Lecture Notes

Petrosky-Nadeau and Zhang (2017, Quantitative Economics, "Solving the Diamond-Mortensen-Pissarides Model Accurately")

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An accurate global projection algorithm is critical for quantifying the basic moments of the Diamond–Mortensen–Pissarides model

- Log linearization understates the mean and volatility of unemployment but overstates the volatility of labor market tightness and the unemployment–vacancy correlation
- Log linearization also understates the impulse responses in unemployment in recessions but overstates the responses in the market tightness in booms

Outline

Outline

A representative household with perfect consumption insurance: The household pools the income of all the members together before choosing per capita consumption and asset holdings

Risk neutral with a time discount factor β

A representative firm uses labor as the single productive input

The matching function:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}$$

in which $\iota > 0$

Define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio

The job finding rate:

$$f_t = f(\theta_t) = \frac{G(U_t, V_t)}{U_t} = \frac{1}{\left(1 + \theta_t^{-\iota}\right)^{1/\iota}}$$

The vacancy filling rate:

$$q_t = q(\theta_t) = \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^{\iota})^{1/\iota}}$$

with $q'(\theta_t) < 0$

The firm uses labor to produce output, Y_t:

$$Y_t = X_t N_t$$

Aggregate labor productivity, X_t, with $x_t \equiv \log(X_t)$, follows:

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}$$

in which $\rho \in (0, 1)$, $\sigma > 0$, and ϵ_{t+1} an i.i.d. standard normal shock

Unit costs in posting vacancies:

$$\kappa_t = \kappa_K X_t + \kappa_W X_t^{\xi}$$

in which $\kappa_K, \kappa_W, \xi > 0$

Employment, N_t, evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t$$

in which vacancies $V_t \geq 0$

The wage rate from a Nash bargaining process between the employed workers and the firm:

$$W_t = \eta\left(X_t + \kappa_t\theta_t\right) + (1 - \eta)b$$

in which $\eta \in (0, 1)$ the workers' relative bargaining weight and b the workers' flow value of unemployment activities

Dividends: $D_t = X_t N_t - W_t N_t - \kappa_t V_t$

The goods market clears:

$$C_t + \kappa_t V_t = X_t N_t$$

The intertemporal job creation condition:

$$\frac{\kappa_t}{q(\theta_t)} - \lambda_t = E_t\left[\beta\left(X_{t+1} - W_{t+1} + (1-s)\left(\frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda_{t+1}\right)\right)\right]$$

The Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0$$

Solve for labor market tightness, $\theta_t = \theta(x_t)$, and the multiplier function, $\lambda_t = \lambda(x_t)$ from the intertemporal job creation condition

$\theta(x_t)$ and $\lambda(x_t)$ must also satisfy the Kuhn-Tucker condition

We approximate the conditional expectation in the right-hand side of the job creation condition as $\mathcal{E}_t \equiv \mathcal{E}(x_t)$

After obtaining $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) \equiv \kappa_t / \mathcal{E}_t$

If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$, and then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$

If $\tilde{q}(\theta_t) \geq 1$, we set $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_t - \mathcal{E}_t$

Approximate the persistent log productivity process, x_t, based on the Rouwenhorst (1995) method

Use 17 grid points to cover the values of x_t, which are precisely within four unconditional standard deviations above and below the unconditional mean of zero

The conditional expectation calculated via matrix multiplication

To obtain an initial guess of the $\mathcal{E}(x_t)$ function, we use the model's loglinear solution via Dynare

Approximate the $\mathcal{E}(x_t)$ function (within four unconditional standard deviations of x_t from its unconditional mean of zero) with tenth-order Chebychev polynomials

The Chebychev nodes obtained with the collocation method

Use the Miranda-Fackler (2002) CompEcon toolbox for function approximation and interpolation

The conditional expectation in the right hand side of the job creation equation computed with the Gauss-Hermite quadrature

The time discount factor, β, $0.99^{1/12}$

The persistence of log productivity, ρ, 0.9895, and its conditional volatility, σ, 0.0034

The workers' bargaining weight, η, 0.052

Flow value of unemployment activities, b, 0.955

The job separation rate, s, 0.0081

The elasticity of the matching function, ι, 0.407

For the vacancy cost function, the capital cost parameter, κ_K, 0.474, the labor cost parameter, κ_W, 0.11, and the exponential parameter in the labor cost, ξ, 0.449

