Review of Finance, 2019, 1–35
doi: 10.1093/rof/rfy032
Advance Access Publication Date: 17 October 2018

Which Factors?*

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Abstract

Many recently proposed, seemingly different factor models are closely related. In spanning tests, the q-factor model largely subsumes the Fama–French five- and six-factor models, and the q^5 model subsumes the Stambaugh–Yuan four-factor model. Their "mispricing" factors are sensitive to the construction procedure, and once replicated via the traditional approach, are close to the q-factors, with correlations of 0.8 and 0.84. Finally, consistent with the investment CAPM, valuation theory predicts a positive relation between the expected investment and the expected return.

JEL classification: G12, G14

Keywords: Factor models, Spanning tests, The investment CAPM, Valuation theory

Received January 5, 2018; accepted September 27, 2018 by Editor Amit Goyal.

* This paper supersedes our prior working papers titled "A Comparison of New Factor Models" and "Motivating Factors." For helpful comments, we thank our discussants Ilan Cooper, Raife Giovinazzo, Serhiy Kozak, Kai Li, Scott Murray, David Ng, Christian Opp, Jay Shanken, Timothy Simin, and Zhenyu Wang, as well as Michael Brennan, David Chapman, Andrei Goncalves, Cam Harvey, Don Keim, Jim Kolari, Dongxu Li, Jim Poterba, Berk Sensoy, Rob Stambaugh, René Stulz, Sheridan Titman, Michael Weisbach, Tong Yao, and other seminar participants at Baruch College, Cheung Kong Graduate School of Business, Georgia Institute of Technology, Guanghua School of Management at Peking University, PBC School of Finance at Tsinghua University, Shanghai University of Finance and Economics, Seoul National University, Texas A&M University, The Ohio State University, University of Iowa, University of Miami, University of Missouri, and University of Southern California, as well as the 2015 Arizona State University Sonoran Winter Finance Conference, the 2015 Chicago Quantitative Alliance Annual Academic Competition, the 2015 Financial Intermediation Research Society Conference, the 2015 Florida State University SunTrust Beach Conference, the 2015 Rodney L. White Center for Financial Research Conference on Financial Decisions and Asset Market at Wharton, the 2015 Society for Financial Studies Finance Cavalcade, the 2015 University of British Columbia Summer Finance Conference, the 2016 Hong Kong University of Science and Technology Finance Symposium, the 27th Annual Conference on Financial Economics and Accounting, and the 7th McGill Global Asset Management Conference. Amit Goyal (the Editor) and two anonymous referees deserve special thanks. All remaining errors are our own.

1. Introduction

A new generation of factor pricing models has emerged in the cross-section of expected returns, including the Hou–Xue–Zhang (2015) four-factor q model and the Hou *et al.* (2018) five-factor q^5 model, the Fama–French (2015, 2018) five- and six-factor models, the Stambaugh–Yuan (2017) four-factor model, the Barillas–Shanken (2018) six-factor model, and the Daniel–Hirshleifer–Sun (2018) three-factor model. In this paper, we compare the new factor models on both empirical and conceptual grounds.

We show that the seemingly different factor models are in fact closely related. In factor spanning tests, the q-factor and q^5 models largely subsume the Fama–French five- and six-factor premiums. From January 1967 to December 2016, the average premiums of the value, investment, profitability, and momentum factors (HML, CMA, RMW, and UMD) are 0.37%, 0.33%, 0.26%, and 0.65% per month ($t = 2.71$, 3.51, 2.5, and 3.61), respectively. However, their q-factor alphas are tiny, 0.07%, –0.00%, 0.01%, and 0.12% ($t = 0.62, -0.02$, 0.08, and 0.5), and the q^5 alphas 0.05%, –0.04%, –0.01%, and –0.16% ($t = 0.48, -0.96, -0.16$, and –0.78), respectively. The cash-based profitability factor, RMWc, earns on average 0.33% ($t = 4.16$), with a q-factor alpha of 0.25% ($t = 3.83$) and a q^5 alpha of 0.14% ($t = 2.18$). The Gibbons, Ross, and Shanken (1989) test cannot reject the q-factor or the q^5 model based on the null that the alphas of HML, CMA, RMW, and UMD are jointly zero. Although the test rejects the q-factor model based on the null that the alphas of HML, CMA, RMWc, and UMD are jointly zero, it fails to reject the q^5 model (p-value = 0.13).

Conversely, the Fama–French five- and six-factor models cannot explain the q and q^5 factor premiums. The investment, return on equity (Roe), and expected growth factors in the q-factor and q^5 models are on average 0.41%, 0.55%, and 0.82% per month ($t = 4.92$, 5.25, and 9.81); their Fama–French five-factor alphas 0.12%, 0.47%, and 0.78% ($t = 3.44$, 5.94, and 11.34); the six-factor alphas 0.11%, 0.3%, and 0.7% ($t = 3.11$, 4.51, and 11.1); and the alphas from the alternative six-factor model with RMWc 0.11%, 0.23%, and 0.61% ($t = 2.78$, 2.8, and 9.33), respectively. The Gibbons–Ross–Shanken test strongly rejects the Fama–French five- and six-factor models based on the null that the alphas of the investment and Roe factors (with or without the expected growth factor) are jointly zero.

Deviating from the traditional approach per Fama and French (1993), Stambaugh and Yuan (2017) use the NYSE, Amex, and NASDAQ breakpoints of the 20 and 80 percentiles when forming their factors, as opposed to the more common NYSE breakpoints of the 30 and 70 percentiles. We reproduce their factors via their exact procedure and also replicate their factors via the traditional approach. The performance of their model is sensitive to the factor construction. While their original factors survive the q-factor model (but not the q^5 model), only the replicated management factor survives the q-factor model. Neither the original nor the replicated Stambaugh–Yuan model can explain the q and q^5 factors in the Gibbons–Ross–Shanken test. However, the q^5 model can explain both their original and replicated models. More important, their replicated factors are close to the q-factors, with correlations of 0.8 and 0.84. As such, the Stambaugh–Yuan cluster analysis essentially rediscovers the q-factors, which are in turn motivated from the investment theory.

Daniel, Hirshleifer, and Sun (2018) also deviate from the traditional approach when constructing their financing and post-earnings-announcement-drift factors. We reproduce their factors via their exact procedure and also replicate their factors via the common approach. Their model's performance is also sensitive to the factor construction. In particular, their financing factor premium is more than halved with the common approach and is explained by the q and q^5 models. However, neither their reproduced nor replicated earnings factor can be explained by our models. Their three-factor model explains the Roe premium but not the investment or expected growth premium. Without a size factor, their model also fails to explain the size premium. Most important, their replicated factors are also close to our q-factors, with correlations of 0.69.

Barillas and Shanken (2018) form a six-factor model by combining the market factor, SMB, the investment and Roe factors from the q-factor model, the Asness–Frazzini (2013) monthly formed HML factor, and UMD. The Brillas–Shanken model cannot explain the expected growth premium, with a large alpha of 0.6% per month ($t = 8.78$). However, neither the q-factor nor the q^5 model can explain the monthly formed HML factor, with alphas of 0.37% ($t = 2.36$) and 0.41% ($t = 2.99$), respectively. Reconstructing the q-factors with all monthly sorts on size, investment-to-assets, and Roe, we show that the monthly formed q and q^5 models deliver insignificant alphas of 0.18% ($t = 0.97$) and 0.26% ($t = 1.64$), respectively, for the monthly formed HML factor.

A comparative advantage of the q-factor and q^5 models is their theoretical foundation from the investment CAPM (Zhang, 2017). In contrast, the Stambaugh–Yuan, Daniel–Hirshleifer–Sun, and Fama–French six-factor models are largely statistical in nature. Fama and French (2015) attempt to motivate their five-factor model from the residual income valuation theory. However, the relations between book-to-market, investment, and profitability with the internal rate of return (IRR) do not necessarily carry over to the one-period-ahead expected return. Empirically, the estimates of the IRRs for RMW differ drastically from their one-period-ahead average returns. In addition, reformulating the valuation equation with the one-period-ahead expected return, we show that the theoretical relation between the expected investment and the expected return is likely positive. In all, the investment CAPM is the only first principles based, theoretical framework that gives rise to the role of accounting variables in forecasting returns.

The rest of the paper is organized as follows. Section 2 describes the construction of all the factors. Section 3 reports the spanning regressions. Section 4 examines asset pricing implications from valuation theory. Finally, Section 5 concludes.

2. Factors

Monthly returns are from Center for Research in Security Prices (CRSP, share codes 10 or 11) and accounting variables from Compustat Annual and Quarterly Fundamental Files.

2.1 The Q-Factor and Q^5 Models

Following Hou, Xue, and Zhang (2015), we construct the size, investment, and Roe factors from independent, triple $2 \times 3 \times 3$ sorts on size, investment-to-assets (I/A), and Roe. Size is the market equity, which is stock price per share times shares outstanding from CRSP. I/A is the annual change in total assets (Compustat annual item AT) divided by one-year-lagged total assets. Roe is income before extraordinary items (Compustat quarterly item IBQ) divided by one-quarter-lagged book equity.[1] We exclude financial firms and firms with negative book equity.

1 Book equity is shareholders' equity, plus balance sheet deferred taxes and investment tax credit (Compustat quarterly item TXDITCQ) if available, minus the book value of preferred stock (item

At the end of June of each year t, we use the NYSE median market equity to split stocks into two groups, small and big. Independently, at the end of June of year t, we split stocks into three I/A groups using the NYSE breakpoints for the low 30%, middle 40%, and high 30% of the I/A values for the fiscal year ending in calendar year t–1. Also, independently, at the beginning of each month, we sort all stocks into three groups based on the NYSE breakpoints for the low 30%, middle 40%, and high 30% of Roe. Earnings data in Compustat quarterly files are used in the months immediately after the most recent public quarterly earnings announcement dates (Compustat quarterly item RDQ). For a firm to enter the factor construction, the end of the fiscal quarter that corresponds to its announced earnings must be within six months prior to the portfolio formation month.

Taking the intersection of the two size, three I/A, and three Roe groups, we form eighteen portfolios. Monthly value-weighted portfolio returns are calculated for the current month, and the portfolios are rebalanced monthly. The size factor, denoted R_{Me}, is the difference (small-minus-big), each month, between the simple average of the returns on the nine small size portfolios and the simple average of the returns on the nine big size portfolios. The investment factor, $R_{\mathrm{I/A}}$, is the difference (low-minus-high), each month, between the simple average of the returns on the six low I/A portfolios and the simple average of the returns on the six high I/A portfolios. Finally, the Roe factor, R_{Roe}, is the difference (high-minus-low), each month, between the simple average of the returns on the six high Roe portfolios and the simple average of the returns on the six low Roe portfolios.[2]

2.1.a. Extending the *q*-factors backward

Hou, Xue, and Zhang (2015) start their sample in January 1972, restricted by the limited earnings announcement dates and book equity in Compustat quarterly files. We follow their exact procedure from January 1972 onward but extend the sample backward to January 1967. To overcome the lack of coverage for quarterly earnings announcement dates, we use the most recent quarterly earnings from the fiscal quarter ending at least four months prior to the portfolio formation month.

To maximize the coverage for quarterly book equity, whenever available we first use quarterly book equity from Compustat quarterly files. We then supplement the coverage for fiscal quarter four with book equity from Compustat annual files.[3] If both approaches

PSTKQ). Depending on availability, we use stockholders' equity (item SEQQ), or common equity (item CEQQ) plus the carrying value of preferred stock (item PSTKQ), or total assets (item ATQ) minus total liabilities (item LTQ) in that order as shareholders' equity.

2 Formally, let R_{ijk}, for $i=1$, 2 and $j,k=1,2,3$, denote the eighteen benchmark portfolios from taking the intersection of the two size, three I/A, and three Roe groups from the independent triple sorts, in which i is the index for the size groups, j the I/A groups, and k the Roe groups. In particular, R_{123} is the returns of the portfolio consisting of all stocks that are simultaneously in the small size group, the middle I/A group, and the high Roe group. The size factor is constructed as $R_{\mathrm{Me}} \equiv (\sum_{j=1}^{3}\sum_{k=1}^{3} R_{1jk})/9 - (\sum_{j=1}^{3}\sum_{k=1}^{3} R_{2jk})/9$, the investment factor, $R_{\mathrm{I/A}} \equiv (\sum_{i=1}^{2}\sum_{k=1}^{3} R_{i1k})/6 - (\sum_{i=1}^{2}\sum_{k=1}^{3} R_{i3k})/6$, and the Roe factor, $R_{\mathrm{Roe}} \equiv (\sum_{i=1}^{2}\sum_{j=1}^{3} R_{ij3})/6 - (\sum_{i=1}^{2}\sum_{j=1}^{3} R_{ij1})/6$. Unlike sequential sorts, the factors from independent sorts do not depend on the order of the three sorting variables.

