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Does *q*-theory with investment frictions explain anomalies in the cross section of returns?[☆]

Dongmei Li[a], Lu Zhang[b,*]

[a] Rady School of Management, University of California, San Diego, USA
[b] Stephen M. Ross School of Business, University of Michigan and NBER, USA

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ABSTRACT

Q-theory predicts that investment frictions steepen the relation between expected returns and firm investment. Using financing constraints to proxy for investment frictions, we show only weak evidence that the investment-to-assets and asset growth effects in the cross section of returns are stronger in financially more constrained firms than in financially less constrained firms. There is no evidence that *q*-theory with investment frictions explains the investment growth, net stock issues, abnormal corporate investment, or net operating assets anomalies. Limits-to-arbitrage proxies dominate *q*-theory with investment frictions in explaining the magnitude of the investment-to-assets and asset growth anomalies in direct comparisons.

1. Introduction

Initiated by Cochrane (1991, 1996), asset pricing based on the *q*-theory of investment argues that real investment explains cross-sectional differences in expected returns. Intuitively, all else equal, low costs of capital imply high net present values of new projects and high investment, and high costs of capital imply low net present values of new projects and low investment. The literature has so far applied the negative expected return–investment relation predicted by *q*-theory to explain a wide range of capital markets anomalies (empirical relations between average stock returns and firm characteristics that cannot be explained by traditional asset pricing models).[1] In this

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* Corresponding author.
E-mail address: zhanglu@umich.edu (L. Zhang).

[1] Cochrane (1991) shows that aggregate investment-to-capital strongly predicts stock market excess returns. Cochrane (1996) uses residential and nonresidential investment growth, and Li, Vassalou, and

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paper we derive and test a novel implication of q-theory on cross-sectional returns—the expected return–investment relation should be steeper in firms with high investment frictions than in firms with low investment frictions. By exploring the previously ignored interaction between the expected return–investment relation and investment frictions, our tests address whether these anomalies can be attributed to q-theory.

With frictions, investment entails deadweight costs, which cause investment to be less elastic to changes in the discount rate than when frictions are absent. Using a simple model, we show that the magnitude of this elasticity decreases with investment costs. The higher are the investment costs that firms face, the less elastic firms' investments are in responding to variation in the discount rate. Equivalently, a given change in investment corresponds to a larger change in the discount rate, meaning that the expected return–investment relation is steeper for firms with high investment frictions than for firms with low investment frictions. If q-theory does explain a particular investment related anomaly, the relation between expected returns and the anomaly variable must satisfy this prediction.

To test this prediction, we identify investment frictions with firm-level proxies of financing constraints. The premise is that if there are investment costs such as adjustment costs of capital, frictions in capital markets induce additional financing costs at the margin. We use three financing constraints proxies: asset size, payout ratio, and bond ratings. Firms with small asset, low payout ratios, and unrated public debt are more financially constrained than firms with big asset, high payout ratios, and rated public debt. We use six investment-related anomaly variables: investment-to-assets (Lyandres, Sun, and Zhang, 2008), asset growth (Cooper, Gulen, and Schill, 2008), investment growth (Xing, 2008), net stock issues (Fama and French, 2008), abnormal corporate investment (Titman, Wei, and Xie, 2004), and net operating assets (Hirshleifer, Hou, Teoh, and Zhang, 2004). We estimate Fama and MacBeth (1973) cross-sectional regressions of returns on a given anomaly variable within extreme subsamples split by a given financing constraints proxy. Under the q-theory logic, the slope should be negative. With investment frictions, the negative slope should be greater in magnitude in the more constrained subsample than in the less constrained subsample.

Overall, the news is not good for q-theory as an explanation of the anomalies. First, we show some evidence in support of the q-theory interpretation of the investment-to-assets and the asset growth effects. Their slopes are significantly greater in magnitude in the more constrained subsample than in the less constrained subsample. For example, the investment-to-assets slope is -0.85 in the small asset tercile and -0.33 in the big asset tercile, and the difference is more than 2.1 standard errors from zero. This slope is -0.93 in the low payout ratio tercile and -0.39 in the high payout ratio tercile, and the difference is more than 2.4 standard errors from zero. The investment-to-assets slope is -0.86 in the subsample without bond ratings and -0.47 in the subsample with bond ratings, and the difference is more than 2.4 standard errors from zero. The difference in the asset growth slope is significant across extreme asset size terciles and across the subsamples with and without bond ratings, but it is insignificant across extreme payout ratio terciles. However, the evidence is not robust to controlling for the January effect and to controlling for size, book-to-market, and momentum in cross-sectional regressions.