The Hagedorn-Manovskii Model

Figure 1: The conditional expectation and labor market tightness

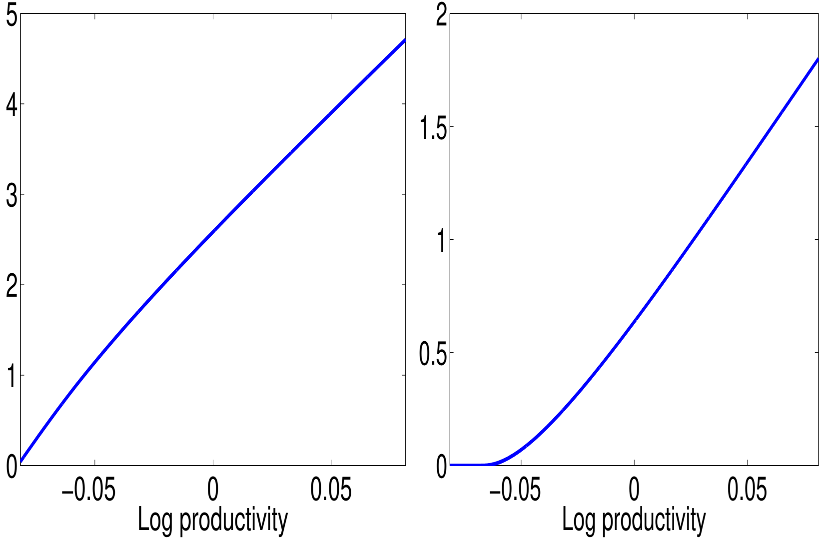


Table 1: Labor market moments

	U	V	θ	X		U	V	θ	X
	HM (2008, Table 4)					Loglinearization			
Std	0.145	0.169	0.292	0.013		0.133	0.144	0.327	0.013
ρ	0.830	0.575	0.751	0.765		0.831	0.681	0.783	0.760
Correlation		−0.724	−0.916	−0.892	U		−0.848	−0.864	−0.927
			0.940	0.904	V			0.858	0.985
				0.967	θ				0.890
	2nd-order perturbation					Projection			
Std	0.164	0.178	0.263	0.013		0.257	0.174	0.267	0.013
ρ	0.831	0.704	0.788	0.760		0.823	0.586	0.759	0.760
Correlation		−0.791	−0.794	−0.795	U		−0.567	−0.662	−0.699
			0.946	0.973	V			0.890	0.909
				0.993	θ				0.996