3 We measure annual book equity per Davis, Fama, and French (2000) as stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (Compustat annual item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by

are unavailable, we apply the clean surplus relation to impute the book equity. If available, we backward impute beginning-of-quarter book equity as end-of-quarter book equity minus quarterly earnings plus quarterly dividends.[4] Because we impose a four-month lag between earnings and the holding period (and the book equity in the denominator of Roe is one-quarter-lagged relative to earnings), all the Compustat data in the backward imputation are at least four-month lagged relative to the portfolio formation month.

If data are unavailable for the backward imputation, we impute the book equity for quarter t forward based on the book equity from prior quarters. Let BEQ_{t-j}, $1 \leq j \leq 4$, denote the latest available quarterly book equity as of quarter t, and $\mathrm{IBQ}_{t-j+1,t}$ and $\mathrm{DVQ}_{t-j+1,t}$ be the sum of quarterly earnings and the sum of quarterly dividends from quarter $t-j+1$ to t, respectively. BEQ_t can be imputed as $\mathrm{BEQ}_{t-j} + \mathrm{IBQ}_{t-j+1,t} - \mathrm{DVQ}_{t-j+1,t}$. We do not use prior book equity from more than four quarters ago ($1 \leq j \leq 4$) to reduce imputation errors. We start the sample in January 1967 to ensure that all the eighteen benchmark portfolios from sorting on size, I/A, and Roe have at least ten firms.

2.1.b. The q^5 model

Hou *et al.* (2018) augment the q-factor model with the expected growth factor, denoted R_{Eg}, to form the q^5 model. The expected growth factor is constructed from independent 2×3 sorts on size and the expected one-year-ahead investment-to-assets change, $E_t[\mathrm{d}^1\mathrm{I/A}]$. Tobin's q, operating cash flow-to-assets, and the change in Roe are used to form $E_t[\mathrm{d}^1\mathrm{I/A}]$.

At the beginning of each month t, Tobin's q is the market equity (price per share times the number of shares outstanding from CRSP) plus long-term debt (Compustat annual item DLTT) and short-term debt (item DLC) scaled by total assets (item AT), all from the fiscal year ending at least four months ago. For firms with multiple share classes, we merge the market equity for all classes. Following Ball *et al.* (2016), we measure operating cash flow-to-assets, denoted Cop, as total revenue (item REVT) minus cost of goods sold (item COGS), minus selling, general, and administrative expenses (item XSGA), plus research and development expenditures (item XRD, zero if missing), minus change in accounts receivable (item RECT), minus change in inventory (item INVT), minus change in prepaid expenses (item XPP), plus change in deferred revenue (item DRC plus item DRLT), plus change in trade accounts payable (item AP), and plus change in accrued expenses (item XACC), scaled by book assets, all from the fiscal year ending at least four months ago. All changes are annual changes, and the missing changes are set to zero.

We measure the change in Roe, denoted dRoe, as Roe minus its value from four quarters ago. We compute dRoe with quarterly earnings from the most recent announcement dates (Compustat quarterly item RDQ), and if not available, from the fiscal quarter ending at

Compustat (item SEQ), if available. Otherwise, we use the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption value (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock.

4 Quarterly earnings are income before extraordinary items (Compustat quarterly item IBQ). Quarterly dividends are zero if dividends per share (item DVPSXQ) are zero. Otherwise, total dividends are dividends per share times beginning-of-quarter shares outstanding adjusted for stock splits during the quarter. Shares outstanding are from Compustat (quarterly item CSHOQ supplemented with annual item CSHO for fiscal quarter four) or CRSP (item SHROUT), and the share adjustment factor is from Compustat (quarterly item AJEXQ supplemented with annual item AJEX for fiscal quarter four) or CRSP (item CFACSHR).

least four months ago (Hou, Xue, and Zhang, 2018). The end of the fiscal quarter corresponding to its most recent dRoe must be within six months prior to the portfolio formation. Missing dRoe values are set to zero in cross-sectional regressions in estimating the expected one-year-ahead investment-to-assets change, $E_t[\mathrm{d}^1\mathrm{I/A}]$.

At the beginning of each month t, we compute $E_t[\mathrm{d}^1\mathrm{I/A}]$ by combining the latest known $\log(q)$, Cop, and dRoe values winsorized at the 1–99% level and the average cross-sectional regression slopes estimated from the prior 120-month rolling window (30 months minimum). In the prior predictive regressions, the dependent variables, $\mathrm{d}^1\mathrm{I/A}$, are from the fiscal year ending at least four months ago as of month t, and the regressors are further lagged accordingly. In particular, the regressors used in the latest monthly cross-sectional regression are further lagged by 12 months relative to the latest known $\log(q)$, Cop, and dRoe values used in calculating $E_t[\mathrm{d}^1\mathrm{I/A}]$. We winsorize both the left- and right-hand side variables in the cross-sectional regressions each month at the 1–99% level. To control for microcaps, we use weighted least squares with the market equity as weights.

At the beginning of each month t, we use the beginning-of-month median NYSE market equity to split stocks into two groups, small and big. Independently, we split all stocks into three groups, low, median, and high, based on the NYSE breakpoints for the low 30%, middle 40%, and high 30% of the ranked values of $E_t[\mathrm{d}^1\mathrm{I/A}]$ calculated at the beginning of the month. Taking the intersection of the two size and three $E_t[\mathrm{d}^1\mathrm{I/A}]$ groups, we form six benchmark portfolios. Monthly value-weighted portfolio returns are calculated for the current month t, and the portfolios are rebalanced at the beginning of month $t+1$. The expected growth factor, R_{Eg}, is the difference (high-minus-low), each month, between the simple average of the returns on the two high $E_t[\mathrm{d}^1\mathrm{I/A}]$ portfolios and the simple average of the returns on the two low $E_t[\mathrm{d}^1\mathrm{I/A}]$ portfolios.

2.2 The Fama–French (2015, 2018) Five- and Six-Factor Models

Subsequent to Hou, Xue, and Zhang (2015), Fama and French (2015) incorporate two factors that resemble the q-factors into their three-factor model to form a five-factor model.[5] RMW is the difference between the returns on portfolios of stocks with robust and weak operating profitability, and CMA the difference between the returns on portfolios of low and high investment stocks. Operating profitability is the total revenue (Compustat annual item REVT) minus cost of goods sold (item COGS, zero if missing), minus selling, general, and administrative expenses (item XSGA, zero if missing), and minus interest expense (item XINT, zero if missing), scaled by book equity. At least one of the three expense items (COGS, XSGA, and XINT) must be non-missing. Investment is measured as I/A, the annual change in total assets divided by one-year-lagged total assets.

Fama and French (2015) construct RMW and CMA from independent 2×3 sorts by interacting size with operating profitability, and separately, with investment-to-assets. At the end of June of year t, stocks are split into two groups, small and big, based on the

5 Hou, Xue, and Zhang (2015) first appear in October 2012 as NBER working paper 18435, which supersedes the prior work with various titles, including "Neoclassical factors" (NBER working paper 13282, July 2007), "An equilibrium three-factor model" (January 2009), "Production-based factors" (April 2009), "A better three-factor model that explains more anomalies" (June 2009), and "An alternative three-factor model" (April 2010). By comparison, the Fama and French (2013, 2015) work is first circulated in June 2013. Their 2013 draft adds only a profitability factor to their three-factor model, and subsequent drafts, starting from November 2013, also add an investment factor.

NYSE median size, and independently into three groups, low, median, and high, based on the 30 and 70 NYSE percentiles of operating profitability, and separately, of investment-to-assets. Taking intersections yields six size-profitability portfolios and six size-I/A portfolios. Monthly value-weighted portfolio returns are calculated from July of year t to June of $t+1$, and the portfolios are rebalanced at the June-end of year $t+1$. RMW is the average of the two high profitability portfolio returns minus the average of the two low profitability portfolio returns. Similarly, CMA is the average of the two low I/A portfolio returns minus the average of the two high I/A portfolio returns.

Fama and French (2018) further incorporate the momentum factor, UMD, from Jegadeesh and Titman (1993), into their five-factor model to form a six-factor model. At the beginning of each month t, stocks are split into two groups, small and big, based on the NYSE median size, and independently into three groups, low, median, and high, based on the 30 and 70 NYSE percentiles of prior 11-month returns from month $t-12$ to $t-2$, skipping month $t-1$. Taking intersections yields six size-momentum portfolios. Monthly value-weighted portfolio returns are calculated for the current month, and the portfolios are rebalanced at the beginning of month $t+1$. UMD is the average of the two winner portfolio returns minus the average of the two loser portfolio returns.

Fama and French (2018) also introduce a cash-based profitability factor, denoted RMWc. At the June end of year t, cash-based operating profitability is revenues (Compustat annual item REVT) minus cost of goods sold (item COGS, zero if missing), minus selling, general, and administrative expenses (item XSGA, zero if missing), minus interest expense (item XINT, zero if missing), minus change in accounts receivable (item RECT), minus change in inventory (item INVT), minus change in prepaid expenses (item XPP), plus change in deferred revenue (item DRC plus item DRLT), plus change in trade accounts payable (item AP), and plus change in accrued expenses (item XACC), scaled by book equity, all from the fiscal year ending in calendar year t–1. At least one of the three expense items (COGS, XSGA, and XINT) must be non-missing. The numerator of this variable is a variant of that in Ball *et al.* (2016), without adding back research and development expenses. The construction of RMWc is analogous to that of RMW.

To facilitate comparison, we obtain all the Fama–French factors except for RMWc from Kenneth French's web site. Because RMWc is not posted online, we follow the exact sample criterion and factor construction in Fama and French (2018) to reproduce RMWc to use in our tests. In particular, the Fama–French sample includes financial firms.

2.3 The Stambaugh–Yuan (2017) Four-Factor Model

Stambaugh and Yuan (2017) start with eleven anomalies, which are grouped into two clusters based on pairwise cross-sectional correlations. The first cluster, labeled MGMT (management), includes net stock issues, composite issues, accruals, net operating assets, asset growth (investment-to-assets in Hou, Xue, and Zhang, 2015), and the annual change in gross property, plant, and equipment plus the annual change in inventories scaled by lagged book assets. The second cluster, labeled PERF (performance), includes failure probability (Campbell, Hilscher, and Szilagyi, 2008), O-score, momentum, gross profitability, and return on assets. We detail the variable definitions in the Online Appendix. Conceptually, MGMT contains different investment measures, and PERF different profitability measures. The individual variables in each cluster are realigned to yield positive average low-minus-high returns. The composite measures, MGMT and PERF, are formed by equal-weighting a stock's percentile rankings across the anomaly variables within a given cluster.

Stambaugh and Yuan (2017) form the MGMT and PERF factors from independent 2×3 sorts on size and MGMT as well as on size and PERF. At the beginning of each month t, stocks (excluding those with prices per share less than \$5) are split by the NYSE median size into two groups, small and big. Independently, stocks are split based on MGMT, and separately, on PERF, into three groups, low, median, and high, with breakpoints of the 20 and 80 percentiles of the NYSE, Amex, and NASDAQ universe. Taking intersections yields six size-MGMT portfolios and six size-PERF portfolios. Monthly value-weighted portfolio returns are calculated for the current month t, and the portfolios are rebalanced at the beginning of month $t+1$. The MGMT factor is the average of the returns on the two low MGMT portfolios minus the average of the returns on the two high MGMT portfolios. The PERF factor is the average of the returns on the two low PERF portfolios minus the average of the returns on the two high PERF portfolios. The size factor is the returns of the portfolio of stocks in the intersection of the small-cap middle portfolios from the double sorts of size with MGMT and with PERF minus the returns of the portfolio of stocks in the intersection of both big-cap middle portfolios from the two double sorts.

Most important, the Stambaugh–Yuan (2017) factor construction deviates from the traditional approach in Fama and French (1993, 2015) and Hou, Xue, and Zhang (2015) in several key aspects. First, when sorting on MGMT and PERF, the breakpoints of the 20 and 80 percentiles are adopted, as opposed to the 30 and 70 percentiles. Second, the NYSE, Amex, and NASDAQ breakpoints are used, instead of the NYSE breakpoints. Finally, the size factor contains stocks only in the middle portfolios of the MGMT and PERF sorts, as opposed to stocks from all three portfolios. To evaluate the sensitivity of their model's performance to its factor construction, we present two sets of results. In the first, we use their original factors series from Yu Yuan's Web site.[6] In the second set, we replicate their factors via the traditional approach.

