Investment-based Costs of Equity

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Abstract

The q^5-characteristics model estimates costs of equity as Lewellen's (2015) out-of-sample forecasts from cross-sectional regressions. The q^5-cost of equity is competitive in evaluation tests, outperforming the accounting implied cost of equity in predicting cross-sectional returns. The q^5-cost of equity is precise at the industry level and aligned with average factor premiums. Its firm-level distribution is weakly left-skewed, whereas the accounting implied cost of equity is right-skewed. However, the accounting cost of equity outperforms in the time series. Factor models perform poorly in out-of-sample tests. Gradient-boosted trees improve on cross-sectional regressions, but not reliably.

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1 Introduction

The cost of equity problem is one of the most foundational problems in finance. Most practical applications hinge on a high-quality cost of equity measure, such as investment management, business valuation, and capital budgeting, with trillions of dollars involved (Pratt and Grabowski 2014).

However, in the academic literature, how to measure a firm's cost of equity is still a largely open question. Despite their popularity, factor models give rise to costs of equity that are "distressingly imprecise" even at the industry level (Fama and French 1997, p. 178). A prominent literature that estimates costs of equity from accounting-based valuation framework has developed over the past 25 years (Gebhardt, Lee, and Swaminathan 2001). Alas, these implied costs of equity exhibit only weak associations with realized returns (Easton and Monahan 2005). And the implied factor premiums often even have the opposite signs to realized premiums (Tang, Wu, and Zhang 2014).

We measure q-theory based costs of equity, denoted QCE, as Lewellen's (2015) out-of-sample forecasts from cross-sectional regressions by combining slopes from prior 120-month rolling windows with the latest known q^5-characteristics. These characteristics undergird the q- and q^5-factor models, including size, investment-to-assets, return on equity, and expected growth (Hou et al. 2021).

Through a battery of evaluation tests, we establish the q^5-characteristics model as competitive for estimating costs of equity. Sorting on a given cost of equity measure is the simplest way to test its association with future returns. QCE outperforms the accounting implied cost of equity (ICE) in this test. The high-minus-low QCE decile earns on average 1.62%, 0.81%, and 0.57% per month ($t = 7.29$, 4.01, and 2.93) at the 1-, 12-, and 24-month horizons post formation, whereas the high-minus-low ICE decile earns −0.03%, 0.03%, and 0.04% ($t = -0.08$, 0.1, and 0.13), respectively.

We also perform the Lewellen (2015) test on whether the predictive slope equals one in cross-sectional regressions of subsequent returns on a given cost of equity proxy. If the average return is an unbiased proxy for the true expected return, this test checks whether the cost of equity measure is unbiased. QCE again outperforms ICE in this test. The QCE slopes are 0.82, 0.63, 0.81, 0.61, and

0.54 at the 1-, 12-, 24-, 36-, and 60-month horizons, whereas the ICE slopes are 0.42, 0.51, 0.49, 0.38, and 0.29, respectively. The corresponding t-values that testing a given slope equals one are 1.63, 2.51, 1.47, 3.1, and 2.92 for QCE, compared with 3.45, 3.8, 5.9, 9.1, and 12.18 for ICE, respectively.

We also use time series and cross-section measurement error variances (MEV) to evaluate costs of equity. Lee, So, and Wang (2021) argue that for treatment effects (how differences in firm policies and characteristics are related to differences in costs of equity), the bias in a given cost of equity measure is less important than the MEVs. We find that QCE underperforms ICE in time series MEV. The QCE-minus-ICE differentials are positive up to the 36-month, albeit turning negative at the 60-month, and all are significant. However, QCE shows an edge over ICE in cross-section MEV. The differentials are negative across all horizons, albeit mostly insignificant.

Inspired by recent advances of machine learning in asset pricing (Gu, Kelly, and Xiu 2020), we explore gradient-boosted trees to capture nonlinear relations between the q^5-characteristics and expected returns. A decision tree predicts returns by sequentially sorting stocks into categories of characteristics. However, a single, deep tree is prone to overfitting. Boosting mitigates overfitting by combining many shallow trees into a single, more reliable forecast.[1] Extensive tests show that gradient-boosted trees improve on the regression-based model in some aspects but worsens its performance in others. Ultimately, in the perennial struggle between interpretability and forecastability, we opt for the time-honored cross-sectional regressions, at least in our applications.

We use the q^5-characteristics model to study firm-level costs of equity, industry costs of equity, expected factor premiums, and the equity premium. At the firm level, the term structure of the QCE mean is largely flat in the January 1977–December 2024 sample. The 1-month QCE is on average 13.83% per annum, which is close to 13.85% at the 12-month and 13.03% at the 36-month. In contrast, the term structure of the QCE volatility is downward sloping: 10.75%, 6.85%, and 4.48% at the 1-, 12-, and 36-month, respectively. Also, the firm-level QCE distributions are weakly

[1]Compared to neural nets, gradient boosting is easier to train, more transparent, and more effective on tabular data that are skewed and heavy-tailed (Grinsztajn, Oyallon, and Varoquaux 2022; McElfresh et al. 2023).

left-skewed across all horizons, while the firm-level ICE distribution is right-skewed.

The q^5-characteristics model largely resolves the Fama-French (1997) imprecision problem. Averaged across 57 nonfinancial industries per North American Industry Classification System (NAICS), the return volatility is 25.94% per annum (relative to the average return of 13.85%). The volatility of the q^5-factor costs of equity is 9.94%, which implies a standard error of 1.43% given a sample length of 48 years (relative to its average of 10.29%). In contrast, the volatility of the 12-month QCE is only 4.4%, which implies a standard error of 0.64% (relative to its average of 10.33%).

The q^5-characteristics model also goes a long way in resolving the weak association problem with realized returns (Tang, Wu, and Zhang 2014). The high-minus-low deciles on momentum (prior 11-month return) and return on equity earn large and significant average realized returns in short horizons, and the high-minus-low expected growth decile does so across all horizons. The QCE premiums are largely aligned with average returns, but the ICE premiums are significantly negative across all horizons. In particular, the high-minus-low expected growth decile earns average realized holding period returns of 0.99%, 9.56%, and 26.96% ($t = 5.38$, 4.85, and 5.18) at the 1-, 12-, and 36-month, the QCE premiums of 1.43%, 9.28%, and 20.68% ($t = 14.11$, 11.98, and 12.57), but the ICE premiums of $-0.14\%, -1.96\%$, and -7.78% ($t = -6.32, -6.43$, and -6.62), respectively.

However, we find that the ICE performance is remarkable in time series predictive regressions of factor premiums, especially the equity premium. The ICE slopes are positive and mostly significant across horizons, with R^2 up to 32.3% for momentum, 12% for the ROE factor, and 11% for the expected growth factor. For the equity premium, the ICE slopes are positive and significant across all horizons, with R^2 up to 24.5%. The QCE performance is fair, but much weaker.

Our estimation of costs of equity follows Lewellen (2015). Lewellen shows that out-of-sample forecasts from cross-sectional regressions provide a simple, yet surprisingly effective, way to form a composite trading strategy on predicted returns. This approach is attractive because the forecasts mimic how an investor could, in real time, combine many firm characteristics to form a stock's expected return. Lewellen examines up to 15 characteristics. We instead focus on the q^5-characteristics that are guided by theory. We compare extensively with ICE and gradient-boosted forecasts. We also detail the properties of QCE at the firm-, industry-, factor-, and aggregate-level.

Our work complements the implied costs of equity literature. We replicate ICE's superior performance in time series tests (Li, Ng, and Swaminathan 2013, 2026; Lee, So, and Wang 2021). However, QCE is relatively unbiased, as shown in cross-sectional predictive regressions. Clearly, bias in a given cost of equity proxy is of first-order importance in business valuation and capital budgeting. QCE is also strongly associated with subsequent returns in portfolio sorts. As such, we echo Lee et al. that time series and cross section entail different pricing information. Intuitively, derived from the accounting valuation framework, ICE is a nonlinear function of valuation ratios, which have traditionally performed well in the time series predictability literature. By comparison, the q^5-characteristics are motivated from economic theory that mostly focuses on the cross section.

Finally, our work expands investment-based asset pricing. Prior studies have mostly focused on in-sample fit (Hou, Xue, and Zhang 2015; Kilic, Yang, and Zhang 2022). We take the all-important step to develop out-of-sample forecasts, which are more directly applicable to industry practice.

The rest of the article is organized as follows. Section 2 describes the different methods of estimating costs of equity. Section 3 presents the evaluation methods and a battery of evaluation tests. Section 4 shows the properties of investment-based costs of equity. Finally, Section 5 summarizes and interprets our findings. A separate Internet Appendix furnishes supplementary results.

2 Estimating Costs of Equity

We study three classes of models: (i) factor models (Section 2.1); (ii) accounting-based implied costs of equity (Section 2.2); and (iii) characteristics-based cross-sectional forecasts (Section 2.3).[2]

[2] We have also studied in-depth a fourth class based on the structural investment model (Liu, Whited, and Zhang 2009). When we started this project back in 2019, our idea was to develop the structural model as a main tool for out-of-sample forecasts. However, this idea has largely failed, as the model's performance leaves much to be desired. As such, we summarize the basic results in Section 5, while delegating the details to the Internet Appendix.

2.1 Factor Models

We study the costs of equity from the Hou et al. (2021) q^5-factor model and the Fama-French (2018) 6-factor model. At the beginning of month t, we estimate betas from prior 60-month rolling window (30 months minimum) and combine the betas with factor premiums averaged over the expanding window from January 1967 to month $t-1$ to compute a stock's expected risk premium. We add back the riskless interest rate at the beginning of month t to obtain its cost of equity. To mitigate outliers, we winsorize factor-based costs of equity at 2.5 and 97.5 percentiles in each cross section.

The estimation is for 1-month forecasts. For longer horizons, because factor models are unconditional, the implied term structure of equity is flat. As such, for 12-month ahead costs of equity, for instance, we compound the 1-month forecast over 12 months: $\text{CE}_{12} = (1+\text{CE}_1)^{12}-1$, in which CE_h denotes the h-month forecast. Estimating betas directly at the h-month horizon requires a substantially longer rolling window to obtain a sufficient number of non-overlapping observations. This procedure reduces the effective sample and adds estimation noise. As such, we opt to keep the monthly frequency to ease comparison with other costs of equity over longer horizons.

2.2 The Accounting-based Implied Cost of Equity

In the dividend discount model, the market value of firm i, P_{it}, is the present value of its expected dividends, $P_{it} = \sum_{\tau=1}^{\infty} E[D_{it+\tau}]/(1+er_i)^{\tau}$, in which $D_{it+\tau}$ is dividends, and er_i is the firm's long-term expected stock return. The clean surplus relation says that dividends equal earnings, $Y_{it+\tau}$, minus the change in book equity, $D_{it+\tau} = Y_{it+\tau} - \triangle B_{it+\tau}$, in which $\triangle B_{it+\tau} \equiv B_{it+\tau} - B_{it+\tau-1}$. The valuation equation then becomes $P_{it}/B_{it} = (1/B_{it})\sum_{\tau=1}^{\infty} E[Y_{it+\tau} - \triangle B_{it+\tau}]/(1+er_i)^{\tau}$.

Following Lee, So, and Wang (2021), we examine the implied costs of equity (ICE) based on Claus and Thomas (2001), Gebhardt, Lee, and Swaminathan (2001), Easton (2004), and Ohlson and Juettner-Nauroth (2005). Although differing in details, these models all back out the cost of equity from the valuation equation. We work with the ICE composite, which is the equal-weighted average of the four ICE variants. Because analyst forecasts are limited to relatively small samples and are likely biased, we follow Hou, van Dijk, and Zhang (2012) to estimate regression-based earnings forecasts. The ICE estimates (in annual magnitude) are updated monthly because earnings forecasts and the market equity vary monthly. Appendix A details the implementation.

Because the ICE estimates are the internal rates of returns with a flat term structure by construction, it is straightforward to adjust the annual ICEs for different horizons. We calculate $\text{ICE}_h = (1 + \text{ICE}_{12})^{h/12} - 1$, before using ICE_h in forecast evaluation at the h-month horizon.

2.3 Characteristics-based Out-of-sample Forecasts

We construct costs of equity as out-of-sample return forecasts from both linear, cross-sectional regressions (Lewellen 2015) and nonlinear gradient-boosted trees via LightGBM (Ke et al. 2017).

2.3.1 Cross-sectional Regressions

Lewellen (2015) shows that out-of-sample forecasts from cross-sectional regressions provide a simple yet effective measure of expected returns, by combining slopes from the prior 120-month rolling window with the latest known characteristics at the beginning of month t.

Lewellen (2015) studies three specifications of cross-sectional regressions. The first specification contains size, book-to-market, and momentum. The second adds accruals, prior 36-month stock issuance, annual return on assets, and asset growth (seven variables in total). The third specification further adds dividend yield, long-term reversal, prior 12-month stock issuance, 36-month rolling market beta, stock volatility, turnover, market leverage, and sales-to-price (15 in total). We implement all three specifications. Appendix B.1 details the exact variable definitions.

We construct investment-based cross-sectional forecasts based on the q^5-factor model, which builds on size, investment-to-assets (I/A), return on equity (ROE), and expected growth (EG). EG is in turn from cross-sectional forecasts of changes in investment-to-assets on the log of Tobin's q, operating cash flow, and changes in ROE. Appendix B.2 details the measurement.

At the beginning of each month t, we construct 1-month ahead costs of equity via cross-sectional regressions of 1-month ahead returns on a given set of characteristics from prior 120-month rolling windows. We combine the average regression coefficients with the latest available winsorized predictors. The most recent accounting information is from the fiscal year (quarter) ending at least four months prior to month t. We use the latest announced quarterly earnings, and if not available, the earnings from the most recent fiscal quarter ending at least four months ago.

Timing-wise, at the beginning of month t, the latest returns in the rolling window regressions run at most up to month $t-1$, and the predictors are further lagged accordingly. When performing the cross-sectional forecasting regressions, we winsorize all the unbounded regressors at the 2.5 and 97.5 percentiles at their respective cross-sectional distributions each month. We do not winsorize returns.

The estimation is for 1-month ahead forecasts. We adapt the procedure analogously for other horizons. For 12-month forecasts, at the beginning of month t, we perform cross-sectional forecasting regressions of returns from month $t-12$ to $t-1$ on a given set of characteristics from prior 120-month rolling windows (with overlapping observations). In the latest rolling window, the returns run at most up to month $t-1$, and the predictors are from the fiscal period ending at least four months prior to month $t-12$. To calculate 12-month costs of capital, we combine the average slopes with the most recent winsorized predictors from the fiscal period ending at least four months prior to month t.

2.3.2 Gradient-boosted Trees

Besides linear regressions, we adopt Gradient Boosting Machine (GBM) to account for the nonlinear relations between q^5-characteristics and expected returns. GBM is a popular tree-based method that combines many decision trees sequentially ("boosting") to improve out-of-sample accuracy.

In our application, a decision tree predicts returns by sequentially splitting stocks into groups based on one of the q^5-characteristics. For instance, we can first split stocks into two groups based on whether a stock's I/A exceeds 10%. Within each I/A group, we can further split stocks into groups based on whether ROE exceeds 6%. The procedure continues until certain limits are reached, such as maximum tree depth. Finally, we compute the average return for each final group (leaf) and use it as the return prediction for stocks with the corresponding combination of characteristics (e.g., size below $1 billion, I/A below 10%, ROE above 6%, and EG above 5%). As a straightforward nonlinear model, a decision tree naturally captures interactions among characteristics.

A single tree is easy to build, but prone to overfitting. A deep tree can perfectly memorize return patterns in sample by creating increasingly narrow groups of stocks, yet fail to generalize out of sample. Boosting mitigates overfitting by combining many shallow trees into a single, more reliable prediction, an approach known as an ensemble. Rather than building one deep tree, boosting builds many shallow trees sequentially, in which each new tree focuses on correcting the errors of previous trees. After the first tree generates return predictions, we compute its errors. We then fit the second tree to the errors, the third tree to the remaining errors of the first two, and so on. The final prediction is the sum of all trees' predictions, each scaled by a small learning rate to control overfitting.

Suppose the first tree splits stocks based on I/A and ROE. The second tree then focuses on stocks that the first tree predicts poorly, perhaps by finding that among high-ROE stocks, EG further differentiates returns in a way the first tree has missed. Over many iterations, the ensemble progressively refines its predictions to capture subtle interactions among size, I/A, ROE, and EG.

Compared to deep learning such as neural nets, GBM models are easier to train, more transparent, and tend to perform better on tabular data of the scale typical in empirical finance (Grinsztajn, Oyallon, and Varoquaux 2022). Grinsztajn et al. compare deep learning against tree-based models across 45 datasets, and find that tree-based models remain state-of-the-art on medium-sized tabular data even without accounting for their faster training speed. One important reason for this relative performance is that neural nets struggle with fitting irregular target functions.

In a larger comparative study, McElfresh et al. (2023) find that gradient-boosted trees are much better than neural nets at handling skewed or heavy-tailed distributions and other data irregularities. In their book "Machine Learning for Tabular Data," Ryan and Massaron (2025) show that gradient boosting is better than deep learning in terms of simplicity and transparency and is very close in terms of efficacy. Because expected returns often have irregular relations with characteristics, with premiums concentrated in specific regions, we opt for gradient boosting.[3]

We use the same monthly cross-sectional data as in our cross-sectional regression models to train our gradient-boosted trees. At the beginning of each month t, we use the latest firm characteristics (size, I/A, ROE, and EG) to forecast h-month ahead cumulative returns over months t through $t+h-1$. We tune hyperparameters via five-fold cross-validation and retrain the final model on the full cross-section for each month. We repeat this procedure each month to yield a monthly series of tree models that are analogous to the monthly series of cross-sectional regression models.

To construct out-of-sample forecasts, we apply the same rolling window procedure as in regression-based forecasts. At the beginning of month t, we apply the latest firm characteristics to each of the preceding 120 monthly tree models to generate 120 return forecasts. The timing is identical to that in cross-sectional regressions to ensure no look-ahead bias. For each stock, the final forecast is the average across the 120 tree-based forecasts. This procedure is analogous to our cross-sectional regression forecasts so as to ease direct comparison.

We are not aware of prior studies that adopt a similar rolling window procedure in machine learning. Besides the ease of comparison with cross-sectional regressions, this procedure has two additional benefits. First, it mitigates the impact of outliers. Outliers only affect a given cross section, whereas our forecasts average across 120 trees. Second, training purely cross-sectional models simplifies data split for cross-validation, with no need to accommodate time series information.

We implement gradient boosting via LightGBM, which is a highly efficient GBM framework that offers substantial speed and memory advantages over alternatives such as XGBoost and CatBoost, making it well-suited for large panel datasets.[4] We implement three versions of the GBM model, each with a different objective function, including mean squared error (MSE), mean absolute error

[3]Another popular tree-based model is random forests, which randomly sample a subset of characteristics at each split to diversify the ensemble. However, with only four (economically motivated) q^5-characteristics, this subsampling procedure offers little diversity and limits its effectiveness.

[4]Originally developed by Microsoft, LightGBM is freely available at https://github.com/lightgbm-org/LightGBM.

(MAE), and Huber loss. MSE penalizes prediction errors quadratically, MAE penalizes errors linearly, and the Huber loss applies a quadratic penalty to errors below a threshold and a linear penalty to errors above it. Doing so combines sensitivity to small errors with robustness to large outliers.[5]

3 Evaluating Costs of Equity

We evaluate the relative performance of the three classes of costs of equity described in Section 2. We present evaluation methods in Section 3.1 and detail evaluation results in Section 3.2.

3.1 Evaluation Methods

We adopt three groups of evaluation methods: (i) portfolio sorts; (ii) cross-sectional predictive regressions (Lewellen 2015); and (iii) measurement error variances (Lee, So, and Wang 2021).

3.1.1 Portfolio Sorts

In portfolio sorts, at the beginning of month t (the end of month $t-1$), we split the NYSE, Amex, and NASDAQ stocks into deciles on each cost of equity. We calculate the monthly value-weighted decile returns for six holding periods (1-, 3-, 12-, 24-, 36-, and 60-month), over month t, from month t to $t+2$, month t to $t+11$, month t to $t+23$, month t to $t+35$, and month t to $t+59$, respectively. The 3-month horizon, for instance, means that for a given decile in each month, there exist three subdeciles, each initiated in a different month in the prior three months. We take the simple average of the subdecile returns as the monthly return for the decile.

We align the holding period with the forecast horizon. For instance, we use the 12-month holding period when forecasting returns with 12-month ahead costs of equity. To ease comparison with Lewellen (2015), we use NYSE-Amex-NASDAQ breakpoints in sorts but present evidence in the full

[5] For all three versions, we tune the following hyperparameters via grid search: maximum tree depth (max_depth: 3, 4, 5), maximum number of leaves (num_leaves: 8, 12, 15, 20), minimum leaf size (min_data_in_leaf: 30, 50, 100), learning rate (learning_rate: 0.01, 0.05, 0.1), L2 regularization on leaf weights (lambda_l2: 0, 5, 20), and row subsampling fraction (bagging_fraction: 0.6, 0.8, 1.0). For the Huber loss, we also tune the threshold parameter (alpha: 0.5, 1, 2), which governs the boundary between the quadratic and linear penalty regions. The full grid yields 972 combinations for MSE and MAE and 2,916 for the Huber loss. The optimal number of boosting rounds is determined jointly within the grid search using early stopping with a patience of 100 rounds and a maximum of 5,000 rounds. To activate row subsampling, we set the bagging frequency to one (bagging_freq = 1).

and all-but-micro samples. In the all-but-micro sample, we retrain all the predictive models without microcaps. Value- rather than equal-weighting returns is more sensitive to microcaps, whereas NYSE-Amex-NASDAQ rather than NYSE breakpoints is less so (Hou, Xue, and Zhang 2020).

We perform four sets of pairwise comparison tests: (i) the q^5-characteristics model via cross-sectional regressions (QCE) vs. the q^5-characteristics models via gradient-boosted trees; (ii) QCE vs. the Lewellen 3-, 7-, and 15-variable models; (iii) QCE vs. the accounting-based implied cost of equity; and (iv) QCE vs. factor models. For each pairwise comparison, we test whether the difference between the two high-minus-low decile returns in question equals zero.

3.1.2 Cross-sectional Predictive Regressions

Following Lewellen (2015), we also perform cross-sectional regressions of subsequent returns on the beginning-of-month costs of equity. We test if the predictive slope equals one. If the average ex post return is an unbiased proxy for the true expected return, this test checks whether a given ex ante expected return proxy is unbiased. We again align the horizon of subsequent returns with the forecast horizon in costs of equity. For example, when evaluating 12-month ahead costs of equity, we use subsequent 12-month returns as the dependent variable in cross-sectional regressions. To ease comparison with Lewellen (2015), we use ordinary least squares but present the evidence in the full and all-but-micro samples to quantify the impact of microcaps.

For each pairwise comparison, we test whether the difference between the two slopes from cross-sectional regressions equals zero. We use time series tests with Newey-West standard errors.

3.1.3 Measurement Error Variances

Lee, So, and Wang (2021) argue that if the goal is to study how differences in corporate policies or firm characteristics are related to differences in costs of equity (treatment effects), the bias in costs of equity is largely irrelevant. Intuitively, if the bias is uncorrelated to the variable in question, taking the difference in costs of equity eliminates the impact of bias. As such, measurement error variances (MEV) emerge as the most relevant criteria of costs of equity for treatment effects.

Let er_{it}^h be firm i's true (but unobservable) h-month ahead expected return at the beginning of month t and $\widehat{er}_{it}^h$ be an empirical proxy of er_{it}^h. The measurement error is defined as $\omega_{it}^h = \widehat{er}_{it}^h - er_{it}^h$. The measurement error should be distinguished from the forecast error, $e_{it+h} \equiv r_{it+h} - \widehat{er}_{it}^h$, for $t = 1, 2, \ldots, T_i^P - h$, in which $T_i^P - h$ is stock i's length of the predictive window (the overall sample minus the initial expanding window, adjusted for the forecast horizon, h). Let $T^P \equiv \max_i\{T_i^P\}$.

Lee, So, and Wang (2021) define the scaled time series variance of an expected return proxy as:

$$\text{SVar}_i^{\text{TS}} = \text{Var}_i\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right), \tag{1}$$

in which $\text{Var}_i(\widehat{er}_{it}^h)$ is time series variance of $\widehat{er}_{it}^h$, and $\text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)$ time series covariance between $\widehat{er}_{it}^h$ and r_{it+h}. Time series MEV is the average $\text{SVar}_i^{\text{TS}}$ across all N firms in the sample:

$$\overline{\text{SVar}}^{\text{TS}} = \frac{1}{N}\sum_i \text{SVar}_i^{\text{TS}}. \tag{2}$$

Following Lee et al., we require 20 observations to compute the time series scaled variance.

Analogously, Lee, So, and Wang (2021) define the scaled cross-sectional variance as:

$$\text{SVar}_t^{\text{CX}} = \text{Var}_t\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right), \tag{3}$$

in which $\text{Var}_t(\widehat{er}_{it}^h)$ is cross-sectional variance of $\widehat{er}_{it}^h$, and $\text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)$ cross-sectional covariance between $\widehat{er}_{it}^h$ and r_{it+h}. The cross-section MEV for a given measure is the time series average of $\text{SVar}_t^{\text{CX}}$ across all the periods, $T^P - h$ (the number of cross-sections):

$$\overline{\text{SVar}}^{\text{CX}} = \frac{1}{T^P - h}\sum_t \text{SVar}_t^{\text{CX}}. \tag{4}$$

We perform two pairwise comparison tests. First, the time series test checks if the time series MEV differential has a mean of zero across the N firms in the sample, using the White (1980) heteroscedasticity-consistent standard errors. Second, the cross-section test checks if the cross-section MEV differential has a mean of zero in the predictive window (with $T^P - h$ months), using the Newey-West standard errors. This test is equivalent to performing the Fama-MacBeth cross-sectional regression of differentials on an intercept and calculating its Newey-West t-value.

3.2 Evaluation Results

3.2.1 Portfolio Sorts

From Panel A of Table 1, sorting on QCE from the q^5-characteristics model via cross-sectional regressions yields a reliable high-minus-low spread in average 1-month ahead return of 1.62% per month ($t = 7.29$) in the full sample. The high-minus-low spread falls to 1.37% ($t = 6.46$) at the 3-month horizon and further to 0.81% ($t = 4.01$) at the 12-month, while still remaining at 0.57% ($t = 2.93$) at the 24-month horizon. The average return spread falls further to 0.26% ($t = 1.54$) at the 36-month before converging to zero at the 60-month horizon.

Gradient-boosted trees improve on the QCE results somewhat, but not significantly so. Panel A of Table 1 shows that gradient-boosted QCE with the MSE loss (QCE-GB) yields a larger high-minus-low spread in average 1-month ahead return of 1.79% per month ($t = 7.02$). The return spread falls to 1.66% ($t = 6.17$) at the 3-month horizon and further to 0.88% ($t = 3.89$) at the 12-month. These estimates are all somewhat higher than their counterparts from cross-sectional regressions. The estimates are close at the 24-month but lower at longer horizons. However, from Panel B, the differences in return spreads are all insignificant.

The results from gradient-boosted QCE with the MAE loss (QCE-GBA) and with the Huber loss (QCE-GBH) are largely similar, as shown in the Internet Appendix. From Panel A of Table A1, the high-minus-low return spread from sorting on QCE-GBA is 1.60% per month ($t = 5.05$) and 0.94% ($t = 3.45$) at the 3- and 12-month horizons, which are higher than 1.37% and 0.81% from sorting on QCE, respectively. The high-minus-low spread from sorting on QCE-GBH is 1.89% ($t = 6.02$) and 1.65% ($t = 5.70$) at the 1- and 3-month horizons, which are higher than 1.62% and 1.37% from sorting on QCE, respectively. However, from Panel B, the differences are again all insignificant.

Among the three Lewellen specifications, LCE (the 7-variable model) performs the best. From

Panel A of Table 1, sorting on LCE yields an average return spread of 1.09% per month ($t = 3.71$) at the 1-month horizon. The spread falls to 0.91% ($t = 3.32$) at the 3-month and further to 0.38% ($t = 1.42$) at the 12-month and 0.31% ($t = 1.31$) at the 36-month horizon. From Panel B, the QCE-minus-LCE differences in the return spread are large and (marginally) significant within short horizons up to 12 months but small and insignificant at longer horizons.

Sorting on LCE-3 (the 3-variable model) and LCE-15 (the 15-variable model) yields lower return spreads than on LCE. From Table A1, the high-minus-low ICE-3 spread is 0.49% per month ($t = 1.46$) at the 1-month, 0.39% ($t = 1.16$) at the 3-month, and 0.04% ($t = 0.15$) at the 12-month horizon. The high-minus-low ICE-15 spread is 0.90% ($t = 2.76$) at the 1-month, 0.81% ($t = 2.41$) at the 3-month, and 0.39% ($t = 1.25$) at the 12-month horizon. These spreads are mostly lower than their LCE counterparts. From Panel B, the QCE-minus-LCE differences are larger and more significant.

The accounting-based implied cost of equity, ICE, performs very poorly. From Panel A of Table 1, the high-minus-low ICE return spread is close to zero across all horizons. In particular, the return spreads are only -0.03%, -0.16%, and 0.03% per month ($t = -0.08, -0.43$, and 0.1) at the 1-, 3-, and 12-month horizons, respectively. The return spreads at longer horizons are largely similar. From Panel B, the QCE-minus-ICE differences are economically large and mostly significant. In particular, the differences are 1.65% ($t = 3.53$), 1.53% ($t = 3.57$), 0.78% ($t = 2.62$), and 0.54% ($t = 2.06$) at the 1-, 3-, 12-, and 24-month horizon, respectively.

Factor models also perform very poorly, echoing Fama and French (1997). The return spread sorted on Q5F (the q^5-factor based cost of equity) yields only 0.13% per month ($t = 0.57$) at the 1-month horizon and remains relatively small and statistically insignificant across all horizons (Table 1). Similarly, the return spread sorted on FF6F (the Fama-French 6-factor based cost of equity) yields -0.34% ($t = -1.3$) and remains negative and mostly insignificant across all horizons (Table A1). The QCE-minus-Q5F differences are large and mostly significant within 12 months, and the QCE-minus-FF6F differences are largely and mostly significant across all horizons.

Table 2 shows portfolio sorts in the all-but-micro sample. Table A2 does the same for additional cost of equity models. Microcaps show nontrivial impact, but the basic inferences are unchanged. First, the regression-based q^5-characteristics model continues to perform well, but the QCE return spread has declined across all horizons. For instance, the 1-month ahead return spread is 1.11% per month ($t = 5.64$) in contrast to 1.62% ($t = 7.29$) with microcaps. The reason is that decile 10 earns an average excess return of 1.29%, which falls from 1.77% with microcaps.