Figure 2: Nonlinear dynamics, projection vs. loglinearization

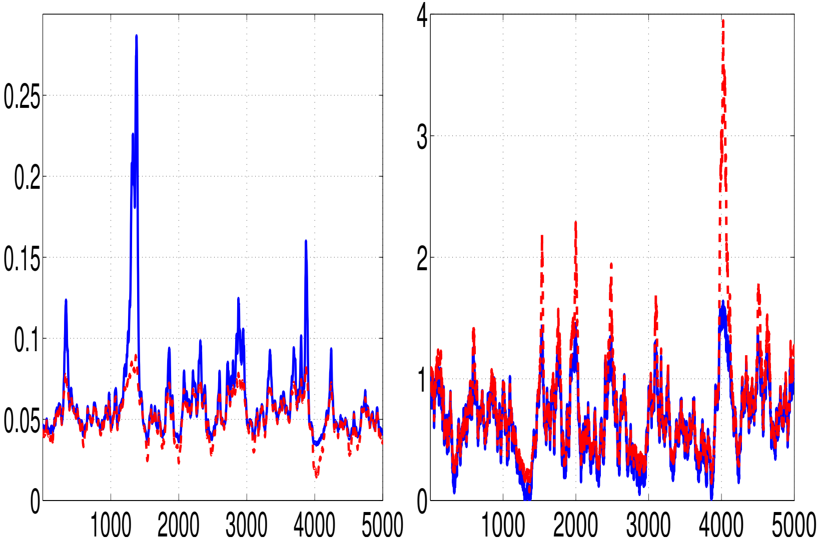
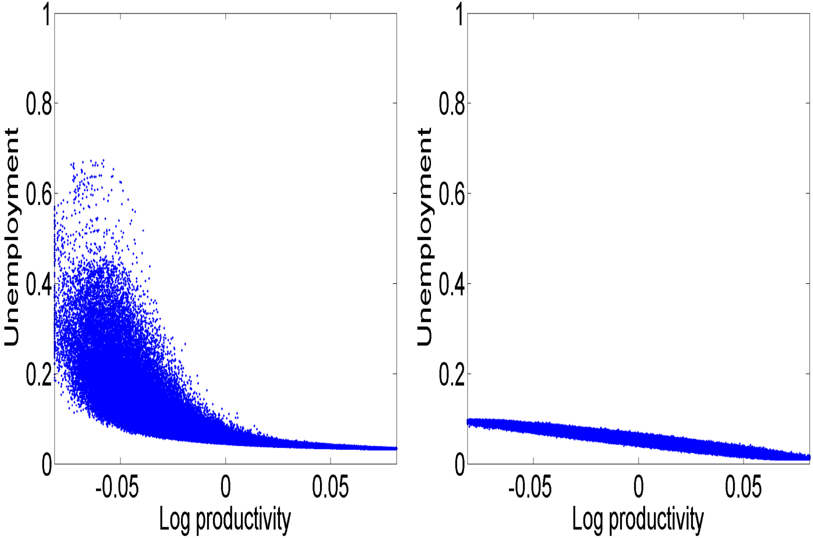
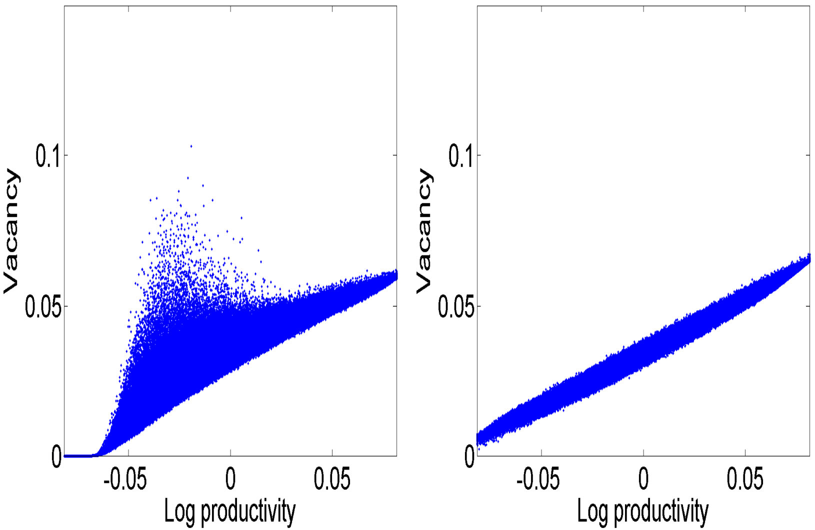