We emphasize the importance of using the replicated Stambaugh–Yuan factors in the model comparison. Formed with the 20–80 breakpoints from the NYSE–Amex–NASDAQ universe, their original factors consist of stocks with more extreme values of the underlying sorting variables than factors formed with the traditional 30–70 breakpoints from the NYSE universe. As such, the original Stambaugh–Yuan factors are more susceptible to microcaps than their replicated factors (Hou, Xue, and Zhang, 2018). While the choice of breakpoints is ultimately an empirical question, using the replicated factors via the traditional approach ensures that we compare apples with apples.

2.4 The Daniel–Hirshleifer–Sun (2018) Three-Factor Model

The Daniel–Hirshleifer–Sun (2018) model contains the market factor, the financing factor (FIN), and the post-earnings-announcement-drift factor (PEAD). FIN is based on two financing measures, the one-year net share issuance from Pontiff and Woodgate (2008) and the five-year composite share issuance from Daniel and Titman (2006). PEAD is based on the four-day cumulative abnormal return, denoted Abr, around the most recent quarterly earnings announcement dates from Chan, Jegadeesh, and Lakonishok (1996). Abr is a

6 We have reproduced the Stambaugh–Yuan factors via their exact procedure and obtained quantitatively similar results.

stock's daily return minus the value-weighted market's daily return cumulated from two trading days before to one trading day after the earnings announcements.

At the end of June of each year t, net share issuance is the natural log of the ratio of split-adjusted shares outstanding for fiscal year ending in calendar year t–1 (the common share outstanding, Compustat annual item CSHO, times the adjustment factor, item AJEX) to the split-adjusted shares outstanding for fiscal year ending in t–2. The composite share issuance is the log growth rate of the market equity not attributable to stock return, $\log(\mathrm{Me}_t/\mathrm{Me}_{t-5}) - r(t-5, t)$, in which $r(t-5, t)$ is the cumulative log stock return from the last trading day of June in year t–5 to the last trading day of June in year t, and Me_t is the market equity from CRSP on the last trading day of June in year t.

Daniel, Hirshleifer, and Sun (2018) construct FIN from annual independent 2×3 sorts on size and the financing variables. The size sort is based on the NYSE median. The composite issuance sort is based on the NYSE breakpoints of the 20 and 80 percentiles. The net share issuance sort is more involved. First, all negative net issuance (repurchasing) firms are split into two groups based on the NYSE median. Second, all positive net issuance (equity issuing) firms are split into three groups based on the NYSE breakpoints of the 30 and 70 percentiles. Finally, firms with the most negative issuance are assigned to the low issuance portfolio, firms with the most positive issuance to the high issuance portfolio, and all the other firms to the middle issuance portfolio.

To combine the net and the composite issuance groups, Daniel, Hirshleifer, and Sun (2018) adopt the following *ad hoc* procedure. If a firm belongs to the high portfolio per both financing measures, or to the high portfolio per one measure, but missing the data for the other, the firm is assigned to the high financing portfolio. If a firm belongs to the low portfolio per both measures, or to the low portfolio per one measure but missing the data for the other, the firm is assigned to the low financing portfolio. In all other cases, the firm is assigned to the middle financing portfolio. The FIN factor is then the simple average of the monthly returns on the two low financing portfolios minus the simple average of the returns on the two high financing portfolios.

The PEAD factor is from monthly independent 2×3 sorts on size and Abr. The size sort is based on the NYSE median, and the Abr sort the NYSE breakpoints of the 20 and 80 percentiles. Value-weighted monthly returns are calculated for the current month, and the portfolios are rebalanced at the beginning of next month. The PEAD factor is the simple average of the returns on the two high Abr portfolios minus the simple average of the returns on the two low Abr portfolios.

We raise three concerns with the factor construction in Daniel, Hirshleifer, and Sun (2018). First, only Abr is picked to form the PEAD factor, even though Chan, Jegadeesh, and Lakonishok (1996) examine simultaneously three PEAD measures that also include standard unexpected earnings (Sue) and revisions in analysts' earnings forecasts (Re). In particular, Sue seems to be more widely used than Abr in the existing literature. Second, the NYSE breakpoints of the 20 and 80 percentiles are used, as opposed to the common 30 and 70 percentiles. Finally, the net issuance and composite issuance sorts are non-traditional, also differing from each other. These concerns suggest that their factors might not be directly comparable to factors that arise from the traditional approach.

To ensure that we compare apples with apples, in addition to reproducing the Daniel–Hirshleifer–Sun factors per their exact procedure, we also replicate their factors per the traditional approach.[7] In particular, we form the PEAD factor by combining Sue, Abr, and Re.[8] At each portfolio formation, we calculate a stock's percentile rankings on each of the three PEAD variables and take their simple average as the stock's ranked PEAD value. When taking the simple average, we use the available percentile rankings. Doing so allows us to extend the sample backward to January 1967. This composite score approach follows Stambaugh and Yuan (2017). We use the same approach to combine the net issuance with the composite issuance in annual sorts. Doing so avoids the Daniel, Hirshleifer, and Sun (2018) *ad hoc*, separate sorts on the two financing measures. Finally, with the composite FIN and PEAD scores, we split stocks based on their NYSE breakpoints of the 30 and 70 percentiles.

2.5 The Barillas–Shanken (2018) Six-Factor Model

Barillas and Shanken (2018) propose a six-factor model that contains the market factor, the Fama–French (2015) SMB, the Hou–Xue–Zhang (2015) investment and Roe factors, the Asness–Frazzini (2013) monthly sorted HML factor, denoted $\mathrm{HML}^{\mathrm{m}}$, and UMD. Asness and Frazzini form $\mathrm{HML}^{\mathrm{m}}$ from monthly sequential sorts on, first, size, and then book-to-market, in which the book equity is from the fiscal year ending at least 6 months ago, but the market equity is updated monthly. We obtain the $\mathrm{HML}^{\mathrm{m}}$ data from the AQR web site. We have reproduced $\mathrm{HML}^{\mathrm{m}}$ via the AQR procedure. We have also replicated $\mathrm{HML}^{\mathrm{m}}$ per independent sorts and obtained quantitatively similar results. As such, we only report the results with the AQR $\mathrm{HML}^{\mathrm{m}}$ factor for brevity.

3. Spanning Regressions

We rely mostly on spanning tests to compare factor models on empirical grounds. This empirical design is largely comparable with Fama and French (2015, 2018) and Barillas and Shanken (2017, 2018). For example, Barillas and Shanken (2017, 2018) argue that for models with traded factors, the extent to which each model is able to price the factors in

7 In our reproduction, we have obtained results that are quantitatively similar to those reported in Daniel, Hirshleifer, and Sun (2018). Because their factors are not available online, we use our reproduced factors in subsequent tests.

8 Sue is calculated as the change in split-adjusted quarterly earnings per share (Compustat quarterly item EPSPXQ divided by item AJEXQ) from its value four quarters ago divided by the standard deviation of this change in quarterly earnings over the prior eight quarters (six quarters minimum). Before 1972, we use the most recent Sue with earnings from fiscal quarters ending at least four months prior to the portfolio formation. Starting from 1972, we use Sue with quarterly earnings from the most recent quarterly earnings announcement dates (Compustat quarterly item RDQ). For a firm to enter our portfolio formation, we require the end of the fiscal quarter that corresponds to its most recent Sue to be within six months prior to the portfolio formation. Because analysts' earnings forecasts from the Institutional Brokers' Estimate System (IBES) are not necessarily revised each month, we construct a six-month moving average of past revisions, $\sum_{\tau=1}^{6}(f_{it-\tau} - f_{it-\tau-1})/p_{it-\tau-1}$, in which $f_{it-\tau}$ is the consensus mean forecast (IBES-unadjusted file, item MEANEST) issued in month $t-\tau$ for firm i's current fiscal year earnings (fiscal period indicator = 1), and $p_{it-\tau-1}$ is the prior month's share price (unadjusted file, item PRICE). We require both earnings forecasts and share prices to be denominated in US dollars (currency code = USD). We also adjust for any stock splits and require a minimum of four monthly forecast changes when constructing Re.

the other model is all that matters for model comparison. They even argue that test assets are irrelevant, regardless of whether the factor models are nested or not. For our purposes, spanning tests provide an informative and concise way to compare factor models.[9]

We detail the spanning tests of the q-factor and q^5 models against the Fama–French five- and six-factor models in Section 3.1, the Stambaugh–Yuan model in Section 3.2, the Daniel–Hirshleifer–Sun model in Section 3.3, and the Barillas–Shanken model in Section 3.4. Finally, we examine pairwise correlations among the factors in Section 3.5.

3.1 The Q-Factor and Q^5 Models versus the Fama–French Five- and Six-Factor Models

The q-factor and q^5 models largely explain the Fama–French five- and six-factor premiums, but their five- and six-factor models cannot explain the q and q^5 factor premiums.

3.1.a. The Fama–French five- and six-factor models cannot explain the q and q^5 factor premiums

In Panel A of Table 1, we regress the q and q^5 factor returns on the Fama–French five- and six-factor models, as well as their alternative six-factor model with RMW replaced by RMWc. From January 1967 to December 2016, the size factor, R_{Me}, in the q-factor model earns on average 0.31% per month ($t = 2.43$). All three Fama–French specifications account for this size premium, with alphas at most 0.05%, due to the presence of SMB.

The investment factor, $R_{\mathrm{I/A}}$, in the q-factor model earns an average return of 0.41% per month ($t = 4.92$). Despite the presence of CMA, the Fama–French five-factor model cannot explain the $R_{\mathrm{I/A}}$ premium, with a significant alpha of 0.12% ($t = 3.44$). The two specifications of the six-factor model yield largely similar results. Our investment factor is stronger than CMA, because $R_{\mathrm{I/A}}$ is based on a joint sort with Roe, whereas the CMA construction does not control for profitability. In the data, I/A and Roe are positively correlated, but forecast returns with opposite signs.

The Roe factor, R_{Roe}, earns an average return of 0.55% per month ($t = 5.25$). The Fama–French five-factor model only reduces the Roe premium slightly to an alpha of 0.47% ($t = 5.94$), despite a large RMW loading of 0.7 ($t = 12.76$). Intuitively, the Roe factor is constructed from monthly sorts on the latest known quarterly earnings, whereas RMW is from annual sorts on the more stale operating profitability from the last fiscal year end. As such, the Roe factor is more powerful than RMW. The Fama–French six-factor model reduces the Roe premium to an alpha of 0.3% ($t = 4.51$), with the help of an UMD loading of 0.24 ($t = 9.94$). Replacing RMW with RMWc in the six-factor model yields a smaller alpha of 0.23%, due to a higher premium of RMWc than RMW, 0.33% versus 0.26%. However, the alpha for the Roe factor is still significant ($t = 2.8$).

The expected growth factor, R_{Eg}, in the q^5 model earns an average return of 0.82% per month ($t = 9.81$). The Fama–French five-factor model reduces the R_{Eg} premium only slightly, with an alpha of 0.78% ($t = 11.34$). Their six-factor model reduces the R_{Eg} premium further to an alpha of 0.7% ($t = 11.1$), with the help of a small UMD loading of 0.12 ($t = 6.42$). Finally, replacing RMW with RMWc in the six-factor model shrinks the R_{Eg}

9 In complementary work, Hou *et al.* (2018) compare asset pricing models with a large set of testing deciles formed on the 158 significant anomalies compiled by Hou, Xue, and Zhang (2018). Their evidence based on the extensive testing assets is consistent with our spanning tests.

premium to an alpha of 0.61%, helped by a large RMWc loading of 0.39 ($t = 6.73$) and a high RMWc premium, but the alpha remains highly significant ($t = 9.33$).

We also perform the Gibbons, Ross, and Shanken (1989, GRS) test on the null hypothesis that the alphas of the key q and q^5 factors in the Fama–French five- and six-factor regressions are jointly zero (Panel C). For the null that the alphas of the investment and Roe factors are jointly zero, the GRS statistic is 22.72 (p-value = 0.00) in the five-factor model, 14.6 (p-value = 0.00) in the six-factor model with RMW, and 8.2 (p-value = 0.00) in the alternative six-factor model with RMWc. For the null that the alphas of the investment, Roe, and expected growth factors are jointly zero, the GRS statistic is 55.14 (p-value = 0.00) in the five-factor model, 48.85 (p-value = 0.00) in the six-factor model, and 36.59 (p-value = 0.00) in the alternative six-factor model. As such, the Fama–French five- and six-factor models cannot explain the q and q^5 factor premiums.