Second, gradient boosting again improves on linear regressions, but the differences are mostly insignificant. Third, the 7-variable version is still the best Lewellen model. Its 1-month average return spread is 0.87% per month ($t = 3.67$), which falls from 1.09% ($t = 3.71$) with microcaps. Fourth, the accounting implied cost of equity continues to struggle. The return spreads are all close to zero and are even negative in five out of six horizons. Fifth, factor models still perform poorly. The return spreads of Q5F range from 0.22% per month to 0.43%, albeit significant at longer horizons. The return spreads from FF6F are all negative and insignificant.

3.2.2 Cross-sectional Predictive Regressions

Table 3 shows cross-sectional predictive regressions of h-month subsequent returns on a given h-month ahead cost of equity at the beginning of month t. From Panel A, the QCE performance is fair. The slopes, which range from 0.54 at the 60-month horizon to 0.82 at the 1-month horizon, are significant from zero across all the horizons. However, their standard errors are small, varying only from 0.11 to 0.16. As a result, the slopes differ significantly from one at the 12-, 36-, and 60-month horizons. For instance, at the 1-month horizon, the slope is 0.82, which is indistinguishable from one (absolute t-value $|t| = 1.63$), with a standard error of 0.11. However, at the 12-month horizon, the slope is 0.63, with a standard error of 0.15. The absolute t-value, which tests the slope equals one, is 2.51.

The QCE performance is competitive with the performance via gradient boosting. With the MSE loss, the QCE-GB performance is mostly comparable (Table 3). At the 12-month, its slope is 0.8, which is insignificant from one, with a standard error of 0.16. However, at the 1-month horizon, its slope overshoots at 1.23, which differs significantly from one, with a standard error of 0.09.

With the MAE loss, the QCE-GBA performance falls far short of the QCE performance. From Table A3, the QCE-GBA slopes differ significantly from one across all horizons, with magnitudes smaller than their counterpart QCE slopes. With the Huber loss, QCE-GBH still underforms QCE somewhat, especially at longer horizons. In particular, the QCE-GBH slopes are 0.47 and 0.28 at the 36- and 60-month horizons, which are lower than the QCE slopes of 0.61 and 0.54, respectively.

The Lewellen 7-variable model is again the best performer among the three specifications. The LCE performance is largely comparable with the QCE performance. From Table 3, for instance, at the 1-month horizon, the LCE slope 0.81 (close to the QCE slope of 0.82), with a standard error of 0.11, and is insignificant from one ($|t| = 1.76$). At the 12-month horizon, the LCE slope is 0.7, which is higher than the QCE slope of 0.63, albeit still significant from one. At the 24-month, however, the LCE slope is 0.72, which is lower than the QCE slope of 0.81, and is significant from one.

Table A3 shows that LCE outperforms the two alternative Lewellen specifications. For instance, the slope of LCE-3 from the 3-variable model is only 0.17 at the 12-month horizon. With a standard error of 0.31, the slope is insignificant from zero but significant from one. With the 15-variable model, the LCE-15 slopes are all significant from one, also with smaller magnitudes. In particular, at the 36-month horizon, the slope is only 0.45, compared with the LCE slope of 0.6.

The ICE performance is poor. From Panel A of Table 3, the ICE slopes differ significantly from one across all six horizons. Their magnitudes are much smaller than the QCE slopes. For instance, the ICE slopes are 0.42, 0.51, and 0.38 at the 1-, 12-, and 36-month horizons, with standard errors of 0.17, 0.13, and 0.07 and t-values of 3.45, 3.8, and 9.1 testing that a given slope equals one, respectively. From Panel B, the QCE-minus-ICE slopes are economically large and statistically significant at the 3-, 24, and 36-month horizons (marginally significant at the 1- and 60-month horizons).

Factor models perform very poorly. Table 3 shows that for the q^5-factor model the Q5F slopes are basically zero, both economically and statistically, across all horizons. The QCE-minus-Q5F differences are large and significant across all horizons. The results for the Fama-French 6-factor model are quantitatively similar (Table A3). Clearly, although popular for style analysis and performance attribution, factor models are simply not built for estimating ex ante costs of equity.

3.2.3 Cross-sectional Predictive Regressions: The Role of Microcaps

Table 4 shows that microcaps are important for costs of equity via cross-sectional forecasts, especially for long-horizon forecasts. As noted, we retrain all the forecast models on the all-but-micro sample. At the 1-month horizon, without microcaps, the QCE slope is 0.74, with a standard error of 0.26, and is indistinguishable from one ($|t| = 0.97$). At the 12-month horizon, the slope falls to 0.48 (versus 0.63 with microcaps) and is significant from one ($|t| = 2.93$). The evidence remains similar at the 24-month but deteriorates further at longer horizons. At the 60-month, the slope is only 0.07 (versus 0.54 with microcaps), with a standard error of 0.11, and is highly significant from one ($|t| = 7.19$). The impact of microcaps on tree-based forecasts is also noteworthy.

The impact of microcaps on the Lewellen 7-variable model is largely similar. At the 1-month horizon, the LCE slope is 0.56, with a standard error of 0.11, which gives rise to an absolute t-value of 3.87 for testing the slope equals one. The 3-month horizon is largely similar. At the 12-month, the slope is only 0.29 (versus 0.7 with microcaps) and significant from one ($|t| = 4.72$). At the 60-month, the slope is only 0.06 and is highly significant from one ($|t| = 7.64$).

Excluding microcaps also materially affects the ICE performance. The ICE slopes all become only weakly positive, and their magnitude varies from 0.04 to 0.15 (versus the 0.29–0.51 range with microcaps). None of the ICE slopes are significant from zero, and all are highly significant from one.

Methodologically, while we value-weight returns in sorts, we follow Lewellen (2015) in using ordinary least squares in cross-sectional forecasts. We do so to ease comparison because QCE follows Lewellen's method. Also, in the ICE construction, the Hou-van Dijk-Zhang (2012) regression-based earnings forecasts also use ordinary least squares. Finally, time series and cross-section MEVs in Lee, So, and Wang (2021) implicitly treat all stocks as equal. We stick with the same practice.

The role of microcaps in forming characteristics-based costs of equity is surprising. Microcaps are plentiful, consisting of about 60% of stocks, but tiny, accounting for only about 3.5% of total market equity (Hou, Xue, and Zhang 2020). It is likely that microcaps contain important information on the relation between characteristics and expected returns, information that transcends their tiny size. First, microcaps exhibit more cross-sectional variations in the discount rate. Second, all else equal, high discount rates over long horizons imply small or tiny market cap today. Excluding microcaps might truncate (at least dilute) the right tail of the expect return distribution. Finally, more important, the economic significance varies over horizons. Studies typically examine short horizons up to 12 months, during which a stock's economic significance does not vary much. However, over long horizons, microcaps can grow into bigcaps, and bigcaps shrink into microcaps.[6]

3.2.4 Measurement Error Variances

Table 5 shows measurement error variances and their components in the full sample, and Table 6 does the same in the all-but-micro sample. Table A7 shows the evidence for additional cost of equity models in the full sample, and Table A8 does the same in the all-but-micro sample.

Several patterns emerge. First, factor models perform poorly. Their MEVs, both time series and cross-section, are often an order of magnitude larger than those of characteristics-based models. Both the large variance and small (even occasionally negative) covariance components contribute to this problem. Excluding microcaps mitigates the problem, but the underperformance remains substantial. Again, factor models are simply not built for estimating ex ante costs of equity.

Second, for time series MEV, ICE is the best performer. This outperformance is due to small variance and large covariance component. For instance, at the 1-month horizon, ICE has a time

[6]Motivated by these considerations, for completeness, we furnish in the Internet Appendix portfolio sorts with equal-weighted returns, with and without microcaps. The results show the clear impact of microcaps on long-horizon returns. From Table A5, with microcaps, the high-minus-low QCE decile earns an average equal-weighted return of 0.84% per month ($t = 6.01$) at the 36-month and 0.55% ($t = 4.54$) at the 60-month horizon. Microcaps are more responsible for the long-horizon returns than equal-weights. Table A6 shows that without microcaps, the average equal-weighted return for the high-minus-low QCE decile is only 0.35% ($t = 3.19$) at the 36-month and 0.11% ($t = 1.08$) at the 60-month horizon. For comparison, the corresponding estimates for the high-minus-low ICE decile are 0.59% ($t = 2.22$) at the 36-month and 0.38% ($t = 1.67$) at the 60-month with microcaps, and 0.23% ($t = 1.25$) at the 36-month and 0.18% ($t = 1.06$) at the 60-month horizon without microcaps.

series MEV of -0.0198, which arises from a small variance of 0.0038 but a large covariance of 0.0237 (Panel A, Table 5). For comparison, QCE's time series MEV is 0.0029, with a variance of 0.0072 and a covariance of 0.0043. The equal MEV on their common sample is strongly rejected. The ICE outperformance is mostly robust across horizons, against diverse models, with and without microcaps.

Third, for cross-section MEV, QCE shows a small edge, especially in short horizons, as shown in negative differentials in cross-section MEV. However, the differentials are mostly insignificant. For instance, at the 1-month horizon, the QCE-minus-ICE differential is -0.0048 ($t = -1.3$) with microcaps and -0.001 ($t = -1.62$) without microcaps. At the 12-month horizon, the QCE-minus-ICE differential is -0.01351 ($t = -0.25$) with microcaps and -0.063 ($t = -0.94$) without microcaps.

The QCE performance is largely comparable with the LCE performance from the Lewellen 7-variable model. However, QCE outperforms the two alternative models, LCE-3 and LCE-15, especially without microcaps. Also, gradient-boosted trees with the MSE loss improve on QCE somewhat, especially with microcaps, but the trees with the MAE and Huber loss worsen the QCE performance for both time series and cross-section MEV, with and without microcaps.

In sum, our extensive evidence on evaluation establishes the regression-based q^5-characteristics model as competitive. It is close to the best performer in portfolio sorts and cross-sectional predictive regressions. Although underperforming the accounting implied cost of equity in time series MEV, the linear q^5-characteristics model compares favorably with all other characteristics-based models in cross-section MEV. Gradient boosting improves the regression-based model in some aspects but worsens its performance in others. Across all evaluation metrics, factor models perform poorly. This evidence highlights the importance of developing the q^5-characteristics model.

4 Investment-based Costs of Equity

After establishing the linear q^5-characteristics model as competitive, we study the properties of QCE for firms (Section 4.1), industries (Section 4.2), factor premiums (Section 4.3), and the equity premium (Section 4.4). We also show the properties of the accounting-based ICE for comparison.

4.1 Firm-level Costs of Equity

Table 7 shows the firm-level properties of costs of equity in the January 1977–December 2024 sample. Because ICE is the internal rate of return, its term structure is flat. As such, we only show (the 12-month ahead) ICE based on annual accounting data. To ease comparison across horizons, we annualize all QCE estimates to align with the unit of (the 12-month ahead) QCE_{12}. For example, annualizing QCE_1 means $(1+\text{QCE}_1)^{12}-1$, and annualizing QCE_{60} means $(1+\text{QCE}_{60})^{1/5}-1$. We annualize each firm-month before calculating time series averages of cross-sectional moments.

Several insights emerge from Table 7. First, the term structure of QCE is largely flat. The 1-month ahead QCE is on average 13.83% per annum, which is close to the 12-month average of 13.85%, which is in turn not far from the 36-month average of 13.03%. The median QCEs show a similar pattern across horizons. The mean QCE estimates are also close to the mean ICE of 13.62%.

Second, the term structure of the QCE volatility is downward sloping. The 1-month ahead QCE volatility is 10.75% per annum, which drops to 6.85% at the 12-month and further to 4.48% at the 36-month horizon. For comparison, the 12-month ICE volatility is 10.26%. Relatedly, the monthly autocorrelations of QCEs vary from 0.91 to 0.95, which are somewhat lower than 0.97 for the ICE.

Third, the QCE distributions are weakly left-skewed across all horizons, with the skewness ranging from −0.6 to −0.9. In contrast, the ICE distribution is right-skewed, with a skewness of 1.91. Relatedly, the excess kurtosis (in excess of three for the normal distribution) of QCEs varies from 2.08 to 2.53 across all horizons, compared to 3.68 for ICE. The histograms in Figure 1 clearly show the distributional differences between QCEs and ICE. In particular, ICE's skewness from pooling the panel data observations together is 4.16, while the skewness for 12-month ahead QCE is −0.48 (untabulated). For excess kurtosis, the comparison is between 28.46 vs. 2.99.

Panel B of Table 7 shows the average slopes and their t-values in cross-sectional regressions underlying the QCEs in the full sample, while Figure 3 shows the time series of 120-month rolling window slopes in calculating the QCEs. The average slopes all have expected signs. The only exception is ROE, which has negative slopes at long horizons. More important, the expected growth slopes increase monotonically with horizon, 0.044 ($t = 6.68$) at the 1-month, 0.327 ($t = 5.3$) at the 12-month, 1.049 ($t = 7.83$) at the 36-month, and 2.064 ($t = 9.54$) at the 60-month horizon. The investment-to-assets slopes also largely increase in magnitude except for the 60-month.

Removing microcaps reduces the cost of equity volatilities. From Table 8, the QCE volatility starts at 6.23% per annum at the 1-month, falls to 3.76% at the 12-month, and to 2.99% at the 36-month. The cross-sectional dispersion also falls. The 5–95 percentiles dispersion is 12.57% for the 12-month QCE, which is only about 56% of 22.32% with microcaps. Also, the mean 12-month QCE is 12.53%, which is higher than 8.94% for the mean ICE. As such, excluding microcaps enlarges the mean QCE-minus-ICE difference. The ICE skewness falls to 0.91 (1.00 with the pulled panel data). The declined skewness is clear in Panel D of Figure 2. From Panel B of Table 8 and Figure 4, the average slopes from the cross-sectional regressions underlying QCE again mostly have expected signs.

4.2 Industry Costs of Equity

Fama and French (1997) show that factor-based industry costs of equity are "distressingly imprecise" (p. 178). QCE goes a long way in addressing this challenge (in the same spirit as ICE). We use NAICS to define sectors and industries. Prior to June 1985, we convert Standard Industry Classification (SIC) codes into NAICS via the meticulous mapping developed by Bai et al. (2024).

Table 9 shows sector and industry costs of equity for 18 nonfinancial NAICS sectors and 57 nonfinancial NAICS industries. For each sector and each industry, we show the times series means and standard deviations of different costs of equity, including the q^5-factor model (Q5F), QCE_1, QCE_3, QCE_{12}, QCE_{24}, QCE_{36}, QCE_{60}, and ICE, as well as realized returns, RR. We value-weight the stock-level costs of equity across all the stocks within a given industry or sector.

The characteristics-based costs of equity are more precise than Q5F, which is in turn more precise than realized returns. From Panel A of Table 9, averaged across 18 sectors, the RR volatility is 21.03% per annum, relative to its mean of only 13.66%. By retaining only common factor variations, Q5F reduces the volatility to 7.73%, relative to a mean of 9.73%. The QCE estimates improve the precision further. At the 12-month horizon, for instance, the QCE volatility is only 4.12%, which drops further to 3.72% at the 60-month. The ICE volatility is 2.82%. Across the 18 sectors, the 12-month ahead QCE volatility ranges from 2.98% to 5.22%, the Q5F volatility from 3.64% to 11.53%, and the ICE volatility from 1.92% to 3.91%.

From Panel B, the characteristics models are again more stable than the q^5-factor model, which is in turn more stable than realized returns. Averaged across 57 nonfinancial industries, the RR volatility is 25.94% per annum, which almost doubles the mean of 13.85%. The Q5F volatility is 9.94%, which is close to its mean of 10.29%. The QCE estimates are again more precise. At the 12-month horizon, the QCE volatility is 4.4%, which is less than one half of its mean of 10.33%. The ICE volatility is 3.37%, which is also less than one half of its mean, 9.04%.

The volatility of industry cost of equity also varies across the 57 industries. The lowest volatility is 2.75% per annum for industry 3270 (nonmetallic mineral products), followed by the second lowest of 2.77% for industry 3260 (plastics and rubber products). On the other extreme, the highest volatility is 7.93% for industry 4930 (warehousing and storage), followed by the second highest of 7.42% for industry 5411 (legal services). For the mean cost of equity, it varies from 6.77% (industry 5130, broadcasting and telecommunications) and 6.8% (industry 5140, information and data processing services) to 13.02% (industry 5310, real estate) and 13.12% (industry 6240, social assistance).

The QCE estimates show substantial cross-industry variation. For the 12-month QCE, it ranges from 7.1% per annum for industry 5140 (information and data processing services) and 7.28% for industry 5130 (broadcasting and telecommunications) to 13.65% for industry 113F (forestry, fishing, and related activities) and 15.47% for industry 5310 (real estate). For ICE, the cost of equity varies from 6.36% for industry 5140 and 6.57% for industry 5110 (publishing industries [including software]) to 12.81% for industry 2200 (utilities) and 17.91% for industry 5310. Although derived very differently, both costs of equity identify information and data processing services as one of the lowest cost of equity industries and real estate as one of the highest cost of equity industries. Although anecdotal, we view the unintended evidence as reassuring cross-validation (consilience).

4.3 Factor Premiums

Tang, Wu, and Zhang (2014) show that ICE estimates differ drastically from average realized returns for many factors. QCE goes a long way in resolving this difficulty. We study size, value, momentum (prior 11-month return, skipping 1 month), investment-to-assets, ROE, and expected growth. We use NYSE breakpoints to form deciles and calculate value-weighted returns, with annual sorts on size, value, and investment-to-assets and monthly sorts on momentum, ROE, and expected growth.

Table 10 shows the factor premiums in the January 1977–December 2024 sample. For the small-minus-big decile, the holding period returns are all insignificant from zero, with the average returns positive up to 12-month but negative afterward. Both the QCE and ICE premiums are all positive and significant across all horizons. For the value-minus-growth decile, the holding period returns are all positive but insignificant. The QCE premiums are negative up to 3-month but turn significantly positive afterward. The ICE premiums are all positive and significant. For the low-minus-high investment decile, the average returns are positive up to 24 months but negative afterward. Both the QCE and ICE premiums are significantly positive across all horizons.

Prior 11-month returns and ROE yield significantly positive premiums in short horizons. For instance, at the 1-month, their average returns are 0.98% ($t = 3.19$) and 0.8% ($t = 3.25$), respectively. Expected growth yields significantly positive premiums across all horizons, 0.99%, 9.56%, and 26.96% ($t = 5.38$, 4.85, and 5.18) at the 1-, 12-, and 36-month, respectively.

The QCE premiums are mostly aligned with average returns of the momentum and ROE factors, significantly positive in short horizons but insignificant in long horizons. In particular, the 1-month ahead QCE premiums are 0.39% ($t = 7.37$) for momentum and 1.27% per month ($t = 13.29$) for ROE. The QCE premiums for expected growth are significantly positive across all horizons, 1.43%, 9.28%, and 20.68% ($t = 14.11$, 11.98, and 12.57) at the 1-, 12-, and 36-month, respectively.

ICE performs poorly in aligning with average returns of the momentum, ROE, and expected growth factors. The ICE premiums are significantly negative across all horizons. The 1-month ICE premium for momentum is -0.35% ($t = -10.97$), and the 1-month ICE premium for ROE is -0.27% ($t = -12.73$). For the expected growth factor, the ICE premiums are $-0.14\%, -1.96\%$, and -7.78% ($t = -6.32, -6.43$, and -6.62) at the 1-, 12-, and 36-month, respectively.

Table 11 shows time series predictive regressions of realized factor premiums over subsequent h months on the h-month ahead expected premiums. For the size, value, and investment premiums, the QCE performance is largely comparable with the ICE performance. The ICE R^2 reaches 20.7% at the 60-month for the size premium. For the value premium, both QCE and ICE slopes are significant across all horizons, but the ICE R^2s are higher, up to 33.8%, at long horizons. For the investment premium, the QCE slopes are positive up to 12 months, with R^2s up to 8.2%. For comparison, the ICE slopes are all insignificant, and the R^2s are all small.

Most impressively, ICE outperforms QCE in predicting momentum, ROE, and expected growth premiums in the time series. For momentum, the ICE slopes are all significantly positive, with R^2s up to 32.3% at the 24-month. In contrast, the QCE slopes are mostly insignificant, albeit all positive, and the R^2s are at most 9.7%. For the ROE premium, the ICE slopes are all positive and mostly significant, especially at long horizons. Although relatively close, the QCE performance is still a bit weaker. Finally, for the EG factor, the ICE slopes are often significantly positive, with R^2s up to 11.1%. For comparison, the QCE slopes are all insignificant, albeit positive, and the R^ss are all tiny.

4.4 The Equity Premium

We value-weight firm-level costs of equity to obtain the aggregate cost of equity over different horizons. We then subtract the risk-free interest rates with matching maturities to obtain the equity premium.[7] From Panel A of Table 12, the term structure of interest rates in the January 1977–

[7]The 1-month Treasury bill rate is from CRSP. The 3-month, 1-year, 2-year, 3-year, and 5-year Treasury yields are from Federal Reserve Economic Data at the St. Louis Fed (series DTB3, DGS1, DGS2, DGS3, and DGS5,

December 2024 sample is upward sloping, 4.29%, 4.72%, and 5.13% per annum at the 1-, 12-, and 36-month horizons, respectively. The QCE equity premium has a weakly downward sloping term structure, 5.69%, 4.35%, and 4.01% at the 1-, 12-, and 36-month, respectively. The ICE equity premium also has a downward sloping term structure, 4.14%, 3.75%, and 3.46% at the 1-, 12-, and 36-month, respectively. This downward slope results from the upward sloping term structure of interest rates. The term structure of the ICE cost of equity is flat by construction.

The equity premium estimates are precise. The 1-month QCE equity premium, 5.69% per annum, has an annual volatility of 5.22%, which implies a standard error of only 0.753%, given the sample length of 48 years. Similarly, the 12-month QCE equity premium, 4.35%, has a volatility of 4.52%, which implies a standard error of 0.652%. The ICE estimates are also precise.

We also show expanding window average returns, denoted EAR, over different horizons, with the expanding window starts in January 1967. For 1-month, EAR is expanding window average monthly market returns minus expanding window average risk-free rate. For 12-month, EAR is the expanding window series of the average prior 12-month excess returns (with overlapping monthly observations). The EAR estimates (all annualized) also have a largely downward sloping term structure, 7.31%, 6.39%, and 5.37% per annum at the 1-, 12-, and 36-month, respectively.

From Panel B, the QCE equity premiums across different horizons are all highly correlated. The ICE equity premium is positively correlated with the QCE estimates up to the 24-month horizon, with the correlations ranging from 0.1 to 0.21, and is uncorrelated at longer horizons. The EAR estimates are positively correlated with both the QCE and ICE estimates. Figure 5 plots the time series of different equity premiums over different horizons.

Panel C of Table 12 performs long-horizon regressions of subsequent h-month ahead realized equity premiums on the h-month ahead expected equity premiums. In addition to horizon, we align the units between the left- and right-hand sides of the regressions. For $h = 60$, for example,

respectively). The 1-year through 5-year yields are constant maturity yields. For the 3-month horizon, we use the secondary market rate because the 3-month constant maturity yield is not available before September 1981.

we convert the ICE aggregate cost of equity from annual to 5-year unit, $(1+\text{ICE})^5-1$, and then subtract the 5-year Treasury bond rate. We use overlapping monthly observations.

ICE shows remarkable performance. Its slopes are all positive and significant up to the 12-month horizon. More important, the in-sample R^2 increases with horizon and reaches 24.5% at the 60-month. More impressively, the out-of-sample R^2 of ICE shows a similar pattern, increasing with horizon and reaches the highest, 24.5%, at the 60-month. We calculate the out-of-sample R^2 per Welch and Goyal (2008), $1-\text{MSE}_i/\text{MSE}_i^A$, in which MSE_i^A is the MSE of the EAR estimate. The time series MEV is also large and negative, meaning that ICE substantially outperforms EAR. For comparison, although fair, the QCE performance falls short of ICE's. The in-sample R^2 is at most 5.1%, the out-of-sample R^2 is occasionally negative, and time series MEV occasionally positive.

5 Summary and Interpretation

This paper develops the q^5-characteristics model for estimating costs of equity as out-of-sample forecasts from cross-sectional predictive regressions by combining slopes from prior 120-month rolling windows with the latest available q^5-characteristics. The q^5-characteristics model is competitive in a battery of evaluation tests, outperforming the accounting implied cost of equity in forecasting returns in the cross section, albeit underperforming in the time series. The q^5-cost of equity is precise at the industry level and aligned with future realized factor premiums. Its firm-level distribution is weakly left-skewed, whereas the accounting cost of equity is right-skewed.

Our work carries broad implications on practice. In their impressive 1,345-page treatise, Pratt and Grabowski (2014) propose a build-up method of estimating a company's cost of equity, which includes the risk-free rate, equity risk premium, size premium, and company-specific risk premium. The company-specific premium is in turn driven by exceptionally small size, industry, return volatility, leverage, and other specific factors, such as dependence on key personnel and pending litigation. This build-up approach balances the CAPM with practical considerations. Alas, because the CAPM is still front and center in their approach, their costs of equity estimates are likely subject to the imprecision and weak association problems that plague the prior literature. The q^5-cost of equity, which is theoretically motivated but less subject to these empirical problems, provides at least an alternative foundation on which one can build costs of equity in practice.

Our work also carries implications on long-standing debates on covariances vs. characteristics, structural vs. reduced form, and regressions vs. machine learning as two cultures in statistics (Brieman 2001). Inspired by Cartwright (1999), we adopt a transcendental argument: We perceive X (empirical knowledge), but without Φ (ontological propositions), X would be inconceivable. Therefore, Φ. In our context, X is our extensive results (and related evidence in empirical finance). Φ is that the world is a process, in which change is fundamental, not an equilibrium, in which change is secondary (Whitehead 1978 [1929]; Deleuze 1994 [1968]; Deleuze and Guattari 1987 [1980]).

On ontology, the economy is not a simple, predictable machine in equilibrium. Rather, the economy is a dynamic, rhizomatic network of multiplicities, assemblages, and processes that are always evolving. An assemblage composes of heterogeneous elements that form a functional whole. Elements (often assemblages themselves) can break away from existing structures, and new elements can re-stabilize into new assemblages. In all, the economy is a processual, complex adaptive system.

In this "dappled" world, there is no "view from nowhere" (Cartwright 1999). Rather, in our epistemology, we must embrace perspectivism, pragmatism, and pluralism, all of which descend from the Kantian idea that all scientific knowledge stems from a certain human vantage point. Perspectivism means that all scientific models must make idealized assumptions, and the assumptions reflect the modeler's respective purpose (Giere 2006). Pragmatism means that if a model works better in practice, it is more likely to be true (Chang 2022). Pluralism means that for real-world complex systems, good science requires integrating diverse, context-sensitive, imperfect models rather than seeking a single, simple, universal model, which is likely nonexistent (Mitchell 2012).

The premise of the covariances vs. characteristics debate is that only covariances matter in explaining returns. While this statement is correct in the standard theory, it reflects a long list of restricted assumptions (e.g., perfect knowledge, complete information, homogeneous expectations). While covariances do affect returns, covariances are unlikely the only causal powers that impact on expected returns in our complex capital markets. Indeed, the investment theory offers different economic mechanisms that involve only characteristics. The "only" bit again reflects a long list of assumptions (e.g., constant returns to scale). Per scientific pluralism, covariances and characteristics both matter, each working at different levels (macro vs. micro), and interact with each other. Factor models and characteristics-based models are not contradictory but complementary, each serving different purposes (in-sample style analysis vs. out-of-sample return forecasts).

On the structural vs. reduced form debate, as noted, when we started this project in 2019, our main idea was to develop the structural investment model as a main tool for out-of-sample forecasts. However, this idea has largely failed, as the model's performance leaves much to be desired. We detail the structural framework adapted from Goncalves, Xue, and Zhang (2020) and its implementation for out-of-sample forecasts in the Internet Appendix A2. Table A9 shows the basic results. The performance in portfolio sorts is fair. The high-minus-low cost of equity decile earns on average 0.54%, 0.68%, and 0.34% per month ($t = 2.3$, 3.19, and 1.64) at the 1-, 12-, and 36-month horizons, respectively. Removing microcaps even improves the results somewhat.

However, in cross-sectional predictive regressions, the structural model performs poorly. Although the slopes are all positive and mostly significant from zero, their magnitudes are relatively small. The null that the slope equals one is strongly rejected across all horizons. For instance, at the 1-month horizon, the slope is only 0.07, with a standard error of 0.03, and is significant from one ($|t| = 33.14$). The largest slope, 0.32, appears at the 36-month horizon, with a standard error of 0.9, and again reliably differs from one ($|t| = 7.2$). The results without microcaps are largely similar.

The process worldview helps make sense of the evidence. The structural model assumes time-invariance for preferences, technologies, and their relations. Its good in-sample fit helps illustrate its underlying economic mechanisms. Alas, as noted, the economy is an evolving process, in which a given empirical phenomenon is a joint manifestation of multiple, often unobservable, causal powers. And their relative strength varies across space and time (Wimsatt 2007). The long list of parametric assumptions in the structural model generalizes poorly and stands in the way of its out-of-sample performance. In contrast, the q^5-characteristics model drops these assumptions, yet still combines the pricing information contained in the key variables identified in the investment theory.

Finally, the process worldview also informs the interpretability vs. forecastability debate in Breiman (2001). The data modeling culture assumes that the data are generated by a pre-specified model, whereas the machine learning culture treats the data model as unknown. While data modeling excels at interpretability at the expense of forecastability, machine learning excels at forecastability at the expense of interpretability. In a processual world, a given pre-specified model is unlikely to perform consistently in out-of-sample forecasts, as the world itself evolves. In our application, we view our regression-based q^5-characteristics model as hitting a goldilocks zone between interpretability and forecastability. The regression model compares well with gradient-boosted trees, yet retains easy interpretability, as we restrict the predictors with the investment theory. Expanding the scope is likely to improve on forecastability, but also lose on interpretability.

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A Estimating Implied Costs of Equity

A.1 The Gebhardt-Lee-Swaminathan (2001, GLS) Model

We skip the firm index i in subscript for notational simplicity. At the beginning of each month t, we estimate firm i's cost of equity as the internal rate of return (IRR) from the nonlinear equation:

$$P_t = B_{y(t)} + \sum_{\tau=1}^{11} \frac{(E_t[\text{ROE}_{y(t)+\tau}] - \text{IRR}) \times B_{y(t)+\tau-1}}{(1+\text{IRR})^{\tau}} + \frac{(E_t[\text{ROE}_{y(t)+12}] - \text{IRR}) \times B_{y(t)+11}}{\text{IRR} \times (1+\text{IRR})^{11}}, \quad \text{(A1)}$$

in which P_t is the market equity at the beginning of month t, $y(t)$ is the firm's latest fiscal year ending at least four months prior to month t. $B_{y(t)}$ is the latest available book equity. $E_t[\text{ROE}_{y(t)+\tau}]$ is the expected return on equity (ROE) for fiscal year $y(t)+\tau$ based on information as of month t.