Figure 3: Ergodic distribution, U_t, projection vs. loglinearization



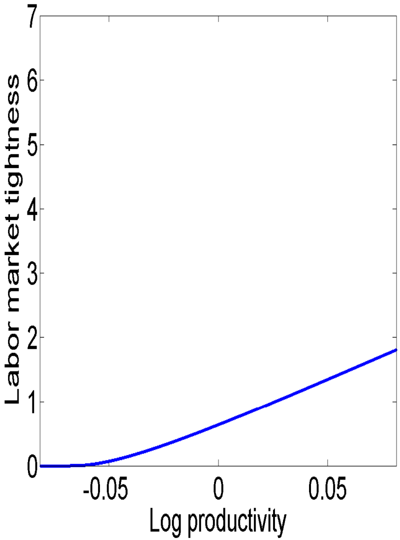
The Hagedorn-Manovskii Model

Figure 3: Ergodic distribution, V_t, projection vs. loglinearization



The Hagedorn-Manovskii Model

Figure 3: Ergodic distribution, θ_t, projection vs. loglinearization



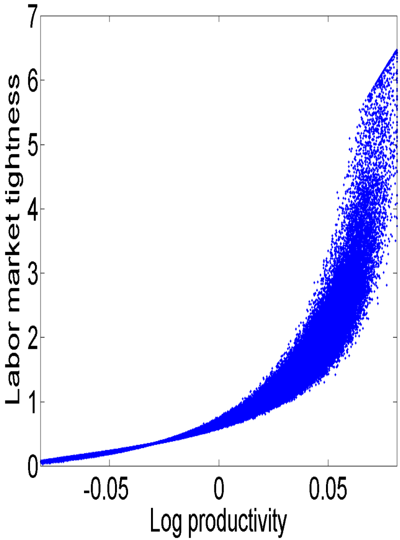


Figure 4: Nonlinear impulse response, U_t, projection vs. loglinearization

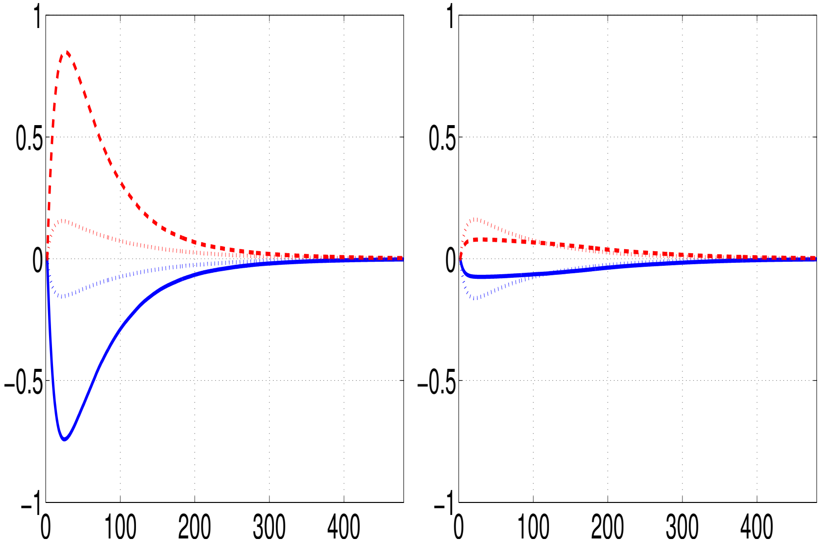


Figure 4: Nonlinear impulse response, θ_t, projection vs. loglinearization

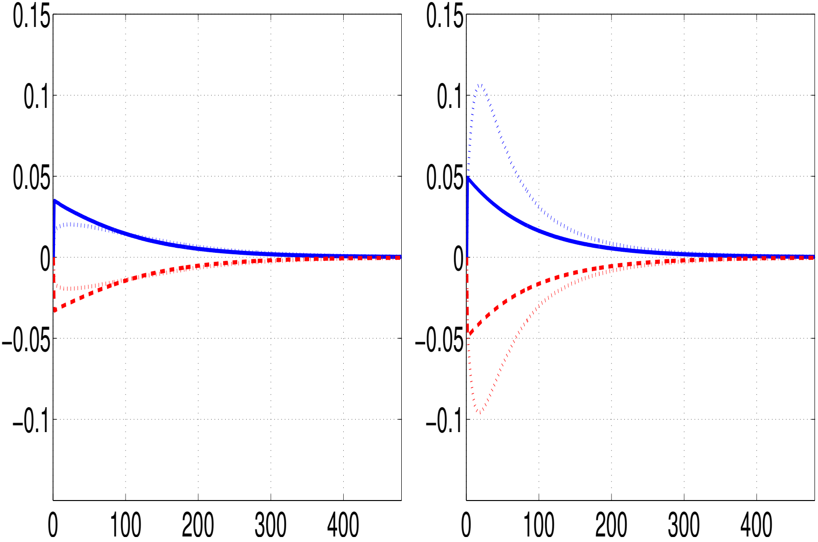


Figure 5: Euler equation errors in the state space

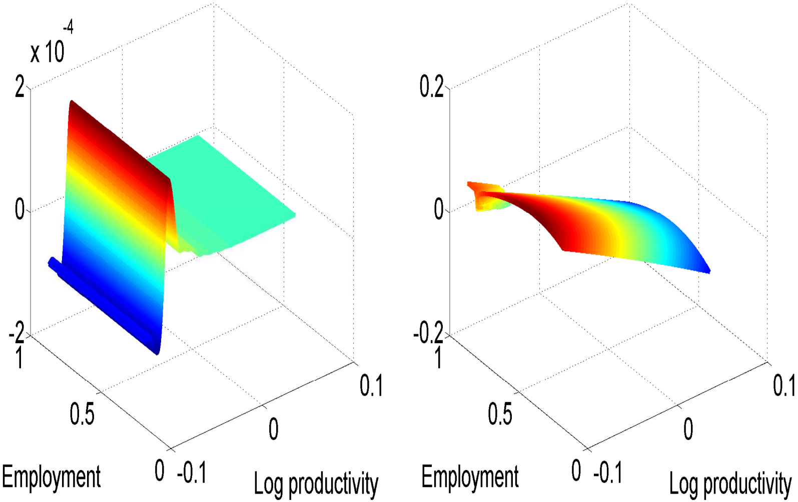
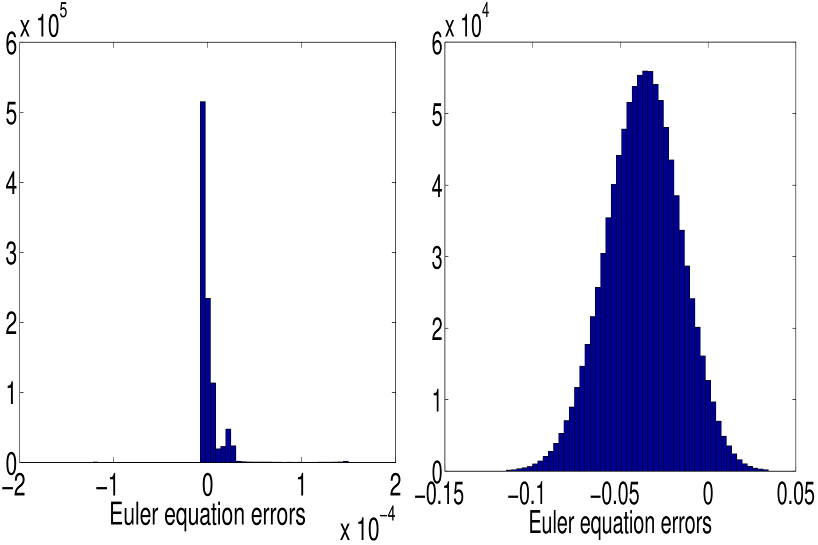


Figure 5: Euler equation errors in simulations



Outline

A representative household with log utility, $\log(C_t)$

A representative firm uses labor, N_t, and capital, K_t, to produce:

$$Y_t = X_t K_t^{\alpha} N_t^{1-\alpha}$$

in which $\alpha \in (0, 1)$ is capital's share

The log productivity, $x_t = \log(X_t)$, follows:

$$x_{t+1} = (1 - \rho)\bar{x} + \rho x_t + \sigma \epsilon_{t+1}$$

in which $\bar{x}$ is the unconditional mean of x_t

Rescale $\bar{x}$ to ensure the average MPL ≈ 1 in simulations

The matching function:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{1/\iota}}$$

Continue to impose $V_t \geq 0$; constant unit cost of vacancy posting

Capital accumulates as:

$$K_{t+1} = (1-\delta)K_t + \Phi(I_t, K_t)$$

in which δ the depreciation rate, I_t investment, and

$$\Phi(I_t, K_t) = \left[a_1 + \frac{a_2}{1-1/\nu}\left(\frac{I_t}{K_t}\right)^{1-1/\nu}\right]K_t, \qquad \nu > 0$$

The equilibrium wage, W_t, follows:

$$W_t = \eta \left[(1-\alpha)\frac{Y_t}{N_t} + \kappa\theta_t \right] + (1-\eta)b$$

Dividends: $D_t \equiv Y_t - W_t N_t - \kappa V_t - I_t$

In equilibrium, the market clears:

$$C_t + I_t + \kappa V_t = Y_t$$

The intertemporal job creation condition:

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t \left[M_{t+1} \left((1-\alpha)\frac{Y_{t+1}}{N_{t+1}} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda_{t+1} \right) \right) \right]$$

The investment Euler equation:

$$\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{1/\nu} = E_t \left[M_{t+1} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1/\nu} (1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}} \right) \right]$$

The Kuhn-Tucker conditions

Solve for $I(N_t, K_t, x_t)$ and $\mathcal{E}(N_t, K_t, x_t)$ from the optimality conditions

Discretize x_t with 17 grid points via the Rouwenhorst procedure

Finite element method, cubic splines, 100 nodes of N_t and on K_t

Tensor product of N_t and K_t on each x_t grid point

The Miranda-Fackler CompEcon toolbox for functional approximation and interpolation

Derivative-free fixed-point iteration with a small damping parameter to solve a system of 340,000 nonlinear equations