3.1.b. Explaining the Fama–French five- and six-factor premiums with the q-factor and q^5 models

From Panel B, the q-factor and q^5 models largely subsume the Fama–French five- and six-factor premiums in spanning regressions, with economically small and mostly insignificant alphas. SMB earns on average 0.25% per month ($t = 1.93$), and its q-factor and q^5 alphas are 0.04% ($t = 1.42$) and 0.07% ($t = 2.29$), respectively. Our size factor, R_{Me}, provides the explanatory power, yielding regression R^2s over 95%. HML has an average return of 0.37% ($t = 2.71$), and its q-factor and q^5 alphas are 0.07% ($t = 0.62$) and 0.05% ($t = 0.48$), respectively. The investment factor, $R_{\mathrm{I/A}}$, delivers the explanatory power. The factor loadings are economically large (about one) and also highly significant (t-values above 11).

The momentum factor, UMD, earns on average 0.65% per month ($t = 3.61$). The q-factor alpha is only 0.12% ($t = 0.5$), helped by a large Roe factor loading of 0.91 ($t = 5.9$). The q^5 alpha is weakly negative, – 0.16% ($t = -0.78$). In addition to a large Roe factor loading of 0.78 ($t = 4.4$), the expected growth factor loading of 0.44 ($t = 2.62$) also helps. Intuitively, momentum winners are both more profitable and are expected to grow faster than momentum losers, both going in the right direction in explaining average returns.

CMA has an average return of 0.33% per month ($t = 3.51$). The q-factor alpha is virtually zero ($t = -0.02$), helped by a large investment factor loading of 0.96 ($t = 33.56$). The q^5 alpha is also tiny, –0.04% ($t = -0.96$), with a similar investment factor loading. RMW has an average return of 0.26% ($t = 2.5$). The q-factor alpha is only 0.01% ($t = 0.08$), with a large Roe factor loading of 0.54 ($t = 8.5$). Similarly, the q^5 alpha is also tiny, – 0.01% ($t = -0.16$), with a large Roe factor loading of 0.53 ($t = 7.85$). Finally, RMWc has an average return of 0.33% ($t = 4.16$). RMWc survives the control of the q-factors, with an alpha of 0.25% ($t = 3.83$). Although the Roe factor loading is significant ($t = 9.88$), its magnitude is only 0.29. The q^5 model reduces the alpha of RMWc further to 0.14%, albeit still significant ($t = 2.18$), helped by both the Roe and expected growth factors.

Panel C reports the GRS tests on the null hypothesis that the alphas of the key Fama–French five- and six-factors are jointly zero in the q-factor and q^5 models. For the null that the alphas of HML, CMA, and RMW are jointly zero, the GRS statistic is 0.2 (p-value = 0.9) in the q-factor model and 0.62 (p-value = 0.6) in the q^5 model. For the null that the alphas of HML, CMA, RMW, and UMD are jointly zero, the GRS statistic is 0.36 (p-value = 0.84) in the q-factor model and 0.65 (p-value = 0.62) in the q^5 model. Finally, for the null that the alphas of HML, CMA, RMWc, and UMD are jointly zero, the GRS statistic is 6.14 (p-value = 0.00) in the q-factor model and 1.81 (p-value = 0.13) in the q^5

model. As such, the q-factor model largely subsumes the Fama–French five- and six-factor models. Although the alternative six-factor model with RMWc survives the q-factor model, it is largely subsumed by the q^5 model.

When constructing the q-factors, we adopt annual sorts on size and investment-to-assets (I/A) at the end of each June but monthly sorts on Roe with the latest known quarterly earnings at the beginning of each month. Using up-to-date quarterly earnings in monthly sorts is critical for the Roe factor's stronger explanatory power than RMW, which is based on more stale operating profitability from the fiscal year ending at least six months ago in annual sorts. We emphasize that monthly sorts aimed to exploit up-to-date information are commonly adopted in the existing literature on, for example, price and earnings momentum. In particular, we should acknowledge that if we instead use annual sorts on annual Roe from the last fiscal year end (Compustat annual item IB scaled by one-year-lagged book equity) in the independent, triple $2 \times 3 \times 3$ sorts on size, investment-to-assets, and Roe, the average Roe premium is only 0.16% per month ($t = 1.66$). The size and investment premiums are also weaker, 0.23% ($t = 1.82$) and 0.32% ($t = 3.64$), respectively.

3.2 The *Q*-Factor and *Q*^{5} Models versus the Stambaugh–Yuan Model

Table II reports the factor spanning tests of the q-factor and q^5 models versus the Stambaugh–Yuan model. As noted, their factor construction deviates from the traditional approach in important ways. As such, we report two sets of results, with one set using their original factors and the other using our replicated factors reconstructed via the traditional approach. The bottomline is that their model's performance is sensitive to the factor construction. While their original factors survive the q-factor and q^5 models, the replicated factors are largely absorbed by the q^5 model. In addition, neither their original nor replicated model can explain the q and q^5 factors.

In Panel A, we use the Stambaugh–Yuan model to explain the q and q^5 factor premiums. Their original model explains the size and investment factors, but not the Roe factor. The alphas of the size and investment factors are –0.04% and 0.08% per month ($t = -0.65$ and 1.26), respectively. However, the alpha of the Roe factor is 0.33% ($t = 3.55$), despite a large PERF factor loading of 0.42 ($t = 11.65$). The expected growth factor also survives the Stambaugh–Yuan model, with an alpha of 0.55% ($t = 9.04$).

The replicated Stambaugh–Yuan factors yield largely similar results. The alphas of the size and investment factors are 0.01% ($t = 0.18$) and 0.07% ($t = 1.41$), but the alphas of the Roe and expected growth factors are 0.32% ($t = 4.71$) and 0.58% ($t = 10.25$), respectively. For the null hypothesis that the investment and Roe factor alphas are jointly zero, the GRS statistic is 8.16 (p-value $= 0.00$) in the original Stambaugh–Yuan model and 12.12 (p-value $= 0.00$) in the replicated model. For the null that the alphas of the investment, Roe, and expected growth factors are jointly zero, the GRS statistic is 30.24 (p-value $= 0.00$) in the original model and 41.27 (p-value $= 0.00$) in the replicated model.

In Panel B, we use the q-factor and q^5 models to explain the Stambaugh–Yuan factors. Their size factor earns on average 0.44% per month ($t = 3.6$), and the replicated version 0.31% ($t = 2.13$). The q-factor and q^5 alphas of the original size factor are significant, about 0.15%. For the replicated size factor, the q-factor alpha is 0.06% ($t = 1.13$), and the q^5 alpha 0.09% ($t = 1.72$). The original MGMT factor earns on average 0.61% ($t = 4.72$), with a q-factor alpha of 0.36% ($t = 4.73$) and a q^5 alpha of 0.12% ($t = 1.64$). The replicated MGMT factor earns on average 0.47% ($t = 4.68$). The q-factor model yields a small

Table I. Spanning tests, the q-factor and q^5 models versus the Fama–French five- and six-factor models (January 1967–December 2016)

$\overline{R}$ is the average return, α the intercept from a spanning regression, and R^2 its goodness-of-fit in percent. R_{Mkt}, R_{Me}, $R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and Roe factors in the q-factor model (q), respectively, and R_{Eg} the expected growth factor in the q^5 model (q^5). MKT, SMB, HML, RMW, and CMA are the market, size, value, profitability, and investment factors in the Fama–French five-factor model (FF5), and UMD the momentum factor in the six-factor model (FF6). The data on MKT, SMB, HML, RMW, CMA, and UMD are from Kenneth French's web site. RMWc is the cash-based profitability factor in the alternative specification of the six-factor model (FF6c), in which RMW is replaced by RMWc. The t-values (in the rows beneath the corresponding estimates) are adjusted for heteroscedasticity and autocorrelations.

	Panel A: Explaining the q and q^5 factors										Panel B: Explaining the Fama–French factors								
	$\overline{R}$	α	MKT	SMB	HML	RMW	CMA	UMD	RMWc	R^2		$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
R_{Me}	0.31	0.05	0.01	0.97	0.04	−0.03	0.02			95	SMB	0.25	0.04	−0.01	0.95	−0.08	−0.09		95
	2.43	1.53	0.88	68.35	1.85	−0.91	0.66					1.93	1.42	−0.82	60.67	−4.48	−6.00		
		0.03	0.01	0.97	0.06	−0.03	0.01	0.03		95			0.07	−0.01	0.94	−0.07	−0.08	−0.04	96
		0.85	1.35	71.18	3.01	−1.28	0.29	2.54					2.29	−1.32	61.42	−3.86	−4.44	−1.95	
		0.05	0.01	0.96	0.05		0.01	0.03	−0.07	95	HML	0.37	0.07	−0.04	0.02	1.02	−0.19		48
		1.37	0.62	74.88	2.92		0.51	2.75	−2.28			2.71	0.62	−0.96	0.24	12.11	−2.61		
$R_{\mathrm{I/A}}$	0.41	0.12	0.01	−0.05	0.04	0.07	0.80			85			0.05	−0.03	0.02	1.01	−0.20	0.03	48
	4.92	3.44	0.91	−3.19	1.63	2.48	29.30						0.48	−0.90	0.26	11.50	−2.42	0.36	
		0.11	0.01	−0.05	0.05	0.06	0.80	0.01		85	UMD	0.65	0.12	−0.08	0.23	−0.00	0.91		28
		3.11	1.09	−3.17	2.12	2.22	30.79	0.82				3.61	0.50	−1.25	1.73	−0.02	5.90		
		0.11	0.01	−0.05	0.05		0.78	0.01	0.06	84			−0.16	−0.03	0.27	−0.11	0.78	0.44	30
		2.78	1.10	−3.13	2.22		27.89	0.81	1.49				−0.78	−0.51	2.00	−0.60	4.40	2.62	
R_{Roe}	0.55	0.47	−0.03	−0.12	−0.24	0.70	0.10			51	CMA	0.33	−0.00	−0.05	0.04	0.96	−0.10		85
	5.25	5.94	−1.20	−2.92	−3.75	12.76	1.01					3.51	−0.02	−3.77	1.91	33.56	−3.57		
		0.30	−0.00	−0.12	−0.10	0.65	−0.02	0.24		66			−0.04	−0.04	0.05	0.94	−0.12	0.06	85
		4.51	−0.01	−3.66	−2.04	14.69	−0.24	9.94					−0.96	−3.14	2.12	35.60	−3.89	2.07	
		0.23	0.03	−0.10	−0.03		−0.18	0.24	0.72	54	RMW	0.26	0.01	−0.03	−0.12	0.03	0.54		48
		2.80	1.41	−2.49	−0.49		−2.05	7.12	8.49			2.50	0.08	−1.17	−1.71	0.38	8.50		

(continued)

Table I. Continued

Panel A: Explaining the q and q^5 factors

	$\overline{R}$	α	MKT	SMB	HML	RMW	CMA	UMD	RMWc	R^2
R_{Eg}	0.82	0.78	−0.10	−0.14	−0.08	0.25	0.28			41
	9.81	11.34	−5.62	−5.36	−2.62	5.19	5.43			
		0.70	−0.09	−0.14	−0.02	0.22	0.22	0.12		48
		11.10	−5.43	−6.43	−0.54	5.43	5.12	6.42		
		0.61	−0.06	−0.10	−0.00		0.18	0.11	0.39	50
		9.33	−3.41	−4.01	−0.01		3.87	5.77	6.73	

Panel B: Explaining the Fama–French factors

	$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
		−0.01	−0.03	−0.12	0.02	0.53	0.03	48
		−0.16	−0.13	−1.59	0.28	7.85	0.42	
RMWc	0.33	0.25	−0.10	−0.18	0.09	0.29		56
	4.16	3.83	−6.00	−5.25	2.02	9.88		
		0.14	−0.09	−0.17	0.05	0.23	0.18	58
		2.18	−5.15	−4.45	0.93	6.55	4.27	

Panel C: GRS statistics and their p-values testing that the alphas of a key set of factors are jointly zero

	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$			$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$			$\alpha_{\mathrm{HML}}, \alpha_{\mathrm{CMA}}, \alpha_{\mathrm{RMW}} = 0$		$\alpha_{\mathrm{RMW}}, \alpha_{\mathrm{UMD}}, \alpha_{\mathrm{RMW}}, \alpha_{\mathrm{UMD}} = 0$		$\alpha_{\mathrm{HML}}, \alpha_{\mathrm{CMA}}, \alpha_{\mathrm{RMWc}}, \alpha_{\mathrm{UMD}} = 0$	
	FF5	FF6	FF6c	FF5	FF6	FF6c	q	q^5	q	q^5	q	q^5
GRS	22.72	14.60	8.20	55.14	48.85	36.59	0.20	0.62	0.36	0.65	6.14	1.81
p	0.00	0.00	0.00	0.00	0.00	0.00	0.90	0.60	0.84	0.62	0.00	0.13

alpha of 0.2%, albeit significant ($t = 3.59$), despite a large investment factor loading of 0.92 ($t = 22.65$). The q^5 model shrinks the alpha further to −0.02% ($t = -0.38$), helped by an expected growth factor loading of 0.36 ($t = 9.79$).