Current book equity, $B_{y(t)}$, uses the latest accounting data from the fiscal year ending at least four months prior to the beginning of month t.[8] We apply clean surplus accounting to construct future book equity as $B_{y(t)+\tau} = B_{y(t)+\tau-1} + B_{y(t)+\tau-1}E_t[\text{ROE}_{y(t)+\tau}](1-k), 1 \leq \tau \leq 11$, in which k is the dividend payout ratio for fiscal year $y(t)$. We calculate k as dividends (item DVC) divided by earnings (item IB) for profitable firms, or dividends divided by 6% of total assets (item AT) for firms with zero or negative earnings. We drop a firm if its book equity is zero or negative.

We construct the expected ROE for the first three years ahead ($\tau \leq 3$) via regression-based earnings forecasts. After fiscal year $y(t) + 3$, we assume that the expected firm-level ROE mean-reverts linearly to the historical industry median ROE by year $y(t) + 12$ and becomes a perpetuity afterwards. As in GLS, we use the Fama-French (1997) 48-industry classification. We use at least five and up to ten years of past ROE data from non-loss firms to compute the industry median ROE.

Following Hou, van Dijk, and Zhang (2012), we estimate costs of equity at the firm level via regression-based earnings forecasts from the pooled cross-sectional regressions, for $1 \leq \tau \leq 3$,

$$Y_{s+\tau} = a + b_1 A_s + b_2 D_s + b_3 DD_s + b_4 Y_s + b_5 Y_s^- + b_6 AC_s + \epsilon_{s+\tau}, \tag{A2}$$

in which Y_s is earnings (item IB) fiscal year s, A_s is total assets (item AT), D_s is dividends (item DVC), and DD_s is a dummy variable that equals one for dividend payers, and zero otherwise. Y_s^- is a dummy that equals one for negative earnings, and zero otherwise, and AC_s is operating accruals.[9]

To mitigate the impact of outliers, we winsorize the unbounded level variables ($Y_{s+\tau}$, Y_s, and AC_s) in equation (A2) at the 2.5 and 97.5 percentiles of their cross-sectional distributions each month. We winsorize the bounded (from below at zero) variables (A_s and D_s) at the 0 and 97.5 percentiles. At the beginning of month t, we estimate the forecasting regressions using the pooled panel data from the prior 120-month rolling window. The latest $Y_{s+\tau}$ data in equation (A2) are from fiscal year $y(t)$ ending at least four months prior to month t, and the right-hand side variables are further lagged accordingly. We form the expected earnings, $E_t[Y_{y(t)+\tau}]$, by combining the estimated regression coefficients with the latest values of the predictors from the fiscal year $y(t)$. We then construct expected ROE recursively as $E_t[\text{ROE}_{y(t)+\tau}] = E_t[Y_{y(t)+\tau}]/B_{y(t)+\tau-1}$.

[8] Following Davis, Fama, and French (2000), we measure book equity as stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if available. If not, we measure stockholders' equity as the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock.

[9] Following Hou, Xue, and Zhang (2020), prior to 1988, we use the balance-sheet approach of Sloan (1996) to measure operating accruals as changes in noncash working capital minus depreciation, in which the noncash working capital is changes in noncash current assets minus changes in current liabilities less short-term debt and taxes payable. In particular, $AC = (\triangle CA - \triangle CASH) - (\triangle CL - \triangle STD - \triangle TP) - DP$, in which $\triangle CA$ is the change in current assets (Compustat annual item ACT), $\triangle CASH$ is the change in cash or cash equivalents (item CHE), $\triangle CL$ is the change in current liabilities (item LCT), $\triangle STD$ is the change in debt included in current liabilities (item DLC, zero if missing), $\triangle TP$ is the change in income taxes payable (item TXP, zero if missing), and DP is depreciation and amortization (item DP, zero if missing). Starting from 1988, we follow Hribar and Collins (2002) to measure AC using the statement of cash flows as net income (item NI) minus net cash flow from operations (item OANCF).

A.2 The Claus-Thomas (2001, CT) Model

At the beginning of each month t, we estimate the internal rate of returns (IRR) from:

$$P_t = B_{y(t)} + \sum_{\tau=1}^{5} \frac{(E_t[\text{ROE}_{y(t)+\tau}] - \text{IRR}) \times B_{y(t)+\tau-1}}{(1+\text{IRR})^{\tau}} + \frac{(E_t[\text{ROE}_{y(t)+5}] - \text{IRR}) \times B_{y(t)+4} \times (1+g)}{(\text{IRR}-g) \times (1+\text{IRR})^5}, \tag{A3}$$

in which P_t is the market equity of month t, $y(t)$ is a firm's latest fiscal year ending at least four months prior to month t. $B_{y(t)+\tau}$ is the book equity for fiscal year $y(t)+\tau$, $E_t[\text{ROE}_{y(t)+\tau}]$ is the expected ROE for fiscal year $y(t)+\tau$ based on information as of month t, and g is the long-term growth rate of abnormal earnings. Abnormal earnings are defined as $(E_t[\text{ROE}_{y(t)+\tau}] - \text{IRR}) \times B_{y(t)+\tau-1}$.

We construct future book equity as $B_{y(t)+\tau} = B_{y(t)+\tau-1} + B_{y(t)+\tau-1}E_t[\text{ROE}_{y(t)+\tau}](1-k), 1 \leq \tau \leq 4$, in which k is the dividend payout ratio for fiscal year $y(t)$. We calculate k as dividends (item DVC) divided by earnings (item IB) for profitable firms, or dividends divided by 6% of total assets (item AT) for firms with zero or negative earnings. We drop a firm if its book equity is zero or negative. We construct the expected ROE, $E_t[\text{ROE}_{y(t)+\tau}]$, for up to five years ahead, using regression-based earnings forecasts. Following CT, we set g to the ten-year Treasury bond rate minus 3%.

A.3 The Ohlson-Juettner-Nauroth (2005, OJ) Model

At the beginning of each month t, we construct the internal rate of returns (IRR) as:

$$\text{IRR} = A_t + \sqrt{A^2 + \frac{E_t[Y_{y(t)+1}]}{P_t} \times (g_t - (\gamma - 1))}, \tag{A4}$$

in which

$$A_t \equiv \frac{1}{2}\left((\gamma - 1) + \frac{E_t[D_{y(t)+1}]}{P_t}\right), \tag{A5}$$

$$g_t \equiv \frac{1}{2}\left(\frac{E_t[Y_{y(t)+3}] - E_t[Y_{y(t)+2}]}{E_t[Y_{y(t)+2}]} + \frac{E_t[Y_{y(t)+5}] - E_t[Y_{y(t)+4}]}{E_t[Y_{y(t)+4}]}\right). \tag{A6}$$

P_t is the market equity of month t, $y(t)$ is a firm's latest fiscal year ending at least four months prior to month t. $E_t[Y_{t+\tau}]$ is the expected earnings for fiscal year $y(t)+\tau$ based on information as of month t, and $E_t[D_{y(t)+1}]$ is the expected dividends for fiscal year $y(t)+1$. Expected earnings are from regression-based earnings forecasts. Expected dividends are expected earnings times the current dividend payout ratio, which is computed as dividends (item DVC) divided by earnings (item IB) for profitable firms, or dividends divided by 6% of total assets (item AT) for firms with zero or negative earnings. We use the average of predicted near-term growth rate and five-year growth rate to estimate g_t. We require $E_t[Y_{y(t)+2}]$ and $E_t[Y_{y(t)+4}]$ to be positive so that g_t is well defined. The perpetual growth rate of abnormal earnings, $\gamma - 1$, is the ten-year Treasury bond rate minus 3%.

A.4 The MPEG Model from Easton (2004)

At the beginning of month t, we estimate the internal rate of returns (IRR) from:

$$P_t = \frac{E_t[Y_{y(t)+2}] + \text{IRR} \times E_t[D_{y(t)+1}] - E_t[Y_{y(t)+1}]}{\text{IRR}^2}, \tag{A7}$$

in which P_t is the market equity of month t, $y(t)$ is a firm's latest fiscal year ending at least four months prior to month t, $E_t[Y_{y(t)+\tau}]$ is the expected earnings for fiscal year $y(t) + \tau$ based on information as of month t, and $E_t[D_{y(t)+1}]$ is the expected dividends for fiscal year $y(t) + 1$. Expected dividends are expected earnings times the current dividend payout ratio, which is computed as dividends (item DVC) divided by earnings (item IB) for profitable firms, or dividends divided by 6% of total assets (item AT) for firms with zero or negative earnings. When equation (A7) has two positive roots (in very few cases), we use the average as the IRR estimate.

B Variable Definitions in Cross-sectional Forecasts

B.1 The Lewellen (2015) Specifications

We implement Lewellen's (2015) three specifications. Lewellen is brief about variable definitions. This appendix provides the exact measurement of all the variables in our implementation.

LogME, log of market equity Market equity, ME, is price per share (CRSP item PRC) times shares outstanding (item SHROUT). At the beginning of each month t, we measure logME as the logarithm of market equity at the end of month $t - 1$.

LogB/M, log of book-to-market At the beginning of each month t, book-to-market, B/M, is the book equity for the latest fiscal year ending at least four months ago divided by the market equity (from CRSP) at the end of month $t - 1$. For firms with more than one share class, we merge the market equity for all share classes before computing B/M. Following Davis, Fama, and French (2000), we measure book equity as stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (Compustat annual item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if it is available. If not, we measure stockholders' equity as the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock.

R11, prior 11-month return At the beginning of each month t, we skip month $t - 1$ and calculate the prior 11-month return from month $t - 12$ to $t - 2$.

LogR24, log of 24-month return from $t - 36$ to $t - 13$ We calculate the sum of log returns from month $t - 36$ to $t - 13$ as the prior 24-month return at the beginning of month t.

LogIssue12 and LogIssue36, growth in shares outstanding At the beginning of each month t, we calculate the log growth of split-adjusted shares outstanding from month $t - 12$ to $t - 1$ (LogIssue12) and from month $t - 36$ to month $t - 1$ (LogIssue36). Split-adjusted shares outstanding equal shares outstanding (CRSP item SHROUT) time the adjustment factor (item CFACSHR).

Acc/A, accrual-to-assets Following Sloan (1996), we measure operating accruals, Acc, as changes in noncash working capital minus depreciation, in which the noncash working capital is changes in noncash current assets minus changes in current liabilities less short-term debt and taxes payable: Acc = (dCA−dCASH)−(dCL−dSTD−dTP)−DP, in which dCA is the change in current assets (item ACT), dCASH is the change in cash or cash equivalents (item CHE), dCL is the change in current liabilities (item LCT), dSTD is the change in debt included in current liabilities (item DLC), dTP is the change in income taxes payable (item TXP), and DP is depreciation and amortization (item DP). Missing changes in income taxes payable are set to zero. At the beginning of each month t, we calculate Acc/A with accruals from the most recent fiscal year ending at least four months ago divided by the average of total assets (item AT) and one-year-lagged total assets.

ROA, return on assets Return on assets, ROA, is income before extraordinary items (item IB) for the most recent fiscal year ending at least four months ago divided by the average of total assets (item AT) and one-year-lagged total assets.

LogAG, log growth of assets At the beginning of each month t, log asset growth, LogAG, is the logarithm of total assets (item AT) from the latest fiscal year ending at least four months ago divided by one-year-lagged total assets.

DY, dividend yield Dividend yield, DY, is the sum of dividends per share (CRSP item DIVAMT, adjusted for stock splits using CFACSHR) from month $t-12$ to $t-1$ divided by price (CRSP item PRC) adjusted for stock splits at the end of month $t-1$, i.e., $[\sum_{s=t-12}^{t-1}(\text{DIVAMT}_s/\text{CFACSHR}_s)]/[\text{PRC}_{t-1}/\text{CFACSHR}_{t-1}]$.

Beta36, market beta Market beta is estimated with weekly returns from month $t-36$ to $t-1$. We calculate weekly returns from daily returns during each calendar week (from Friday market close to next Friday market close) and then estimate the market beta using 156 weekly returns (50 weeks minimum). The last return comes from the latest week ending before the beginning of month t.

StdDev, monthly standard deviation Monthly standard deviation, StdDev, is calculated with daily returns from month $t-12$ to month $t-1$, i.e., $\text{StdDev}_t = \sqrt{21/(N-1)\sum_{d\in[t-12,\ t-1]} r_d^2}$, in which d is the index of trading days from month $t-12$ to $t-1$, r_d is the daily return, and N is the total number of trading days in the prior 12-month period. We require a minimum of 100 daily returns.

Turnover, average monthly turnover Turnover is the average monthly turnover from month $t-12$ to month $t-1$. Monthly turnover is the shares traded during a given month (CRSP item VOL) divided by total shares outstanding (item SHROUT) at the end of that month. We adjust volume for Nasdaq stocks per Gao and Ritter (2010).

D/P, debt-to-price At the beginning of each month t, debt-to-price, D/P, is the ratio of total debt (item DLC plus item DLTT) for the latest fiscal year ending at least four months ago divided by the market equity at the end of month $t-1$ from CRSP. For firms with more than one share class, we merge the market equity for all share classes.

S/P, sales-to-price At the beginning of each month t, sales-to-price, S/P, is the ratio of sales (item SALE) from the latest fiscal year ending at least four months ago divided by the market equity at the end of month $t-1$ from CRSP. For firms with more than one share class, we merge the market equity for all share classes.

Winsorization In cross-sectional regressions of returns, we winsorize logME, logB/M, LogIssue12, LogIssue36, Acc/A, ROA, LogAG, R11, logR24, and Beta36 at the 2.5 and 97.5 percentiles at their respective cross-sectional distributions each month. We winsorize DY, StdDev, Turnover, D/P, and S/P at the 0 and 97.5 percentiles because these variables are bounded from below at zero.

B.2 The q^5 Characteristics

The q^5 characteristics include size, investment-to-assets, return on equity, and expected growth. We follow Hou, Xue, and Zhang (2020) in variable definitions.

Size, ME Size (market equity, ME) is price per share (CRSP item PRC) times shares outstanding (item SHROUT). At the beginning of each month t, ME is measured at the end of month $t-1$. We further take logarithm of ME when using it in cross-sectional regressions.

Investment-to-assets, I/A At the beginning of each month t, I/A is total assets (item AT) from the fiscal year ending at least four months ago divided by one-year-lagged total assets minus one.

Return on equity, ROE Return on equity, ROE, is income before extraordinary items (item IBQ) divided by one-quarter-lagged book equity. Book equity is shareholders' equity, plus balance sheet deferred taxes and investment tax credit (item TXDITCQ) if available, minus the book value of preferred stock (item PSTKQ). Depending on availability, we use stockholders' equity (item SEQQ), or common equity (item CEQQ) plus the book value of preferred stock, or total assets (item ATQ) minus total liabilities (item LTQ) in that order as shareholders' equity.

Before 1972, the sample coverage is limited for quarterly book equity in Compustat quarterly files. We expand the coverage with book equity from Compustat annual files as well as by imputing quarterly book equity with clean surplus accounting. Whenever available, we first use quarterly book equity from Compustat quarterly files. We then supplement the coverage for fiscal quarter four with annual book equity from Compustat annual files. Annual book equity is stockholders' book equity, plus balance sheet deferred taxes and investment tax credit (item TXDITC) if available, minus the book value of preferred stock. Stockholders' equity is the value reported by Compustat (item SEQ), if available. If not, stockholders' equity is the book value of common equity (item CEQ) plus the par value of preferred stock (item PSTK), or the book value of assets (item AT) minus total liabilities (item LT). Depending on availability, we use redemption (item PSTKRV), liquidating (item PSTKL), or par value (item PSTK) for the book value of preferred stock.

If both are unavailable, we apply the clean surplus relation to impute the book equity. First, if available, we backward impute the beginning-of-quarter book equity as the end-of-quarter book equity minus quarterly earnings plus quarterly dividends. Quarterly earnings are income before extraordinary items (item IBQ). Quarterly dividends are zero if dividends per share (item DVPSXQ) are zero. Otherwise, dividends are dividends per share times beginning-of-quarter shares outstanding adjusted for stock splits during the quarter. Shares outstanding are from Compustat (item CSHOQ supplemented with annual item CSHO for fiscal quarter four) or CRSP (item SHROUT), and the share adjustment factor is from Compustat (item AJEXQ supplemented with annual item AJEX for fiscal quarter four) or CRSP (item CFACSHR).

If data are unavailable for the backward imputation, we impute the book equity for quarter t forward based on book equity from prior quarters. Let BEQ_{t-j}, $1 \leq j \leq 4$ denote the latest available quarterly book equity as of quarter t, and $\text{IBQ}_{t-j+1,t}$ and $\text{DVQ}_{t-j+1,t}$ be the sum of quarterly earnings and quarterly dividends from quarter $t-j+1$ to t, respectively. BEQ_t can then be imputed as $\text{BEQ}_{t-j} + \text{IBQ}_{t-j+1,t} - \text{DVQ}_{t-j+1,t}$. We do not use prior book equity from more than four quarters ago (i.e., $1 \leq j \leq 4$) to reduce imputation errors.

At the beginning of each month t, we use the most recent available ROE. Before 1972, we use the most recent ROE computed with quarterly earnings from fiscal quarters ending at least four months ago. Starting from 1972, we use ROE computed with quarterly earnings from the most recent quarterly earnings announcements (item RDQ). We require the end of the fiscal quarter corresponding to its most recent ROE to be within prior six months. To avoid potentially erroneous records, we also require the earnings announcement date to be after the corresponding fiscal quarter end.

Expected growth, EG We estimate monthly cross-sectional forecasting regressions of one-year-ahead investment-to-assets change, $\text{d}^1\text{I/A}$, on the log of Tobin'q, $\log(q)$, operating cash flows, Cop, and the change in return on equity, dROE. At the beginning of each month t, we measure current investment-to-assets as total assets (item AT) from the most recent fiscal year ending at least four months ago minus the total assets from one year prior, scaled by the one-year-prior total assets. The one-year ahead investment-to-assets change, $\text{d}^1\text{I/A}$, is the investment-to-assets from the first year after the most recent fiscal year end minus the current investment-to-assets.

At the beginning of each month t, Tobin's q is the market equity (from CRSP) plus total debt (item DLTT plus item DLC, zero if missing) scaled by book assets (item AT), all from the most recent fiscal year ending at least four months ago. For firms with multiple share classes, we merge the market equity for all classes. We measure operating cash flows, Cop, as total revenue (item REVT) minus cost of goods sold (item COGS), minus selling, general, and administrative expenses (item XSGA), plus research and development expenditures (item XRD, zero if missing), minus change in accounts receivable (item RECT), minus change in inventory (item INVT), minus change in prepaid expenses (item XPP), plus change in deferred revenue (item DRC plus item DRLT), plus change in trade accounts payable (item AP), and plus change in accrued expenses (item XACC), scaled by book assets, all from the fiscal year ending at least four months ago. Missing annual changes are set to zero. dROE is ROE minus the four-quarter-lagged ROE. Missing dROE values are set to zero in cross-sectional growth forecasting regressions. We follow exactly Hou et al.'s (2021) procedure in winsorizing all the variables on both sides of the regressions at the 1 and 99 percentiles of their cross-sectional distributions each month.

At the beginning of each month t, we construct expected one-year-ahead investment-to-assets changes, denoted $E_t[\text{d}^1\text{I/A}]$, by combining the most recent winsorized predictors with the average slopes estimated from the prior 120-month rolling window (30 months minimum). The most recent predictors, $\log(q)$ and Cop, are from the most recent fiscal year ending at least four months ago as of month t. dROE is computed using the latest announced quarterly earnings, and if not available, the earnings from the most recent fiscal quarter ending at least four months ago. To avoid look-ahead bias, the average slopes in calculating $E_t[\text{d}^1\text{I/A}]$ are estimated from the prior rolling window regressions, in which $\text{d}^1\text{I/A}$ is from the most recent fiscal year ending at least four months ago as of month t, and the predictors are further lagged by 12 months.

Winsorization In cross-sectional forecasting regressions of returns, we winsorize the log market equity, investment-to-assets, return on equity, and expected growth at the 2.5 and 97.5 percentiles at their respective cross-sectional distributions each month.

Table 1 : Portfolio Sorts, January 1977–December 2024

In Panel A, at the beginning of month t, we split the NYSE, Amex, and NASDAQ stocks on each cost of equity into deciles and calculate the monthly value-weighted returns for different holding periods, h (in months). For instance, the 3-month horizon means that for a given decile in each month, there exist 3 subdeciles, each initiated in a different month in the prior 3 months. We take the simple average of the subdecile returns as the monthly return for the decile. We align the holding period with the forecast horizon in costs of equity, e.g., we use the 12-month holding period when forecasting returns with 12-month ahead costs of equity. We use NYSE-Amex-NASDAQ breakpoints and value-weighted returns. Each decile return is in excess of the 1-month Treasury bill rate. Beneath each row of average return estimates, we show the corresponding Newey-West t-statistics. Panel B shows pairwise comparison tests of QCE (the q^5-characteristics model via cross-sectional regressions) versus alternative models. For each pairwise comparison, we test whether the difference between the two high-minus-low (H−L) decile returns is on average zero.

Panel A: Portfolio sorts

h	1	2	3	4	5	6	7	8	9	10	H−L
				QCE (the q^5-characteristics model via cross-sectional regressions)							
1	0.15	0.59	0.69	0.88	0.90	0.98	1.10	1.05	1.28	1.77	1.62
	0.53	2.81	3.60	4.67	4.54	4.84	4.97	4.41	5.52	6.57	7.29
3	0.19	0.59	0.66	0.82	0.89	0.99	0.92	1.07	1.17	1.56	1.37
	0.68	2.75	3.39	4.34	4.49	5.06	4.14	4.95	5.18	6.08	6.46
12	0.45	0.75	0.80	0.88	0.91	0.90	0.95	1.00	1.12	1.26	0.81
	1.98	3.92	4.09	4.33	4.43	4.18	4.13	4.11	4.37	4.17	4.01
24	0.51	0.68	0.82	0.83	0.87	0.96	0.99	1.05	1.17	1.08	0.57
	2.41	3.76	4.03	4.08	4.10	4.25	4.21	4.22	4.34	3.45	2.93
36	0.65	0.72	0.75	0.78	0.83	0.82	0.89	0.87	0.92	0.91	0.26
	3.11	3.82	3.61	3.72	3.78	3.69	3.76	3.48	3.54	3.11	1.54
60	0.68	0.79	0.77	0.74	0.78	0.83	0.84	0.85	0.88	0.69	0.01
	3.28	4.04	3.79	3.36	3.52	3.69	3.58	3.54	3.50	2.50	0.07
				QCE-GB (the q^5-characteristics model via gradient boosting)							
1	−0.21	0.34	0.60	0.41	0.66	0.78	0.85	0.98	1.23	1.59	1.79
	−0.63	1.28	2.64	2.06	3.34	4.10	4.63	4.83	5.42	5.88	7.02
3	−0.12	0.37	0.47	0.63	0.65	0.78	0.91	1.04	1.20	1.54	1.66
	−0.34	1.44	2.09	3.06	3.36	4.07	4.67	5.04	5.17	5.46	6.17
12	0.32	0.49	0.58	0.75	0.74	0.74	0.83	0.97	1.13	1.20	0.88
	1.07	2.01	2.74	3.93	3.88	3.95	4.43	4.83	4.61	3.80	3.89
24	0.43	0.61	0.72	0.65	0.65	0.72	0.77	0.86	0.97	0.98	0.55
	1.61	2.75	3.58	3.30	3.34	3.75	3.99	4.15	4.00	3.15	3.03
36	0.58	0.66	0.73	0.73	0.69	0.70	0.72	0.73	0.76	0.78	0.20
	2.20	3.05	3.64	3.82	3.65	3.67	3.66	3.51	3.25	2.91	1.16
60	0.71	0.75	0.77	0.80	0.76	0.76	0.79	0.76	0.72	0.66	−0.06
	2.74	3.53	3.84	4.09	3.85	3.91	3.93	3.66	3.36	2.66	−0.34

h	1	2	3	4	5	6	7	8	9	10	H−L
					LCE (the Lewellen 7-variable model)						
1	0.39	0.73	0.88	0.75	0.89	0.96	1.07	1.10	1.23	1.48	1.09
	1.64	3.71	4.80	4.12	4.81	4.69	4.31	4.51	4.35	4.12	3.71
3	0.33	0.65	0.76	0.83	0.87	0.98	0.97	1.09	1.12	1.24	0.91
	1.41	3.37	4.20	4.66	4.66	4.80	4.36	4.32	3.91	3.61	3.32
12	0.59	0.83	0.80	0.85	0.87	0.85	0.95	0.99	0.96	0.97	0.38
	2.52	4.57	4.55	4.61	4.57	4.20	4.27	3.92	3.26	2.78	1.42
24	0.63	0.78	0.80	0.80	0.78	0.85	0.91	0.95	1.00	1.04	0.41
	2.66	4.31	4.57	4.40	4.12	4.19	4.14	3.86	3.57	3.24	1.64
36	0.64	0.80	0.80	0.79	0.82	0.80	0.84	0.86	0.93	0.95	0.31
	2.67	4.30	4.47	4.19	4.21	3.96	3.87	3.70	3.61	3.32	1.31
60	0.73	0.86	0.86	0.84	0.77	0.76	0.77	0.75	0.79	0.79	0.06
	3.10	4.37	4.60	4.40	3.99	3.79	3.65	3.29	3.16	2.91	0.28
					ICE (the accounting-based implied cost of equity)						
1	0.79	0.58	0.68	0.72	0.70	0.69	0.77	0.75	0.83	0.75	−0.03
	2.98	2.90	3.76	4.05	4.01	3.92	3.89	3.28	2.87	1.65	−0.08
3	0.80	0.63	0.64	0.70	0.70	0.68	0.75	0.74	0.89	0.64	−0.16
	3.07	3.11	3.53	3.99	4.10	3.77	4.01	3.24	3.26	1.50	−0.43
12	0.75	0.63	0.72	0.70	0.76	0.76	0.73	0.75	0.95	0.78	0.03
	2.90	3.24	3.98	4.03	4.56	4.44	3.98	3.50	3.77	2.06	0.10
24	0.73	0.67	0.76	0.73	0.77	0.78	0.78	0.79	0.95	0.76	0.04
	2.87	3.50	4.20	4.31	4.67	4.69	4.44	3.95	4.01	2.27	0.13
36	0.75	0.70	0.76	0.74	0.76	0.77	0.78	0.77	0.94	0.70	−0.05
	2.98	3.64	4.23	4.37	4.64	4.66	4.56	4.02	4.17	2.25	−0.18
60	0.79	0.73	0.77	0.75	0.75	0.75	0.74	0.75	0.85	0.71	−0.09
	3.21	3.80	4.32	4.44	4.54	4.48	4.40	4.16	4.04	2.46	−0.39
					Q5F (the q^5-factor model)						
1	0.71	0.76	0.66	0.64	0.65	0.66	0.81	0.76	0.89	0.84	0.13
	2.24	2.89	3.03	3.31	3.88	3.75	4.31	3.77	4.09	2.93	0.57
3	0.67	0.84	0.62	0.68	0.64	0.71	0.78	0.78	0.85	0.86	0.19
	2.11	3.14	2.85	3.58	3.75	4.16	4.25	3.96	3.89	2.99	0.83
12	0.67	0.75	0.61	0.66	0.64	0.69	0.76	0.83	0.90	0.99	0.32
	2.14	2.94	2.90	3.67	3.78	4.16	4.25	4.35	4.09	3.54	1.57
24	0.70	0.71	0.58	0.65	0.66	0.66	0.76	0.81	0.93	1.08	0.38
	2.28	2.95	2.90	3.66	3.96	3.99	4.36	4.29	4.17	3.85	1.95
36	0.76	0.67	0.58	0.64	0.67	0.68	0.75	0.79	0.95	1.08	0.32
	2.59	2.86	2.97	3.71	4.12	4.11	4.37	4.22	4.21	3.82	1.72
60	0.85	0.67	0.61	0.65	0.68	0.68	0.72	0.77	0.94	1.03	0.18
	3.02	2.95	3.21	3.80	4.25	4.16	4.26	4.14	4.04	3.64	1.08

Panel B: Pairwise comparison tests with QCE

	h	Diff	t	h	Diff	t
QCE-GB	1	−0.17	−0.74	24	0.02	0.11
LCE	1	0.53	1.90	24	0.16	0.86
ICE	1	1.65	3.53	24	0.54	2.06
Q5F	1	1.49	5.44	24	0.19	0.70
QCE-GB	3	−0.29	−1.50	36	0.06	0.31
LCE	3	0.46	1.66	36	−0.05	−0.25
ICE	3	1.53	3.57	36	0.31	1.14
Q5F	3	1.18	4.65	36	−0.06	−0.23
QCE-GB	12	−0.07	−0.34	60	0.07	0.36
LCE	12	0.43	2.18	60	−0.05	−0.27
ICE	12	0.78	2.62	60	0.10	0.43
Q5F	12	0.49	1.76	60	−0.17	−0.73

Table 2 : Portfolio Sorts, the All-but-micro Sample, January 1977–December 2024

Starting from the full sample, we exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. In Panel A, at the beginning of month t, we split all stocks on each cost of equity into deciles and calculate the monthly value-weighted returns for different holding periods, h (in months). For instance, the 3-month horizon means that for a given decile in each month, there exist 3 subdeciles, each initiated in a different month in the prior 3 months. We take the simple average of the subdecile returns as the monthly return for the decile. We align the holding period with the forecast horizon in costs of equity, e.g., we use the 12-month holding period when forecasting returns with 12-month ahead costs of equity. We use all-but-micro breakpoints and value-weighted returns. Each decile return is in excess of the 1-month Treasury bill rate. Beneath each row of average return estimates, we show the corresponding Newey-West t-statistics. Panel B shows pairwise comparison tests of QCE (the q^5-characteristics model via cross-sectional regressions) versus alternative models. For each pairwise comparison, we test whether the difference between the two high-minus-low (H−L) decile returns is on average zero.