Second, no evidence exists that q-theory with investment frictions explains the investment growth, net stock issues, abnormal corporate investment, or net operating assets anomalies. Their slopes do not differ significantly across extreme financing constraints subsamples. For example, the difference in the investment growth slope is only -0.04 across the extreme asset size terciles and is within 0.9 standard errors from zero. The difference in the net stock issues slope across the subsamples with and without bond ratings is -0.04, which is within 0.2 standard errors from zero. The difference in the abnormal corporate investment slope across extreme payout ratio terciles is -0.05, which is within 1.3 standard errors from zero. The slope difference sometimes even goes in the wrong direction from the prediction of q-theory. In particular, the net operating assets slope in the high payout ratio tercile is higher in magnitude than that in the low payout ratio tercile by 0.06, although the difference is insignificant.

Third, and more important, limits-to-arbitrage proxies dominate financing constraints measures in explaining the magnitude of the investment-to-assets and asset growth anomalies.[2] We show that proxies for investment frictions are correlated with those for limits-to-arbitrage (idiosyncratic volatility and dollar trading volume). Firms with stocks that are more costly to trade face higher investment frictions. However, in direct comparisons financing constraints proxies are largely insignificant after we control for limits-to-arbitrage, but limits-to-arbitrage proxies (in particular, idiosyncratic volatility) remain

(*footnote continued*)
Xing (2006) use sectoral investment growth to price the cross section of returns. Zhang (2005), Li, Livdan, and Zhang (2009), and Livdan, Sapriza, and Zhang (2009) use dynamic investment models to understand the value anomaly, external financing anomalies, and the relation between average returns and financing constraints, respectively. Anderson and Garcia-Feijóo (2006) show that investment growth is correlated with size and book-to-market. Lyandres, Sun, and Zhang (2008) show that adding an investment factor into the capital asset pricing model and the Fama and French (1993) three-factor model substantially reduces the magnitude of the underperformance following initial public offerings, seasoned equity offerings, and convertible bond offerings. Xing (2008) shows that an investment growth factor explains the book-to-market effect approximately as well as Fama and French's value factor. Liu, Whited, and Zhang (2009) derive and test implications of investment Euler equations for cross-sectional returns. Finally, Wu, Zhang, and Zhang (2010) show that capital investment helps explain the accrual anomaly.

[2] Shleifer and Vishny (1997) argue that anomalies can persist if arbitrage costs outweigh arbitrage benefits, and a sizable empirical literature shows that anomalies are stronger in firms with high limits-to-arbitrage than in firms with low limits-to-arbitrage (e.g., Pontiff, 1996; Ali, Hwang, and Trombley, 2003; Mashruwala, Rajgopal, and Shevlin, 2006).

significant after we control for financing constraints. If the empirical proxies have sufficiently high quality, the overall evidence suggests that the q-theory explanation for the investment-to-assets and asset growth anomalies is not robust to controlling for limits-to-arbitrage and that the mispricing hypothesis seems to better explain the anomalies in question. However, no evidence exists that arbitrage costs affect the magnitude of the investment growth, net stock issues, or abnormal corporate investment anomalies from the prediction of the mispricing hypothesis.

The rest of the paper is organized as follows. Section 2 develops the investment frictions hypothesis from q-theory and sets up limits-to-arbitrage as an alternative hypothesis. Section 3 describes our data. Section 4 presents our empirical results. Finally, Section 5 concludes.

2. Hypothesis development

We develop the investment frictions hypothesis based on q-theory in Section 2.1, and set up limits-to-arbitrage as an alternative hypothesis in Section 2.2.

2.1. A model of investment frictions

There are two periods, 0 and 1, and heterogeneous firms, indexed by i. Firms use capital and costlessly adjustable inputs to produce a perishable good. The levels of these inputs are chosen each period to maximize the firms' operating profits, defined as revenues minus the expenditures on these inputs. Firm i's operating profits are given by ΠK_{i0} in period 0 and ΠK_{i1} in period 1, in which Π is the long-term average return on assets. We assume that Π is time-invariant and constant across firms to focus on the role of investment costs. K_{i0} and K_{i1} are firm i's capital in periods 0 and 1, respectively. The profit function exhibits constant returns to scale, meaning that Π is both the marginal product of capital and the average product of capital. Taking the operating profits as given, firms choose optimal investment to maximize their market value.