The original PERF factor earns on average 0.68% per month ($t = 4.2$). The q-factor model yields an alpha of 0.34% ($t = 2$), with the help of a large Roe factor loading of 0.95 ($t = 10.42$). The q^5 model yields a tiny alpha of 0.01% ($t = 0.05$), helped by both the Roe and expected growth factor loadings, 0.79 ($t = 8.4$) and 0.53 ($t = 4.8$), respectively. The replicated PERF factor earns on average 0.49% ($t = 3.67$). The q-factor and q^5 alphas are both insignificant, 0.03% ($t = 0.28$) and –0.19% ($t = -1.87$), respectively. The Roe and expected growth factors again pull their weight.

For the GRS tests, the null hypothesis that the alphas of the original MGMT and PERF factors are jointly zero has a test statistic of 17.16 (p-value = 0.00) in the q-factor model and 1.46 (p-value = 0.23) in the q^5 model. The null that the alphas of the replicated MGMT and PERF factors are jointly zero has a test statistic of 7.96 (p-value = 0.00) in the q-factor model and 2.38 (p-value = 0.09) in the q^5 model. As such, the q^5 model subsumes both the original and replicated Stambaugh–Yuan factors.

Stambaugh and Yuan (2017) include financial firms and firms with negative book equity, but impose a $5 price screen in their sample selection. For comparison, we exclude financial firms and firms with negative book equity, without imposing the price screen. Without going through the details, we can report that the sample differences have little impact on our spanning regressions. Panel A of Table A1 in the Online Appendix, which is based on their sample criterion, shows largely similar results as Table II.

3.3 The *Q*-Factor and *Q*^{5} Models versus the Daniel–Hirshleifer–Sun Model

Table III reports the spanning tests of the q-factor and q^5 models versus the Daniel–Hirshleifer–Sun model. As noted, their factor construction also deviates from the traditional approach in important ways. As such, we report two sets of results, with one set using the reproduced factors via their exact procedure and the other set using the replicated factors via the traditional procedure.

The bottomline is that the Daniel–Hirshleifer–Sun model's performance is sensitive to the factor construction. Their FIN factor premium is more than halved with the traditional construction and is explained by both the q and q^5 models. However, their PEAD factor (reproduced or replicated) cannot be explained by the q and q^5 models. Their three-factor model explains the Roe premium but not the investment or expected growth premium. Finally, without a size factor, their model fails to explain the size premium.

In Panel A, we use the Daniel–Hirshleifer–Sun model to explain the q and q^5 factors. Without a size factor, their model cannot explain the size premium, with an alpha of 0.46% per month ($t = 3.11$) with the reproduced factors and 0.63% ($t = 4.25$) with the replicated factors. Their model also fails to explain the investment factor, with an alpha of 0.18% ($t = 2.56$) with the reproduced factors and 0.32% ($t = 4.34$) with the replicated factors, as well as the expected growth factor, with an alphas of 0.56% ($t = 7.42$) and 0.54% ($t = 7.45$), respectively. The Daniel–Hirshleifer–Sun model does subsume the Roe premium, with an alpha of 0.1% ($t = 0.83$) from the reproduced model and –0.14% ($t = -1.91$) from the replicated model. However, the GRS tests all reject the null that their model can explain the key q and q^5 factor premiums jointly.

Table II. Spanning tests, the q-factor and q^5 models versus the Stambaugh–Yuan model (January 1967–December 2016)

$\overline{R}$ is the average return, α the intercept, and R^2 its goodness-of-fit in percent. $R_{\mathrm{Mkt}}, R_{\mathrm{Me}}, R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and Roe factors in the q-factor and q^5 models, respectively, and R_{Eg} the expected growth factor in the q^5 model. MKT, SMB, MGMT, and PERF are the market, size, management, and performance factors in the Stambaugh–Yuan model. The t-values (in the rows beneath the corresponding estimates) are adjusted for heteroscedasticity and autocorrelations. The original Stambaugh–Yuan factors data are from Yu Yuan's web site.

	Panel A: Explaining the q and q^5 factors							Panel B: Explaining the Stambaugh–Yuan factors								
	$\overline{R}$	α	MKT	SMB	MGMT	PERF	R^2		$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
								The original Stambaugh–Yuan model								
R_{Me}	0.31	−0.04	−0.01	0.97	−0.06	−0.06	87	SMB	0.44	0.16	0.01	0.86	−0.01	0.01		86
	2.43	−0.65	−0.67	25.97	−1.71	−2.98			3.60	3.37	0.57	31.16	−0.23	0.45		
$R_{\mathrm{I/A}}$	0.41	0.08	0.01	0.05	0.53	−0.02	61			0.14	0.01	0.87	−0.02	−0.00	0.04	86
	4.92	1.26	0.52	2.35	15.99	−1.06				2.43	0.81	30.92	−0.50	−0.03	0.97	
R_{Roe}	0.55	0.33	0.02	−0.20	0.02	0.42	46	MGMT	0.61	0.36	−0.17	−0.15	1.00	−0.06		69
	5.25	3.55	0.73	−3.44	0.42	11.65			4.72	4.73	−7.95	−5.02	18.59	−1.33		
R_{Eg}	0.82	0.55	−0.03	−0.10	0.29	0.21	53			0.12	−0.13	−0.11	0.90	−0.18	0.38	73
	9.81	9.04	−1.76	−3.92	12.19	10.72				1.64	−6.70	−4.15	18.76	−3.91	7.61	
								PERF	0.68	0.34	−0.18	0.11	−0.30	0.95		45
									4.20	2.00	−4.22	1.35	−2.02	10.42		
										0.01	−0.12	0.15	−0.44	0.79	0.53	49
										0.05	−3.17	1.95	−3.06	8.40	4.80	
		$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$				$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$				$\alpha_{\mathrm{MGMT}}, \alpha_{\mathrm{PERF}} = 0$ in q				$\alpha_{\mathrm{MGMT}}, \alpha_{\mathrm{PERF}} = 0$ in q^5		
GRS		8.16				30.24				17.16				1.46		
p		0.00				0.00				0.00				0.23		

(continued)

Table II. Continued

The replicated Stambaugh–Yuan model

Panel A: Explaining the q and q^5 factors

	$\overline{R}$	α	MKT	SMB	MGMT	PERF	R^2
R_{Me}	0.31	0.01	−0.04	0.95	−0.03	0.10	92
	2.43	0.18	−2.51	29.43	−1.00	4.23	
$R_{\mathrm{I/A}}$	0.41	0.07	0.00	0.05	0.70	−0.02	71
	4.92	1.41	−0.08	2.77	26.78	−0.85	
R_{Roe}	0.55	0.32	0.01	−0.16	−0.04	0.59	67
	5.25	4.71	0.50	−4.54	−0.82	20.03	
R_{Eg}	0.82	0.58	−0.05	−0.09	0.35	0.25	57
	9.81	10.25	−3.29	−4.48	13.57	9.03	

	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$
GRS	12.12	41.27
p	0.00	0.00

Panel B: Explaining the Stambaugh–Yuan factors

	$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
SMB	0.31	0.06	0.06	0.94	0.04	−0.16		93
	2.13	1.13	3.37	18.96	0.86	−4.94		
		0.09	0.06	0.93	0.05	−0.15	−0.05	93
		1.72	3.28	18.52	1.08	−3.94	−1.54	
MGMT	0.47	0.20	−0.09	−0.10	0.92	−0.06		75
	4.68	3.59	−5.82	−4.10	22.65	−1.68		
		−0.02	−0.05	−0.07	0.83	−0.17	0.36	79
		−0.38	−4.21	−3.30	23.50	−5.28	9.79	
PERF	0.49	0.03	−0.08	0.08	−0.15	1.00		65
	3.67	0.28	−2.87	1.85	−1.72	13.97		
		−0.19	−0.05	0.11	−0.24	0.89	0.35	68
		−1.87	−1.62	2.63	−2.91	11.57	4.85	

	$\alpha_{\mathrm{MGMT}}, \alpha_{\mathrm{PERF}} = 0$ in q	$\alpha_{\mathrm{MGMT}}, \alpha_{\mathrm{PERF}} = 0$ in q^5
GRS	7.96	2.38
p	0.00	0.09

Table III. Spanning tests, the q-factor and q^5 models versus the Daniel–Hirshleifer–Sun (2018) model (January 1967–December 2016)

$\overline{R}$ is the average return, α the intercept, and R^2 its goodness-of-fit coefficient in percent. $R_{\mathrm{Mkt}}, R_{\mathrm{Me}}, R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and Roe factors in the q-factor and q^5 models, respectively, and R_{Eg} the expected growth factor in the q^5 model. MKT, FIN, and PEAD are the market, financing, and post-earnings-announcement-drift factors in the Daniel–Hirshleifer–Sun model. The t-values (reported in the rows beneath the corresponding estimates) are adjusted for heteroscedasticity and autocorrelations.

Panel A: Explaining the q and q^5 factors							Panel B: Explaining the Daniel–Hirshleifer–Sun factors								
	$\overline{R}$	α	MKT	FIN	PEAD	R^2		$\overline{R}$	α	MKT	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
The reproduced Daniel–Hirshleifer–Sun model (July 1972–December 2016)															
R_{Me}	0.27	0.46	0.06	−0.24	−0.04	13	FIN	0.83	0.33	−0.17	−0.21	1.15	0.33		59
	2.03	3.11	1.10	−2.23	−0.28			4.55	2.67	−4.11	−2.36	11.45	3.89		
$R_{\mathrm{I/A}}$	0.41	0.18	−0.03	0.29	−0.01	43			0.14	−0.14	−0.19	1.08	0.24	0.30	60
	4.69	2.56	−1.33	10.21	−0.21				1.12	−3.47	−2.02	10.77	2.57	3.50	
R_{Roe}	0.54	0.10	0.01	0.24	0.38	20	PEAD	0.62	0.56	−0.04	0.05	−0.08	0.19		8
	4.80	0.83	0.17	4.15	3.66			7.73	5.66	−1.64	0.84	−1.06	3.53		
R_{Eg}	0.83	0.56	−0.08	0.22	0.21	37			0.47	−0.03	0.06	−0.11	0.15	0.15	9
	9.44	7.42	−4.49	8.36	5.20				5.32	−1.17	1.02	−1.42	2.15	1.95	
		$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$			$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$				$\alpha_{\mathrm{FIN}}, \alpha_{\mathrm{PEAD}} = 0$ in q				$\alpha_{\mathrm{FIN}}, \alpha_{\mathrm{PEAD}} = 0$ in q^5		
GRS		4.89			23.90				29.67				14.99		
p		0.01			0.00				0.00				0.00		

(continued)

Table III. Continued

	Panel A: Explaining the q and q^5 factors							Panel B: Explaining the Daniel–Hirshleifer–Sun factors							
	$\overline{R}$	α	MKT	FIN	PEAD	R^2		$\overline{R}$	α	MKT	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
The replicated Daniel–Hirshleifer–Sun model (January 1967–December 2016)															
R_{Me}	0.31	0.63	0.00	−0.46	−0.24	23	FIN	0.32	0.00	−0.16	−0.22	0.86	0.22		69
	2.43	4.25	0.07	−3.76	−3.20			2.53	0.01	−6.90	−3.94	14.01	4.23		
$R_{\mathrm{I/A}}$	0.41	0.32	0.00	0.44	−0.07	48			−0.05	−0.15	−0.22	0.84	0.19	0.09	69
	4.92	4.34	−0.14	8.97	−1.99				−0.65	−6.97	−3.61	12.37	3.26	1.45	
R_{Roe}	0.55	−0.14	0.04	0.32	0.78	58	PEAD	0.72	0.43	0.00	0.02	−0.11	0.61		48
	5.25	−1.91	1.65	5.98	18.90			7.78	5.13	0.00	0.52	−1.71	11.76		
R_{Eg}	0.82	0.54	−0.08	0.28	0.31	47			0.31	0.02	0.03	−0.15	0.55	0.18	49
	9.81	7.45	−4.64	8.26	8.59				4.07	0.96	0.98	−2.36	8.98	2.89	
	$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}} = 0$			$\alpha_{\mathrm{I/A}}, \alpha_{\mathrm{Roe}}, \alpha_{\mathrm{Eg}} = 0$				$\alpha_{\mathrm{FIN}}, \alpha_{\mathrm{PEAD}} = 0$ in q				$\alpha_{\mathrm{FIN}}, \alpha_{\mathrm{PEAD}} = 0$ in q^5			
GRS	14.27			35.37				20.44				8.67			
p	0.00			0.00				0.00				0.00			

In Panel B, we use the q and q^5 models to explain the FIN and PEAD factors in the Daniel–Hirshleifer–Sun model. The reproduced FIN factor earns an average return of 0.83% per month ($t = 4.55$), a q-factor alpha of 0.33% ($t = 2.67$), but an insignificant q^5 alpha of 0.14% ($t = 1.12$). In contrast, the replicated FIN factor premium is only 0.32% ($t = 2.53$), and its q-factor and q^5 alphas are both close to zero. The investment factor is the main source behind the models' explanatory power for FIN. The reproduced PEAD factor earns an average return of 0.62% ($t = 7.73$). Both the q and q^5 models fail to explain this premium, with alphas of 0.56% ($t = 5.66$) and 0.47% ($t = 5.32$), respectively. The replicated PEAD factor earns an even higher average return of 0.72% ($t = 7.78$), although its q and q^5 alphas are smaller, 0.43% ($t = 5.13$) and 0.31% ($t = 4.07$), respectively. Finally, the GRS tests indicate that the q and q^5 models cannot explain FIN and PEAD jointly.