Panel A: Portfolio sorts

h	1	2	3	4	5	6	7	8	9	10	H−L
	QCE (the q^5-characteristics model via cross-sectional regressions)										
1	0.18	0.55	0.57	0.61	0.86	0.75	0.91	1.07	0.94	1.29	1.11
	0.68	2.59	2.55	3.12	4.61	3.79	4.45	5.30	4.62	5.74	5.64
3	0.27	0.49	0.67	0.60	0.75	0.78	0.83	0.96	1.00	1.15	0.88
	1.02	2.34	3.08	3.02	4.01	4.17	4.37	4.92	4.72	5.19	4.56
12	0.33	0.63	0.63	0.73	0.71	0.75	0.83	0.82	0.89	0.98	0.65
	1.24	2.93	3.04	3.81	3.77	3.87	4.44	4.41	4.46	4.29	3.70
24	0.44	0.58	0.63	0.69	0.72	0.72	0.79	0.77	0.82	0.91	0.47
	1.82	2.65	3.12	3.56	3.74	3.68	4.11	4.16	4.24	3.93	3.08
36	0.62	0.70	0.73	0.63	0.69	0.73	0.74	0.73	0.77	0.81	0.19
	2.53	3.42	3.81	3.37	3.71	3.89	3.97	3.84	3.99	3.50	1.25
60	0.75	0.78	0.77	0.76	0.75	0.75	0.71	0.73	0.73	0.80	0.05
	3.06	3.82	3.82	3.87	4.02	4.04	3.78	3.83	3.71	3.50	0.35
	QCE-GB (the q^5-characteristics model via gradient boosting)										
1	0.05	0.32	0.48	0.69	0.70	0.62	0.92	1.02	1.08	1.18	1.13
	0.18	1.44	2.25	3.39	3.84	3.28	4.88	5.08	5.11	4.91	6.22
3	0.18	0.25	0.57	0.63	0.71	0.81	0.87	0.96	1.02	1.19	1.01
	0.64	1.07	2.80	3.43	3.78	4.24	4.49	4.87	4.87	4.99	5.80
12	0.46	0.55	0.60	0.65	0.73	0.70	0.74	0.80	0.93	1.05	0.59
	1.72	2.38	2.96	3.49	3.87	3.70	3.90	4.18	4.79	4.56	3.48
24	0.55	0.70	0.70	0.72	0.73	0.72	0.75	0.76	0.85	0.96	0.41
	2.26	3.31	3.57	3.93	4.00	3.95	3.86	3.92	4.17	4.28	2.59
36	0.67	0.70	0.75	0.73	0.70	0.73	0.70	0.75	0.77	0.83	0.16
	2.75	3.32	3.76	3.96	3.83	4.00	3.81	4.02	3.92	3.77	1.08
60	0.75	0.77	0.78	0.82	0.77	0.77	0.73	0.71	0.72	0.76	0.01
	3.13	3.70	3.99	4.38	4.18	4.17	3.96	3.80	3.69	3.58	0.09

h	1	2	3	4	5	6	7	8	9	10	H−L
					LCE (the Lewellen 7-variable model)						
1	0.30	0.44	0.56	0.69	0.71	0.81	0.85	0.89	1.03	1.17	0.87
	1.24	2.17	2.91	3.64	3.80	4.37	4.45	4.37	4.67	4.11	3.67
3	0.27	0.47	0.62	0.58	0.75	0.79	0.85	0.87	1.02	1.06	0.78
	1.18	2.41	3.44	3.17	4.15	4.22	4.63	4.49	4.50	3.80	3.58
12	0.47	0.57	0.67	0.74	0.77	0.78	0.76	0.79	0.80	0.90	0.43
	2.14	3.00	3.79	4.20	4.42	4.42	4.19	4.22	3.72	3.47	2.34
24	0.59	0.64	0.71	0.68	0.74	0.78	0.76	0.77	0.79	0.82	0.23
	2.68	3.43	3.95	3.98	4.23	4.36	4.30	4.07	4.08	3.48	1.45
36	0.64	0.68	0.69	0.74	0.74	0.73	0.75	0.76	0.80	0.81	0.17
	2.84	3.50	3.79	4.19	4.23	4.19	4.27	4.20	4.27	3.70	1.17
60	0.68	0.75	0.77	0.77	0.74	0.77	0.79	0.78	0.72	0.68	0.00
	2.95	3.96	4.19	4.23	4.17	4.43	4.52	4.39	4.06	3.27	0.00
					ICE (the accounting-based implied cost of equity)						
1	0.84	0.61	0.55	0.62	0.77	0.71	0.64	0.76	0.77	0.84	0.01
	3.12	3.02	2.89	3.31	4.34	3.85	3.52	4.15	4.12	3.59	0.03
3	0.87	0.59	0.62	0.64	0.70	0.70	0.69	0.75	0.67	0.85	−0.02
	3.27	2.83	3.26	3.46	4.04	3.91	3.90	4.11	3.76	3.73	−0.08
12	0.83	0.60	0.63	0.70	0.70	0.75	0.77	0.75	0.66	0.80	−0.03
	3.18	2.98	3.40	3.90	4.00	4.40	4.53	4.30	3.68	3.86	−0.12
24	0.78	0.61	0.68	0.75	0.74	0.78	0.79	0.77	0.70	0.78	−0.01
	3.07	3.01	3.77	4.22	4.27	4.62	4.64	4.49	4.03	3.97	−0.03
36	0.77	0.64	0.70	0.74	0.76	0.78	0.77	0.76	0.71	0.70	−0.07
	3.08	3.18	3.88	4.20	4.41	4.63	4.52	4.48	4.22	3.73	−0.33
60	0.81	0.65	0.72	0.74	0.77	0.77	0.78	0.75	0.66	0.67	−0.14
	3.30	3.31	3.97	4.19	4.45	4.57	4.56	4.39	4.04	3.75	−0.69
					Q5F (the q^5-factor model)						
1	0.71	0.72	0.65	0.59	0.74	0.68	0.79	0.74	0.85	0.96	0.25
	2.51	3.29	3.23	3.31	4.53	3.99	4.46	3.77	4.20	3.72	1.21
3	0.74	0.66	0.65	0.65	0.75	0.68	0.76	0.77	0.82	0.96	0.22
	2.58	3.10	3.35	3.73	4.62	4.01	4.48	4.02	3.99	3.67	1.09
12	0.71	0.63	0.67	0.63	0.68	0.67	0.76	0.81	0.80	1.04	0.33
	2.59	2.91	3.54	3.74	4.17	4.09	4.52	4.45	3.97	4.13	1.84
24	0.68	0.59	0.64	0.63	0.67	0.65	0.75	0.80	0.84	1.11	0.43
	2.57	2.85	3.47	3.78	4.18	4.02	4.51	4.47	4.14	4.33	2.49
36	0.68	0.59	0.64	0.64	0.68	0.67	0.76	0.76	0.87	1.09	0.40
	2.64	2.89	3.58	3.91	4.30	4.12	4.58	4.28	4.30	4.20	2.54
60	0.74	0.62	0.65	0.65	0.70	0.69	0.74	0.70	0.87	1.06	0.31
	2.98	3.12	3.73	4.06	4.43	4.33	4.49	3.96	4.29	4.01	2.11

Panel B: Pairwise comparison tests with QCE						
	h	Diff	t	h	Diff	t
QCE-GB	1	−0.02	−0.18	24	0.06	0.58
LCE	1	0.24	0.86	24	0.23	1.53
ICE	1	1.10	3.33	24	0.47	1.64
Q5F	1	0.85	3.44	24	0.04	0.18
QCE-GB	3	−0.13	−1.09	36	0.03	0.39
LCE	3	0.10	0.38	36	0.02	0.14
ICE	3	0.90	2.72	36	0.25	0.91
Q5F	3	0.66	2.92	36	−0.22	−1.01
QCE-GB	12	0.05	0.44	60	0.04	0.53
LCE	12	0.22	0.98	60	0.05	0.43
ICE	12	0.67	2.22	60	0.19	0.73
Q5F	12	0.32	1.45	60	−0.27	−1.32

Table 3 : Cross-sectional Predictive Regressions, January 1977–December 2024

Panel A shows cross-sectional regressions of subsequent h-month returns on a given h-month ahead cost of equity at the beginning of month t. s is the average slope, ste its Newey-West standard error, and $|t_{s=1}|$ the absolute t-value that tests the slope equals one. We examine five cost of equity measures, including QCE (the q^5-characteristics model via cross-sectional regressions); QCE-GB (the q^5-characteristics model via gradient boosting); LCE (the Lewellen 7-variable model); ICE (the accounting-based implied cost of equity); and Q5F (the q^5-factor model). Panel B shows pairwise comparison tests of QCE against the alternative models. For each pairwise comparison, we test whether the slope difference is on average zero. We use time series tests with Newey-West t-values.

Panel A: Cross-sectional predictive regressions

	h	s	ste	$\|t_{s=1}\|$	h	s	ste	$\|t_{s=1}\|$	h	s	ste	$\|t_{s=1}\|$
QCE	1	0.82	0.11	1.63	12	0.63	0.15	2.51	36	0.61	0.13	3.10
QCE-GB	1	1.23	0.09	2.51	12	0.80	0.16	1.28	36	0.69	0.14	2.26
LCE	1	0.81	0.11	1.76	12	0.70	0.15	2.01	36	0.60	0.16	2.51
ICE	1	0.42	0.17	3.45	12	0.51	0.13	3.80	36	0.38	0.07	9.10
Q5F	1	0.01	0.03	35.38	12	0.00	0.02	42.25	36	0.01	0.01	90.11
QCE	3	0.81	0.13	1.54	24	0.81	0.13	1.47	60	0.54	0.16	2.92
QCE-GB	3	1.07	0.14	0.55	24	0.94	0.14	0.41	60	0.51	0.13	3.70
LCE	3	0.77	0.12	1.83	24	0.72	0.14	2.05	60	0.57	0.18	2.33
ICE	3	0.33	0.15	4.35	24	0.49	0.09	5.90	60	0.29	0.06	12.18
Q5F	3	0.00	0.03	34.41	24	0.01	0.01	66.00	60	0.00	0.00	203.77

Panel B: Pairwise comparison tests with QCE

	h	Diff	t	h	Diff	t	h	Diff	t
QCE-GB	1	−0.40	−5.32	12	−0.16	−1.68	36	−0.08	−1.16
LCE	1	0.02	0.17	12	−0.07	−0.72	36	0.02	0.17
ICE	1	0.40	1.81	12	0.13	0.78	36	0.23	2.47
Q5F	1	0.81	7.88	12	0.63	4.24	36	0.60	4.66
QCE-GB	3	−0.27	−3.40	24	−0.13	−2.07	60	0.03	0.24
LCE	3	0.03	0.30	24	0.10	1.10	60	−0.04	−0.49
ICE	3	0.47	2.12	24	0.33	2.74	60	0.24	1.90
Q5F	3	0.80	6.56	24	0.80	6.15	60	0.53	3.32

Table 4 : Cross-sectional Predictive Regressions, the All-but-micro Sample, January 1977–December 2024

Starting from the full sample, we exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. Panel A shows cross-sectional regressions of subsequent h-month returns on a given h-month ahead cost of equity at the beginning of month t. s is the average slope, ste its Newey-West standard error, and $|t_{s=1}|$ the absolute t-value that tests the slope equals one. We examine five cost of equity measures, including QCE (the q^5-characteristics model via cross-sectional regressions); QCE-GB (the q^5-characteristics model via gradient boosting); LCE (the Lewellen 7-variable model); ICE (the accounting-based implied cost of equity); and Q5F (the q^5-factor model). Panel B shows pairwise comparison tests of QCE against the alternative models. Each comparison tests whether the slope difference is on average zero with Newey-West t-values.

Panel A: Cross-sectional predictive regressions

	h	s	ste	$\|t_{s=1}\|$	h	s	ste	$\|t_{s=1}\|$	h	s	ste	$\|t_{s=1}\|$
QCE	1	0.74	0.26	0.97	12	0.48	0.18	2.93	36	0.22	0.10	7.55
QCE-GB	1	0.79	0.14	1.49	12	0.58	0.16	2.58	36	0.22	0.12	6.23
LCE	1	0.56	0.11	3.87	12	0.29	0.15	4.72	36	0.18	0.11	7.71
ICE	1	0.15	0.27	3.16	12	0.10	0.20	4.40	36	0.07	0.13	7.21
Q5F	1	0.03	0.03	28.84	12	0.05	0.02	41.99	36	0.06	0.01	66.30
QCE	3	0.66	0.21	1.67	24	0.41	0.12	5.10	60	0.07	0.13	7.19
QCE-GB	3	0.76	0.16	1.48	24	0.42	0.14	4.18	60	0.02	0.15	6.29
LCE	3	0.49	0.12	4.27	24	0.23	0.16	4.89	60	0.06	0.12	7.64
ICE	3	0.04	0.25	3.79	24	0.12	0.15	5.71	60	0.05	0.11	8.35
Q5F	3	0.03	0.03	30.61	24	0.07	0.02	49.06	60	0.02	0.01	128.91

Panel B: Pairwise comparison tests with QCE

	h	Diff	t	h	Diff	t	h	Diff	t
QCE-GB	1	−0.04	−0.23	12	−0.10	−1.11	36	0.00	0.02
LCE	1	0.18	0.70	12	0.19	1.32	36	0.05	0.48
ICE	1	0.59	1.56	12	0.38	1.36	36	0.15	0.89
Q5F	1	0.72	2.79	12	0.43	2.38	36	0.17	1.54
QCE-GB	3	−0.10	−0.78	24	−0.02	−0.24	60	0.05	0.66
LCE	3	0.17	0.82	24	0.18	1.15	60	0.01	0.08
ICE	3	0.62	1.87	24	0.28	1.44	60	0.02	0.14
Q5F	3	0.62	3.13	24	0.34	2.73	60	0.05	0.39

Table 5 : Measurement Error Variances (Times 100), January 1977–December 2024

In Panel A, $\overline{\text{SVar}}^{\text{TS}} = (1/N)\sum_i \left(\text{Var}_i\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)\right)$ is the time series measurement error variance (MEV) of a given expected return proxy, $\widehat{er}_{it}^h$, $\text{Var}^{\text{TS}} = (1/N)\sum_i \text{Var}_i\left(\widehat{er}_{it}^h\right)$ the variance component, and $\text{Cov}^{\text{TS}} = (1/N)\sum_i \text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)$ the covariance component. $\overline{\text{SVar}}^{\text{CX}} = (1/(T^p - h))\sum_t \left(\text{Var}_t\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)\right)$ is the cross-section MEV of $\widehat{er}_{it}^h$, in which $T^p - h$ is the number of cross-sections in the predictive window. $\text{Var}^{\text{CX}} = (1/(T^p - h))\sum_t \text{Var}_t\left(\widehat{er}_{it}^h\right)$ is the variance component, and $\text{Cov}^{\text{CX}} = (1/(T^p - h))\sum_t \text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)$ the covariance component. We examine five cost of equity measures, including QCE (the q^5-characteristics model via cross-sectional regressions); QCE-GB (the q^5-characteristics model via gradient boosting); LCE (the Lewellen 7-variable model); ICE (the accounting-based implied cost of equity); and Q5F (the q^5-factor model). Panel B shows pairwise comparison tests with QCE. Diff^{TS} is the time series MEV differential, and t_{TS} its White heteroscedasticity-adjusted t-value. Diff^{CX} is the cross-section MEV differential, and t_{CX} its Newey-West t-value.

Panel A: Measurement error variances

	h	$\overline{\text{SVar}}^{\text{TS}}$	Var^{TS}	Cov^{TS}	$\overline{\text{SVar}}^{\text{CX}}$	Var^{CX}	Cov^{CX}	h	$\overline{\text{SVar}}^{\text{TS}}$	Var^{TS}	Cov^{TS}	$\overline{\text{SVar}}^{\text{CX}}$	Var^{CX}	Cov^{CX}
QCE	1	0.0029	0.0072	0.0043	−0.0033	0.0079	0.0112	24	−1.8373	1.3018	3.1391	−0.6282	1.6281	2.2563
QCE-GB	1	−0.0063	0.0049	0.0112	−0.0071	0.0052	0.0123	24	−2.2231	0.9961	3.2193	−1.0816	1.0918	2.1734
LCE	1	0.0012	0.0051	0.0039	−0.0038	0.0062	0.0100	24	−3.2462	1.6852	4.9314	−1.1976	2.1540	3.3516
ICE	1	−0.0198	0.0038	0.0237	0.0015	0.0059	0.0044	24	−8.5829	3.8721	12.4551	0.6826	5.6091	4.9265
Q5F	1	0.0237	0.0202	−0.0035	0.0312	0.0317	0.0004	24	23.20	21.96	−1.2398	31.01	31.73	0.7205
QCE	3	0.0306	0.0541	0.0236	−0.0239	0.0571	0.0811	36	−1.3427	2.6343	3.9770	−0.2653	3.1630	3.4283
QCE-GB	3	−0.0162	0.0365	0.0527	−0.0419	0.0377	0.0796	36	−1.1691	2.1457	3.3148	−0.9205	2.1069	3.0274
LCE	3	0.0381	0.0429	0.0048	−0.0233	0.0473	0.0706	36	−3.7170	3.8659	7.5829	−1.5122	4.6619	6.1741
ICE	3	−0.1618	0.0357	0.1975	0.0205	0.0558	0.0353	36	−12.66	14.04	26.70	7.97	18.47	10.50
Q5F	3	0.2317	0.1861	−0.0456	0.2927	0.2904	−0.0022	36	93.20	93.69	0.4880	122.04	125.04	2.9978
QCE	12	−0.2009	0.4546	0.6555	−0.1996	0.5234	0.7230	60	−5.8606	7.9166	13.78	1.6146	9.6297	8.0151
QCE-GB	12	−0.4339	0.3363	0.7702	−0.3843	0.3621	0.7464	60	−4.0086	6.8684	10.88	−0.1898	6.7147	6.9045
LCE	12	−0.3560	0.4758	0.8318	−0.2228	0.5898	0.8126	60	−7.9222	11.80	19.72	0.3232	13.91	13.58
ICE	12	−2.6246	0.7093	3.3340	−0.0644	1.0922	1.1567	60	29.93	139.00	109.07	89.28	141.05	51.77
Q5F	12	4.3364	3.5564	−0.7801	5.4646	5.4208	−0.0438	60	1501.82	1496.04	−5.7771	1735.74	1730.84	−4.9075

Panel B: Pairwise comparison tests with QCE

	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}
QCE-GB	1	0.0092	11.17	0.0038	4.11	12	0.2330	4.50	0.1847	2.74	36	−0.1736	−0.97	0.6553	1.85
LCE	1	0.0017	2.27	0.0006	0.49	12	0.2433	5.53	0.0232	0.27	36	2.5401	8.60	1.2469	2.18
ICE	1	0.0189	18.99	−0.0048	−1.30	12	2.4538	16.65	−0.1351	−0.25	36	11.08	10.81	−8.2397	−1.83
Q5F	1	−0.0219	−18.44	−0.0345	−16.33	12	−4.6231	−31.59	−5.6641	−13.54	36	−93.52	−34.89	−122.30	−8.82
QCE-GB	3	0.0468	11.30	0.0180	2.84	24	0.3859	1.45	0.4534	2.65	60	−1.8520	−3.26	1.8044	1.63
LCE	3	−0.0035	−0.78	−0.0006	−0.07	24	1.6269	10.96	0.5694	1.97	60	3.7340	3.56	1.2914	0.68
ICE	3	0.1639	25.43	−0.0444	−1.24	24	7.0845	13.61	−1.3109	−0.91	60	−31.71	−4.05	−87.67	−2.40
Q5F	3	−0.2162	−23.46	−0.3166	−15.31	24	−25.32	−34.61	−31.64	−11.99	60	−1502.00	−22.25	−1734.13	−4.77

Table 6 : Measurement Error Variances (Times 100), the All-but-micro Sample, January 1977–December 2024

Starting from the full sample, we exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. In Panel A, $\overline{\text{SVar}}^{\text{TS}} = (1/N)\sum_i \left(\text{Var}_i\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)\right)$ is the time series measurement error variance (MEV) of a given expected return proxy, $\widehat{er}_{it}^h$, $\text{Var}^{\text{TS}} = (1/N)\sum_i \text{Var}_i\left(\widehat{er}_{it}^h\right)$ the variance component, and $\text{Cov}^{\text{TS}} = (1/N)\sum_i \text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)$ the covariance component. $\overline{\text{SVar}}^{\text{CX}} = (1/(T^p-h))\sum_t \left(\text{Var}_t\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)\right)$ is the cross-section MEV of $\widehat{er}_{it}^h$, in which $T^p - h$ is the number of cross-sections in the predictive window. $\text{Var}^{\text{CX}} = (1/(T^p-h))\sum_t \text{Var}_t\left(\widehat{er}_{it}^h\right)$ is the variance component, and $\text{Cov}^{\text{CX}} = (1/(T^p-h))\sum_t \text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)$ the covariance component. We examine five cost of equity measures, including QCE (the q^5-characteristics model via cross-sectional regressions); QCE-GB (the q^5-characteristics model via gradient boosting); LCE (the Lewellen 7-variable model); ICE (the accounting-based implied cost of equity); and Q5F (the q^5-factor model). Panel B shows pairwise comparison tests with QCE. Diff^{TS} is the time series MEV differential, and t_{TS} its White heteroscedasticity-adjusted t-value. Diff^{CX} is the cross-section MEV differential, and t_{CX} its Newey-West t-value.

Panel A: Measurement error variances

	h	$\overline{\text{SVar}}^{\text{TS}}$	Var^{TS}	Cov^{TS}	$\overline{\text{SVar}}^{\text{CX}}$	Var^{CX}	Cov^{CX}	h	$\overline{\text{SVar}}^{\text{TS}}$	Var^{TS}	Cov^{TS}	$\overline{\text{SVar}}^{\text{CX}}$	Var^{CX}	Cov^{CX}
QCE	1	0.0022	0.0023	0.0001	−0.0012	0.0027	0.0040	24	0.6637	0.5633	−0.1005	0.0925	0.5828	0.4903
QCE-GB	1	0.0021	0.0016	−0.0005	−0.0013	0.0016	0.0030	24	0.7076	0.4617	−0.2459	0.1029	0.3894	0.2865
LCE	1	0.0051	0.0043	−0.0008	−0.0002	0.0029	0.0031	24	1.0084	0.7074	−0.3010	0.3105	0.6041	0.2936
ICE	1	−0.0039	0.0004	0.0043	0.0000	0.0006	0.0006	24	−1.5924	0.3588	1.9512	0.1656	0.5326	0.3670
Q5F	1	0.0133	0.0104	−0.0029	0.0148	0.0157	0.0009	24	9.3795	9.9863	0.6068	13.01	14.45	1.4354
QCE	3	0.0167	0.0173	0.0006	−0.0089	0.0199	0.0289	36	2.9130	1.6992	−1.2137	0.9458	1.4669	0.5211
QCE-GB	3	0.0176	0.0129	−0.0047	−0.0092	0.0129	0.0221	36	2.5373	1.2606	−1.2767	0.6000	0.8907	0.2907
LCE	3	0.0419	0.0337	−0.0081	0.0013	0.0223	0.0210	36	2.7725	1.5812	−1.1913	0.7410	1.3957	0.6547
ICE	3	−0.0342	0.0039	0.0381	0.0016	0.0058	0.0042	36	−3.0219	0.9918	4.0137	0.6889	1.4898	0.8009
Q5F	3	0.1197	0.0966	−0.0231	0.1366	0.1452	0.0086	36	32.29	33.89	1.5989	43.48	46.93	3.4496
QCE	12	0.2086	0.1586	−0.0500	−0.0317	0.1711	0.2028	60	10.07	8.7487	−1.3223	6.7921	6.5318	−0.2603
QCE-GB	12	0.2506	0.1302	−0.1204	−0.0169	0.1188	0.1357	60	6.5855	4.9308	−1.6547	3.2916	3.2485	−0.0431
LCE	12	0.6499	0.2720	−0.3779	0.1298	0.1987	0.0689	60	7.2778	6.2085	−1.0692	4.3508	5.0192	0.6684
ICE	12	−0.4811	0.0728	0.5539	0.0317	0.1081	0.0764	60	−9.7377	4.2425	13.9803	3.0888	6.4230	3.3341
Q5F	12	2.0189	1.8371	−0.1818	2.4955	2.7162	0.2207	60	256.35	266.74	10.39	331.28	339.36	8.0839

Panel B: Pairwise comparison tests with QCE

	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}
QCE-GB	1	0.0000	0.20	0.0000	0.24	12	−0.0420	−3.55	−0.0150	−0.62	36	0.3757	4.62	0.3460	2.18
LCE	1	−0.0039	−7.53	−0.0010	−0.83	12	−0.4339	−12.22	−0.1610	−1.84	36	0.2996	1.75	0.2050	0.66
ICE	1	0.0059	16.02	−0.0010	−1.62	12	0.7008	24.96	−0.0630	−0.94	36	6.0985	26.43	0.2570	0.49
Q5F	1	−0.0119	−15.51	−0.0160	−14.28	12	−1.8688	−21.08	−2.5270	−13.73	36	−30.74	−27.51	−42.53	−11.77
QCE-GB	3	−0.0009	−0.65	0.0000	0.09	24	−0.0439	−1.62	−0.0100	−0.16	60	3.4856	7.74	3.5010	2.86
LCE	3	−0.0328	−8.65	−0.0100	−1.08	24	−0.2570	−3.26	−0.2180	−1.51	60	3.7143	6.18	2.4410	1.44
ICE	3	0.0492	16.86	−0.0110	−1.65	24	2.3097	24.73	−0.0730	−0.34	60	20.63	22.73	3.7030	1.41
Q5F	3	−0.1093	−17.37	−0.1460	−14.42	24	−9.0840	−26.40	−12.92	−12.80	60	−259.50	−27.08	−324.49	−8.85

Table 7 : Firm-level Properties of Costs of Equity, January 1977–December 2024

In Panel A, #obs. is the number of firm-month observations. The summary statistics are time series averages of cross-sectional moments, including mean, standard deviation (std), skewness (skew), excess kurtosis (kurt), autocorrelation (ρ), and 1, 5, 25, 50, 75, 95, and 99 percentiles. QCE is the cost of equity from the q^5-characteristics model via cross-sectional regressions, ICE is the accounting-based implied cost of equity, and QCE_{12} minus ICE is the difference between 12-month ahead QCE and ICE. Panel B shows the average slopes and their t-values in cross-sectional regressions in forming QCE. The panel uses the full sample to illustrate the slopes, while the QCE estimates uses latest available 120-month rolling window slopes (Figure 3).

Panel A: Firm-level properties of QCE, ICE, and their differences

h	#obs.	mean	std	skew	kurt	ρ	p1	p5	p25	p50	p75	p95	p99
						QCE							
1	1,488,265	13.83	10.75	−0.81	2.51	0.91	−19.96	−6.18	9.17	14.77	19.96	29.23	38.11
3	1,488,265	13.69	9.55	−0.90	2.53	0.92	−16.72	−4.51	9.72	14.73	19.18	27.00	34.34
12	1,488,265	13.85	6.85	−0.71	2.08	0.93	−6.71	1.51	10.44	14.34	18.07	23.83	28.45
24	1,488,265	13.50	5.42	−0.74	2.33	0.94	−2.57	4.02	10.64	13.87	16.92	21.44	24.92
36	1,488,265	13.03	4.48	−0.76	2.23	0.95	0.37	5.13	10.59	13.36	15.90	19.64	22.44
60	1,488,263	12.96	3.50	−0.60	2.34	0.95	3.47	6.77	10.98	13.22	15.26	18.12	20.24
						ICE							
12	1,991,510	13.62	10.26	1.91	3.68	0.97	4.06	4.72	7.33	10.14	15.44	38.25	50.34
						QCE_{12} minus ICE							
12	1,464,185	1.17	11.07	−1.50	3.80	0.95	−38.87	−22.39	−1.77	3.81	7.67	13.04	17.44

Panel B: Average slopes and their t-values in cross-sectional return regressions

	Slopes				t-values			
h	logME	I/A	ROE	EG	logME	I/A	ROE	EG
1	−0.002	−0.010	0.057	0.044	−4.36	−6.91	6.18	6.68
3	−0.004	−0.031	0.145	0.117	−3.44	−7.91	5.03	6.57
12	−0.019	−0.107	0.188	0.327	−3.43	−7.16	1.85	5.30
24	−0.035	−0.160	0.063	0.667	−3.96	−5.80	0.42	6.89
36	−0.048	−0.191	−0.071	1.049	−3.69	−4.91	−0.37	7.83
60	−0.091	−0.147	−0.627	2.064	−3.75	−1.83	−2.12	9.54

Table 8 : Firm-level Properties of Costs of Equity, the All-but-micro Sample, January 1977–December 2024

We exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. In Panel A, #obs. is the number of firm-month observations. The summary statistics are time series averages of cross-sectional moments, including mean, standard deviation (std), skewness (skew), excess kurtosis (kurt), autocorrelation (ρ), and 1, 5, 25, 50, 75, 95, and 99 percentiles. QCE is the cost of equity from the q^5-characteristics model via cross-sectional regressions, ICE is the accounting-based implied cost of equity, and QCE_{12} minus ICE is the difference between 12-month ahead QCE and ICE. Panel B shows the average slopes and their t-values in cross-sectional return regressions in QCE. The panel uses the full sample to illustrate the slopes, while the QCE estimates uses latest available 120-month rolling window slopes (Figure 4).