The time discount factor, $\beta = 0.99^{1/3}$

The persistence of log productivity, $\rho_x = 0.95^{1/3}$

Capital's weight, $\alpha = 1/3$, the depreciation rate, $\delta = 0.01$, and the separation rate, $s = 0.035$

The elasticity of the matching function, ι, 1.25

Choose the conditional volatility of the log productivity, $\sigma = 0.0065$, to match the output volatility of 2.17% per annum in the model

Choose the elasticity in the installation function, $\nu = 2$, to match the consumption volatility of 1.78% in the data

The workers' bargaining weight, η, 0.04

Flow value of unemployment activities, b, 0.95

The cost of vacancy posting, κ, 0.45

These values imply an average unemployment rate of 5.75% in the model, which is close to 5.87% in the data, and an unemployment volatility of 0.133, which is close to 0.132 in the data

Unit-free job creation equation errors:

$$e_t^V \equiv \left[\frac{\frac{\kappa}{q(\theta_t)} - \lambda_t}{E_t \left[\frac{\beta}{C_{t+1}} \left((1-\alpha)\frac{Y_{t+1}}{N_{t+1}} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda_{t+1} \right) \right) \right]} - C_t \right] / C_t$$

Unit-free investment Euler equation errors:

$$e_t^I \equiv \left[\frac{\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{1/\nu}}{E_t \left[\frac{\beta}{C_{t+1}} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1/\nu} (1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}} \right) \right]} - C_t \right] / C_t.$$

Figure 9: Job creation equation errors in simulations, projection vs. loglinearization

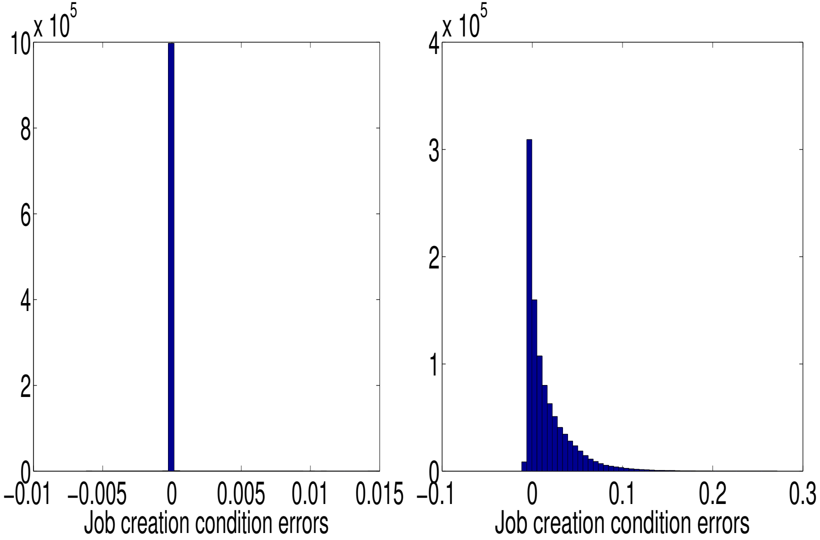


Figure 9: Investment Euler equation errors in simulations, projection vs. loglinearization