Daniel, Hirshleifer, and Sun (2018) exclude financial firms but include firms with negative book equity in their sample selection. For comparison, we exclude financial firms and firms with negative book equity. Without going through the details, we can report that the sample differences have little impact. Panel B of Table A1 in the Online Appendix, which is based on their sample criterion, yields largely similar results as Table III.

3.4 The Q-Factor and Q^5 Models versus the Barillas–Shanken Model

Table IV reports the spanning tests of the q-factor and q^5 models versus the Barillas–Shanken model. Because their model includes the investment and Roe factors in the q-factor model, we only study whether their model can explain the expected growth factor in the q^5 model. From Panel A, the answer is no. The Barillas–Shanken alpha of the expected growth factor is 0.6% per month ($t = 8.78$).

Panel B shows that the monthly formed HML factor, HML^{m}, earns an average premium of 0.34% per month ($t = 2.13$). Neither the q-factor nor the q^5 model can explain the HML^{m} premium, leaving alphas of 0.37% ($t = 2.36$) and 0.41% ($t = 2.99$), respectively. The investment factor loadings are economically large, 0.93 and 0.95, going in the right direction in explaining the HML^{m} premium. However, their impact is mostly offset by the large but negative Roe factor loadings, –0.69 and –0.67, respectively, going in the wrong direction in explaining the HML^{m} premium.[10]

3.5 Correlation Matrix

To shed further light on the relations between the myriad of factors, Table V reports their correlation matrix. The size factor in the q-factor model and SMB in the Fama–French models are largely equivalent, with a correlation of 0.97. The investment factor, $R_{\text{I/A}}$, in the q-factor model has high correlations of 0.67 with HML, 0.91 with CMA, 0.84 with the replicated MGMT factor, 0.69 with the replicated FIN factor, and 0.49 with the monthly formed HML. As such, HML contains similar pricing information as the investment factor, and MGMT and FIN are also closely related factors.

10 In untabulated results, we reconstruct the q-factors via monthly sorts on all three characteristics, including size and investment-to-assets. The monthly formed size, I/A, and Roe factor premiums are on average 0.33%, 0.5%, and 0.57% per month ($t = 2.49, 5.73$, and 5.23), and their correlations with the original q-factors are 0.96, 0.92, and 0.98, respectively. The monthly formed q and q^5 models do a better job in explaining the HML^{m} premium, with alphas of 0.18% ($t = 0.97$) and 0.26% ($t = 1.64$), respectively.

Table IV. Spanning tests, the q-factor and q^5 models versus the Barillas–Shanken (2018) six-factor model (January 1967–December 2016)

$\overline{R}$ is the average return, α the intercept, and R^2 its goodness-of-fit in percent. R_{Mkt}, R_{Me}, $R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and Roe factors in the q-factor and q^5 models, respectively, and R_{Eg} the expected growth factor in the q^5 model. MKT, SMB, UMD, and HML$^{\mathrm{m}}$ are the market, size, momentum, and the Asness–Frazzini monthly formed HML factor in the Barillas–Shanken model. The HML$^{\mathrm{m}}$ data are from AQR's web site. The t-values (in the rows beneath the corresponding estimates) are adjusted for heteroscedasticity and autocorrelations.

Panel A: Regressing the q^5 factors on the Barillas–Shanken factors

	$\overline{R}$	α	MKT	SMB	$R_{\mathrm{I/A}}$	R_{Roe}	UMD	HML$^{\mathrm{m}}$	R^2
R_{Me}	0.31	−0.04	0.02	1.00	0.03	0.09	0.02	0.05	95
	2.43	−1.08	1.79	60.21	1.11	2.98	1.85	2.01	
R_{Eg}	0.82	0.60	−0.10	−0.11	0.18	0.25	0.09	0.06	50
	9.81	8.78	−5.80	−4.77	4.50	5.90	3.54	2.00	

Panel B: Regressing the Asness–Frazzini HML factor on the q-factor and q^5 models

	$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	R^2
HML$^{\mathrm{m}}$	0.34	0.37	−0.01	−0.10	0.93	−0.69		48
	2.13	2.36	−0.12	−0.95	8.18	−6.78		
		0.41	−0.01	−0.10	0.95	−0.67	−0.08	48
		2.99	−0.30	−0.98	7.72	−5.61	−0.72	

The Roe factor, R_{Roe}, has high correlations of 0.67 with RMW and 0.57 with RMWc. Intuitively, R_{Roe}, RMW, and RMWc are all based on different measures of profitability. The Roe factor also has a high correlation of 0.5 with UMD, suggesting that momentum contains some pricing information of Roe. More important, the Roe factor has high correlations of 0.8 with the replicated PERF factor and 0.69 with the replicated PEAD factor. As such, PERF and PEAD are closely related to the Roe factor.

The expected growth factor, R_{Eg}, has a high correlation of 0.59 with RMWc. Intuitively, firms with more cash available for investments tend to have high expected investment growth than firms with less cash available for investments. Cash- and accruals-based profitability measures are related, giving rise to correlations of R_{Eg} with the Roe factor, 0.52; with RMW, 0.43; with the replicated PERF factor, 0.51; and with the replicated PEAD factor, 0.4. Cash flows are also related to investment, giving rise to correlations of R_{Eg} with $R_{\mathrm{I/A}}$, 0.38; with CMA, 0.33; with the replicated MGMT factor, 0.54; and with the replicated FIN factor, 0.54. In all, the seemingly different factors are closely related.

4. Asset Pricing Implications from Valuation Theory

In this section, we turn to the economic foundation of factor models. The q and q^5 models stand out in that the investment, Roe, and expected growth factors are motivated from the first principle of real investment (Hou, Xue, and Zhang, 2015; Hou *et al.*, 2018). For comparison, the Stambaugh–Yuan (2017) model and the Fama–French (2018) six-factor model

Table V. Correlation matrix (January 1967–December 2016)

$R_{\mathrm{Mkt}}, R_{\mathrm{Me}}, R_{\mathrm{I/A}}$, and R_{Roe} are the market, size, investment, and Roe factors in the q and q^5 models, respectively, and R_{Eg} the expected growth factor in the q^5 model. SMB, HML, RMW, and CMA are the size, value, profitability, and investment factors in the Fama–French five- and six-factor models, respectively, and UMD the momentum factor in the six-factor model. RMWc is the cash-based profitability factor in the alternative six-factor model. MGMT and PERF are the management and performance factors in the replicated Stambaugh–Yuan model in our sample, and FIN and PEAD the financing and post-earnings-announcement drift factors in the replicated Daniel–Hirshleifer–Sun model, respectively. The p-values testing that a given correlation equals zero are in the rows beneath the correlations. The data on SMB, HML, RMW, CMA, and UMD are from Kenneth French's web site. The data on $\mathrm{HML}^{\mathrm{m}}$ from the AQR web site.

	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	SMB	HML	RMW	CMA	UMD	RMWc	MGMT	PERF	FIN	PEAD	$\mathrm{HML}^{\mathrm{m}}$
R_{Mkt}	0.27	–0.38	–0.21	–0.47	0.28	–0.27	–0.24	–0.40	–0.15	–0.48	–0.49	–0.23	–0.57	–0.11	–0.12
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
R_{Me}		–0.15	–0.31	–0.37	0.97	–0.04	–0.37	–0.05	–0.02	–0.53	–0.28	–0.20	–0.44	–0.18	0.00
		0.00	0.00	0.00	0.00	0.30	0.00	0.21	0.59	0.00	0.00	0.00	0.00	0.00	0.94
$R_{\mathrm{I/A}}$			0.04	0.38	–0.19	0.67	0.10	0.91	0.03	0.26	0.84	–0.02	0.69	–0.07	0.49
			0.34	0.00	0.00	0.00	0.02	0.00	0.53	0.00	0.00	0.55	0.00	0.10	0.00
R_{Roe}				0.52	–0.37	–0.14	0.67	–0.09	0.50	0.57	0.05	0.80	0.34	0.69	–0.45
				0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.25	0.00	0.00	0.00	0.00
R_{Eg}					–0.42	0.19	0.43	0.33	0.35	0.59	0.54	0.51	0.54	0.40	–0.06
					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18
SMB						–0.09	–0.37	–0.09	–0.05	–0.53	–0.31	–0.22	–0.47	–0.22	–0.01
						0.04	0.00	0.03	0.21	0.00	0.00	0.00	0.00	0.00	0.77
HML							0.10	0.69	–0.19	0.17	0.66	–0.23	0.64	–0.29	0.78
							0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RMW								–0.02	0.11	0.76	0.17	0.54	0.53	0.25	–0.05
								0.70	0.01	0.00	0.00	0.00	0.00	0.00	0.21
CMA									0.00	0.18	0.83	–0.08	0.63	–0.14	0.51
									0.97	0.00	0.00	0.04	0.00	0.00	0.00

(continued)

Table V. Continued

	R_{Me}	$R_{I/A}$	R_{Roe}	R_{Eg}	SMB	HML	RMW	CMA	UMD	RMWc	MGMT	PERF	FIN	PEAD	HML^m
UMD										0.17	0.03	0.71	0.03	0.70	−0.65
										0.00	0.46	0.00	0.46	0.00	0.00
RMWc											0.37	0.46	0.63	0.27	0.00
											0.00	0.00	0.00	0.00	0.96
MGMT												0.04	0.80	−0.05	0.49
												0.36	0.00	0.18	0.00
PERF													0.22	0.70	−0.63
													0.00	0.00	0.00
FIN														0.02	0.43
														0.59	0.00
PEAD															−0.61
															0.00

are largely statistical in nature.[11] Although Daniel, Hirshleifer, and Sun (2018) attempt to motivate their FIN factor from long-term overreaction and the PEAD factor from short-term underreaction, the conceptual linkage between specific psychological biases and anomalies in question seems tenuous.[12]

Fama and French (2015) attempt to provide an economic foundation for their five-factor model based on the residual income valuation model (Preinreich, 1938; Miller and Modigliani, 1961; Ohlson, 1995). In the dividend discounting model, a firm's market equity is the present value of its dividends:

$$P_{it} = \sum_{\tau=1}^{\infty} \frac{E[D_{it+\tau}]}{(1+r_i)^{\tau}} \tag{1}$$

in which P_{it} is the market equity, D_{it} dividends, and r_i the long-term average expected return, or the internal rate of return (IRR) (Williams, 1938). The clean surplus relation says that dividends equal earnings minus the change in book equity, $D_{it+\tau}=Y_{it+\tau} - \triangle\text{Be}_{it+\tau}$, in which $Y_{it+\tau}$ is earnings, and $\triangle\text{Be}_{it+\tau} \equiv \text{Be}_{it+\tau} - \text{Be}_{it+\tau-1}$ the change in book equity. The dividend discounting model becomes:

$$\frac{P_{it}}{\text{Be}_{it}} = \frac{\sum_{\tau=1}^{\infty} E[Y_{it+\tau} - \triangle\text{Be}_{it+\tau}]/(1+r_i)^{\tau}}{\text{Be}_{it}}. \tag{2}$$

Fama and French (2015) make three predictions based on Equation (2). First, fixing everything except the current market value, P_{it}, and the expected stock return, r_i, a low P_{it}, or a high book-to-market equity, Be_{it}/P_{it}, implies a high expected return. Second, fixing everything except the expected profitability and the expected stock return, high expected profitability implies a high expected return. Finally, fixing everything except the expected book equity growth (expected investment) and the expected return, high expected book equity growth implies a low expected return.