Panel A: Firm-level properties of QCE, ICE, and their differences

h	#obs.	mean	std	skew	kurt	ρ	p1	p5	p25	p50	p75	p95	p99
						QCE							
1	677,967	13.29	6.23	−0.39	1.32	0.92	−4.07	1.94	10.10	13.63	16.92	23.01	28.38
3	677,967	13.13	5.52	−0.49	1.38	0.92	−2.61	2.93	10.37	13.51	16.40	21.56	25.97
12	677,967	12.53	3.76	−0.53	1.32	0.94	1.67	5.60	10.61	12.84	14.86	18.17	20.71
24	677,967	12.12	3.11	−0.54	1.27	0.94	3.25	6.41	10.47	12.37	14.08	16.79	18.69
36	677,967	11.76	2.99	−0.57	1.61	0.95	3.15	6.40	10.19	11.98	13.61	16.23	18.11
60	677,967	11.70	2.77	−0.66	2.08	0.95	3.59	6.80	10.28	11.92	13.40	15.78	17.48
						ICE							
12	849,932	8.94	3.24	0.91	0.68	0.97	4.05	4.62	6.60	8.37	10.63	15.52	18.00
						QCE_{12} minus ICE							
12	670,908	3.92	4.96	−0.70	1.21	0.95	−10.93	−5.26	1.28	4.46	7.16	11.08	13.93

Panel B: Average slopes and their t-values in cross-sectional return regressions

	Slopes				t-values			
h	logME	I/A	ROE	Eg	logME	I/A	ROE	Eg
1	−0.001	−0.006	0.047	0.049	−2.50	−3.48	3.58	5.15
3	−0.003	−0.018	0.116	0.132	−2.30	−3.90	2.93	5.23
12	−0.007	−0.060	0.155	0.391	−1.51	−3.54	1.08	4.75
24	−0.012	−0.104	0.135	0.722	−1.42	−3.20	0.52	4.92
36	−0.016	−0.113	0.041	1.133	−1.31	−2.09	0.12	5.18
60	−0.037	−0.036	−0.452	2.341	−1.54	−0.30	−0.88	7.07

Table 9 : Industry Costs of Equity, January 1977–December 2024

For each nonfinancial NAICS sector or industry, we calculate its sector- or industry-level cost of equity by value-weighting the costs of equity across all the firms within the sector or industry. There are 19 nonfinancial sectors and 58 industries. However, one industry (sector) "Management of companies and enterprises" drops out because it contains only a few firms in each cross section, firms that do not survive our sample criteria. In addition to realized returns (RR), we report the cost of equity from the q^5-factor model (Q5F); 1-, 3-, 12-, 24-, 36-, and 60-month ahead QCEs; as well as ICE. We value-weight firm-month costs of equity across all the firms within a given industry to obtain industry costs of equity. We then annualize each industry-month before taking the time series means and standard deviations for industry costs of equity. The only exception is RR, which renders our annualization procedure infeasible because of outliers even at the industry level. Instead, after we value-weight firm-level RR to obtain industry RR, we take time series means and standard deviations of (monthly) industry RRs first before annualizing them. Annualized mean equals $(1 + \text{monthly mean})^{12} - 1$, and annualized standard deviation equals monthly standard deviation times $\sqrt{12}$. The sector code is as follows: 11 Agriculture, forestry, fishing, and hunting; 21 Mining; 22 Utilities; 23 Construction; 31ND Nondurable goods; 33DG Durable goods; 42 Wholesale trade; 44 Retail trade; 48TW Transportation and warehousing; 51 Information; 53 Real estate and rental and leasing; 54 Professional, scientific, and technical services; 56 Administrative and waste management services; 61 Educational services; 62 Health care and social assistance; 71 Arts, entertainment, and recreation; 72 Accommodation and food services; 81 Other services, except government. The industry code is as follows: 110C Farms; 113F Forestry, fishing, and related activities; 2110 Oil and gas extraction; 2120 Mining, except oil and gas; 2130 Support activities for mining; 2200 Utilities; 2300 Construction; 311A Food and beverage and tobacco products; 313T Textile mills and textile product mills; 315A Apparel and leather and allied products; 3210 Wood products; 3220 Paper products; 3230 Printing and related support activities; 3240 Petroleum and coal products; 3250 Chemical products; 3260 Plastics and rubber products; 3270 Nonmetallic mineral products; 3310 Primary metals; 3320 Fabricated metal products; 3330 Machinery; 3340 Computer and electronic products; 3350 Electrical equipment, appliances, and components; 336M Motor vehicles, bodies and trailers, and parts; 336O Other transportation equipment; 3370 Furniture and related products; 338A Miscellaneous manufacturing; 4200 Wholesale trade; 44RT Retail trade; 4810 Air transportation; 4820 Railroad transportation; 4830 Water transportation; 4840 Truck transportation; 4850 Transit and ground passenger transportation; 4860 Pipeline transportation; 487S Other transportation and support activities; 4930 Warehousing and storage; 5110 Publishing industries (includes software); 5120 Motion picture and sound recording industries; 5130 Broadcasting and telecommunications; 5140 Information and data processing services; 5310 Real estate; 5320 Rental and leasing services and lessors of intangible assets; 5411 Legal services; 5412 Miscellaneous professional, scientific, and technical services; 5415 Computer systems design and related services; 5610 Administrative and support services; 5620 Waste management and remediation services; 6100 Educational services; 6210 Ambulatory health care services; 622H Hospitals; 6230 Nursing and residential care facilities; 6240 Social assistance; 711A Performing arts, spectator sports, museums, and related activities; 7130 Amusements, gambling, and recreation industries; 7210 Accommodation; 7220 Food services and drinking places; 8100 Other services, except government.

	Panel A: Sector costs of equity																	
	Time series mean									Time series standard deviation								
Sector	Q5F	QCE_1	QCE_3	QCE_{12}	QCE_{24}	QCE_{36}	QCE_{60}	ICE	RR	Q5F	QCE_1	QCE_3	QCE_{12}	QCE_{24}	QCE_{36}	QCE_{60}	ICE	RR
11	12.01	10.70	11.41	10.33	10.02	9.79	9.59	7.65	15.65	7.51	5.61	5.28	4.92	4.60	4.45	4.62	3.00	21.36
21	7.10	8.07	9.04	8.46	8.59	8.83	9.13	9.28	11.37	11.53	4.03	3.93	4.07	3.90	3.92	4.09	2.54	26.70
22	9.58	8.07	9.01	8.73	8.75	9.10	9.66	12.81	11.86	4.64	6.22	5.75	5.22	4.57	4.13	3.86	3.91	13.84
23	7.94	9.30	9.92	9.26	9.15	9.23	9.60	8.89	15.62	8.97	4.21	4.24	4.45	3.99	3.88	3.35	2.59	24.94
31ND	11.35	10.78	11.72	9.44	8.93	8.84	8.80	8.41	12.96	4.32	4.25	4.63	4.65	4.33	4.06	4.15	2.86	13.60
33DG	11.50	10.32	11.10	9.34	9.17	9.22	9.31	8.10	14.70	3.64	3.90	3.84	3.97	3.78	3.59	3.81	2.90	20.51
42	10.44	9.78	10.38	9.26	9.03	9.08	9.42	8.53	13.38	5.10	3.72	3.54	3.71	3.48	3.52	3.56	2.63	18.13
44	7.95	8.92	9.83	8.28	8.01	8.10	8.29	7.59	15.56	8.64	4.06	4.03	4.33	4.00	3.73	3.61	2.91	18.65
48TW	8.48	9.46	10.34	9.51	9.39	9.38	9.40	9.42	13.39	4.52	3.42	3.47	3.61	3.51	3.58	3.70	2.96	19.20
51	9.16	8.21	9.25	7.97	8.04	8.35	8.63	8.71	14.40	6.23	4.74	4.58	4.64	4.26	3.78	3.80	3.70	17.61
53	6.73	10.84	11.27	10.41	10.57	10.60	10.81	9.78	16.19	11.20	4.95	4.63	4.47	4.18	3.87	4.07	3.09	25.70
54	9.83	11.11	11.46	9.62	9.42	9.49	9.91	7.86	12.04	5.39	3.90	3.65	4.23	4.12	3.97	4.11	2.91	19.91
56	11.10	10.64	11.14	9.66	9.46	9.44	9.65	7.53	12.66	6.72	4.43	4.06	3.77	3.25	2.90	2.95	2.39	19.92
61	11.78	13.33	13.38	12.02	11.62	11.38	11.55	7.44	12.41	10.81	4.00	3.74	3.35	2.76	2.86	3.04	2.67	25.45
62	12.67	10.22	10.52	9.73	9.81	9.88	10.44	8.15	13.85	10.44	3.55	3.35	3.38	3.04	3.08	3.42	1.92	23.79
71	8.86	10.66	11.07	10.60	10.59	10.68	11.03	7.95	12.62	11.22	4.58	4.35	3.91	3.49	3.50	3.74	2.31	25.58
72	8.76	10.99	11.47	9.52	9.19	9.15	9.26	7.24	15.08	7.83	3.87	4.05	4.49	4.24	4.10	4.10	2.27	20.58
81	9.94	12.36	12.72	11.80	11.47	11.27	11.41	8.98	12.06	10.44	3.46	2.98	2.98	2.77	2.81	3.00	3.29	23.11
Average	9.73	10.21	10.83	9.66	9.51	9.54	9.77	8.57	13.66	7.73	4.27	4.12	4.12	3.79	3.65	3.72	2.82	21.03
Industry	Panel B: Industry costs of equity																	
110C	11.68	10.71	11.41	10.30	9.97	9.75	9.54	7.58	15.71	8.15	5.73	5.35	4.91	4.58	4.43	4.60	3.02	21.91
113F	27.96	13.20	13.47	13.65	13.29	12.69	12.33	11.62	17.77	28.59	9.44	8.85	6.19	4.21	4.19	4.44	7.18	56.51
2110	8.48	7.92	8.90	8.36	8.56	8.82	9.08	10.00	12.58	10.76	4.49	4.38	4.23	4.08	4.11	4.36	2.56	29.00
2120	5.67	8.25	9.18	8.64	8.74	8.89	9.16	8.49	10.72	16.19	5.44	5.10	4.71	4.23	4.06	3.88	2.67	27.67
2130	3.86	7.84	8.80	8.63	8.81	9.20	9.65	8.76	13.16	11.23	3.96	3.51	3.59	3.62	3.82	4.07	3.33	35.67
2200	9.58	8.07	9.01	8.73	8.75	9.10	9.66	12.81	11.86	4.64	6.22	5.75	5.22	4.57	4.13	3.86	3.91	13.84
2300	7.94	9.30	9.92	9.26	9.15	9.23	9.60	8.89	15.62	8.97	4.21	4.24	4.45	3.99	3.88	3.35	2.59	24.94
311A	11.09	11.09	11.91	9.24	8.53	8.35	8.33	8.06	14.54	7.09	4.37	4.98	5.06	4.56	4.23	4.08	2.60	14.49
313T	12.66	12.42	12.78	11.97	11.57	11.26	11.27	9.48	11.39	7.49	4.02	3.98	3.70	3.36	3.49	3.65	3.35	28.12
315A	11.62	12.53	12.84	10.86	10.43	10.25	10.26	7.98	14.56	7.43	3.64	3.70	3.78	3.51	3.54	3.77	3.14	22.08
3210	13.32	10.44	11.31	10.76	10.45	10.40	10.69	8.86	13.86	7.88	4.78	4.23	3.65	3.38	3.30	3.22	3.01	28.64
3220	14.09	11.14	11.97	9.85	9.37	9.24	9.22	8.61	10.92	7.77	3.95	3.86	3.85	3.75	3.64	3.85	2.19	18.04
3230	10.79	14.30	14.66	13.20	12.39	11.82	11.59	10.09	10.06	8.71	4.41	4.40	3.47	2.78	2.79	3.00	2.58	23.70
3240	11.92	8.94	10.20	8.52	8.16	8.25	8.27	10.38	14.19	6.27	3.93	4.25	4.63	4.41	4.14	4.30	3.26	19.98
3250	10.43	11.21	12.13	9.68	9.16	9.04	8.97	7.67	12.90	4.09	4.94	5.25	4.91	4.50	4.18	4.29	2.57	14.79
3260	9.83	12.86	13.44	11.84	11.24	10.92	10.76	8.69	12.46	5.03	3.19	3.21	2.89	2.77	2.84	3.22	2.57	20.78
3270	7.35	12.58	13.28	12.04	11.49	11.12	10.94	9.41	15.74	10.08	3.79	3.67	2.96	2.85	3.01	3.23	2.85	25.13
3310	10.17	9.52	10.45	9.79	9.69	9.72	9.90	10.22	11.15	9.88	4.92	4.54	3.89	3.64	3.52	3.42	3.13	28.34

	Time series mean									Time series standard deviation								
Industry	Q5F	QCE_1	QCE_3	QCE_{12}	QCE_{24}	QCE_{36}	QCE_{60}	ICE	RR	Q5F	QCE_1	QCE_3	QCE_{12}	QCE_{24}	QCE_{36}	QCE_{60}	ICE	RR
3320	10.69	11.94	12.60	10.94	10.40	10.27	10.38	8.11	15.13	5.38	3.88	3.97	3.86	3.53	3.41	3.31	2.84	19.69
3330	9.35	10.14	10.88	9.35	9.06	9.09	9.16	8.26	14.05	6.30	3.76	3.77	3.99	3.81	3.70	3.70	2.82	22.18
3340	13.67	10.57	11.29	9.24	9.15	9.26	9.36	7.15	15.61	4.41	4.58	4.37	4.26	4.04	3.80	4.08	2.57	23.65
3350	9.21	10.19	11.01	9.71	9.25	9.12	9.07	8.53	15.57	5.28	4.35	4.22	3.68	3.40	3.41	3.63	2.66	22.26
336M	6.74	9.86	10.79	9.45	9.17	9.03	9.05	12.61	16.59	7.33	5.10	4.91	4.01	3.86	3.75	3.92	4.23	29.23
336O	10.58	9.82	10.73	8.77	8.43	8.40	8.52	9.03	13.91	7.78	4.56	4.65	4.78	4.59	4.35	4.33	3.34	19.76
3370	13.41	12.78	13.30	11.88	11.24	10.96	11.02	8.58	12.00	6.73	3.00	2.77	2.76	2.58	2.68	2.89	2.87	24.15
338A	9.71	10.37	11.21	9.89	9.63	9.63	9.81	7.05	12.38	4.36	3.52	3.60	3.49	3.28	3.24	3.52	2.70	17.38
4200	10.44	9.78	10.38	9.26	9.03	9.08	9.42	8.53	13.38	5.10	3.72	3.54	3.71	3.48	3.52	3.56	2.63	18.13
44RT	7.95	8.92	9.83	8.28	8.01	8.10	8.29	7.59	15.56	8.64	4.06	4.03	4.33	4.00	3.73	3.61	2.91	18.65
4810	11.51	9.23	9.82	9.66	9.89	10.01	10.21	10.88	10.76	11.69	6.66	5.81	4.49	4.01	4.01	4.29	3.26	30.94
4820	8.74	9.75	11.25	10.88	10.42	10.35	10.55	9.49	17.42	5.46	5.09	4.99	5.15	4.46	4.01	3.62	3.43	21.95
4830	7.31	12.00	12.49	11.94	11.51	11.15	11.06	10.00	10.78	11.78	3.75	3.59	3.12	2.82	3.09	3.24	2.56	23.80
4840	10.07	14.16	14.18	12.52	11.97	11.49	11.21	8.34	13.65	7.36	4.89	4.58	3.56	3.13	3.42	3.64	3.07	23.38
4850	1.83	12.17	12.30	12.02	11.79	11.09	10.66	10.11	24.08	18.49	7.92	7.33	4.86	3.26	3.18	3.28	7.46	42.51
4860	3.44	7.70	8.41	8.15	8.31	8.47	8.84	11.00	15.46	12.25	7.10	6.81	5.34	5.02	5.12	5.40	3.43	25.08
487S	11.43	10.21	10.80	9.75	9.70	9.75	9.96	8.22	12.03	8.01	4.60	4.44	4.40	4.36	4.46	4.59	2.65	22.50
4930	7.29	11.87	11.99	11.08	10.89	10.92	11.20	10.19	20.50	13.17	9.25	8.41	7.60	6.66	6.07	4.95	4.68	31.16
5110	11.44	10.20	10.99	9.09	8.93	9.01	9.20	6.57	18.68	7.12	4.06	3.87	3.97	3.70	3.38	3.78	2.58	22.87
5120	6.52	9.69	10.38	9.60	9.50	9.38	9.69	8.75	16.43	10.92	5.42	4.87	4.59	4.21	3.80	3.86	3.13	31.35
5130	8.91	6.93	8.18	7.28	7.41	7.82	8.16	10.77	11.40	6.29	5.43	5.32	5.05	4.57	4.08	4.05	4.20	16.97
5140	8.52	7.85	8.50	7.10	7.55	8.07	8.77	6.36	18.50	11.66	7.62	7.38	7.06	6.07	4.95	4.32	2.46	24.85
5310	19.06	11.91	12.63	15.47	16.30	17.76	18.33	17.91	−8.26	16.33	8.16	7.79	5.81	4.11	3.60	2.47	14.15	32.29
5320	6.71	10.86	11.28	10.41	10.57	10.58	10.79	9.78	16.28	11.21	4.95	4.64	4.46	4.17	3.86	4.06	3.09	25.79
5411	13.60	13.62	13.40	13.35	12.89	12.26	12.09	10.18	13.42	23.96	6.06	5.48	4.49	3.61	3.71	4.06	5.93	52.44
5412	9.76	10.55	10.92	9.81	9.72	9.89	10.38	7.87	12.28	5.61	4.79	4.43	4.22	3.77	3.50	3.70	2.97	20.10
5415	12.64	11.91	11.96	9.67	9.51	9.49	10.08	7.73	11.36	11.35	5.13	4.64	4.86	4.79	4.75	4.69	2.97	25.15
5610	10.17	12.64	12.81	11.09	10.67	10.58	10.73	7.67	13.09	5.48	4.17	3.96	3.78	3.36	3.32	3.40	2.69	20.41
5620	11.44	8.68	9.52	8.29	8.27	8.39	8.76	7.75	14.55	9.31	5.95	5.57	4.93	4.02	3.21	3.18	2.19	23.36
6100	11.78	13.33	13.38	12.02	11.62	11.38	11.55	7.44	12.41	10.81	4.00	3.74	3.35	2.76	2.86	3.04	2.67	25.45
6210	12.53	9.45	9.83	9.64	10.10	10.34	11.03	7.90	11.30	9.00	4.40	4.05	3.89	3.68	3.62	3.90	2.46	26.87
622H	12.42	11.58	11.76	10.93	10.59	10.24	10.34	9.09	18.19	14.25	4.95	4.73	3.88	3.31	3.30	3.24	2.47	29.92
6230	9.93	11.47	11.84	11.37	11.06	10.89	11.26	8.86	12.86	10.97	6.34	5.86	4.69	3.51	3.30	3.33	2.14	31.13
6240	12.95	12.96	12.66	13.20	13.33	13.45	14.04	8.32	20.29	20.08	9.06	8.29	5.84	4.44	3.74	3.58	5.35	50.46
711A	11.93	10.41	10.84	10.79	10.76	10.86	11.26	8.57	10.73	18.01	5.83	5.39	4.68	4.10	3.97	4.26	2.80	25.46
7130	9.31	10.70	11.04	10.65	10.75	10.85	11.23	8.32	11.95	11.62	5.42	5.04	4.15	3.61	3.69	3.70	2.77	29.79
7210	4.48	10.16	10.57	9.27	9.15	9.25	9.47	8.05	18.51	14.62	6.58	6.59	6.32	5.66	5.37	5.09	2.66	32.03
7220	10.92	11.51	11.97	9.80	9.37	9.27	9.33	6.94	15.21	7.59	3.42	3.53	4.04	3.88	3.77	3.90	2.28	18.94
8100	9.94	12.36	12.72	11.80	11.47	11.27	11.41	8.98	12.06	10.44	3.46	2.98	2.98	2.77	2.81	3.00	3.29	23.11
Average	10.29	10.75	11.34	10.33	10.12	10.09	10.26	9.04	13.85	9.94	5.10	4.86	4.40	3.92	3.77	3.80	3.37	25.94

Table 10 : Expected Factor Premiums, January 1977–December 2024

We form one-way deciles with annual sorts on size (ME), book-to-market (BM), and investment-to-assets (I/A) as well as monthly sorts on prior 11-month returns (R^{11}), return on equity (ROE), and expected growth (EG). We use NYSE breakpoints and value-weighted returns. For each decile, we calculate subsequently realized holding period returns over 1-, 3-, 12-, 24-, 36-, and 60-month horizons as well as ex ante expected returns measured by QCE and ICE over the corresponding horizons. We report the differences between the extreme deciles and their Newey-West t-values.

	1m	3m	12m	24m	36m	60m	1m	3m	12m	24m	36m	60m
	Average realized returns						t-values of average realized returns					
ME, 1−10	0.00	0.19	0.50	−3.83	−9.02	−21.73	0.00	0.24	0.17	−0.81	−1.27	−1.95
BM, 10−1	0.22	0.74	4.48	8.10	10.52	13.21	0.83	0.98	1.61	1.61	1.38	0.94
I/A, 1−10	0.18	0.62	1.32	0.52	−2.38	−15.86	0.90	1.04	0.62	0.15	−0.45	−1.51
R11, 10−1	0.98	2.55	4.38	−0.01	5.74	14.44	3.19	2.71	1.39	0.00	0.88	1.34
ROE, 10−1	0.80	1.94	4.00	6.99	10.74	19.96	3.25	2.72	1.73	1.99	2.28	2.64
EG, 10−1	0.99	2.94	9.56	17.65	26.96	52.83	5.38	5.50	4.85	5.16	5.18	5.34
	Average QCEs						t-values of average QCEs					
ME, 1−10	0.41	0.95	7.35	15.77	22.55	44.11	5.39	4.26	7.73	8.30	7.21	7.52
BM, 10−1	−0.17	−0.40	1.24	3.25	4.03	6.87	−2.88	−2.50	2.16	3.19	2.91	3.84
I/A, 1−10	0.72	2.18	7.72	12.10	14.78	17.43	13.05	14.33	14.54	13.56	12.99	8.88
R11, 10−1	0.39	1.00	1.03	0.14	−1.39	−6.40	7.37	6.70	2.21	0.19	−1.33	−3.21
ROE, 10−1	1.27	3.17	3.79	2.53	0.54	−6.74	13.29	10.91	3.75	1.68	0.28	−2.94
EG, 10−1	1.43	3.88	9.28	15.43	20.68	28.33	14.11	14.25	11.98	11.62	12.57	8.20
	Average ICEs						t-values of average ICEs					
ME, 1−10	0.36	1.11	5.01	11.87	21.37	54.27	7.68	7.66	7.58	7.40	7.14	6.29
BM, 10−1	0.64	1.95	8.52	19.20	32.65	71.34	28.89	28.59	27.18	25.14	23.01	18.62
I/A, 1−10	0.24	0.74	3.23	7.29	12.42	27.49	14.20	14.13	13.78	13.24	12.60	10.96
R11, 10−1	−0.35	−1.07	−4.71	−10.74	−18.54	−42.74	−10.97	−10.92	−10.68	−10.28	−9.76	−8.36
ROE, 10−1	−0.27	−0.83	−3.66	−8.40	−14.59	−33.62	−12.73	−12.67	−12.37	−11.89	−11.33	−9.90
EG, 10−1	−0.14	−0.44	−1.96	−4.48	−7.78	−17.99	−6.32	−6.34	−6.43	−6.54	−6.62	−6.67

Table 11 : Predicting Factor Premiums, January 1977–December 2024

Factors are the differences between the extreme deciles from one-way annual sorts on size (ME), book-to-market (BM), and investment-to-assets (I/A), as well as monthly sorts on prior 11-month returns (R^{11}), return on equity (ROE), and expected growth (EG). We align the factors to yield positive average premiums. We perform predictive regressions of realized factor premiums over subsequent h months on h-month ahead expected premiums. We use overlapping monthly observations, in which $h = 1, 3, 12, 24, 36$, and 60 months. We show the slopes, Newey-West t-values, and R^2s. QCE is the q^5-characteristics model via cross-sectional regressions, and ICE is the accounting-based implied cost of equity.

h	Slope	t-value	R^2	Slope	t-value	R^2	Slope	t-value	R^2	Slope	t-value	R^2
	QCE			ICE			QCE			ICE		
	Panel A: ME						Panel B: BM					
1	1.020	1.905	0.011	0.625	0.658	0.002	1.271	2.353	0.013	1.166	0.819	0.001
3	0.745	1.412	0.012	0.889	0.846	0.007	1.523	2.804	0.039	1.641	1.259	0.008
12	0.437	1.131	0.014	0.877	0.968	0.025	1.352	2.709	0.066	2.665	2.616	0.079
24	0.623	1.840	0.054	0.605	0.914	0.029	2.170	5.196	0.184	2.950	4.307	0.209
36	0.128	0.323	0.003	0.758	1.252	0.062	2.414	3.437	0.196	2.652	4.821	0.304
60	−0.278	−0.880	0.020	0.974	3.353	0.207	1.779	1.983	0.054	1.783	6.250	0.338
	Panel C: I/A						Panel D: R^{11}					
1	1.409	2.879	0.027	1.402	1.148	0.003	0.400	0.617	0.000	4.161	2.353	0.021
3	1.234	2.510	0.051	1.478	1.268	0.008	1.200	1.769	0.011	4.910	2.560	0.085
12	1.107	2.228	0.082	1.544	1.850	0.032	2.101	1.926	0.065	4.498	4.328	0.280
24	0.666	1.364	0.030	0.598	1.047	0.010	2.520	2.362	0.097	3.241	4.980	0.323
36	0.549	1.131	0.017	0.201	0.697	0.002	0.866	0.999	0.017	1.799	6.485	0.273
60	−0.373	−0.587	0.006	0.052	0.302	0.000	0.735	0.941	0.017	0.922	5.505	0.186
	Panel E: ROE						Panel F: EG					
1	0.423	1.367	0.003	2.488	1.365	0.007	0.129	0.530	0.000	2.109	1.974	0.007
3	0.409	1.314	0.008	2.935	1.849	0.025	0.220	0.822	0.003	1.808	1.849	0.016
12	0.593	2.211	0.042	2.256	2.538	0.067	0.197	0.541	0.004	2.547	3.345	0.111
24	0.788	2.538	0.083	1.655	3.659	0.101	0.319	1.153	0.012	1.518	2.306	0.083
36	0.580	1.525	0.040	1.195	2.740	0.100	0.177	0.569	0.003	0.937	1.487	0.046
60	1.200	2.882	0.118	0.760	4.282	0.120	0.520	1.307	0.032	0.746	1.252	0.038

Table 12 : The Equity Premium, January 1977–December 2024

Panel A shows the (annualized) means and volatilities of the QCE equity premium across horizons, including the 1-, 3-, 12-, 24-, 36-, and 60-month. Rf is the risk-free rate. The panel also reports the ICE equity premium and the average realized return from an expanding window, denoted EAR. For 1-month (1m), EAR is the average of the expanding window series of the average monthly returns. The expanding window starts in January 1967. For 12-monnth (12m), EAR is the average of the expanding window series of the average prior 12-month returns (with overlapping monthly observations). Panel B shows the correlation matrix of the equity premium estimates. The upper triangular shows the Pearson correlations, and the lower triangular the Spearman correlations. ICE_{12} is the 12-month ICE equity premium, and EAR_1 is the 1-month EAR equity premium. Panel C show the predictive regressions of subsequent h-month realized equity premium on the ex ante h-month ahead equity premium. For the ICE equity premium, we align the units between the left- and right-hand sides of the regression. For the 1-month regression, for instance, we convert the ICE aggregate cost of equity from annual to monthly unit, i.e., $(1+\text{ICE})^{1/12}-1$, then subtract the 1-month treasury bill rate. For the 60-month regression, we convert the ICE aggregate cost of equity from annual to 5-year unit, i.e., $(1+\text{ICE})^5-1$, then subtract the 5-year treasury bond rate. We use overlapping monthly observations. The t-values are Newey-West t-values. Oos-R^2 is the Welch-Goyal (2008) out-of-sample R^2, calculated as $1-\text{MSE}_h/\text{MSE}_h^A$, in which MSE_h is the mean squared forecast error (MSE) at the h-month horizon, and MSE_h^A is the MSE of the expanding window average return. We align the EAR horizon with the horizon, h, in the left-hand side. For the 12-month, for instance, EAR is the time series of prior 12-month returns averaged from the expanding window. $\overline{\text{SVar}}^{\text{TS}}$ is time series MEV.

Panel A: Means and volatilities of the equity premium and risk-free rates

		1m	3m	12m	24m	36m	60m
QCE	mean	5.69	6.62	4.35	3.96	4.01	3.93
	std	5.22	4.78	4.52	4.52	4.72	5.07
ICE	mean	4.14	4.19	3.75	3.56	3.46	3.31
	std	2.54	2.38	2.40	2.29	2.18	1.99
EAR	mean	7.31	7.20	6.39	5.80	5.37	5.30
	std	4.56	4.48	4.98	5.18	5.46	5.56
Rf	mean	4.29	4.25	4.72	4.97	5.13	5.42
	std	3.70	3.50	3.76	3.74	3.67	3.53

Panel B: Correlation matrix of the equity premium estimates

QCE	1m	3m	12m	24m	36m	60m	ICE_{12}	EAR_1
1m	1	0.96	0.92	0.91	0.87	0.73	0.21	0.70
3m	0.95	1	0.95	0.92	0.89	0.77	0.12	0.65
12m	0.91	0.94	1	0.97	0.90	0.71	0.13	0.56
24m	0.90	0.89	0.95	1	0.97	0.80	0.10	0.61
36m	0.83	0.84	0.87	0.96	1	0.92	0.03	0.69
60m	0.64	0.68	0.64	0.77	0.89	1	−0.06	0.75
ICE_{12}	0.17	0.07	0.11	0.07	−0.02	−0.15	1	0.43
EAR_1	0.55	0.45	0.39	0.45	0.48	0.48	0.54	1

Panel C: Predictive regressions of subsequent h-month realized equity premiums

	QCE					ICE				
h	Slope	t-value	R^2	Oos-R^2	$\overline{\text{SVar}}^{\text{TS}}$	Slope	t-value	R^2	Oos-R^2	$\overline{\text{SVar}}^{\text{TS}}$
1	0.501	1.064	0.002	−0.001	0.000	2.510	2.500	0.013	0.001	−0.002
3	0.345	0.648	0.002	0.000	0.004	1.935	1.997	0.020	0.003	−0.009
12	0.662	1.363	0.032	0.017	−0.067	1.781	2.390	0.063	0.022	−0.145
24	0.558	1.921	0.051	0.025	−0.112	1.731	2.592	0.113	0.053	−0.550
36	0.381	2.087	0.038	0.036	0.617	1.864	3.026	0.178	0.112	−1.399
60	0.037	0.187	0.001	−0.010	10.518	1.995	4.492	0.245	0.188	−4.849

Figure 1 : The Firm-level Cost of Equity Distribution, QCE vs. ICE, January 1977–December 2024

This figure shows the histograms of firm-level costs of equity based on the samples in Table 7. We show the distributions of 1-, 12-, and 36-month ahead QCEs as well as the (12-month) ICE.