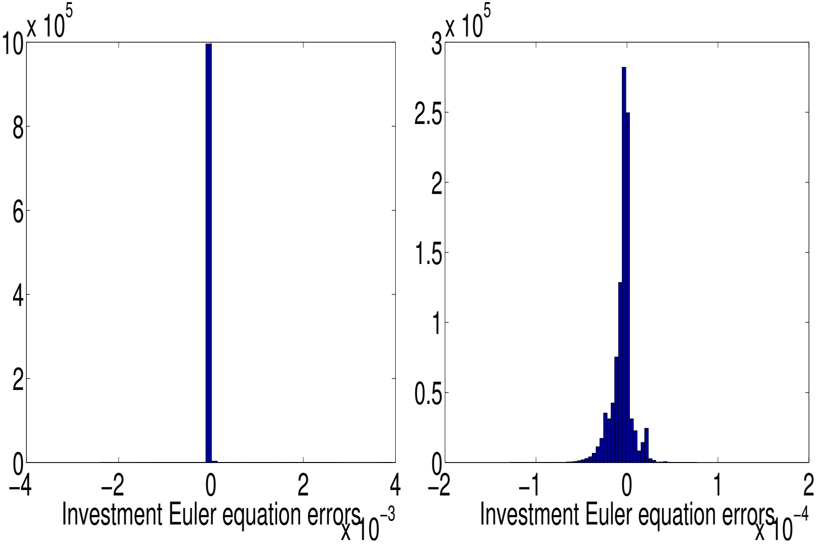


Figure 10: Ergodic distribution, projection vs. loglinearization

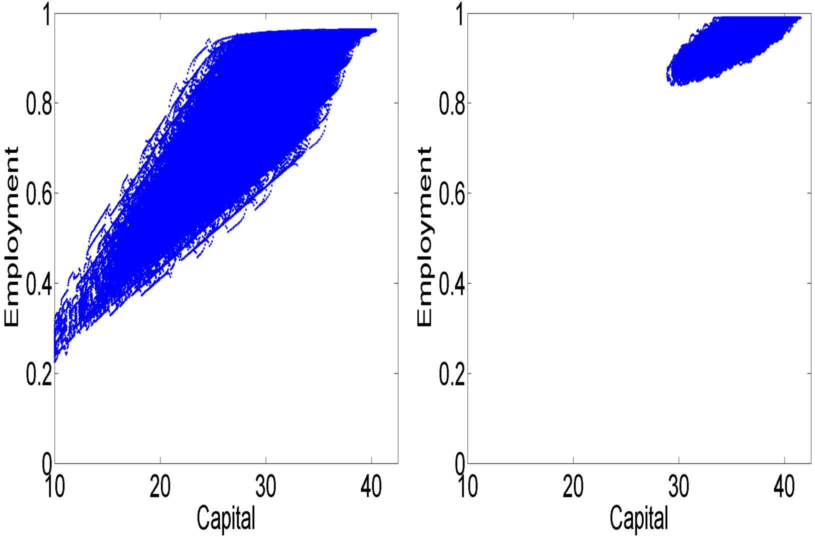


Figure 11: Ergodic distribution, U_t, projection vs. loglinearization

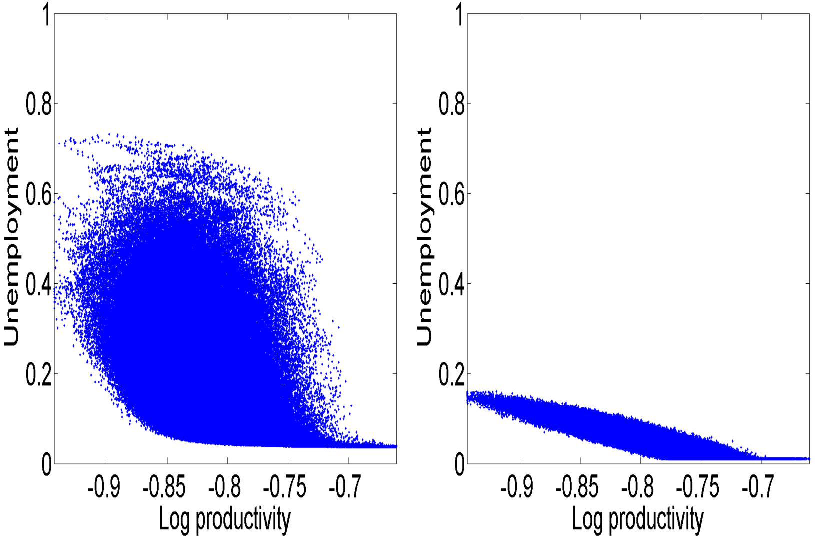


Figure 11: Ergodic distribution, V_t, projection vs. loglinearization

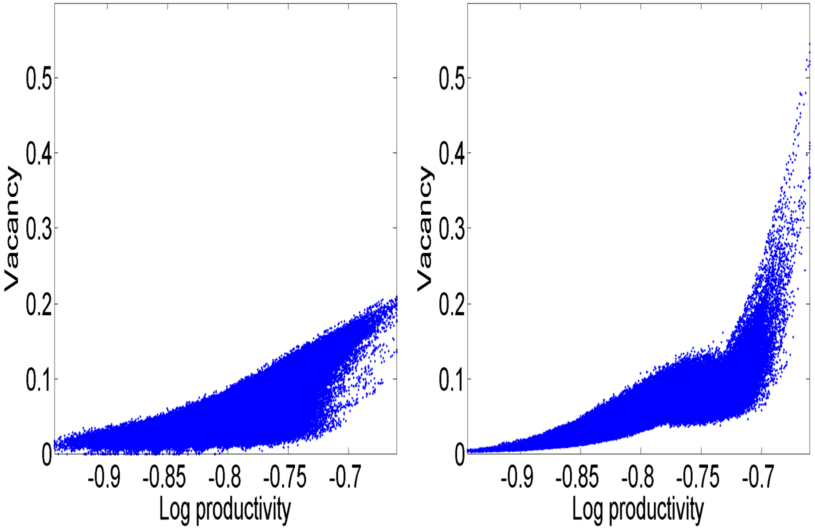
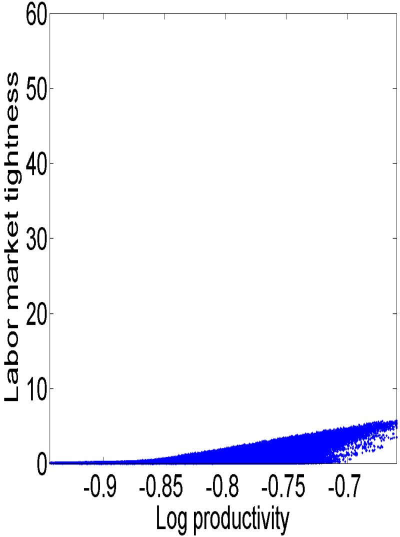


Figure 11: Ergodic distribution, θ_t, projection vs. loglinearization



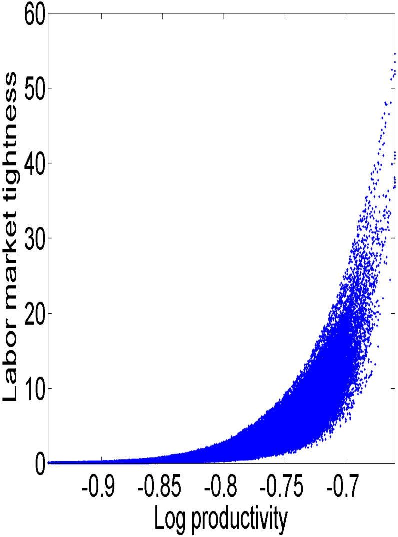


Table 3: Business cycle moments

	σ_Y	ρ_1^Y	ρ_2^Y	ρ_3^Y	ρ_4^Y	σ_C	ρ_1^C	ρ_2^C	ρ_3^C	ρ_4^C
Data	1.78	0.34	0.07	−0.05	0.06	2.17	0.15	0.01	−0.06	0.02
Loglinear	1.72	0.19	−0.07	−0.06	−0.06	2.41	0.18	−0.08	−0.07	−0.07
2nd-order	3.08	0.23	−0.07	−0.07	−0.06	8.38	0.18	−0.12	−0.09	−0.07
Projection	3.26	0.21	−0.08	−0.06	−0.06	2.60	0.23	−0.06	−0.05	−0.05