Firm i starts with capital stock, K_{i0}, invests in period 0, and produces in both periods. The firm exits at the end of period 1 with a liquidation value of $(1-\delta)K_{i1}$, in which $0 \leq \delta \leq 1$ is the rate of capital depreciation. Capital evolves as $K_{i1} = I_{i0} + (1-\delta)K_{i0}$, in which I_{i0} is capital investment over period 0. When investing, firms incur deadweight costs due to investment frictions. The cost function, denoted $C(I_{i0},K_{i0})$, is increasing and convex in I_{i0} and decreasing in K_{i0}. In particular, we assume that the cost of investment frictions per dollar of capital is quadratic in capital growth:

$$C(I_{i0},K_{i0}) = \frac{\lambda_i}{2}\left(\frac{I_{i0}}{K_{i0}}\right)^2 K_{i0}. \tag{1}$$

We use the cost parameter $\lambda_i > 0$ to model the magnitude of the investment costs. Firms with higher λ_i face more investment frictions than firms with lower λ_i.

There is no restriction that I_{i0} is positive. The total cost of investment is $I_{i0}+C(I_{i0},K_{i0})$, in which I_{i0} is the purchasing cost of the capital good when $I_{i0} \geq 0$ and is the resale value of the capital good when $I_{i0} < 0$ (negative cost). When $I_{i0} \geq 0$, the marginal (total) cost of investment is $1+\partial C(I_{i0},K_{i0})/\partial I_{i0} = 1+\lambda_i(I_{i0}/K_{i0})$, which is greater than or equal to one. When $I_{i0} < 0$, the marginal (total) revenue of disinvestment continues to be $1+\lambda_i(I_{i0}/K_{i0})$, which is less than one (the marginal resale value of the capital good) because of investment frictions.

Firm i has a gross discount rate, denoted R_i. The discount rate varies across firms due to, for example, firm-specific loadings on macroeconomic risk factors. The firm chooses optimal investment, I^*_{i0}, to maximize its market value at the beginning of period 0:

$$\max_{\{I_{i0}\}} \Pi K_{i0} - I_{i0} - \frac{\lambda_i}{2}\left(\frac{I_{i0}}{K_{i0}}\right)^2 K_{i0} + \frac{1}{R_i}[\Pi K_{i1} + (1-\delta)K_{i1}]. \tag{2}$$

The market value of firm i is the sum of period 0's free cash flow, $\Pi K_{i0} - I_{i0} - (\lambda_i/2)(I_{i0}/K_{i0})^2 K_{i0}$, and the discounted value of date 1's cash flow, $(\Pi K_{i1} + (1-\delta)K_{i1})/R_i$. In this two-period setup, firm i does not invest in the second period, $I^*_{i1}=0$, meaning that date 1's cash flow is the sum of the operating profits and the liquidation value of the capital.

The trade-off of firm i when making investment decisions is between foregoing free cash flows today in exchange for higher free cash flows tomorrow (when $I^*_{i0} \geq 0$) or increasing free cash flows today at the expense of lower free cash flows tomorrow (when $I^*_{i0} < 0$). Setting the first-order derivative of the objective function with respect to I_{i0} to zero yields

$$R_i = \frac{\Pi + 1 - \delta}{1+\lambda_i(I^*_{i0}/K_{i0})}. \tag{3}$$

This optimality condition is intuitive. When $I^*_{i0} \geq 0$, the numerator in the right-hand side of Eq. (3) is the marginal benefit of investment, $\Pi + 1 - \delta$, including the marginal product of capital, Π, and the marginal liquidation value of capital, $1-\delta$. The denominator is the marginal (total) cost of investment that includes the marginal purchasing cost of the capital good and the marginal investment cost. The marginal benefit of investment is in date 1's dollar terms, and the marginal cost of investment is in date 0's dollar terms. As such, the optimality condition says that the marginal benefit of investment, discounted in date 0's dollar terms, should be equal to the marginal cost of investment. Equivalently, the investment return (the ratio of the marginal benefit of investment in date 1's dollar terms divided by the marginal cost of investment in date 0's dollar terms) should equal the discount rate, as in Cochrane (1991).

The economic interpretation of Eq. (3) when $I^*_{i0} < 0$ is parallel. In particular, the numerator in the right-hand side of the equation is the foregone marginal benefit of investment in period 1, and the denominator is the period 0's marginal benefit of disinvestment that includes the marginal resale value of the capital good, net of the marginal disinvestment cost due to frictions. The optimality condition says that the foregone marginal benefit of investment in period 1, discounted in date 0's dollar terms, should equal the marginal benefit of disinvestment in period 0. Equivalently, the investment return should equal the discount rate, even when $I^*_{i0} < 0$.