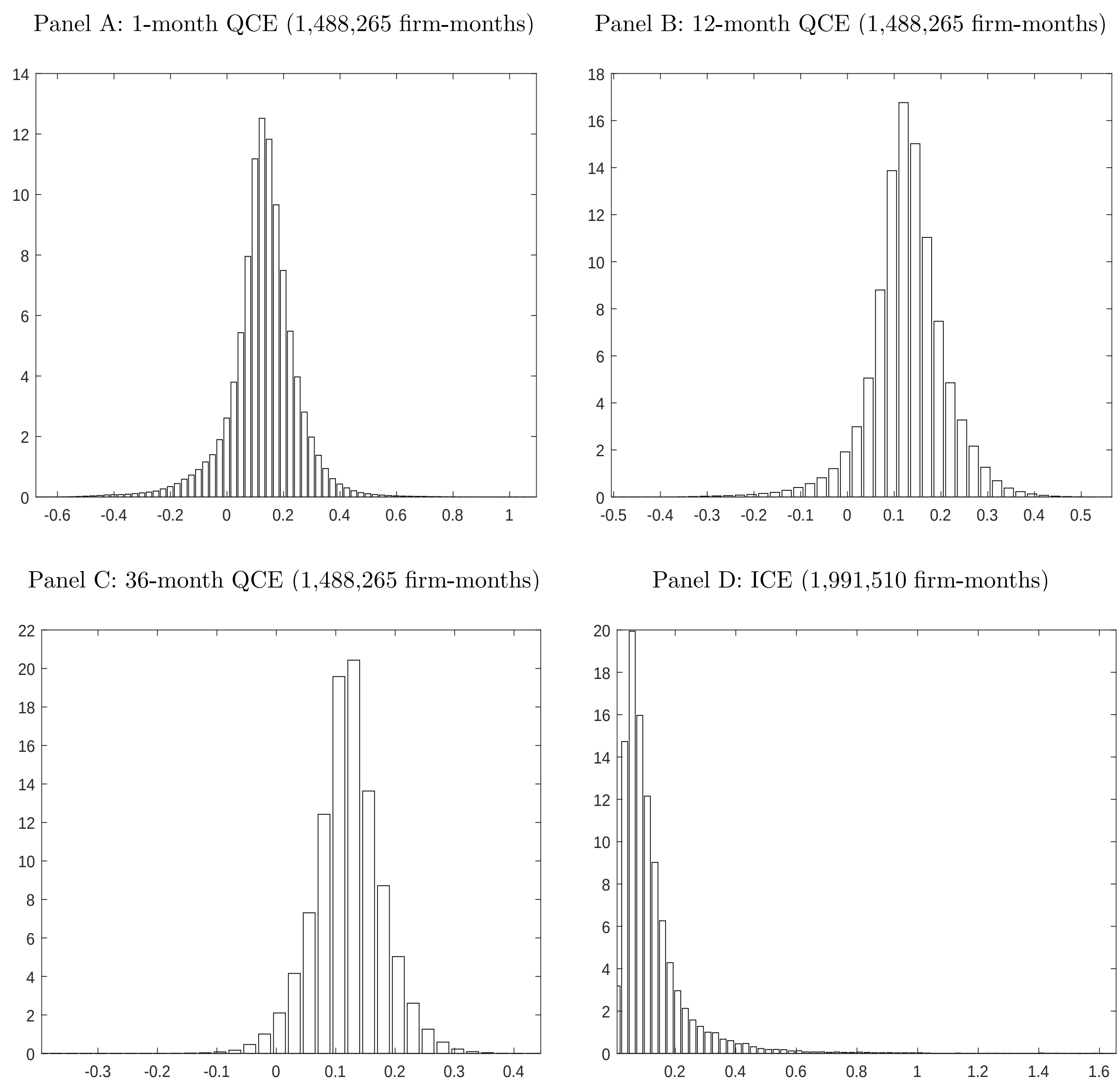


Figure 2 : The Firm-level Cost of Equity Distribution, QCE vs. ICE, the All-but-micro Sample, January 1977–December 2024

This figure shows the histograms of firm-level costs of equity based on the all-but-micro sample. We show the distributions of 1-, 12-, and 36-month ahead QCEs as well as the (12-month) ICE.

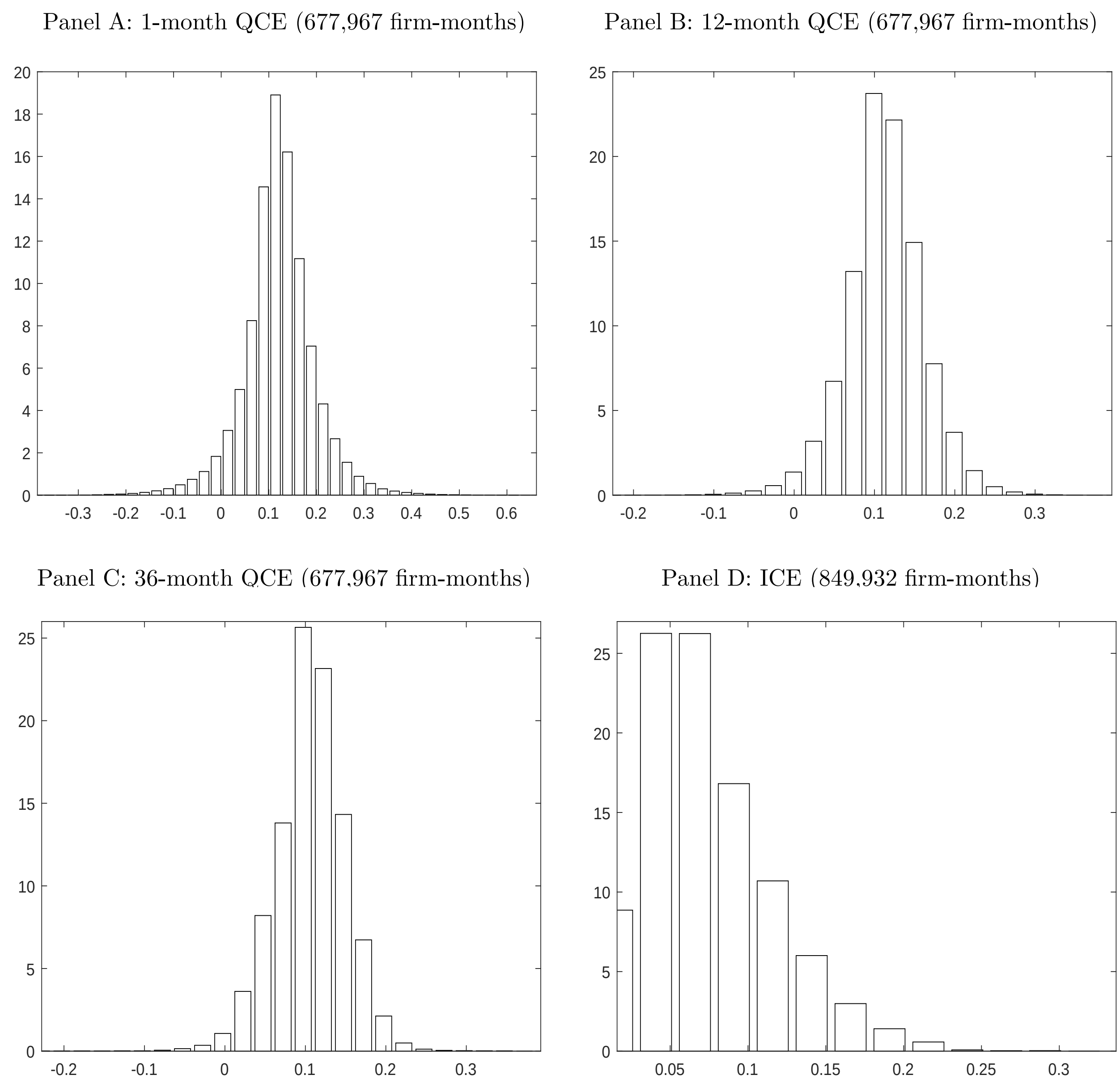


Figure 3 : The 120-month Rolling Window Slopes in Cross-sectional Regressions of 1-, 12-, and 36-month Subsequent Returns Underlying the QCE Estimates, January 1977–December 2024

We show the time series plots of 120-month rolling window slopes in cross-sectional regressions of 1-, 12-, and 36-month subsequent returns on the latest available q^5-characteristics, including the logarithm of size (logME), investment-to-assets (I/A), return on equity (ROE), and expected growth (Eg). In each panel, the solid blue line is for the 1-month slope times 12, the red broken line is for the 12-month slope, and the black dotted line the 36-month slope divided by three.

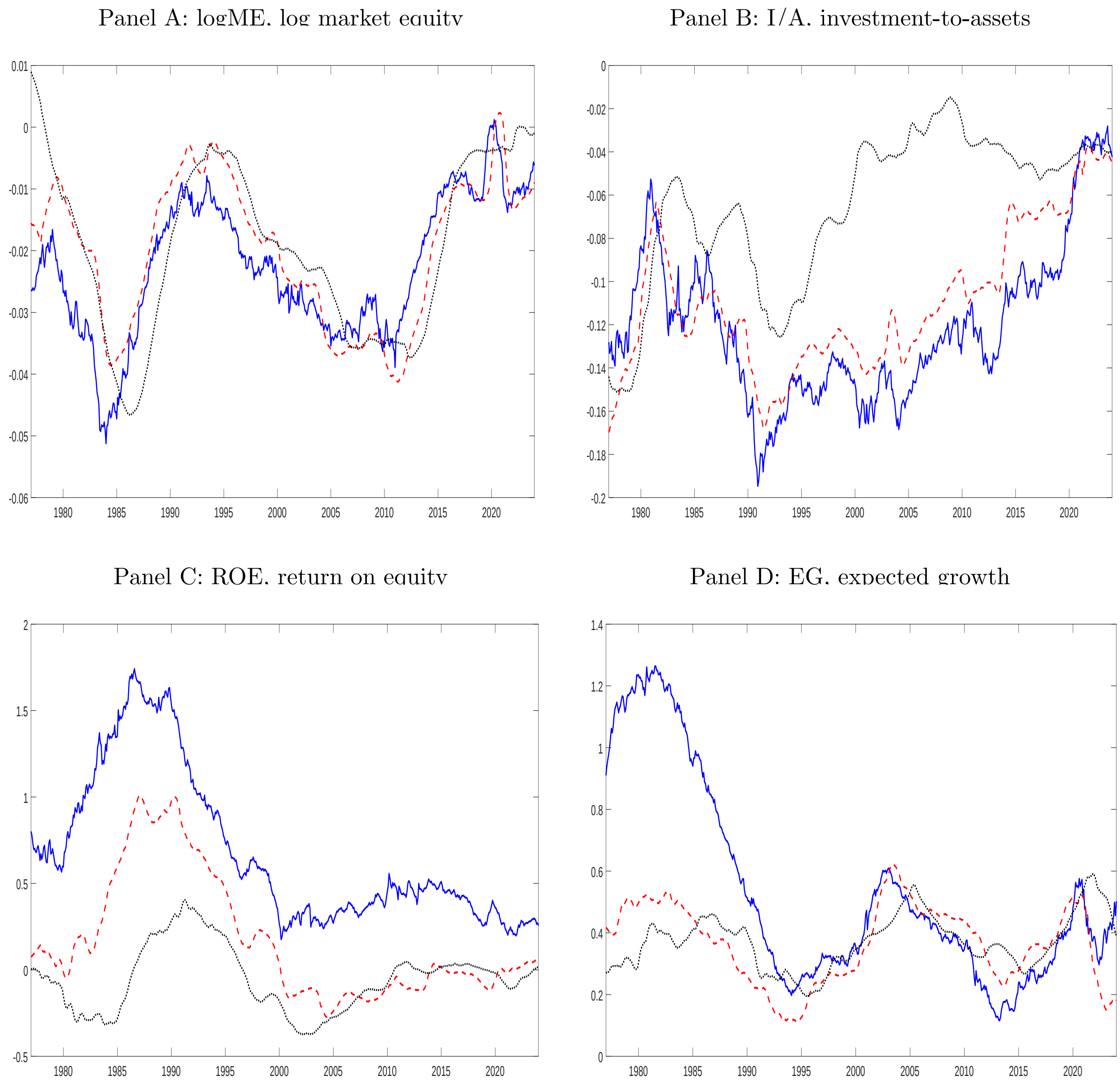


Figure 4 : The 120-month Rolling Window Slopes in Cross-sectional Regressions of 1-, 12-, and 36-month Subsequent Returns Underlying the QCE Estimates, the All-but-micro Sample, January 1977–December 2024

We exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. We plot the time series of 120-month rolling window slopes in cross-sectional regressions of 1-, 12-, and 36-month subsequent returns on the latest available q^5-characteristics, including the logarithm of size (logME), investment-to-assets (I/A), return on equity (ROE), and expected growth (Eg). In each panel, the solid blue line is for the 1-month slope times 12, the red broken line is for the 12-month slope, and the black dotted line the 36-month slope divided by three.

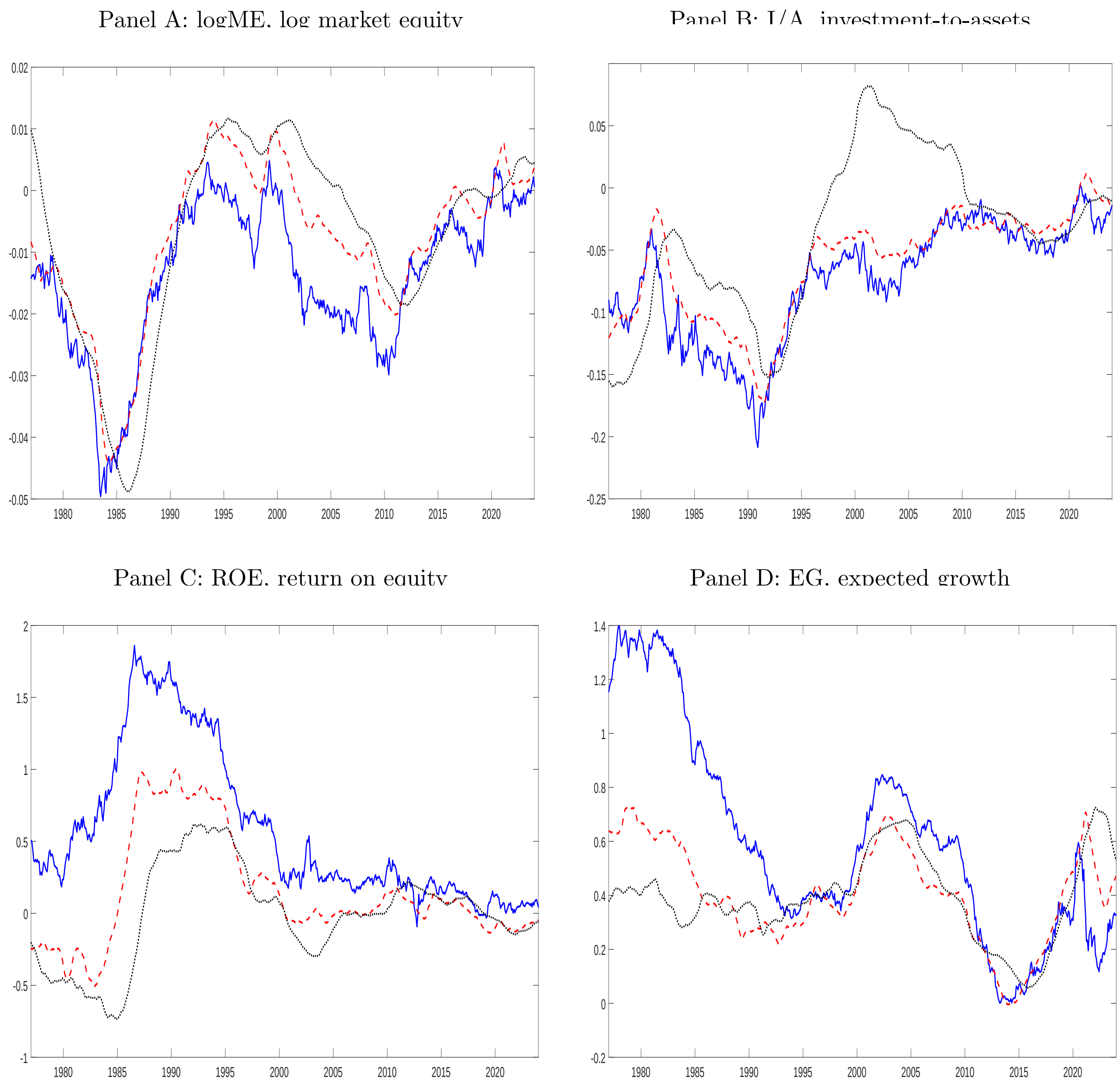
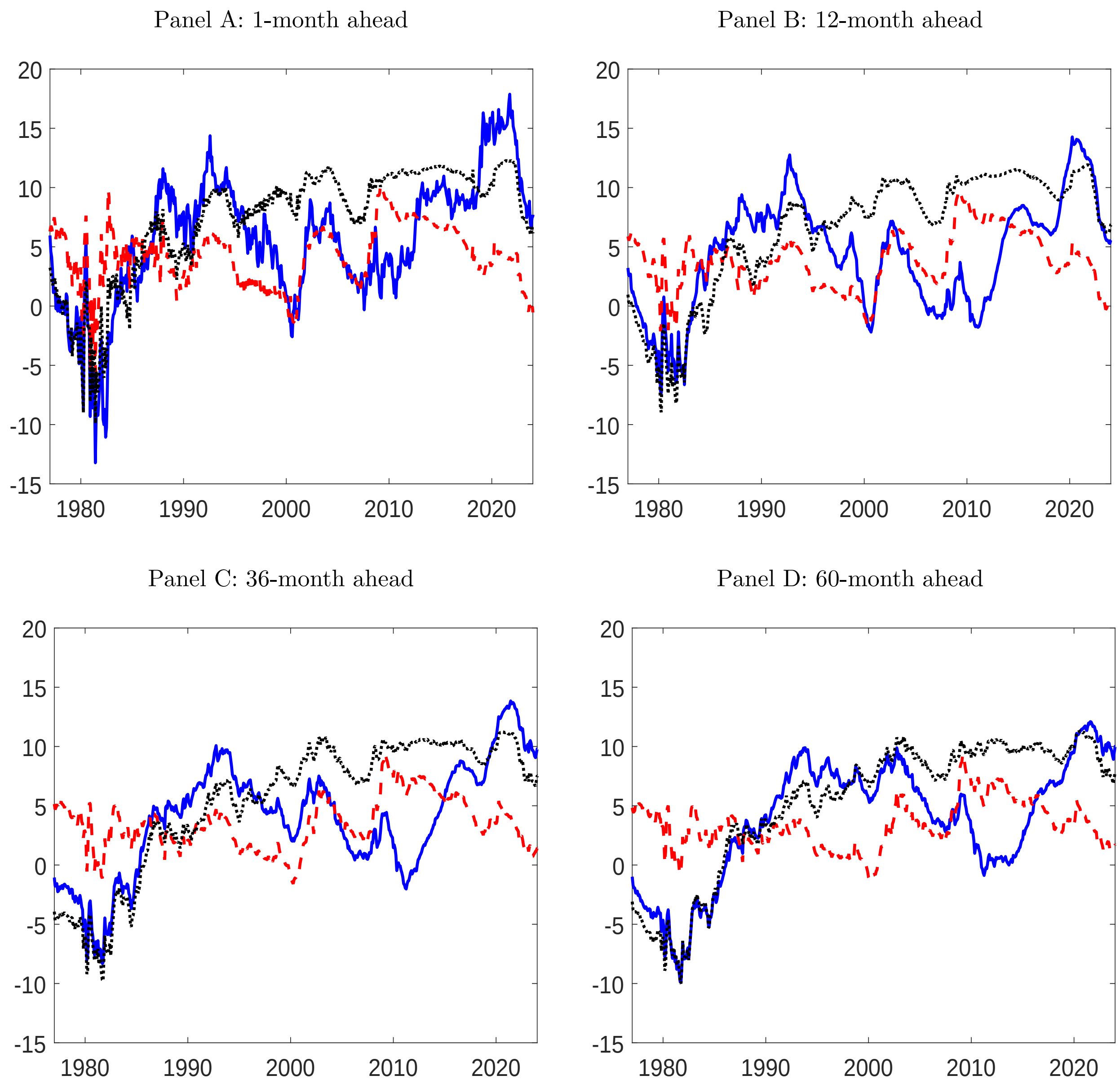


Figure 5 : The Equity Premium, January 1977–December 2024

In each panel, the time series in blue solid line is the QCE equity premium, the series in red broken link is the ICE equity premium, and the series in black dotted line is the historical average realized returns, denoted EAR, from an expanding window that starts in January 1967. All series are annualized. For 1-month, EAR is the average of the expanding window series of the average monthly returns. For 12-month, EAR is the average of the expanding window series of the average prior 12-month returns (with overlapping monthly observations).



The Internet Appendix for "Investment-based Costs of Equity" (for Online Publication Only)

Appendix A1 furnishes supplementary results from five additional costs of equity, including QCE-GBA (the q^5-characteristics model with gradient boosting using mean absolute errors as the loss function); QCE-GBH (the q^5-characteristics model via gradient boosting using the Huber loss function); LCE-3 (the Lewellen 3-variable model); LCE-15 (the Lewellen 15-variable model); and FF6F (the Fama-French 6-factor model). We also report portfolio sorts with equal-weighted returns. Appendix A2 details the structural investment model and shows its results.

A1 Supplementary Results

Table A1 shows the results from portfolio sorts for the additional costs of equity in the full sample, and Table A2 does the same for the all-but-micro sample. Table A3 shows cross-sectional predictive regressions for the additional measures in the full sample, and Table A4 does the same for the all-but-micro sample. Table A5 shows portfolio sorts with equal-weighted returns in the full sample, and Table A6 does the same for the all-but-micro sample. Finally, Table A7 shows measurement error variances in the full sample, and Table A8 does the same for the all-but-micro sample.

A2 The Structural Investment Model

We adopt the Goncalves-Xue-Zhang (2020) 2-capital model:

$$r_{it+1} = \frac{w_{it}^K r_{it+1}^K + (1 - w_{it}^K) r_{it+1}^W - w_{it}^B r_{it+1}^{Ba}}{1 - w_{it}^B}, \tag{A1}$$

in which r_{it+1} is stock i's return in month t; $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$ after-tax cost of debt; r_{it+1}^B pre-tax cost of debt; τ_{t+1} corporate tax rate; $w_{it}^K \equiv q_{it}K_{it+1}/(q_{it}K_{it+1} + W_{it+1})$ the weight of the market equity due to physical capital; K_{it+1} physical capital; W_{it+1} working capital; $q_{it} = 1 + (1 - \tau_t)a(I_{it}/K_{it})$ the marginal q of physical capital with $a > 0$; I_{it} physical capital investment; $w_{it}^B \equiv B_{it+1}/(P_{it} + B_{it+1})$ the market leverage; B_{it+1} debt; and P_{it} the market equity.

The physical investment return is denoted r_{it+1}^K:

$$r_{it+1}^K \equiv \frac{(1 - \tau_{t+1})\left[\gamma_K \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2}\left(\frac{I_{it+1}}{K_{it+1}}\right)^2\right] + \tau_{t+1}\delta_{it+1} + (1 - \delta_{it+1})\left[1 + (1 - \tau_{t+1})a\left(\frac{I_{it+1}}{K_{it+1}}\right)\right]}{1 + (1 - \tau_t)a\left(\frac{I_{it}}{K_{it}}\right)}; \tag{A2}$$

r_{it+1}^W is the working capital investment return:

$$r_{it+1}^W \equiv 1 + (1 - \tau_{t+1})\gamma_W \frac{Y_{it+1}}{W_{it+1}}; \tag{A3}$$

Y_{it+1} sales; δ_{it+1} physical capital depreciation rate; and $\gamma_K, \gamma_W > 0$ are technological parameters.

To form costs of capital from equation (A1), we need to form several auxiliary expectations. Because the after-tax cost of debt and sales are relatively persistent in the data, to reduce estimation errors, we set $E_t[r^{Ba}_{it+1}]$ and $E_t[Y_{it+1}]$ to their respective values from the most recent fiscal year ending at least four months ago. Analogously, we set the future tax rate, τ_{t+1}, to its most recent value. The physical investment rate is less persistent. Following Goncalves, Xue, and Zhang (2020), we forecast I_{it+1}/K_{it+1} on the log of Tobin's q_{it}, sales-to-total capital, $Y_{it}/(K_{it}+W_{it})$, and lagged physical investment rate, I_{it}/K_{it}. Tobin's q is the market equity from CRSP plus total debt (item DLTT plus item DLC, zero if missing) scaled by book assets (item AT).

At the beginning of each month t, we use the prior 120-month rolling window to estimate cross-sectional forecasting regressions. We winsorize I_{it+1}/K_{it+1}, the log of Tobin's q_{it}, and I_{it}/K_{it} at the 2.5 and 97.5 percentiles of their respective cross-sectional distributions each month. We winsorize $Y_{it}/(K_{it}+W_{it})$ (bounded below at zero) at the 0 and 97.5 percentiles each month. The I_{it+1} data are from the most recent fiscal year ending at least four months ago as of month t, and the predictors are further lagged accordingly. Because of time-to-build in the model, although dated $t+1$, K_{it+1} and W_{it+1} are one-year-lagged relative to I_{it+1}. We then combine the average slopes with the latest known predictors from at least four months ago to calculate $E_t[I_{it+1}/K_{it+1}]$.

With the auxiliary expectations in hand, plugging them into the right-hand side of equation (A1) yields the cost-of-equity function, denoted $er(\mathcal{I}_{it};\gamma,a)$, in which $\mathcal{I}_{it}$ is the conditioning information as of month t, and $\gamma=\gamma_K+\gamma_W$ and a are two structural parameters.[10] Equation (A1) becomes:

$$r_{it+1} = er(\mathcal{I}_{it};\gamma,a)+\epsilon_{it+1}, \tag{A4}$$

in which $E_t[\epsilon_{it+1}]=0$. At the beginning of month t, we align firm i's annual stock returns from t to $t+11$, which corresponds to the left-hand side of equation (A4). In the right-hand side, $er(\mathcal{I}_{it};\gamma,a)$ is based on the latest available $\mathcal{I}_{it}$ (lagged by at least four months prior to month t).

We estimate γ and a from equation (A4) via nonlinear least squares (nonlinear cross-sectional regressions). At the beginning of each month from January 1977 to December 2024, we recursively estimate γ and a using the expanding window that starts in January 1967. The starting point means that the conditioning information, $\mathcal{I}_{it}$, for January 1967 can come from as early as July 1965. However, the endpoint of the first recursive window, January 1977, means that the annual window of stock returns runs up to December 1976 and the $\mathcal{I}_{it}$ in the forecasting regression is further lagged accordingly. We expand the recursive window one month at a time until December 2024 to yield the time series of structural parameters. We obtain the cost of equity for each stock i at the beginning of month t, $er(\mathcal{I}_{it};\widehat{\gamma}_t,\widehat{a}_t)$, by combining the latest recursive parameters with the latest $\mathcal{I}_{it}$ (from the fiscal year ending at least four months prior to month t).

The benchmark estimation above is for 12-month forecasts, which match the annual accounting variables contained in $\mathcal{I}_{it}$. We adapt the procedure for other forecasting horizons. For 1-month forecasts, the returns in the left-hand side of equation (A4) are 1-month ahead, but the cost-of-equity function in the right-hand side becomes $er(\mathcal{I}_{it};\gamma,a)^{1/12}$. Analogously, for 60-month forecasts, the returns in the left-hand side are 60-month ahead (with overlapping monthly observations), but the right-hand side becomes $er(\mathcal{I}_{it};\gamma,a)^5$. As such, we allow γ and a to vary with horizons. While often

[10] In equation (A1), γ_K and γ_W enter only in the form of $\gamma_K+\gamma_W$. From equations (A2) and (A3):

$$\begin{aligned} w^K_{it}r^K_{it+1}+(1-w^K_{it})r^W_{it+1} &= \frac{(1-\tau_{t+1})(\gamma_K+\gamma_W)Y_{it+1}/(K_{it+1}+W_{it+1})}{q_{it}K_{it+1}/(K_{it+1}+W_{it+1})+W_{it+1}/(K_{it+1}+W_{it+1})}+ \\ & \quad w^K_{it}\frac{(1-\tau_{t+1})(a/2)\left(I_{it+1}/K_{it+1}\right)^2+\tau_{t+1}\delta_{it+1}+(1-\delta_{it+1})q_{it+1}}{q_{it}}+(1-w^K_{it}). \end{aligned}$$

As such, the structural model contains only two parameters, γ and a (Goncalves, Xue, and Zhang 2020).

labeled “structural” parameters, their estimates do vary in practice because the data generating process is vastly more complex than our economic model, which is inevitably misspecified.

Data are from Center for Research in Security Prices (CRSP) monthly stock files and annual Compustat industrial files. We exclude firms with gross property, plant, and equipment or sales either zero or negative. Both are necessary to calculate the right-hand side of equation (A1).

In the model, time-t stock variables are at the beginning of period t, and time-t flow variables are over the course of period t. In Compustat both stock and flow variables are recorded at the end of period t. As such, for the year 2010, for example, we take time-t stock variables from the 2009 balance sheet, and time-t flow variables from the 2010 income or cash flow statement.

Output, Y_{it}, is sales (item SALE), working capital is current assets (item ACT). Debt, B_{it+1}, is long-term debt (item DLTT) plus short-term debt (item DLC). Missing values are set to zero. The market leverage, w_{it}^B, is the ratio of debt to the sum of debt and market equity. The tax rate, τ_t, is the statutory corporate income tax rate from the Commerce Clearing House’s annual publications. The pre-tax cost of debt, r_{it+1}^B, is total interest and related expenses (item XINT) scaled by debt.

We measure investment as change in net property, plant, and equipment (item PPENT) plus depreciation and amortization (item DP) minus amortization (item AM, zero if missing). However, we measure physical capital, K_{it}, as gross property, plant, and equipment (item PPEGT) and set the depreciate rate of physical capital, δ_{it}, to zero. Ideally, in a perfect world, we ought to use the replacement cost of capital as K_{it} and the economic depreciation rate as δ_{it}. Integrating national accounting in Bureau of Economic Analysis (BEA) with financial accounting in Compustat, Bai et al. (2024) construct firm-specific replacement costs of capital and economic depreciation rates for the entire Compustat universe. However, the Bai et al. data cannot be used to forecast returns. The capital and investment price deflators and industry-specific economic depreciation rates are from the BEA data for calendar years, but the fiscal years of firms in Compustat do not always end in December. To deal with this misalignment, Bai et al. use linear interpolation to impute the price deflators and depreciation rates for all possible fiscal year ending months.

More important, for our purpose, Bai et al. (2024) show that gross PPE is much closer to the replacement cost of capital than net PPE. The ratio of the replacement cost to gross PPE is on average 0.98, but the ratio to net PPE is 2.11. Intuitively, gross PPE ignores both depreciation and price inflation. Ignoring the former induces an upward bias, but ignoring the latter induces a downward bias. The two largely offset each other on average. In contrast, net PPE uses aggressive (accounting) depreciation and ignores price inflation. Both induce downward biases. As such, although both are free of look-ahead bias, gross PPE is a better proxy for the replacement cost than net PPE.