	σ_I	ρ_1^I	ρ_2^I	ρ_3^I	ρ_4^I	$E[U]$
Data	8.93	0.02	−0.16	−0.19	−0.10	5.87
Loglinear	3.26	0.16	−0.11	−0.09	−0.08	5.75
2nd-order	5.65	0.20	−0.10	−0.09	−0.07	16.40
Projection	4.45	0.19	−0.10	−0.08	−0.07	10.75

Table 4: Labor market moments

	U	V	θ	Y/N		U	V	θ	Y/N
	Data					Loglinearization			
Std	0.132	0.134	0.263	0.012		0.133	0.167	0.355	0.011
ρ	0.901	0.909	0.881	0.773		0.815	0.537	0.759	0.746
Correlation		−0.887	−0.830	−0.158	U		−0.536	−0.696	−0.881
			0.930	0.350	V			0.566	0.782
				0.240	θ				0.821
	2nd-order perturbation					Projection			
Std	0.238	1.222	0.770	0.031		0.158	0.158	0.254	0.010
ρ	0.852	0.611	0.720	0.779		0.844	0.588	0.763	0.657
Correlation		0.061	−0.153	0.346	U		−0.359	−0.473	−0.337
			0.859	0.795	V			0.899	0.983
				0.692	θ				0.930

An accurate global projection algorithm is critical for quantifying the basic moments of the Diamond–Mortensen–Pissarides model