Firms choose investment taking R_i and λ_i as given, meaning that I^*_{i0}/K_{i0} is a function of R_i and λ_i. We totally differentiate Eq. (3) with respect to R_i to obtain

$$\frac{d(I^*_{i0}/K_{i0})}{dR_i} = -\frac{[1+\lambda_i(I^*_{i0}/K_{i0})]^2}{\lambda_i(\Pi+1-\delta)} < 0. \tag{4}$$

As such, investment and the discount rate are negatively related. Investment is related to average returns with a negative slope (e.g., Cochrane, 1991; Xing, 2008; Liu, Whited, and Zhang, 2009).

We are interested in knowing how λ_i affects the magnitude of the expected return–investment relation. To this end, we totally differentiate the absolute value of $d(I^*_{i0}/K_{i0})/dR_i$ with respect to λ_i to obtain

$$d\left|\frac{d(I^*_{i0}/K_{i0})}{dR_i}\right| \Big/ d\lambda_i = \frac{2[1+\lambda_i(I^*_{i0}/K_{i0})]}{\lambda_i(\pi+1-\delta)}\left[\frac{I^*_{i0}}{K_{i0}}+\lambda_i\frac{d(I^*_{i0}/K_{i0})}{d\lambda_i}\right] - \frac{[1+\lambda_i(I^*_{i0}/K_{i0})]^2}{\lambda_i^2(\pi+1-\delta)} \tag{5}$$

$$= -\frac{[1+\lambda_i(I^*_{i0}/K_{i0})]^2}{\lambda_i^2(\pi+1-\delta)} < 0. \tag{6}$$

The second equality follows because

$$\frac{I^*_{i0}}{K_{i0}}+\lambda_i\frac{d(I^*_{i0}/K_{i0})}{d\lambda_i} = 0, \tag{7}$$

from totally differentiating both sides of Eq. (3) with respect to λ_i.

Fig. 1 illustrates the economic mechanism at work. We let I^*_{i0}/K_{i0} vary from -20% to 80% per annum with $\delta = 0$ and $\Pi = 15\%$ per annum. We plot the monthly R_i implied by Eq. (3) against monthly I^*_{i0}/K_{i0} for three parameter values of λ_i: zero (no frictions, the black dotted line); 10 (low frictions, the blue solid line), and 30 (high frictions, the red dashed line). As we gradually increase λ_i, the investment–discount rate relation, $d(I^*_{i0}/K_{i0})/dR_i$, becomes flatter. With higher costs, investment is less elastic to the discount rate. Equivalently, the expected return–investment relation, $dR_i/d(I^*_{i0}/K_{i0})$, becomes steeper. In particular, when investment approaches being frictionless, $\lambda_i \to 0$, I^*_{i0}/K_{i0} becomes vertical in the discount rate, and the expected return becomes flat in I^*_{i0}/K_{i0}.

The economic intuition is as follows. The derivative $d(I^*_{i0}/K_{i0})/dR_i$ measures the elasticity of optimal investment with respect to the discount rate. When investment approaches being frictionless, $\lambda_i \to 0$, investment becomes infinitely elastic to changes in the discount rate. With frictions, $\lambda_i > 0$, investment entails deadweight costs, and higher magnitude investment-to-capital entails higher deadweight costs. As such, investment is less elastic to the discount rate. The crucial observation for our empirical tests is that the magnitude of this elasticity decreases with λ_i. The higher is λ_i, the less elastically investment responds to changes in the discount rate. That is, the higher is λ_i, a given magnitude change in investment-to-capital corresponds to a higher magnitude change in the discount rate. This effect means that the negative expected return–investment relation is steeper for firms with high investment frictions than for firms with low investment frictions. Our empirical analysis is centered around this investment frictions hypothesis.