Table A1 : Portfolio Sorts, Additional Costs of Equity, January 1977–December 2024

In Panel A, at the beginning of month t, we split the NYSE, Amex, and NASDAQ stocks on each cost of equity into deciles and calculate the monthly value-weighted returns for different holding periods, h (in months). For instance, the 3-month horizon means that for a given decile in each month, there exist 3 subdeciles, each initiated in a different month in the prior three months. We take the simple average of the subdecile returns as the monthly return for the decile. We align the holding period with the forecast horizon in costs of equity, e.g., we use the 12-month holding period when forecasting returns with 12-month ahead costs of equity. We use NYSE-Amex-NASDAQ breakpoints and value-weighted returns. Each decile return is in excess of the 1-month Treasury bill rate. Beneath each row of average return estimates, we show the corresponding Newey-West t-statistics. Panel B shows pairwise comparison tests of QCE (the q^5-characteristics model via cross-sectional regressions) versus alternative models. For each pairwise comparison, we test whether the difference between the two high-minus-low (H−L) decile returns is on average zero.

Panel A: Portfolio sorts

h	1	2	3	4	5	6	7	8	9	10	H−L
	QCE-GBA (the q^5 characteristics model with gradient boosting, mean absolute errors)										
1	−0.60	0.24	−0.01	0.23	0.46	0.59	0.60	0.63	0.83	0.98	1.59
	−1.39	0.62	−0.02	0.76	1.80	2.67	2.98	3.16	4.55	4.91	4.71
3	−0.54	0.06	0.09	0.46	0.45	0.71	0.60	0.70	0.90	1.06	1.60
	−1.30	0.15	0.28	1.61	1.75	3.22	2.99	3.67	4.92	5.08	5.05
12	0.01	0.37	0.52	0.54	0.68	0.69	0.70	0.75	0.81	0.96	0.94
	0.03	1.17	1.84	2.05	2.85	3.21	3.42	3.90	4.22	4.91	3.45
24	0.32	0.58	0.54	0.68	0.70	0.69	0.72	0.78	0.80	0.88	0.55
	0.95	2.08	1.99	2.73	3.12	3.14	3.47	3.95	4.20	4.57	2.37
36	0.46	0.68	0.70	0.72	0.74	0.81	0.76	0.76	0.81	0.79	0.33
	1.40	2.55	2.66	3.02	3.15	3.71	3.69	3.73	4.08	4.19	1.44
60	0.60	0.79	0.82	0.81	0.81	0.82	0.85	0.84	0.80	0.74	0.14
	2.00	2.95	3.27	3.40	3.44	3.67	3.87	3.94	3.87	3.96	0.68
	QCE-GBH (the q^5 characteristics model with gradient boosting, the Huber loss)										
1	−0.41	0.18	0.44	0.56	0.58	0.61	0.90	0.92	1.14	1.48	1.89
	−1.01	0.59	1.73	2.52	2.82	3.10	4.96	4.86	5.33	6.01	6.02
3	−0.44	0.22	0.34	0.48	0.68	0.59	0.86	0.95	1.11	1.21	1.65
	−1.11	0.69	1.29	2.07	3.26	2.96	4.36	4.93	5.48	5.14	5.70
12	0.02	0.37	0.45	0.69	0.62	0.71	0.73	0.82	0.94	1.04	1.02
	0.05	1.18	1.68	2.76	2.83	3.38	3.69	4.35	4.88	5.16	3.73
24	0.35	0.53	0.54	0.70	0.69	0.65	0.76	0.82	0.82	0.90	0.55
	1.08	1.90	2.09	2.83	3.16	3.08	3.65	4.15	4.28	4.57	2.53
36	0.50	0.69	0.68	0.70	0.78	0.76	0.77	0.78	0.82	0.79	0.29
	1.58	2.54	2.57	3.01	3.43	3.52	3.72	3.81	4.10	4.14	1.36
60	0.58	0.77	0.82	0.81	0.81	0.84	0.87	0.84	0.80	0.75	0.17
	1.96	2.87	3.26	3.36	3.45	3.77	3.93	3.95	3.83	4.00	0.84

h	1	2	3	4	5	6	7	8	9	10	H−L
					LCE-3 (the Lewellen 3-variable model)						
1	0.76	0.82	0.85	0.81	0.86	0.94	0.97	1.11	1.20	1.25	0.49
	3.01	3.81	4.14	3.92	4.30	4.31	4.25	4.19	3.85	3.37	1.46
3	0.76	0.81	0.90	0.80	0.83	0.90	0.97	0.97	1.03	1.15	0.39
	3.04	3.72	4.39	3.81	4.07	4.30	4.33	3.91	3.44	3.16	1.16
12	0.81	0.88	0.86	0.82	0.81	0.83	0.84	0.82	0.92	0.85	0.04
	3.77	4.49	4.28	4.02	3.87	3.79	3.54	3.16	3.09	2.41	0.15
24	0.75	0.73	0.79	0.77	0.76	0.79	0.85	0.92	0.96	1.00	0.25
	3.76	3.98	4.19	4.01	3.76	3.71	3.76	3.66	3.36	2.98	0.93
36	0.72	0.70	0.76	0.77	0.75	0.76	0.78	0.81	0.83	0.81	0.08
	3.48	3.61	3.92	3.82	3.62	3.58	3.53	3.35	3.11	2.67	0.35
60	0.89	0.85	0.81	0.76	0.75	0.75	0.71	0.72	0.75	0.72	−0.17
	4.13	4.40	4.19	3.81	3.61	3.59	3.30	3.13	3.00	2.60	−0.75
					LCE-15 (the Lewellen 15-variable model)						
1	0.49	0.70	0.75	0.74	0.79	0.75	0.93	1.11	1.26	1.39	0.90
	1.61	2.87	3.74	3.90	4.17	3.62	4.10	4.36	4.34	4.33	2.76
3	0.52	0.63	0.70	0.70	0.67	0.80	0.84	1.02	1.15	1.33	0.81
	1.57	2.81	3.74	3.77	3.57	3.94	3.76	4.21	4.05	4.08	2.41
12	0.66	0.74	0.70	0.72	0.80	0.86	0.91	0.89	0.97	1.05	0.39
	2.31	3.69	3.83	3.81	4.07	4.12	3.97	3.56	3.45	2.96	1.25
24	0.66	0.74	0.75	0.73	0.80	0.77	0.77	0.87	0.93	1.00	0.34
	2.51	3.74	4.12	3.96	4.22	3.81	3.54	3.63	3.45	3.08	1.27
36	0.71	0.77	0.76	0.75	0.75	0.74	0.74	0.78	0.82	0.88	0.17
	2.78	3.71	4.07	4.04	3.95	3.74	3.47	3.47	3.29	3.11	0.71
60	0.80	0.80	0.78	0.78	0.78	0.76	0.78	0.74	0.75	0.74	−0.06
	3.27	3.85	3.98	4.02	4.08	3.89	3.95	3.61	3.43	3.08	−0.25
					FF6F (the Fama-French 6-factor model)						
1	0.86	0.84	0.76	0.76	0.75	0.71	0.72	0.76	0.67	0.52	−0.34
	2.86	3.63	3.76	4.08	4.21	3.85	3.81	3.70	2.77	1.58	−1.30
3	0.87	0.84	0.74	0.80	0.71	0.72	0.69	0.75	0.70	0.56	−0.31
	2.91	3.65	3.64	4.37	4.04	4.02	3.75	3.64	2.86	1.74	−1.22
12	0.94	0.78	0.70	0.81	0.68	0.75	0.70	0.75	0.75	0.63	−0.31
	3.12	3.39	3.46	4.59	3.90	4.28	3.83	3.70	3.11	2.04	−1.28
24	0.95	0.76	0.71	0.76	0.69	0.73	0.67	0.76	0.79	0.71	−0.24
	3.25	3.34	3.58	4.43	4.08	4.23	3.79	3.84	3.34	2.39	−1.12
36	0.97	0.75	0.70	0.74	0.70	0.75	0.69	0.76	0.80	0.70	−0.27
	3.39	3.38	3.66	4.32	4.14	4.37	3.95	3.81	3.48	2.38	−1.36
60	1.01	0.80	0.71	0.75	0.69	0.71	0.69	0.73	0.75	0.67	−0.34
	3.70	3.75	3.85	4.45	4.15	4.14	3.92	3.67	3.30	2.33	−1.97

Panel B: Pairwise comparison tests with QCE						
	h	Diff	t	h	Diff	t
QCE-GBA	1	0.04	0.09	24	0.02	0.06
QCE-GBH	1	−0.27	−0.97	24	0.03	0.08
LCE-3	1	1.13	3.51	24	0.33	1.70
LCE-15	1	0.72	2.40	24	0.23	1.16
FF6F	1	1.96	6.02	24	0.81	3.04
QCE-GBA	3	−0.23	−0.68	36	−0.07	−0.21
QCE-GBH	3	−0.28	−1.04	36	−0.03	−0.10
LCE-3	3	0.99	2.85	36	0.18	0.90
LCE-15	3	0.56	1.71	36	0.09	0.50
FF6F	3	1.68	5.55	36	0.53	2.07
QCE-GBA	12	−0.13	−0.36	60	−0.13	−0.45
QCE-GBH	12	−0.21	−0.58	60	−0.16	−0.57
LCE-3	12	0.77	3.65	60	0.18	0.95
LCE-15	12	0.42	1.65	60	0.07	0.31
FF6F	12	1.12	3.82	60	0.35	1.55

Table A2 : Portfolio Sorts, Additional Costs of Equity, the All-but-micro Sample, January 1977–December 2024

We exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. In Panel A, at the beginning of month t, we split all stocks on each cost of equity into deciles and calculate the monthly value-weighted returns for different holding periods, h (in months). For instance, the 3-month horizon means that for a given decile in each month, there exist 3 subdeciles, each initiated in a different month in the prior three months. We take the simple average of the subdecile returns as the monthly return for the decile. We align the holding period with the forecast horizon in costs of equity, e.g., we use the 12-month holding period when forecasting returns with 12-month ahead costs of equity. We use NYSE-Amex-NASDAQ breakpoints and value-weighted returns. Each decile return is in excess of the 1-month Treasury bill rate. Beneath each row of average return estimates, we show the corresponding Newey-West t-statistics. Panel B shows pairwise comparison tests of QCE (the q^5-characteristics model via cross-sectional regressions) versus alternative models. For each pairwise comparison, we test whether the difference between the two high-minus-low (H−L) decile returns is on average zero.

Panel A: Portfolio sorts

h	1	2	3	4	5	6	7	8	9	10	H−L
	QCE-GBA (the q^5 characteristics model with gradient boosting, mean absolute errors)										
1	0.10	0.26	0.42	0.56	0.69	0.65	0.75	0.88	0.83	1.07	0.97
	0.30	0.96	1.90	2.67	3.58	3.41	4.03	4.58	4.29	4.89	4.29
3	0.06	0.32	0.50	0.69	0.64	0.65	0.73	0.81	0.96	1.09	1.02
	0.20	1.19	2.17	3.32	3.29	3.64	4.07	4.35	4.90	4.93	4.65
12	0.41	0.56	0.61	0.61	0.66	0.74	0.77	0.76	0.81	0.93	0.52
	1.37	2.13	2.58	2.87	3.33	3.88	4.23	4.04	4.32	4.58	2.88
24	0.57	0.57	0.75	0.65	0.77	0.70	0.70	0.73	0.83	0.84	0.27
	2.03	2.29	3.20	3.06	3.79	3.61	3.68	4.05	4.38	4.36	1.65
36	0.61	0.72	0.74	0.74	0.76	0.80	0.75	0.73	0.75	0.81	0.20
	2.18	2.90	3.20	3.58	3.78	4.05	3.95	3.90	4.03	4.28	1.20
60	0.79	0.80	0.86	0.81	0.81	0.81	0.82	0.76	0.74	0.74	−0.05
	2.87	3.30	3.80	3.78	3.92	3.91	4.14	3.86	3.89	3.98	−0.30
	QCE-GBH (the q^5 characteristics model with gradient boosting, the Huber loss)										
1	0.02	0.35	0.47	0.63	0.72	0.61	0.90	0.96	1.11	1.18	1.16
	0.07	1.54	2.24	3.15	3.64	3.26	4.80	4.73	5.52	5.07	6.04
3	0.07	0.33	0.52	0.60	0.66	0.78	0.83	0.97	1.02	1.20	1.13
	0.23	1.39	2.47	3.21	3.51	4.28	4.42	4.92	5.06	5.22	5.86
12	0.44	0.52	0.57	0.56	0.67	0.74	0.81	0.79	0.84	0.98	0.55
	1.47	2.04	2.37	2.63	3.51	3.97	4.34	4.35	4.42	4.75	2.90
24	0.54	0.58	0.71	0.61	0.76	0.73	0.74	0.75	0.82	0.88	0.33
	1.95	2.35	3.06	2.90	3.82	3.81	3.95	3.98	4.28	4.46	2.01
36	0.63	0.73	0.71	0.75	0.78	0.80	0.74	0.74	0.76	0.80	0.18
	2.25	2.99	3.12	3.65	3.89	4.03	3.84	3.98	4.03	4.21	1.06
60	0.79	0.80	0.85	0.79	0.84	0.80	0.79	0.77	0.74	0.73	−0.05
	2.87	3.34	3.70	3.70	4.06	3.96	3.96	3.94	3.93	3.97	−0.33

h	1	2	3	4	5	6	7	8	9	10	H−L
					LCE-3 (the Lewellen 3-variable model)						
1	0.37	0.49	0.56	0.55	0.70	0.71	0.76	0.88	0.85	1.12	0.75
	1.62	2.44	2.89	2.91	3.69	3.59	3.75	4.24	3.57	3.63	2.74
3	0.48	0.52	0.55	0.65	0.65	0.69	0.77	0.86	0.93	0.99	0.52
	2.09	2.70	2.92	3.57	3.47	3.76	4.09	4.27	3.86	3.24	1.98
12	0.66	0.66	0.67	0.69	0.76	0.78	0.76	0.74	0.68	0.86	0.20
	2.81	3.36	3.69	3.93	4.34	4.35	4.14	3.86	3.19	3.17	0.93
24	0.76	0.76	0.81	0.73	0.74	0.77	0.73	0.70	0.76	0.82	0.06
	3.26	3.81	4.17	4.07	4.23	4.39	4.07	3.88	3.97	3.55	0.36
36	0.83	0.80	0.76	0.72	0.74	0.75	0.78	0.74	0.75	0.77	−0.06
	3.50	3.88	3.83	3.76	3.94	4.07	4.28	4.09	4.04	3.75	−0.35
60	0.94	0.87	0.83	0.80	0.79	0.77	0.75	0.71	0.70	0.63	−0.31
	3.84	4.20	4.24	4.25	4.27	4.20	4.17	4.00	3.90	3.21	−2.18
					LCE-15 (the Lewellen 15-variable model)						
1	0.43	0.57	0.56	0.60	0.71	0.60	0.74	0.90	0.99	1.20	0.77
	1.65	2.82	2.93	3.16	3.90	3.02	3.68	4.27	4.30	4.45	2.86
3	0.49	0.57	0.60	0.56	0.67	0.72	0.73	0.83	0.91	1.11	0.62
	1.96	2.96	3.39	3.07	3.70	3.82	3.70	3.93	3.84	4.01	2.39
12	0.62	0.69	0.68	0.67	0.69	0.72	0.74	0.67	0.79	0.87	0.26
	2.69	3.60	3.83	3.82	3.90	3.90	3.81	3.06	3.21	3.07	1.11
24	0.65	0.68	0.68	0.71	0.72	0.74	0.73	0.76	0.76	0.80	0.15
	3.05	3.59	3.81	3.96	3.95	3.95	3.77	3.63	3.35	2.91	0.69
36	0.69	0.73	0.66	0.72	0.72	0.74	0.76	0.78	0.73	0.82	0.12
	3.15	3.82	3.69	4.09	3.96	4.09	4.13	4.06	3.44	3.19	0.60
60	0.79	0.82	0.75	0.75	0.73	0.69	0.70	0.72	0.70	0.73	−0.06
	3.47	4.25	4.17	4.24	3.98	3.87	3.87	3.82	3.48	3.22	−0.31
					FF6F (the Fama-French 6-factor model)						
1	0.83	0.83	0.73	0.65	0.68	0.76	0.80	0.70	0.70	0.63	−0.20
	3.12	4.08	3.95	3.66	3.89	4.21	4.30	3.63	3.20	2.21	−0.92
3	0.86	0.84	0.77	0.63	0.68	0.74	0.75	0.74	0.70	0.64	−0.22
	3.27	4.13	4.19	3.65	3.94	4.19	4.14	3.88	3.13	2.26	−1.03
12	0.86	0.73	0.77	0.65	0.68	0.74	0.73	0.73	0.69	0.72	−0.14
	3.31	3.60	4.24	3.92	3.99	4.27	4.11	3.91	3.18	2.64	−0.73
24	0.87	0.69	0.75	0.68	0.67	0.73	0.69	0.73	0.73	0.78	−0.09
	3.41	3.45	4.15	4.15	4.02	4.29	3.97	3.93	3.47	2.99	−0.50
36	0.88	0.68	0.74	0.69	0.69	0.71	0.69	0.75	0.76	0.78	−0.10
	3.53	3.46	4.21	4.23	4.14	4.26	3.96	4.05	3.64	3.02	−0.64
60	0.92	0.71	0.76	0.69	0.68	0.69	0.67	0.72	0.73	0.71	−0.21
	3.88	3.79	4.43	4.35	4.10	4.12	3.80	3.87	3.54	2.82	−1.51

	Panel B: Pairwise comparison tests with QCE					
	h	Diff	t	h	Diff	t
QCE-GBA	1	0.14	0.88	24	0.19	1.42
QCE-GBH	1	−0.05	−0.39	24	0.13	1.07
LCE-3	1	0.36	1.12	24	0.40	2.37
LCE-15	1	0.34	1.12	24	0.32	1.36
FF6F	1	1.31	4.52	24	0.56	2.47
QCE-GBA	3	−0.14	−0.94	36	−0.02	−0.12
QCE-GBH	3	−0.25	−2.10	36	0.01	0.07
LCE-3	3	0.37	1.19	36	0.24	1.64
LCE-15	3	0.26	0.88	36	0.06	0.32
FF6F	3	1.11	3.87	36	0.29	1.24
QCE-GBA	12	0.12	0.93	60	0.10	0.78
QCE-GBH	12	0.10	0.77	60	0.10	0.83
LCE-3	12	0.45	1.78	60	0.36	2.76
LCE-15	12	0.39	1.39	60	0.11	0.52
FF6F	12	0.79	3.00	60	0.26	1.14

Table A3 : Cross-sectional Predictive Regressions, Additional Costs of Equity, January 1977–December 2024

Panel A shows cross-sectional regressions of subsequent h-month returns on a given h-month ahead cost of equity at the beginning of month t. s is the average slope, ste its Newey-West standard error, and $|t_{s=1}|$ the absolute t-value that tests the slope equals one. We examine five additional cost of equity measures, including QCE-GBA (the q^5-characteristics model with gradient boosting using mean absolute errors as the loss function); QCE-GBH (the q^5-characteristics model via gradient boosting using the Huber loss function); LCE-3 (the Lewellen 3-variable model); LCE-15 (the Lewellen 15-variable model); and FF6F (the Fama-French 6-factor model). Panel B shows pairwise comparison tests of QCE (the q^5-characteristics model via cross-sectional regressions) against these alternative models. For each pairwise comparison, we test whether the slope difference is on average zero. We use time series tests with Newey-West t-values.

Panel A: Cross-sectional predictive regressions

	h	s	ste	$\lvert t_{s=1}\rvert$	h	s	ste	$\lvert t_{s=1}\rvert$	h	s	ste	$\lvert t_{s=1}\rvert$
QCE-GBA	1	0.53	0.11	4.12	12	0.57	0.15	2.85	36	0.45	0.13	4.28
QCE-GBH	1	0.95	0.12	0.43	12	0.67	0.16	2.09	36	0.47	0.13	4.06
LCE-3	1	0.76	0.19	1.27	12	0.17	0.31	2.65	36	0.61	0.23	1.66
LCE-15	1	0.73	0.09	2.97	12	0.62	0.12	3.04	36	0.45	0.10	5.62
FF6F	1	0.01	0.04	23.16	12	0.01	0.05	21.43	36	−0.02	0.02	49.42
QCE-GBA	3	0.64	0.14	2.60	24	0.53	0.14	3.43	60	0.27	0.17	4.24
QCE-GBH	3	0.84	0.14	1.16	24	0.58	0.13	3.18	60	0.28	0.17	4.22
LCE-3	3	0.61	0.24	1.61	24	0.62	0.24	1.59	60	0.52	0.23	2.09
LCE-15	3	0.71	0.10	2.86	24	0.55	0.10	4.50	60	0.35	0.10	6.28
FF6F	3	0.01	0.04	22.11	24	−0.01	0.03	36.16	60	−0.03	0.01	75.78

Panel B: Pairwise comparison tests with QCE

	h	Diff	t	h	Diff	t	h	Diff	t
QCE-GBA	1	0.30	2.85	12	0.06	0.40	36	0.16	1.10
QCE-GBH	1	−0.12	−1.65	12	−0.04	−0.28	36	0.14	1.03
LCE-3	1	0.07	0.47	12	0.46	2.14	36	0.00	0.02
LCE-15	1	0.09	1.13	12	0.01	0.09	36	0.17	2.34
FF6F	1	0.81	7.39	12	0.62	3.93	36	0.63	4.90
QCE-GBA	3	0.17	1.36	24	0.28	1.76	60	0.27	1.25
QCE-GBH	3	−0.03	−0.30	24	0.23	1.55	60	0.26	1.25
LCE-3	3	0.20	1.18	24	0.20	0.90	60	0.02	0.17
LCE-15	3	0.09	0.97	24	0.26	3.18	60	0.19	1.72
FF6F	3	0.79	6.19	24	0.82	6.37	60	0.57	3.49

Table A4 : Cross-sectional Predictive Regressions, Additional Costs of Equity, the All-but-micro Sample, January 1977–December 2024

We exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. Panel A shows cross-sectional regressions of subsequent h-month returns on a given h-month ahead cost of equity at the beginning of month t. s is the average slope, ste its Newey-West standard error, and $|t_{s=1}|$ the absolute t-value that tests the slope equals one. We examine five additional cost of equity measures, including QCE-GBA (the q^5-characteristics model with gradient boosting using mean absolute errors as the loss function); QCE-GBH (the q^5-characteristics model via gradient boosting using the Huber loss function); LCE-3 (the Lewellen 3-variable model); LCE-15 (the Lewellen 15-variable model); and FF6F (the Fama-French 6-factor model). Panel B shows pairwise comparison tests of QCE (the q^5-characteristics model via cross-sectional regressions) against these alternative models. For each pairwise comparison, we test whether the slope difference is on average zero. We use time series tests with Newey-West t-values.

Panel A: Cross-sectional predictive regressions

	h	s	ste	$\lvert t_{s=1}\rvert$	h	s	ste	$\lvert t_{s=1}\rvert$	h	s	ste	$\lvert t_{s=1}\rvert$
QCE-GBA	1	0.67	0.17	1.99	12	0.51	0.17	2.92	36	0.30	0.11	6.34
QCE-GBH	1	0.81	0.18	1.08	12	0.54	0.16	2.83	36	0.29	0.11	6.41
LCE-3	1	0.58	0.25	1.68	12	0.28	0.21	3.38	36	−0.34	0.26	5.26
LCE-15	1	0.41	0.13	4.50	12	0.24	0.14	5.50	36	0.07	0.09	10.44
FF6F	1	0.01	0.06	16.88	12	0.01	0.05	19.57	36	−0.01	0.03	36.21
QCE-GBA	3	0.75	0.18	1.36	24	0.38	0.13	4.84	60	0.13	0.17	5.19
QCE-GBH	3	0.79	0.17	1.21	24	0.39	0.13	4.83	60	0.12	0.17	5.20
LCE-3	3	0.46	0.20	2.77	24	0.14	0.26	3.37	60	−0.46	0.21	6.85
LCE-15	3	0.37	0.14	4.53	24	0.09	0.12	7.32	60	0.04	0.09	10.77
FF6F	3	0.01	0.06	17.07	24	0.00	0.04	27.75	60	−0.04	0.02	47.48

Panel B: Pairwise comparison tests with QCE

	h	Diff	t	h	Diff	t	h	Diff	t
QCE-GBA	1	0.08	0.40	12	−0.03	−0.30	36	−0.08	−0.91
QCE-GBH	1	−0.07	−0.42	12	−0.06	−0.57	36	−0.07	−0.82
LCE-3	1	0.17	0.49	12	0.20	1.02	36	0.56	2.49
LCE-15	1	0.33	1.27	12	0.24	1.50	36	0.15	1.54
FF6F	1	0.73	2.77	12	0.47	2.45	36	0.23	1.96
QCE-GBA	3	−0.09	−0.55	24	0.03	0.26	60	−0.06	−0.65
QCE-GBH	3	−0.13	−1.08	24	0.01	0.16	60	−0.05	−0.56
LCE-3	3	0.20	0.74	24	0.27	1.06	60	0.53	3.78
LCE-15	3	0.29	1.39	24	0.31	2.23	60	0.04	0.32
FF6F	3	0.64	3.06	24	0.41	3.00	60	0.11	0.82

Table A5 : Portfolio Sorts, Equal-weighted Returns, January 1977–December 2024

In Panel A, at the beginning of month t, we split the NYSE, Amex, and NASDAQ stocks on each cost of equity into deciles and calculate the monthly equal-weighted returns for different holding periods, h (in months). For instance, the 3-month horizon means that for a given decile in each month, there exist 3 subdeciles, each initiated in a different month in the prior 3 months. We take the simple average of the subdecile returns as the monthly return for the decile. We align the holding period with the forecast horizon in costs of equity, e.g., we use the 12-month holding period when forecasting returns with 12-month ahead costs of equity. We use NYSE-Amex-NASDAQ breakpoints and equal-weighted returns. Each decile return is in excess of the 1-month Treasury bill rate. Beneath each row of average return estimates, we show the corresponding Newey-West t-statistics. Panel B shows pairwise comparison tests of QCE (the q^5-characteristics model via cross-sectional regressions) versus alternative models. For each pairwise comparison, we test whether the difference between the two high-minus-low (H−L) decile returns is on average zero.

Panel A: Portfolio sorts											
h	1	2	3	4	5	6	7	8	9	10	H−L
	QCE (the q^5-characteristics model via cross-sectional regressions)										
1	−0.34	0.28	0.60	0.71	0.93	0.92	1.08	1.21	1.61	2.06	2.40
	−0.86	0.89	2.11	2.73	3.71	3.61	4.11	4.57	5.77	6.76	12.02
3	−0.28	0.39	0.68	0.76	0.94	0.96	1.04	1.23	1.39	1.84	2.13
	−0.74	1.25	2.45	2.91	3.74	3.75	3.90	4.65	5.09	6.09	11.02
12	0.15	0.63	0.79	0.88	0.88	0.91	0.98	1.07	1.27	1.55	1.41
	0.44	2.25	3.00	3.41	3.39	3.46	3.64	3.83	4.41	4.73	8.32
24	0.31	0.60	0.76	0.82	0.88	0.96	1.05	1.14	1.32	1.51	1.20
	1.05	2.32	2.98	3.24	3.38	3.59	3.83	4.00	4.41	4.48	7.84
36	0.48	0.77	0.82	0.88	0.92	0.97	1.02	1.08	1.18	1.32	0.84
	1.64	2.91	3.16	3.44	3.53	3.66	3.78	3.90	4.16	4.13	6.01
60	0.68	0.85	0.89	0.91	0.94	0.97	0.99	1.05	1.11	1.23	0.55
	2.58	3.35	3.48	3.51	3.60	3.69	3.72	3.84	3.96	3.86	4.54
	QCE-GB (the q^5-characteristics model via gradient boosting)										
1	−0.59	0.33	0.49	0.71	0.83	1.06	1.08	1.21	1.47	2.48	3.06
	−1.50	1.02	1.73	2.71	3.35	4.28	4.37	4.71	5.43	7.36	16.16
3	−0.43	0.36	0.63	0.78	0.87	1.03	1.07	1.18	1.42	2.05	2.47
	−1.10	1.17	2.21	3.01	3.53	4.20	4.31	4.60	5.25	6.12	12.36
12	0.12	0.61	0.75	0.85	0.92	0.98	1.04	1.07	1.17	1.61	1.49
	0.36	2.09	2.83	3.42	3.63	3.88	4.05	4.03	4.10	4.55	8.67
24	0.26	0.70	0.79	0.85	0.90	0.97	1.01	1.06	1.16	1.64	1.38
	0.81	2.58	3.06	3.35	3.55	3.84	3.93	3.94	3.97	4.66	9.35
36	0.45	0.82	0.88	0.91	0.93	0.97	1.01	1.01	1.06	1.42	0.97
	1.40	3.06	3.40	3.54	3.61	3.75	3.86	3.76	3.81	4.36	7.62
60	0.62	0.89	0.94	0.93	0.93	0.96	0.99	1.02	1.05	1.28	0.65
	2.09	3.41	3.70	3.62	3.62	3.75	3.80	3.85	3.87	4.04	5.30

h	1	2	3	4	5	6	7	8	9	10	H−L
					LCE (the Lewellen 7-variable model)						
1	−0.35	0.47	0.75	0.85	0.92	1.01	1.08	1.27	1.52	1.95	2.30
	−1.01	1.67	2.97	3.43	3.77	4.15	4.30	4.66	5.14	5.45	9.38
3	−0.21	0.57	0.81	0.93	0.97	1.00	1.11	1.21	1.38	1.68	1.90
	−0.62	2.06	3.17	3.80	4.04	4.14	4.46	4.50	4.71	4.80	7.99
12	0.07	0.71	0.85	0.92	0.95	0.99	1.07	1.13	1.27	1.58	1.51
	0.21	2.73	3.50	3.85	3.99	4.06	4.22	4.11	4.25	4.47	6.73
24	0.22	0.71	0.84	0.90	0.92	0.98	1.05	1.16	1.31	1.57	1.35
	0.69	2.74	3.52	3.87	3.91	4.02	4.13	4.25	4.45	4.52	6.16
36	0.37	0.80	0.91	0.93	0.97	0.97	1.03	1.10	1.22	1.44	1.06
	1.16	3.06	3.69	3.84	4.07	3.97	4.09	4.19	4.34	4.47	5.19
60	0.64	0.87	0.95	0.96	0.96	0.98	1.00	1.06	1.17	1.32	0.68
	2.09	3.40	3.88	3.95	3.98	4.01	4.00	4.07	4.26	4.27	3.55
					ICE (the accounting-based implied cost of equity)						
1	0.52	0.65	0.73	0.73	0.81	0.83	0.90	0.85	1.05	1.50	0.98
	1.69	2.56	3.05	3.02	3.26	3.32	3.40	2.80	2.85	3.06	2.64
3	0.56	0.69	0.76	0.77	0.82	0.86	0.87	0.89	1.01	1.38	0.82
	1.83	2.73	3.14	3.21	3.34	3.39	3.33	2.92	2.77	2.86	2.29
12	0.49	0.66	0.81	0.82	0.86	0.94	0.97	1.00	1.08	1.36	0.87
	1.64	2.64	3.36	3.42	3.51	3.70	3.66	3.31	3.03	2.97	2.68
24	0.50	0.72	0.86	0.88	0.92	0.96	1.00	1.06	1.15	1.29	0.79
	1.73	2.87	3.54	3.62	3.71	3.75	3.77	3.61	3.34	2.98	2.68
36	0.59	0.77	0.89	0.90	0.94	0.96	1.01	1.08	1.16	1.18	0.59
	2.02	3.06	3.66	3.74	3.80	3.79	3.84	3.77	3.46	2.87	2.22
60	0.69	0.82	0.91	0.94	0.96	0.98	1.02	1.11	1.15	1.07	0.38
	2.41	3.28	3.75	3.87	3.93	3.93	3.98	4.02	3.64	2.80	1.67
					Q5F (the q^5-factor model)						
1	0.77	1.03	0.93	0.93	0.94	0.92	0.94	0.94	1.01	0.80	0.03
	2.03	3.35	3.40	3.70	4.11	4.01	3.95	3.74	3.62	2.33	0.20
3	0.79	1.02	0.93	0.94	0.93	0.94	0.94	0.95	1.01	0.81	0.02
	2.10	3.34	3.47	3.79	4.07	4.10	3.96	3.80	3.59	2.36	0.16
12	0.82	0.97	0.92	0.92	0.96	0.96	0.98	0.96	1.01	0.86	0.04
	2.23	3.25	3.49	3.77	4.23	4.22	4.14	3.82	3.66	2.53	0.33
24	0.82	0.92	0.91	0.91	0.96	0.97	1.01	1.01	1.03	0.91	0.10
	2.30	3.16	3.51	3.81	4.22	4.29	4.28	4.05	3.79	2.71	0.84
36	0.82	0.92	0.92	0.92	0.96	0.98	1.02	1.02	1.04	0.94	0.12
	2.36	3.20	3.61	3.90	4.23	4.31	4.34	4.11	3.86	2.82	1.10
60	0.87	0.95	0.92	0.93	0.96	0.98	1.01	1.02	1.05	0.99	0.11
	2.60	3.36	3.68	3.99	4.25	4.36	4.38	4.18	3.94	3.03	1.21

Panel B: Pairwise comparison tests with QCE						
	h	Diff	t	h	Diff	t
QCE-GB	1	−0.66	−4.59	24	−0.18	−2.15
LCE	1	0.10	0.47	24	−0.15	−1.01
ICE	1	1.42	2.96	24	0.41	1.62
Q5F	1	2.37	10.78	24	1.10	5.46
QCE-GB	3	−0.35	−2.93	36	−0.13	−1.41
LCE	3	0.23	1.18	36	−0.22	−1.52
ICE	3	1.31	2.86	36	0.25	0.94
Q5F	3	2.10	9.87	36	0.72	4.04
QCE-GB	12	−0.08	−0.81	60	−0.10	−1.19
LCE	12	−0.10	−0.66	60	−0.13	−0.76
ICE	12	0.53	1.65	60	0.17	0.89
Q5F	12	1.37	6.07	60	0.44	2.72

Table A6 : Portfolio Sorts, Equal-weighted Returns, the All-but-micro Sample, January 1977–December 2024

Starting from the full sample, we exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. In Panel A, at the beginning of month t, we split all stocks on each cost of equity into deciles and calculate the monthly equal-weighted returns for different holding periods, h (in months). For instance, the 3-month horizon means that for a given decile in each month, there exist 3 subdeciles, each initiated in a different month in the prior 3 months. We take the simple average of the subdecile returns as the monthly return for the decile. We align the holding period with the forecast horizon in costs of equity, e.g., we use the 12-month holding period when forecasting returns with 12-month ahead costs of equity. We use all-but-micro breakpoints and equal-weighted returns. Each decile return is in excess of the 1-month Treasury bill rate. Beneath each row of average return estimates, we show the corresponding Newey-West t-statistics. Panel B shows pairwise comparison tests of QCE (the q^5-characteristics model via cross-sectional regressions) versus alternative models. For each pairwise comparison, we test whether the difference between the two high-minus-low (H$-$L) decile returns is on average zero.