Equation (2) connects book-to-market, investment, and profitability to the IRR. However, Fama and French (2015) argue that the difference between the one-period-ahead expected return and the IRR is unimportant.[13] Empirically, Fama and French use current

11 In particular, Fama and French (2018) acknowledge: "We include momentum factors (somewhat reluctantly) now to satisfy insistent popular demand. We worry, however, that opening the game to factors that seem empirically robust but lack theoretical motivation has a destructive downside: the end of discipline that produces parsimonious models and the beginning of a dark age of data dredging that produces a long list of factors with little hope of sifting through them in a statistically reliable way (p. 237)."

12 For example, in a recent survey from the behavioral perspective, Lee and So (2015) acknowledge: "Be forewarned: none of these [behavioral] studies will provide a clean one-to-one mapping between the investor psychology literature and specific market anomalies. Rather, their goal is to simply set out the experimental evidence from psychology, sociology, and anthropology. The hope is that, thus armed, financial economists would be more attuned to, and more readily recognize, certain market phenomena as manifestations of these enduring human foibles (p. 69)."

13 In particular, Fama and French (2015) argue: "Most asset pricing research focuses on short-horizon returns—we use a one-month horizon in our tests. If each stock's short-horizon expected return is positively related to its IRR—if, for example, the expected return is the same for all horizons—the valuation equation implies that the cross-section of expected returns is determined by the combination of current prices and expectations of future dividends. The decomposition of

profitability as a proxy for the expected profitability to form RMW and current asset growth as a proxy for the expected investment to form CMA.

We raise four concerns on the Fama–French (2015) reasoning. First, the IRR can differ drastically from, and can even correlate negatively with, the one-period-ahead expected return (Section 4.1). Second, HML is a separate factor from CMA in the Fama–French setup but is redundant in explaining average returns in the data (Section 4.2). Third, CMA can only arise from the market-to-book term, P_{it}/Be_{it}, in Equation (2). In contrast, the expected book equity growth is positively correlated with the one-period-ahead expected return (Section 4.3). Finally, past investment is a poor proxy for the expected investment (Section 4.4).

4.1 The IRR Is Not Equal to the One-Period-Ahead Expected Return

The Fama–French (2015) assumption that the expected return is the same for all horizons contradicts the notion of time-varying expected returns. The IRR can differ greatly from the one-period-ahead expected return. The difference is most striking in the context of price and earnings momentum. Chan, Jegadeesh, and Lakonishok (1996) show that momentum profits are short-lived, large, and positive for up to 12 months, but negative afterward. In contrast, Tang, Wu, and Zhang (2014) estimate price and earnings momentum to be significantly negative, once measured as the IRR per Gebhardt, Lee, and Swaminathan (2001).

To quantify how the IRRs deviate from one-period-ahead average returns, we estimate the IRRs for the Fama–French (2015) SMB, HML, RMW, and CMA per Claus and Thomas (2001); Gebhardt, Lee, and Swaminathan (2001); Easton (2004); and Ohlson and Juettner-Nauroth (2005). Although differing in implementation details, these methods all share the basic idea of backing out the IRRs from different versions of the valuation Equation (2). The baseline versions of these accounting methods use analysts' forecasts as expected cash flows. Because analysts' forecasts are limited to a relatively small sample of large, mature firms, and are likely even biased, we also implement two alternative procedures. Hou, van Dijk, and Zhang (2012) use pooled cross-sectional regressions to forecast future earnings, and Tang, Wu, and Zhang (2014) use annual cross-sectional regressions to forecast future profitability. We detail the estimation procedures in the Online Appendix.

Empirically, we take one period to be one year, and compare the average factor IRRs at the June end of each year t with the annual average factor returns from the July of year t to the June of year $t+1$. Panel A of Table VI reports that the IRRs estimated with analysts' earnings forecasts for RMW differ significantly from their one-period-ahead average returns. The differences for RMW are significant in twelve out of the twelve experiments from intersecting the three expected Roe estimation procedures with the four accounting models. The IRRs of RMW are even significantly negative in eight experiments, in contrast to the average returns that are significantly positive in all twelve.

Averaging across the four IRR models implemented with analysts' earnings forecasts, the IRR of RMW is –1.58% per annum ($t=-9.66$), whereas its one-period-ahead average return is 4.52% ($t=2.88$). The contrast from implementing the accounting models with cross-sectional earnings forecasts is largely similar, –1.84% ($t=-9.41$) versus 3.61% ($t=2.66$). With cross-sectional Roe forecasts, the comparison is between –2.47% ($t=-21.47$) versus 3.14% ($t=2.54$).

cash flows then implies that each stock's relevant expected return is determined by its price-to-book ratio and expectations of its future profitability and investment (p. 2)."

Table VI. Estimates of the IRRs for the Fama–French (2015) factors (1967–2016)

AR, IRR, and Diff (all in annual percent) are the average return, the IRR, and AR minus IRR, respectively. SMB, HML, RMW, and CMA are the Fama–French (2015) size, value, profitability, and investment factors, respectively. IRR is measured at the June of each year t, and AR from the July of year t to June of $t+1$. Panel A uses the analysts' earnings forecasts, Panel B the Hou–van Dijk–Zhang (2012) cross-sectional earnings forecasts, and Panel C the Tang–Wu–Zhang (2014) cross-sectional Roe forecasts in estimating the IRRs. GLS denotes the Gebhardt–Lee–Swaminathan model, Easton the Easton model, CT the Claus–Thomas model, OJ the Ohlson–Juettner–Nauroth model, and Average the averages across the models. The t-values (in the rows beneath the IRRs) are adjusted for heteroscedasticity and autocorrelations.

	Panel A: IBES earnings forecasts (1979–2016)						Panel B: Cross-sectional earnings forecasts (1967–2016)						Panel C: Cross-sectional Roe forecasts (1967–2016)					
	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff
		GLS			Easton			GLS			Easton			GLS			Easton	
SMB	1.51	0.88	0.64	1.57	2.51	−0.94	2.57	1.52	1.04	2.38	5.11	−2.72	2.91	0.06	2.84	2.71	1.14	1.58
	0.76	4.62	0.33	0.82	14.85	−0.50	1.25	4.02	0.53	1.18	6.96	−1.44	1.49	0.26	1.49	1.38	4.82	0.80
HML	2.98	3.50	−0.52	2.90	3.26	−0.36	3.36	5.57	−2.20	3.27	7.23	−3.96	3.64	5.21	−1.57	3.87	7.60	−3.73
	1.39	18.91	−0.25	1.29	7.45	−0.17	1.83	27.23	−1.24	1.77	15.32	−2.08	1.98	32.87	−0.89	2.04	16.10	−2.08
RMW	3.72	−1.19	4.91	4.48	−3.27	7.75	3.46	−1.43	4.89	4.06	−3.66	7.72	3.13	−1.35	4.47	3.35	−6.41	9.76
	2.64	−8.27	3.58	2.82	−9.44	4.75	2.72	−6.54	4.00	2.77	−10.22	5.00	2.54	−7.81	3.77	2.65	−19.72	7.19
CMA	3.46	0.64	2.82	3.58	2.45	1.13	3.72	1.59	2.13	4.46	4.06	0.40	3.50	1.09	2.41	3.63	4.52	−0.88
	2.87	4.69	2.42	3.14	7.91	1.08	3.17	9.09	1.87	4.34	11.56	0.37	3.19	5.75	2.24	3.10	9.97	−0.78
		CT			OJ			CT			OJ			CT			OJ	
SMB	1.55	1.11	0.43	1.70	0.68	1.02	2.65	2.71	−0.06	3.73	3.21	0.52	3.23	−1.17	4.41	3.21	−1.40	4.62
	0.78	6.84	0.22	0.86	4.57	0.53	1.29	4.23	−0.03	1.91	5.39	0.28	1.65	−3.65	2.29	1.59	−3.86	2.26
HML	3.12	0.14	2.98	2.05	0.73	1.32	3.14	3.68	−0.54	2.85	4.74	−1.89	2.95	2.64	0.31	3.02	4.23	−1.22
	1.42	0.42	1.33	0.98	3.77	0.63	1.71	16.21	−0.29	1.69	17.64	−1.10	1.74	5.90	0.18	1.58	9.36	−0.63

(continued)

Table VI. Continued

	Panel A: IBES earnings forecasts (1979–2016)						Panel B: Cross-sectional earnings forecasts (1967–2016)						Panel C: Cross-sectional Roe forecasts (1967–2016)					
	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff	AR	IRR	Diff
RMW	3.66	0.29	3.38	4.07	–0.05	4.12	3.18	–0.01	3.19	3.16	–1.57	4.72	2.57	–0.04	2.61	2.07	–2.11	4.17
	2.68	1.94	2.48	3.41	–0.27	3.41	3.01	–0.03	3.04	3.14	–5.50	4.45	2.56	–0.20	2.59	2.04	–9.38	3.85
CMA	3.41	0.11	3.30	3.15	0.08	3.07	3.32	1.52	1.81	3.89	2.12	1.77	2.91	0.62	2.29	3.26	0.66	2.60
	2.88	0.79	2.78	2.79	0.48	2.70	2.92	11.29	1.54	3.35	11.11	1.54	2.68	4.32	2.06	2.67	3.06	2.04
	Average						Average						Average					
SMB	1.44	1.72	–0.28				2.53	3.22	–0.69				2.90	–0.25	3.15			
	0.76	10.74	–0.15				1.23	5.60	–0.35				1.49	–0.93	1.64			
HML	2.90	2.04	0.86				3.52	5.31	–1.79				3.60	5.14	–1.54			
	1.28	9.07	0.39				1.88	25.28	–0.97				1.96	17.72	–0.85			
RMW	4.52	–1.58	6.10				3.61	–1.84	5.45				3.14	–2.47	5.61			
	2.88	–9.66	3.90				2.66	–9.41	4.07				2.54	–21.47	4.52			
CMA	3.40	1.16	2.24				3.81	2.64	1.17				3.44	2.02	1.43			
	2.92	7.09	2.02				3.34	19.06	1.04				3.17	13.47	1.34			

Table VI also reports important IRR-average-return differences for CMA, although not as drastic as the differences for RMW. The differences for CMA are significant for six out of twelve experiments. Finally, without going through the details, we can report that, consistent with Tang, Wu, and Zhang (2014), the IRR-average-return differences for SMB and HML are mostly insignificant.

4.2 The Relation between Investment and Book-to-Market

Fama and French (2015) argue that market-to-book, expected profitability, and expected investment give rise to three separate factors in Equation (2). However, empirically, once RMW and CMA are added to their three-factor model, Fama and French report that HML becomes redundant in describing average returns in the data. This evidence contradicts their conceptual argument.

However, the evidence accords well with the investment CAPM underlying the q-factor model. Intuitively, the marginal cost of investment (which increases with investment-to-assets) equals marginal q (the value of an extra unit of capital). With constant returns to scale, marginal q equals average q (Hayashi, 1982), which is in turn highly correlated with market-to-book equity. This tight economic linkage between investment and value implies that HML should be highly correlated with the investment factor. From January 1967 to December 2016, the correlation between HML and CMA is 0.69, and the correlation between HML and the investment factor in the q-factor model is 0.67 (Table V). The economic linkage between investment and value also means that CMA can be motivated from the market-to-book term in the valuation Equation (2), barring the difference between the IRR and the one-period-ahead expected return (Section 4.1).