A natural test of whether q-theory explains investment-related anomalies is to check how the magnitude of the expected return–investment relation varies across different subsamples of firms categorized by firm-level investment costs. As such, our primary test is to estimate univariate Fama and MacBeth (1973) cross-sectional regressions of monthly percent excess returns on a given investment-related anomaly variable within each subsample, defined as having high, medium, and low investment costs. If q-theory explains the anomaly, the magnitude of the slope on the anomaly variable should be

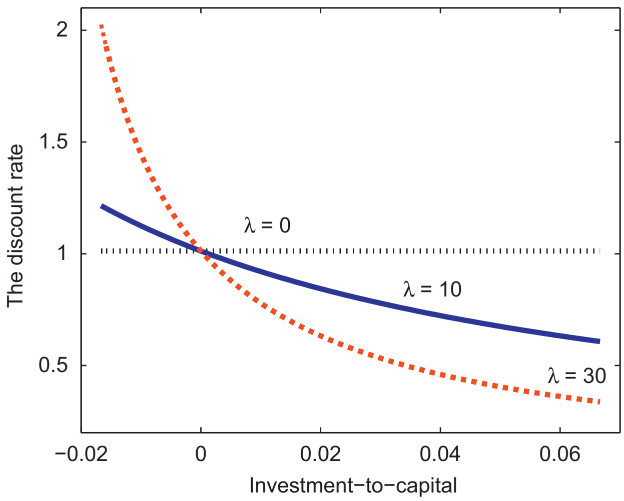

Fig. 1. The discount rate versus investment-to-capital in the two-period q-theory model. This figure plots the discount rate, R_i, against the optimal investment-to-capital ratio, I^*_{i0}/K_{i0}, based on Eq. (3). We plot the relation for three parameter values of λ_i: zero (no frictions, the black dotted line), 10 (low frictions, the blue solid line), and 30 (high frictions, the red dashed line). We set $\pi = 0.15/12$ per month, $\delta = 0$, and let I^*_{i0}/K_{i0} vary from $-0.20/12$ to $0.80/12$ per month.

higher in firms with high investment frictions than in firms with low investment frictions.

2.2. Limits-to-arbitrage as an alternative to q-theory

Anomalies are empirical relations between average returns and firm characteristics, relations that cannot be explained by traditional asset pricing models. Many empirical studies interpret anomalies as driven by systematic mispricing. If anomalies are due to mispricing, why do professional arbitrageurs not exploit the trading opportunities to eliminate the mispricing? Shleifer and Vishny (1997) argue that, because of trading frictions, arbitrage can be costly and limited. When the costs of arbitrage outweigh the benefits of arbitrage, mispricing might not be quickly and entirely traded away.

While the q-theory explanation stresses the importance of investment frictions from the firms' side, the limits-to-arbitrage explanation stresses the importance of trading frictions from the investors' side. Because the two theories depend on different types of frictions that coexist in the data, they are unlikely to be mutually exclusive. Investment frictions and trading frictions can be related. Firms with stocks that are more costly to trade could also face higher investment costs. As such, it is not inconceivable that the evidence that has been exclusively attributed to limits-to-arbitrage in prior studies might be driven partly by investment frictions per q-theory. It also means that the effect of investment frictions on anomalies might be due to limits-to-arbitrage. We address these possibilities in Section 4.3.

3. Data

We obtain accounting data from Compustat and stock returns data from the Center for Research in Security Prices (CRSP). All domestic common shares trading on NYSE, Amex, and Nasdaq with accounting and returns data available are included except for financial firms, which have four-digit standard industrial classification (SIC) codes between 6000 and 6999. Following Fama and French (1993), we exclude closed-end funds, trusts, American Depository Receipts, Real Estate Investment Trusts, units of beneficial interest, and firms with negative book value of equity. To mitigate backfilling biases, we require firms to be listed on Compustat for two years before including them in our sample. We use the one-month Treasury bill rate from Kenneth French's website as the risk-free rate. The sample is from 1963 to 2008.

3.1. Proxies for investment frictions

The investment frictions hypothesis is derived under a general formulation of the investment-cost function. Empirically, we identify investment frictions with firm-level measures of financing constraints. We assume that more constrained firms face higher investment costs. This identification strategy is straightforward to implement. In recent years the corporate finance literature has developed firm-level proxies for financing constraints that are reasonably well accepted. We employ three measures of financing constraints: asset size, payout ratio, and bond ratings. Firms with small asset size, low payout ratios, or unrated corporate bonds are financially more constrained than firms with big asset size, high payout ratios, or rated corporate bonds.

Asset size. We measure asset size as book value of total assets (Compustat annual item AT). At the end of June of each year t, we sort all firms into terciles based on total assets for the fiscal year ending in calendar year $t-1$ using the breakpoints for all public firms traded on NYSE, Amex, and Nasdaq. We assign firms in the small asset tercile of the annual asset size distribution to the more constrained subsample and firms in the big asset tercile to the less constrained subsample. Asset size is a standard measure of financing constraints (e.g., Gilchrist and Himmelberg, 1995; Erickson and Whited, 2000; Almeida and Campello, 2007). Small asset firms are usually young and less familiar to investors than big asset firms. It seems reasonable to assume that small asset firms are more affected by financial frictions than big asset firms.