Panel A: Portfolio sorts

h	1	2	3	4	5	6	7	8	9	10	H$-$L
			QCE (the q^5-characteristics model via cross-sectional regressions)								
1	0.15	0.49	0.60	0.76	0.84	0.85	0.93	1.11	1.15	1.34	1.18
	0.45	1.89	2.53	3.17	3.67	3.90	4.12	4.93	4.95	5.55	5.98
3	0.19	0.54	0.71	0.80	0.81	0.87	0.92	1.07	1.14	1.20	1.01
	0.58	2.06	2.92	3.34	3.61	3.98	4.12	4.80	4.84	4.99	5.39
12	0.40	0.65	0.74	0.81	0.86	0.88	0.91	0.95	0.99	1.06	0.66
	1.30	2.49	3.03	3.44	3.79	3.91	4.02	4.20	4.24	4.38	4.52
24	0.51	0.71	0.75	0.80	0.85	0.86	0.92	0.93	0.98	1.07	0.56
	1.79	2.82	3.10	3.43	3.68	3.72	3.97	3.97	4.15	4.33	4.81
36	0.67	0.80	0.80	0.83	0.85	0.86	0.86	0.91	0.93	1.03	0.35
	2.41	3.23	3.34	3.53	3.70	3.74	3.70	3.93	3.91	3.93	3.19
60	0.82	0.89	0.88	0.88	0.86	0.87	0.86	0.85	0.90	0.94	0.11
	3.06	3.62	3.71	3.74	3.73	3.76	3.69	3.62	3.75	3.61	1.08
			QCE-GB (the q^5-characteristics model via gradient boosting)								
1	0.12	0.44	0.66	0.81	0.79	0.98	0.92	1.04	1.17	1.29	1.17
	0.38	1.65	2.76	3.52	3.57	4.29	4.20	4.53	5.04	5.15	7.52
3	0.21	0.50	0.74	0.76	0.86	0.90	0.95	1.05	1.08	1.21	1.00
	0.69	1.86	2.99	3.36	3.81	4.08	4.32	4.61	4.66	4.81	6.78
12	0.46	0.65	0.75	0.80	0.87	0.89	0.87	0.92	0.97	1.07	0.61
	1.52	2.50	3.12	3.49	3.88	3.98	3.83	4.07	4.23	4.22	4.59
24	0.54	0.74	0.76	0.81	0.86	0.87	0.91	0.91	0.95	1.03	0.49
	1.91	2.98	3.21	3.51	3.75	3.85	3.89	3.86	4.01	4.05	4.52
36	0.65	0.79	0.84	0.86	0.86	0.89	0.88	0.90	0.92	0.95	0.30
	2.35	3.21	3.50	3.73	3.75	3.89	3.76	3.83	3.86	3.68	2.81
60	0.76	0.89	0.89	0.92	0.90	0.88	0.87	0.87	0.86	0.89	0.13
	2.85	3.65	3.77	3.96	3.88	3.80	3.75	3.69	3.62	3.59	1.40

h	1	2	3	4	5	6	7	8	9	10	H−L
					LCE (the Lewellen 7-variable model)						
1	0.16	0.53	0.70	0.79	0.85	0.96	0.93	1.00	1.06	1.26	1.10
	0.54	2.21	3.10	3.68	4.10	4.61	4.60	4.92	4.77	4.26	5.60
3	0.27	0.63	0.73	0.79	0.88	0.92	0.95	0.99	1.04	1.11	0.84
	0.94	2.57	3.28	3.71	4.25	4.50	4.73	4.90	4.66	3.87	4.27
12	0.49	0.77	0.81	0.84	0.88	0.91	0.91	0.93	0.90	0.84	0.35
	1.69	3.22	3.59	3.83	4.18	4.49	4.48	4.56	4.19	3.11	2.02
24	0.56	0.77	0.83	0.85	0.89	0.90	0.89	0.89	0.91	0.88	0.32
	2.04	3.33	3.85	4.01	4.27	4.33	4.31	4.13	4.12	3.26	2.29
36	0.65	0.78	0.84	0.88	0.90	0.89	0.88	0.88	0.91	0.91	0.27
	2.33	3.38	3.84	4.14	4.27	4.23	4.17	4.13	4.12	3.59	1.95
60	0.78	0.84	0.90	0.91	0.92	0.90	0.89	0.88	0.82	0.83	0.05
	2.84	3.59	4.12	4.25	4.34	4.34	4.21	4.15	3.82	3.39	0.38
					ICE (the accounting-based implied cost of equity)						
1	0.69	0.61	0.72	0.76	0.84	0.79	0.83	0.86	0.84	0.92	0.23
	2.41	2.55	3.19	3.40	3.81	3.44	3.58	3.69	3.56	3.11	0.93
3	0.73	0.66	0.77	0.78	0.81	0.85	0.84	0.82	0.78	0.85	0.13
	2.56	2.77	3.41	3.47	3.67	3.72	3.63	3.56	3.36	2.94	0.52
12	0.62	0.69	0.78	0.82	0.84	0.85	0.86	0.83	0.80	0.83	0.20
	2.25	2.94	3.50	3.67	3.78	3.75	3.74	3.60	3.40	2.95	0.95
24	0.58	0.71	0.81	0.86	0.88	0.88	0.89	0.86	0.83	0.85	0.27
	2.14	3.03	3.63	3.84	3.93	3.85	3.87	3.73	3.53	3.08	1.38
36	0.61	0.74	0.83	0.87	0.90	0.91	0.91	0.89	0.86	0.85	0.23
	2.31	3.17	3.71	3.90	4.04	4.00	3.95	3.82	3.70	3.10	1.25
60	0.68	0.75	0.84	0.89	0.91	0.92	0.93	0.92	0.88	0.86	0.18
	2.62	3.28	3.80	4.00	4.05	4.04	4.06	3.97	3.90	3.26	1.06
					Q5F (the q^5-factor model)						
1	0.71	0.82	0.83	0.84	0.84	0.79	0.85	0.85	0.85	0.87	0.16
	2.38	3.38	3.73	4.01	4.30	3.93	4.19	3.92	3.64	3.06	1.14
3	0.70	0.80	0.85	0.83	0.84	0.82	0.86	0.90	0.85	0.86	0.15
	2.37	3.30	3.83	4.07	4.23	4.11	4.26	4.12	3.62	3.03	1.12
12	0.65	0.76	0.80	0.82	0.87	0.85	0.87	0.89	0.88	0.86	0.20
	2.21	3.14	3.66	4.02	4.40	4.32	4.28	4.12	3.74	3.08	1.63
24	0.62	0.75	0.79	0.82	0.87	0.86	0.90	0.93	0.92	0.90	0.28
	2.11	3.07	3.63	4.00	4.37	4.32	4.35	4.30	3.92	3.25	2.33
36	0.65	0.76	0.80	0.83	0.87	0.88	0.91	0.93	0.95	0.92	0.27
	2.25	3.14	3.70	4.05	4.37	4.38	4.41	4.31	4.03	3.33	2.34
60	0.75	0.77	0.80	0.83	0.87	0.88	0.91	0.92	0.96	0.95	0.20
	2.65	3.23	3.73	4.10	4.34	4.40	4.43	4.31	4.07	3.46	1.94

Panel B: Pairwise comparison tests with QCE						
	h	Diff	t	h	Diff	t
QCE-GB	1	0.01	0.09	24	0.07	1.18
LCE	1	0.08	0.34	24	0.24	1.65
ICE	1	0.95	2.91	24	0.29	1.22
Q5F	1	1.02	4.93	24	0.28	1.73
QCE-GB	3	0.01	0.09	36	0.05	1.06
LCE	3	0.17	0.75	36	0.09	0.74
ICE	3	0.88	2.79	36	0.12	0.50
Q5F	3	0.85	4.37	36	0.08	0.48
QCE-GB	12	0.05	0.76	60	−0.02	−0.50
LCE	12	0.31	1.88	60	0.06	0.53
ICE	12	0.46	1.68	60	−0.07	−0.33
Q5F	12	0.45	2.67	60	−0.09	−0.59

Table A7 : Measurement Error Variances (Times 100), Additional Costs of Equity, January 1977–December 2024

In Panel A, $\overline{\text{SVar}}^{\text{TS}} = (1/N)\sum_i \left(\text{Var}_i\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)\right)$ is the time series measurement error variance (MEV) of a given expected return proxy, $\widehat{er}_{it}^h$, $\text{Var}^{\text{TS}} = (1/N)\sum_i \text{Var}_i\left(\widehat{er}_{it}^h\right)$ the variance component, and $\text{Cov}^{\text{TS}} = (1/N)\sum_i \text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)$ the covariance component. $\overline{\text{SVar}}^{\text{CX}} = (1/(T^p - h))\sum_t \left(\text{Var}_t\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)\right)$ is the cross-section MEV of $\widehat{er}_{it}^h$, in which $T^p - h$ is the number of cross-sections in the predictive window. $\text{Var}^{\text{CX}} = (1/(T^p - h))\sum_t \text{Var}_t\left(\widehat{er}_{it}^h\right)$ is the variance component, and $\text{Cov}^{\text{CX}} = (1/(T^p - h))\sum_t \text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)$ the covariance component. We examine five additional cost of equity measures, including QCE-GBA (the q^5-characteristics model with gradient boosting using mean absolute errors as the loss function); QCE-GBH (the q^5-characteristics model via gradient boosting using the Huber loss function); LCE-3 (the Lewellen 3-variable model); LCE-15 (the Lewellen 15-variable model); and FF6F (the Fama-French 6-factor model). Panel B shows pairwise comparison tests with QCE (the q^5-characteristics model from cross-sectional regressions). Diff^{TS} is the time series MEV differential, and t_{TS} its White heteroscedasticity-adjusted t-value. Diff^{CX} is the cross-section MEV differential, and t_{CX} its Newey-West t-value.

Panel A: Measurement error variances

	h	$\overline{\text{SVar}}^{\text{TS}}$	Var^{TS}	Cov^{TS}	$\overline{\text{SVar}}^{\text{CX}}$	Var^{CX}	Cov^{CX}	h	$\overline{\text{SVar}}^{\text{TS}}$	Var^{TS}	Cov^{TS}	$\overline{\text{SVar}}^{\text{CX}}$	Var^{CX}	Cov^{CX}
QCE-GBA	1	0.0136	0.0052	−0.0083	0.0025	0.0102	0.0077	24	1.0959	0.8086	−0.2873	0.1708	1.0680	0.8973
QCE-GBH	1	0.0046	0.0048	0.0002	−0.0041	0.0064	0.0105	24	0.8507	0.8175	−0.0332	0.0449	1.0449	1.0000
LCE-3	1	0.0013	0.0049	0.0036	−0.0034	0.0053	0.0088	24	−3.2599	1.1971	4.4570	−0.8305	1.6271	2.4576
LCE-15	1	0.0074	0.0069	−0.0005	−0.0019	0.0083	0.0102	24	−3.6605	2.1610	5.8215	−1.1720	2.4511	3.6230
FF6F	1	0.0131	0.0091	−0.0040	0.0136	0.0134	−0.0002	24	10.88	9.0960	−1.7831	13.3088	13.0248	−0.2840
QCE-GBA	3	0.0938	0.0354	−0.0583	0.0015	0.0587	0.0572	36	2.9450	1.6999	−1.2452	0.4651	1.9499	1.4847
QCE-GBH	3	0.0620	0.0358	−0.0262	−0.0201	0.0478	0.0680	36	2.6764	1.6975	−0.9788	0.3758	1.8957	1.5199
LCE-3	3	0.0510	0.0383	−0.0127	−0.0132	0.0363	0.0495	36	−4.1733	2.9507	7.1241	−0.8511	3.6528	4.5039
LCE-15	3	0.0683	0.0566	−0.0117	−0.0135	0.0621	0.0757	36	−4.4821	4.7721	9.2542	−0.7289	5.3369	6.0658
FF6F	3	0.1220	0.0844	−0.0376	0.1272	0.1242	−0.0030	36	34.80	31.43	−3.3641	44.67	43.50	−1.1756
QCE-GBA	12	0.7069	0.2822	−0.4248	0.1045	0.4026	0.2981	60	5.6406	4.7564	−0.8841	2.5273	4.5431	2.0158
QCE-GBH	12	0.6056	0.2852	−0.3204	0.0305	0.3758	0.3453	60	5.3022	4.8327	−0.4695	2.4872	4.6753	2.1881
LCE-3	12	−0.0321	0.3321	0.3643	−0.0524	0.4228	0.4752	60	−11.21	10.32	21.53	0.9915	12.71	11.72
LCE-15	12	−0.4533	0.6233	1.0767	−0.1952	0.7219	0.9171	60	−2.7158	16.75	19.47	6.7850	18.28	11.50
FF6F	12	2.2476	1.6218	−0.6258	2.4013	2.3700	−0.0314	60	262.62	253.38	−9.2376	345.23	331.84	−13.40

Panel B: Pairwise comparison tests with QCE

	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}
QCE-GBA	1	−0.0107	−18.99	−0.0058	−2.59	12	−0.9079	−20.85	−0.3041	−1.50	36	−4.2877	−22.50	−0.7304	−0.81
QCE-GBH	1	−0.0017	−3.19	0.0008	0.76	12	−0.8065	−19.56	−0.2301	−1.24	36	−4.0191	−21.65	−0.6411	−0.74
LCE-3	1	0.0018	2.53	0.0001	0.12	12	−0.0488	−1.10	−0.1472	−1.94	36	3.2981	10.58	0.5858	1.05
LCE-15	1	−0.0045	−5.14	−0.0014	−0.70	12	0.3062	4.60	−0.0044	−0.02	36	3.2559	8.15	0.4637	0.59
FF6F	1	−0.0108	−11.47	−0.0169	−11.01	12	−2.4855	−22.44	−2.6009	−8.94	36	−37.04	−32.42	−44.94	−10.98
QCE-GBA	3	−0.0632	−18.39	−0.0254	−1.64	24	−2.9331	−22.31	−0.7990	−1.65	60	−11.50	−15.43	−0.9127	−0.35
QCE-GBH	3	−0.0314	−9.87	−0.0038	−0.36	24	−2.6880	−21.14	−0.6732	−1.49	60	−11.16	−15.21	−0.8726	−0.35
LCE-3	3	−0.0113	−2.55	−0.0108	−1.23	24	1.7324	11.33	0.2022	0.68	60	7.7242	7.29	0.6230	0.37
LCE-15	3	−0.0363	−6.22	−0.0104	−0.62	24	1.9925	10.25	0.5437	1.12	60	−2.1593	−1.43	−5.1704	−1.40
FF6F	3	−0.0966	−13.30	−0.1512	−10.77	24	−13.10	−28.31	−13.94	−10.83	60	−264.38	−34.88	−343.62	−8.27

Table A8 : Measurement Error Variances (Times 100), Additional Costs of Equity, the All-but-micro Sample, January 1977–December 2024

We exclude microcaps (stocks with market equity below the NYSE 20 percentile) from each cross section. In Panel A, $\overline{\text{SVar}}^{\text{TS}} = (1/N)\sum_i \left(\text{Var}_i\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)\right)$ is the time series measurement error variance (MEV) of a given expected return proxy, $\widehat{er}_{it}^h$, $\text{Var}^{\text{TS}} = (1/N)\sum_i \text{Var}_i\left(\widehat{er}_{it}^h\right)$ the variance component, and $\text{Cov}^{\text{TS}} = (1/N)\sum_i \text{Cov}_i\left(\widehat{er}_{it}^h, r_{it+h}\right)$ the covariance component. $\overline{\text{SVar}}^{\text{CX}} = (1/(T^p-h))\sum_t \left(\text{Var}_t\left(\widehat{er}_{it}^h\right) - 2\text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)\right)$ is the cross-section MEV of $\widehat{er}_{it}^h$, in which $T^p - h$ is the number of cross-sections in the predictive window. $\text{Var}^{\text{CX}} = (1/(T^p-h))\sum_t \text{Var}_t\left(\widehat{er}_{it}^h\right)$ is the variance component, and $\text{Cov}^{\text{CX}} = (1/(T^p-h))\sum_t \text{Cov}_t\left(\widehat{er}_{it}^h, r_{it+h}\right)$ the covariance component. We examine five additional cost of equity measures, including QCE-GBA (the q^5-characteristics model with gradient boosting using mean absolute errors as the loss function); QCE-GBH (the q^5-characteristics model via gradient boosting using the Huber loss function); LCE-3 (the Lewellen 3-variable model); LCE-15 (the Lewellen 15-variable model); and FF6F (the Fama-French 6-factor model). Panel B shows pairwise comparison tests with QCE (the q^5-characteristics model from cross-sectional regressions). Diff^{TS} is the time series MEV differential, and t_{TS} its White heteroscedasticity-adjusted t-value. Diff^{CX} is the cross-section MEV differential, and t_{CX} its Newey-West t-value.

Panel A: Measurement error variances

	h	$\overline{\text{SVar}}^{\text{TS}}$	Var^{TS}	Cov^{TS}	$\overline{\text{SVar}}^{\text{CX}}$	Var^{CX}	Cov^{CX}	h	$\overline{\text{SVar}}^{\text{TS}}$	Var^{TS}	Cov^{TS}	$\overline{\text{SVar}}^{\text{CX}}$	Var^{CX}	Cov^{CX}
QCE-GBA	1	0.0045	0.0017	−0.0028	−0.0006	0.0019	0.0026	24	1.1035	0.5357	−0.5679	0.1462	0.5801	0.4339
QCE-GBH	1	0.0025	0.0016	−0.0009	−0.0013	0.0017	0.0029	24	1.0498	0.5313	−0.5185	0.1305	0.5790	0.4486
LCE-3	1	0.0065	0.0049	−0.0016	0.0000	0.0027	0.0027	24	1.0172	0.4963	−0.5209	0.3508	0.2984	−0.0524
LCE-15	1	0.0089	0.0053	−0.0036	0.0013	0.0038	0.0025	24	1.9399	1.1792	−0.7606	1.1607	1.1261	−0.0346
FF6F	1	0.0079	0.0046	−0.0032	0.0071	0.0067	−0.0004	24	5.1058	4.3910	−0.7148	6.6904	6.1581	−0.5323
QCE-GBA	3	0.0309	0.0129	−0.0180	−0.0058	0.0133	0.0191	36	2.8894	1.1985	−1.6909	0.5431	1.1897	0.6465
QCE-GBH	3	0.0210	0.0138	−0.0073	−0.0088	0.0142	0.0230	36	2.8290	1.1805	−1.6485	0.5444	1.1718	0.6274
LCE-3	3	0.0579	0.0379	−0.0200	0.0051	0.0205	0.0154	36	2.1713	1.0153	−1.1561	0.7085	0.6137	−0.0948
LCE-15	3	0.0764	0.0445	−0.0319	0.0148	0.0305	0.0157	36	4.2629	2.7132	−1.5497	2.7682	2.7437	−0.0245
FF6F	3	0.0680	0.0432	−0.0248	0.0670	0.0623	−0.0047	36	15.14	13.81	−1.3301	19.84	18.63	−1.2128
QCE-GBA	12	0.3702	0.1539	−0.2162	0.0034	0.1578	0.1544	60	6.8003	3.6244	−3.1759	2.8848	3.2161	0.3313
QCE-GBH	12	0.3374	0.1528	−0.1846	−0.0061	0.1536	0.1597	60	6.5968	3.6052	−2.9916	2.8687	3.2087	0.3400
LCE-3	12	0.7379	0.2549	−0.4830	0.1807	0.1450	−0.0357	60	5.1594	4.5332	−0.6262	3.9116	2.6312	−1.2805
LCE-15	12	1.0705	0.3980	−0.6725	0.3588	0.3119	−0.0469	60	19.04	14.41	−4.6301	16.11	13.23	−2.8764
FF6F	12	1.1327	0.8302	−0.3025	1.2722	1.1827	−0.0895	60	84.48	83.48	−1.0039	112.80	104.47	−8.3262

Panel B: Pairwise comparison tests with QCE

	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}	h	Diff^{TS}	t_{TS}	Diff^{CX}	t_{CX}
QCE-GBA	1	−0.0024	−10.79	−0.0010	−1.62	12	−0.1616	−11.84	−0.0350	−1.24	36	0.0236	0.20	0.4030	1.30
QCE-GBH	1	−0.0003	−1.74	0.0000	0.13	12	−0.1289	−10.06	−0.0260	−1.05	36	0.0839	0.74	0.4010	1.31
LCE-3	1	−0.0042	−7.66	−0.0010	−0.89	12	−0.5141	−16.57	−0.2120	−1.98	36	0.6802	4.41	0.2370	0.93
LCE-15	1	−0.0075	−12.36	−0.0030	−1.49	12	−0.8775	−15.70	−0.3900	−2.23	36	−1.1432	−3.67	−1.8220	−3.58
FF6F	1	−0.0061	−10.55	−0.0080	−7.20	12	−0.9347	−14.34	−1.3040	−6.45	36	−12.28	−21.44	−18.89	−8.89
QCE-GBA	3	−0.0142	−9.27	−0.0030	−1.13	24	−0.4398	−10.65	−0.0540	−0.52	60	3.2707	6.07	3.9070	2.22
QCE-GBH	3	−0.0044	−3.23	0.0000	−0.08	24	−0.3860	−9.89	−0.0380	−0.40	60	3.4742	6.44	3.9230	2.23
LCE-3	3	−0.0403	−9.56	−0.0140	−1.23	24	−0.3781	−5.91	−0.2580	−1.70	60	4.9461	7.98	2.8800	1.83
LCE-15	3	−0.0670	−14.30	−0.0240	−1.59	24	−1.1298	−7.79	−1.0680	−3.32	60	−8.8998	−6.73	−9.3170	−3.21
FF6F	3	−0.0542	−11.23	−0.0760	−6.86	24	−4.4626	−19.75	−6.5980	−7.48	60	−74.55	−23.45	−106.01	−8.47

Table A9 : Test Results for the Structural Investment Model, January 1977–December 2024

In Panel A, at the beginning of month t, we split all stocks on the structural cost of equity into deciles and calculate the monthly value-weighted returns for different holding periods, h (in months). For instance, the 3-month horizon means that for a given decile in each month, there exist 3 subdeciles, each initiated in a different month in the prior 3 months. We take the simple average of the subdecile returns as the monthly return for the decile. We align the holding period with the forecast horizon in costs of equity, e.g., we use the 12-month holding period when forecasting returns with 12-month ahead costs of equity. We use NYSE-Amex-NASDAQ breakpoints and value-weighted returns. Each decile return is in excess of the 1-month Treasury bill rate. Beneath each row of average returns, we show the corresponding Newey-West t-values. Panel B shows cross-sectional predictive regressions of subsequent h-month returns on the structural h-month ahead cost of equity at the beginning of month t. s is the average slope, ste its Newey-West standard error, $t_{s=0}$ the t-value that tests the slope equals zero, and $|t_{s=1}|$ the absolute t-value that tests the slope equals one.

Panel A: Portfolio sorts											
h	1	2	3	4	5	6	7	8	9	10	H−L
The full sample											
1	0.38	0.68	0.77	0.72	0.73	0.80	0.80	0.77	0.93	0.92	0.54
	1.45	3.06	3.72	3.48	4.29	4.69	4.85	3.97	5.41	4.40	2.30
3	0.20	0.58	0.72	0.81	0.68	0.85	0.93	0.82	0.88	0.97	0.77
	0.73	3.12	3.45	4.02	3.75	4.90	5.06	4.63	4.75	4.88	3.45
12	0.38	0.55	0.71	0.79	0.83	0.79	0.86	0.83	0.83	1.06	0.68
	1.51	3.04	3.48	3.95	4.09	4.32	4.63	4.22	4.30	5.54	3.19
24	0.47	0.59	0.72	0.73	0.84	0.80	0.81	0.79	0.88	0.91	0.44
	1.86	3.33	3.49	3.43	4.07	4.19	4.29	4.01	4.35	4.89	2.13
36	0.55	0.68	0.83	0.84	0.91	0.83	0.85	0.85	0.85	0.89	0.34
	2.11	3.79	4.56	4.19	4.69	4.71	4.65	4.61	4.36	5.05	1.64
60	0.60	0.72	0.81	0.88	0.86	0.82	0.84	0.85	0.82	0.84	0.24
	2.40	3.89	4.00	4.29	4.43	4.29	4.54	4.48	3.96	4.58	1.23
The all-but-micro sample											
1	0.20	0.67	0.72	0.73	0.66	0.80	0.87	0.80	0.77	0.90	0.70
	0.81	3.65	3.52	3.58	3.53	4.12	4.85	4.29	4.17	4.71	3.31
3	0.25	0.62	0.75	0.80	0.63	0.86	0.79	0.86	0.82	0.89	0.64
	1.01	3.38	3.50	3.80	3.36	4.59	4.46	4.61	4.38	4.81	2.97
12	0.35	0.60	0.76	0.81	0.74	0.81	0.84	0.90	0.77	0.92	0.57
	1.39	3.32	3.78	3.68	4.27	4.15	4.50	5.13	3.74	4.81	2.63
24	0.44	0.66	0.75	0.78	0.73	0.79	0.81	0.86	0.77	0.88	0.44
	1.81	3.53	3.72	3.56	3.77	4.15	4.11	4.62	3.72	4.69	2.14
36	0.52	0.79	0.82	0.88	0.82	0.85	0.88	0.89	0.80	0.89	0.37
	2.06	4.25	4.63	4.26	4.31	4.79	4.79	4.99	4.16	5.14	1.80
60	0.57	0.81	0.86	0.83	0.78	0.88	0.80	0.88	0.77	0.83	0.26
	2.25	4.28	4.66	3.85	4.03	4.80	4.31	4.70	3.91	4.63	1.30

Panel B: Cross-sectional predictive regressions									
h	s	ste	$t_{s=0}$	$\lvert t_{s=1}\rvert$	h	s	ste	$t_{s=0}$	$\lvert t_{s=1}\rvert$
The full sample					The all-but-micro sample				
1	0.07	0.03	2.58	33.14	1	0.06	0.03	1.88	27.70
3	0.15	0.06	2.67	15.16	3	0.11	0.06	2.02	16.13
12	0.26	0.14	1.91	5.36	12	0.14	0.11	1.33	8.10
24	0.31	0.12	2.71	5.93	24	0.19	0.08	2.47	10.65
36	0.32	0.09	3.45	7.20	36	0.23	0.07	3.09	10.53
60	0.31	0.10	3.18	7.03	60	0.17	0.06	2.67	13.23