshleifer-Sun 3-factor model

Monthly Sharpe ratios of factor models, 1/1967–12/2018

$q$	$q^5$	FF5	FF6	FF6c	BS6	SY4	DHS
0.42	0.63	0.32	0.37	0.43	0.48	0.41	0.42

150 anomalies with NYSE breakpoints and value-weighted returns significant at the 5% level (Hou, Xue, and Zhang 2019)

- Momentum: 39
- Value-versus-growth: 15
- Investment: 26
- Profitability: 40
- Intangibles: 27
- Trading frictions: 3

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	All (150)				
$q$	0.28	52	25	0.11	101
$q^5$	0.19	23	6	0.10	57
FF5	0.43	100	69	0.13	112
FF6	0.30	74	37	0.11	91
FF6 <sub>c</sub>	0.27	59	25	0.11	71
BS6	0.29	63	37	0.13	132
SY4	0.29	64	25	0.11	87
DHS	0.37	70	33	0.14	97

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	Momentum (39)				
$q$	0.25	11	3	0.10	24
$q^5$	0.17	4	1	0.09	15
FF5	0.62	37	29	0.15	36
FF6	0.27	19	6	0.10	21
FF6 <sub>c</sub>	0.24	14	5	0.09	18
BS6	0.23	12	4	0.12	33
SY4	0.32	19	6	0.10	23
DHS	0.25	10	3	0.14	26

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	Value-versus-growth (15)				
$q$	0.21	1	0	0.11	8
$q^5$	0.22	3	0	0.13	7
FF5	0.15	2	0	0.10	7
FF6	0.19	4	0	0.10	9
FF6 <sub>c</sub>	0.17	3	0	0.10	6
BS6	0.23	6	2	0.13	14
SY4	0.24	4	1	0.12	9
DHS	0.78	15	13	0.23	15

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{\text{GRS}}$
	Investment (26)				
$q$	0.22	9	4	0.10	19
$q^5$	0.10	1	0	0.08	6
FF5	0.24	10	7	0.09	17
FF6	0.22	10	6	0.09	16
FF6 <sub>c</sub>	0.18	8	2	0.08	7
BS6	0.22	8	6	0.11	24
SY4	0.19	8	3	0.09	17
DHS	0.34	20	4	0.10	22

# Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{ \alpha_{H-L} }$	$\#_{ t  \geq 1.96}$	$\#_{ t  \geq 3}$	$\overline{ \alpha }$	$\#_{p < 5\%}^{GRS}$
	Profitability (40)				
$q$	0.25	16	6	0.10	28
$q^5$	0.14	5	1	0.09	14
FF5	0.43	32	23	0.12	32
FF6	0.31	26	13	0.10	25
FF6 <sub>c</sub>	0.26	18	7	0.10	21
BS6	0.31	20	12	0.12	37
SY4	0.29	20	9	0.10	24
DHS	0.18	6	1	0.09	13

# Stress Tests

Explaining the composite score deciles, 1/1967–12/2018

	$\alpha_{H-L}$	$t_{H-L}$	$ \bar{\alpha} $	$p_{GRS}$
	All (150): $\bar{R} = 1.69$ ( $t = 9.62$ )			
$q$	0.86	5.64	0.16	0.00
$q^5$	0.37	2.62	0.10	0.01
FF5	1.33	7.94	0.25	0.00
FF6	0.94	7.46	0.16	0.00
FF6c	0.82	6.77	0.14	0.00
BS6	0.68	4.85	0.13	0.00
SY4	0.90	7.61	0.16	0.00
DHS	0.74	4.98	0.14	0.00



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# Individual Factor Regressions

Examples, 1/1967–12/2018

	Sue1	$R^6$	Bm	Oa	dFin	Dac	Rdm
$\bar{R}$	0.45	0.83	0.43	-0.29	0.27	-0.45	0.73
$t_{\bar{R}}$	3.50	3.66	2.14	-2.36	2.43	-3.47	2.96
$\alpha_q$	0.05	0.30	0.11	-0.57	0.41	-0.74	0.81
$\alpha_{q^5}$	-0.07	-0.16	0.05	-0.20	0.14	-0.31	0.27
$t_q$	0.39	1.04	0.71	-4.25	2.97	-5.33	3.64
$t_{q^5}$	-0.52	-0.64	0.32	-1.30	0.97	-2.16	1.24
$\alpha_{FF6}$	0.26	0.19	-0.09	-0.48	0.46	-0.69	0.68
$\alpha_{FF6c}$	0.22	0.16	-0.09	-0.32	0.34	-0.59	0.79
$t_{FF6}$	2.23	1.92	-0.82	-3.49	3.81	-5.08	3.24
$t_{FF6c}$	1.84	1.57	-0.74	-2.13	2.63	-4.12	3.64

The  $q^5$  model outperforms the Fama-French 6-factor model in a large-scale empirical horse race