Payout ratio. The payout ratio also is a common measure of financing constraints (e.g., Fazzari, Hubbard, and Peterson, 1988; Almeida, Campello, and Weisbach, 2004; Almeida and Campello, 2007). The payout ratio is the ratio of total distributions including dividends for preferred stocks (Compustat annual item DVP), dividends for common stocks (item DVC), and share repurchases (item PRSTKC) divided by operating income before depreciation (item OIBDP). At the end of June of each year t, we sort all firms into terciles on their payout ratios for the fiscal year ending in calendar year $t-1$ using the breakpoints for all public firms traded on NYSE, Amex, and Nasdaq. We assign firms in the low payout ratio tercile to the more constrained subsample and firms in the high payout ratio tercile to the less constrained subsample.

A complication arises when firms have negative earnings that makes the payout ratio ill-defined. In total about 18% of firm-year observations have negative earnings and about 5.4% of firm-year observations have negative earnings as well as positive distributions (the sum of Compustat annual items DVP, DVC, and PRSTKC). The existing literature does not provide clear guidance on how to deal with firms with negative earnings. We assign firms with negative earnings and positive distributions to the less constrained subsample, and firms with negative earnings and zero distribution to the more constrained subsample.

Bond rating. We retrieve data on firms' bond ratings from Standard & Poor's and identify the firms that never had their public debt rated in our sample period. These firms are assigned to the more constrained subsample in years when they report positive public debt. The less constrained subsample contains firms whose public debt has been rated during the sample period and firms without public debt outstanding. We assume that a firm has public debt if its long-term debt (Compustat annual item DLTT) is nonzero. This approach has been used extensively in corporate finance (e.g., Kashyap, Lamont, and Stein, 1994; Cummins, Hassett, and Oliner, 1999; Almeida, Campello, and Weisbach, 2004; Almeida and

Campello, 2007). We experiment with commercial paper (short-term debt) ratings as an alternative measure as in Almeida, Campello, and Weisbach (2004), and the results are largely similar (not reported).[3]

3.2. Anomaly variables related to real investment

We consider six anomaly variables that have been linked to real investment in prior studies.

Investment-to-assets, *I/A*. Lyandres, Sun, and Zhang (2008) use this variable as the primary investment variable motivated by *q*-theory. *I/A* is the change in gross property, plant, and equipment (Compustat annual item PPEGT) plus the change in inventories (item INVT) divided by lagged total assets (item AT). Property, plant, and equipment represent long-lived assets for operations over many years such as buildings, machinery, furniture, and other equipment. Inventories represent short-lived assets within a normal operating cycle such as merchandise, raw materials, supplies, and work in progress.

Asset growth, $\Delta A/A$. Asset growth is measured as the change in total assets (Compustat annual item AT) divided by lagged total assets, and it is the most comprehensive measure of investment-to-assets, in which investment is the change in total assets. Cooper, Gulen, and Schill (2008) show that asset growth strongly predicts future abnormal returns and interpret the evidence by saying that "bias in the capitalization of new investments leads to a host of potential investment policy distortions" and that "such potential distortions are present and economically meaningful" (p. 1648). Our tests can address whether *q*-theory explains the asset growth effect.

Investment growth, $\Delta I/I$. Xing (2008) shows that firms with low investment growth earn significantly higher average returns than firms with high investment growth and interprets the evidence as consistent with *q*-theory. Xing also shows that an investment growth factor, defined as the difference in returns between stocks with low investment growth and stocks with high investment growth, can account for the book-to-market effect approximately as well as the Fama and French (1993) value factor. We use Xing's definition of investment growth as the growth rate of capital expenditures (Compustat annual item CAPX). Including investment growth in our tests can address whether Xing's evidence is explained by *q*-theory.

Net stock issues, *NSI*. Combining evidence that returns following equity issues are low (e.g., Ritter, 1991; Loughran and Ritter, 1995) and that returns following stock repurchases are high (e.g., Ikenberry, Lakonishok, and Vermaelen, 1995), Daniel and Titman (2006), Fama and French (2008), and Pontiff and Woodgate (2008) show that net stock issues and average returns are negatively correlated. *NSI* is the natural log of the ratio of the split-adjusted shares outstanding (Compustat annual item CSHO times item ADJEX_C) at the fiscal year ending in calendar $t-1$ divided by the split-adjusted shares outstanding at the fiscal year ending in $t-2$.