4.3 The Relations among Past Investment, the Expected Investment, and the Expected Return

Fama and French (2015) argue that Equation (2) predicts a negative relation between the expected investment and the IRR. However, this negative relation does not apply to the one-period-ahead expected return, $E_t[r_{it+1}]$. From the definition of return, $P_{it} = (E_t[D_{it+1}] + E_t[P_{it+1}])/(1 + E_t[r_{it+1}])$, and the clean surplus relation, we can reformulate the valuation Equation (2) in terms of the one-period-ahead expected return:

$$P_{it} = \frac{E_t[Y_{it+1} - \triangle \mathrm{Be}_{it+1}] + E_t[P_{it+1}]}{1 + E_t[r_{it+1}]}. \tag{3}$$

Dividing both sides of Equation (3) by Be_{it} and rearranging, we obtain:

$$\frac{P_{it}}{\mathrm{Be}_{it}} = \frac{E_t\left[\frac{Y_{it+1}}{\mathrm{Be}_{it}}\right] - E_t\left[\frac{\triangle \mathrm{Be}_{it+1}}{\mathrm{Be}_{it}}\right] + E_t\left[\frac{P_{it+1}}{\mathrm{Be}_{it+1}}\left(1 + \frac{\triangle \mathrm{Be}_{it+1}}{\mathrm{Be}_{it}}\right)\right]}{1 + E_t[r_{it+1}]}, \tag{4}$$

$$\frac{P_{it}}{\mathrm{Be}_{it}} = \frac{E_t\left[\frac{Y_{it+1}}{\mathrm{Be}_{it}}\right] + E_t\left[\frac{\triangle \mathrm{Be}_{it+1}}{\mathrm{Be}_{it}}\left(\frac{P_{it+1}}{\mathrm{Be}_{it+1}} - 1\right)\right] + E_t\left[\frac{P_{it+1}}{\mathrm{Be}_{it+1}}\right]}{1 + E_t[r_{it+1}]}. \tag{5}$$

Fixing everything except $E_t[\triangle \mathrm{Be}_{it+1}/\mathrm{Be}_{it}]$ and $E_t[r_{it+1}]$, high $E_t[\triangle \mathrm{Be}_{it+1}/\mathrm{Be}_{it}]$ implies *high* $E_t[r_{it+1}]$, because $P_{it+1}/\mathrm{Be}_{it+1} - 1$ is likely positive in the data. This prediction is consistent with the weakly positive $E_t[\triangle \mathrm{Be}_{it+1}/\mathrm{Be}_{it}]$ - $E_t[r_{it+1}]$ relation documented in Fama and French (2006).

The relation between the expected investment and the expected return is also positive in the investment CAPM, providing the motivation for the expected growth factor (Hou *et al.*, 2018). As such, the prediction from the valuation Equation (2), once reformulated in terms of the one-period-ahead expected return, is consistent with the investment theory.

4.4 Past Investment Is a Poor Proxy for the Expected Investment

After motivating CMA from the expected investment effect, Fama and French (2015) use past investment as a proxy for the expected investment. This procedure is problematic. Whereas past profitability is a good proxy for the expected profitability, past investment is a poor proxy for the expected investment. A large economics literature on lumpy investment emphasizes the lack of persistence of micro-level investment data (Dixit and Pindyck, 1994; Doms and Dunne, 1998; Whited, 1998).

To show the poor quality of past investment as a proxy for the expected investment, we adopt the Fama–French (2006) setup and perform annual cross-sectional regressions of future book equity growth rates, $\triangle \text{Be}_{it+\tau}/\text{Be}_{it+\tau-1} \equiv (\text{Be}_{it+\tau} - \text{Be}_{it+\tau-1})/\text{Be}_{it+\tau-1}$, for $\tau = 1, 2, \ldots, 10$, on the current asset growth, $\triangle A_{it}/A_{it-1} = (A_{it} - A_{it-1})/A_{it-1}$, and, separately, on book equity growth, $\triangle \text{Be}_{it}/\text{Be}_{it-1}$. For comparison, we also report annual cross-sectional regressions of future operating profitability, $\text{Op}_{it+\tau}$, on current Op_{it}.

Following Fama and French (2006), we include all stocks from 1963 to 2016, including financial firms. We measure book equity per Davis, Fama, and French (2000) (footnote 3) and operating profitability per Fama and French (2015). Variables dated *t* are from the fiscal year ending in calendar year *t*. Firms with total assets below \$5 million or book equity below \$2.5 million in year *t* are excluded in Panel A of Table VII. The cutoffs are \$25 and \$12.5 million, respectively, in Panel B. The right- and left-hand side variables in the regressions are winsorized each year at the 1–99% level.

Asset growth does not predict future book equity growth. In Panel A in Table VII, the slope starts at 0.22 at the one-year horizon and falls to 0.06 in year three and to 0.04 in year five. The average R^2 of the cross-sectional regressions starts at 5% in year one, drops to zero in year four, and stays at zero for the remaining years. Book equity growth does not predict future book equity growth either. The slope starts at 0.2 at the one-year horizon and drops to 0.06 in year three and to 0.02 in year five. The average R^2 of the cross-sectional regressions starts at 6% in year one, drops to zero in year four, and stays at zero for the remaining years. The results with the more stringent sample criterion in Panel B are largely similar. The evidence casts doubt on the motivation of CMA from the expected investment effect, but it lends support to our reinterpretation of CMA as the substitute for the value effect via the market-to-book term in the valuation Equation (2).

The last five columns in Table VII show that operating profitability forecasts future operating profitability. In Panel A, the slope in the annual cross-sectional regressions starts with 0.8 in year one, drops to 0.59 in year three and 0.49 in year five, and remains at 0.38 even in year ten. The average R^2 starts at 54% in year one, drops to 27% in year three and 19% in year five, and remains above 10% in year ten. The evidence with the more stringent sample criterion in Panel B is largely similar. As such, using past profitability as a proxy for the expected profitability is reasonable. However, using past investment as a proxy for the expected investment as in Fama and French (2015) is problematic.

Table VII. Annual cross-sectional regressions of future book equity growth rates and operating profitability (1963–2016)

The sample contains stocks on NYSE, Amex, and Nasdaq. We do not exclude financial firms per Fama and French (2015). All the regressions are annual cross-sectional regressions. A_{it} is total assets for firm i at year t, $\triangle A_{it} \equiv A_{it} - A_{it-1}$, Be_{it} is book equity for firm i at year t, $\triangle \mathrm{Be}_{it} \equiv \mathrm{Be}_{it} - \mathrm{Be}_{it-1}$, and Op_{it} is operating profitability for firm i at year t. Book equity is measured as in Davis, Fama, and French (2000), and operating profitability is measured as in Fama and French (2015). Variables dated t are measured at the end of the fiscal year ending in calendar year t. To avoid the excess influence of small firms, we follow Fama and French (2006) and exclude those with total assets below \$5 million or book equity below \$2.5 million in year t in Panel A. The cutoffs are \$25 million and \$12.5 million in Panel B. We winsorize all regression variables at the 1 and 99 percentiles of the cross-sectional distribution each year.

		$\frac{\triangle \mathrm{Be}_{it+\tau}}{\mathrm{Be}_{it+\tau-1}} = \gamma_0 + \gamma_1 \frac{\triangle A_{it}}{A_{it-1}} + \epsilon_{t+\tau}$					$\frac{\triangle \mathrm{Be}_{it+\tau}}{\mathrm{Be}_{it+\tau-1}} = \gamma_0 + \gamma_1 \frac{\triangle \mathrm{Be}_{it}}{\mathrm{Be}_{it-1}} + \epsilon_{t+\tau}$					$\mathrm{Op}_{it+\tau} = \gamma_0 + \gamma_1 \mathrm{Op}_{it} + \epsilon_{t+\tau}$				
τ	Number of firms	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2
Panel A: Firms with assets ≥\$5 million and book equity ≥\$2.5 million																
1	3,105	0.09	14.46	0.22	13.94	0.05	0.09	13.10	0.20	8.47	0.06	0.03	4.73	0.80	43.30	0.54
2	2,843	0.10	14.32	0.10	7.60	0.01	0.10	14.43	0.10	5.21	0.02	0.05	6.24	0.67	27.12	0.36
3	2,624	0.10	14.71	0.06	6.31	0.01	0.10	14.70	0.06	4.05	0.01	0.07	7.84	0.59	24.18	0.27
4	2,431	0.10	15.78	0.05	5.53	0.00	0.10	15.88	0.05	3.69	0.00	0.09	9.32	0.53	22.64	0.22
5	2,259	0.10	14.76	0.04	3.44	0.00	0.10	15.71	0.02	1.92	0.00	0.10	11.18	0.49	22.78	0.19
6	2,103	0.10	14.99	0.05	4.57	0.00	0.10	14.71	0.03	2.27	0.00	0.11	13.03	0.45	23.22	0.16
7	1,961	0.09	15.15	0.04	4.43	0.00	0.10	15.26	0.03	2.68	0.00	0.11	14.62	0.43	21.87	0.15
8	1,828	0.09	15.07	0.03	4.14	0.00	0.10	15.35	0.01	1.71	0.00	0.12	15.86	0.40	19.23	0.13
9	1,706	0.09	15.09	0.03	3.37	0.00	0.10	15.16	0.01	1.19	0.00	0.12	15.08	0.39	17.63	0.12
10	1,593	0.09	14.47	0.04	4.32	0.00	0.09	14.61	0.02	2.13	0.00	0.12	14.14	0.38	16.68	0.11

(continued)

Table VII. Continued

τ	Number of firms	$\frac{\triangle \mathrm{Be}_{it+\tau}}{\mathrm{Be}_{it+\tau-1}} = \gamma_0 + \gamma_1 \frac{\triangle A_{it}}{A_{it-1}} + \epsilon_{t+\tau}$					$\frac{\triangle \mathrm{Be}_{it+\tau}}{\mathrm{Be}_{it+\tau-1}} = \gamma_0 + \gamma_1 \frac{\triangle \mathrm{Be}_{it}}{\mathrm{Be}_{it-1}} + \epsilon_{t+\tau}$					$\mathrm{Op}_{it+\tau} = \gamma_0 + \gamma_1 \mathrm{Op}_{it} + \epsilon_{t+\tau}$				
		γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2	γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	R^2
Panel B: Firms with assets $\geq$\$25 million and book equity $\geq$\$12.5 million																
1	2,492	0.08	15.77	0.23	16.94	0.05	0.08	14.11	0.24	10.36	0.07	0.03	7.15	0.82	58.34	0.61
2	2,284	0.09	15.41	0.13	10.30	0.02	0.09	15.84	0.12	7.08	0.02	0.06	8.95	0.70	34.86	0.42
3	2,109	0.09	16.25	0.08	7.65	0.01	0.09	16.18	0.08	5.75	0.01	0.08	10.74	0.62	30.44	0.32
4	1,956	0.09	16.71	0.07	6.75	0.01	0.09	16.69	0.07	4.41	0.01	0.09	13.11	0.56	29.61	0.26
5	1,821	0.09	16.35	0.05	3.94	0.01	0.09	16.97	0.04	2.85	0.01	0.10	15.92	0.52	30.60	0.22
6	1,699	0.09	16.24	0.05	5.50	0.00	0.09	16.04	0.04	3.37	0.00	0.11	17.80	0.48	31.61	0.19
7	1,588	0.09	16.48	0.05	4.96	0.00	0.09	16.58	0.04	3.12	0.00	0.12	19.76	0.45	31.23	0.17
8	1,485	0.09	15.84	0.03	3.99	0.00	0.09	15.83	0.02	2.62	0.00	0.13	20.33	0.43	27.16	0.15
9	1,388	0.09	15.68	0.03	3.84	0.00	0.09	15.80	0.02	2.10	0.00	0.13	18.98	0.42	22.62	0.14
10	1,298	0.09	14.50	0.05	5.23	0.00	0.09	14.71	0.03	2.98	0.00	0.13	17.77	0.41	21.77	0.13

5. Conclusion

Many recently proposed, seemingly different factor models are closely related. Empirically, the q-factor model largely subsumes the Fama–French (2015, 2018) five- and six-factor models in spanning regressions. The Stambaugh–Yuan (2017) factors are sensitive to their construction, and once replicated via the traditional approach, are close to the q-factors, with correlations of 0.8 and 0.84. Neither the original nor the replicated Stambaugh–Yuan model can explain the q and q^5 factors in the Gibbons–Ross–Shanken (1989) test, but the q^5 model can explain both their original and replicated factors. The Daniel–Hirshleifer–Sun (2018) factors are also sensitive to their construction, and once replicated via the traditional approach, are close to the q-factors, with correlations of 0.69. Their three-factor model cannot explain the size, investment, and expected growth factors, and the q and q^5 models cannot explain their earnings factor. Finally, the Barillas–Shanken (2018) model, which embeds the investment and Roe factors from the q-factor model, cannot explain the expected growth factor in the q^5 model. Although the q-factor model cannot explain the Asness–Frazzini (2013) monthly formed HML factor in the Barillas–Shanken specification, the monthly formed q-factor model can.

Conceptually, a unique advantage of the q-factor and q^5 models over the competing models is their economic foundation based on the first principle of real investment. In contrast, the Stambaugh–Yuan (2017), Daniel–Hirshleifer–Sun (2018), and Fama–French (2018) six-factor models are mostly *ad hoc* and statistical in nature. We also show that the Fama–French (2015) five-factor model cannot be motivated from valuation theory as originally advertised. In particular, once reformulated with the one-period-ahead expected return, valuation theory also implies a positive relation between the expected investment and the expected return, consistent with the investment CAPM.

Supplementary Material

Supplementary data are available at *Review of Finance* online.

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