The interpretation of the *NSI* effect is controversial. Ritter (1991), Loughran and Ritter (1995), and Ikenberry, Lakonishok, and Vermaelen (1995) argue that the evidence suggests behavioral market timing. Managers can create value for existing shareholders by timing financing and payout decisions to exploit market inefficiencies, and investors underreact to the pricing implications of this market timing behavior. In contrast, Li et al. (2009) argue that the *NSI* effect is connected to investment. The balance-sheet constraint of firms implies that the uses of funds must equal the sources of funds. As such, net issuers should invest more and earn lower expected returns than nonissuers. Lyandres, Sun, and Zhang (2008) show that equity issuers invest more than nonissuers and that adding an investment factor into standard factor models substantially reduces the amount of long-term underperformance following equity issues. We include *NSI* into our tests to examine whether the *NSI* effect can be explained by *q*-theory with investment frictions.

Abnormal corporate investment, *ACI*. Following Titman, Wei, and Xie (2004), we measure *ACI* used for the portfolio formation year t as $ACI_{t-1} = 3CE_{t-1}/(CE_{t-2}+CE_{t-3}+CE_{t-4})-1$, in which CE_{t-1} is capital expenditures (Compustat annual item CAPX) divided by sales (item SALE) for the fiscal year ending in calendar year $t-1$. The prior three-year moving average of capital expenditures is designed to project the benchmark level of investment for the fiscal year $t-1$. An *ACI* value greater than zero indicates that the past fiscal year's investment is greater than the average over the prior three years. In this sense, *ACI* can be interpreted as a measure of abnormal investment. Titman, Wei, and Xie (2004) show that firms with high *ACI* values earn significantly lower average returns than firms with low *ACI* values, and they interpret the evidence as suggesting that "investors tend to underreact to the empire building implications of increased investment expenditures" (p. 677). We use *ACI* in our tests to see whether the negative *ACI*-return relation can be interpreted as an investment effect consistent with *q*-theory.

Net operating assets, *NOA*. Hirshleifer, Hou, Teoh, and Zhang (2004) show that the ratio of net operating assets scaled by lagged total assets strongly predicts cross-sectional returns with a negative slope. Net operating assets measure the cumulation over time of the difference between net operating income (accounting-value added) and free cash flow (cash-value added). Hirshleifer, Hou, Teoh, and Zhang (2004) argue that an accumulation of accounting earnings without a commensurate

[3] We experiment with the Kaplan and Zingales (1997) index, but the index is weakly correlated with the other measures. Several studies cast doubt on this index as a valid measure of financing constraints (e.g., Almeida, Campello, and Weisbach, 2004; Whited and Wu, 2006; Hennessy and Whited, 2007; Hadlock and Pierce, 2010). Reestimating Kaplan and Zingales's ordered logit model on a larger, more recent sample, Hadlock and Pierce find that only two out of five components in the index have signs consistent with the original index. As such, we do not use the Kaplan and Zingales index. Whited and Wu (2006) propose another financing constraints index by combining cash flow-to-assets, a cash dividend dummy, long-term debt-to-assets, total assets, and industry and firm-level sales growth. The cross-sectional Spearman's correlation between asset size and their index is -0.94 in our sample. We opt to use asset size because it is simpler and is less likely to be affected by specification errors (see also Hadlock and Pierce, 2010).

accumulation of free cash flows casts doubt on the sustainability of future profitability. In addition, investors have limited attention and fail to discount for this unsustainability. As such, high *NOA* firms are overvalued and should earn negative long-run abnormal returns, and low *NOA* firms are undervalued and should earn positive long-run abnormal returns.

We ask whether *q*-theory explains the negative *NOA*-return relation. Hirshleifer, Hou, Teoh, and Zhang (2004) show that the cumulative difference between operating income and free cash flow (*NOA*) equals the sum of the cumulative difference between operating income before depreciation and operating cash flow (cumulative operating accruals) and the cumulative investment. The latter results from fixed capital investing activities; the former from working capital investing activities (e.g., Fairfield, Whisenant, and Yohn, 2003). Wu, Zhang, and Zhang (2010) show that controlling for investment-to-assets substantially reduces the predictive power of *NOA* for future returns and interpret the evidence as consistent with *q*-theory.

We examine whether the *NOA* effect varies with investment costs. This test is more stringent because the *q*-theory prediction based on investment costs is not likely to hold under alternative (e.g., behavioral) explanations. Following Hirshleifer, Hou, Teoh, and Zhang (2004), we define *NOA* as $OA-OL$ scaled by lagged total assets (Compustat annual item AT). *OA* is operating assets: total assets minus cash and short-term investment (item CHE). *OL* is operating liabilities: $TA-STD-LTD-MI-PS-CE$, in which *TA* is total assets, *STD* is debt included in current liabilities (item DLC), *LTD* is long-term debt (item DLTT), *MI* is minority interests (item MIB), *PS* is preferred stocks (item PSTK), and *CE* is common equity (item CEQ).

3.3. Proxies for limits-to-arbitrage

We employ two proxies from Ali, Hwang, and Trombley (2003): idiosyncratic volatility and dollar trading volume. Stocks with high idiosyncratic volatility or low trading volume are more costly to arbitrage than stocks with low idiosyncratic volatility or high trading volume, respectively.

Idiosyncratic volatility. Because arbitrage strategies are not diversified, arbitrageurs must take idiosyncratic volatility without being compensated with higher expected returns. As such, high idiosyncratic volatility implies that arbitrage is more costly and limited, and low idiosyncratic volatility implies that arbitrage is less costly and limited. We regress daily stock returns on a value-weighted market portfolio over a maximum of 250 days ending on June 30 of year *t* and calculate idiosyncratic volatility as the standard deviation of the residuals. At the end of June of each year *t*, we sort all firms into terciles on their idiosyncratic volatilities using the breakpoints for all public firms traded on NYSE, Amex, and Nasdaq. We assign firms in the low idiosyncratic volatility tercile to the low limits-to-arbitrage subsample and firms in the high idiosyncratic volatility tercile to the high limits-to-arbitrage subsample.

Dollar trading volume. When stocks are mispriced, transaction costs limit the extent to which arbitrageurs can exploit the trading opportunities to eliminate the mispricing. If stocks are heavily traded, trades are more likely to be completed quickly and are less likely to have adverse price impact. If stocks are thinly traded, trades are less likely to be completed quickly and are more likely to have adverse price impact. Arbitrages are more limited for stocks with low trading volume than for stocks with high trading volume.

Dollar trading volume is the annual trade volume in a firm's shares from July 1 of year $t-1$ to June 30 of year *t*. At the end of each June, we compute dollar volume for each firm as the sum of the last 12 months' daily dollar volume, which is the product of share volume and daily closing price from CRSP. At the end of June of each year *t*, we sort all firms into terciles based on trading volume on June 30 of year *t* using the breakpoints for all public firms traded on NYSE, Amex, and Nasdaq. We assign firms in the low trading volume tercile to the high limits-to-arbitrage subsample and firms in the high trading volume tercile to the low limits-to-arbitrage subsample.

4. Empirical results

Section 4.1 presents descriptive statistics, Section 4.2 tests the investment frictions hypothesis, and Section 4.3 examines the incremental effect of investment frictions relative to limits-to-arbitrage.

4.1. Descriptive analysis

Table 1 reports descriptive statistics. To alleviate the effect of outliers, we winsorize all variables at 1% and 99% before including them in our tests. From Panel A, the asset size distribution is highly skewed toward small firms. The median asset size is 85.5 million dollars, but the mean asset size is almost 10 times larger at 846.1 million dollars. The payout ratio has a mean of 0.14, a median of 0.04, and a standard deviation of 0.27. (In calculating these descriptive statistics, we do not include firm-year observations with negative earnings but positive distributions.) We define the bond rating dummy to take the value of one when firms report positive but unrated public debt and zero otherwise. On average, 53% of firms belong to the more constrained group per the bond rating criterion.

We also calculate pairwise cross-sectional Spearman's rank correlations for each year and report time series averaged correlations. From Panel B, the correlations are 0.45 between asset size and payout ratio, -0.37 between asset size and bond rating dummy, and -0.21 between payout ratio and bond rating dummy. Evaluated with time series standard errors, all the correlations are significant at the 1% level. The evidence suggests that, sensibly, small asset firms are more likely to have low payout ratios and unrated public debt issues than big asset firms and that firms with low payout ratios are more likely to have unrated public debt than firms with high payout ratios.

Table 1
Descriptive statistics (July 1963–December 2008).

Asset size (in millions of dollars) is book assets (Compustat annual item AT). The payout ratio is total distributions including dividends for preferred stocks (item DVP), dividends for common stocks (item DVC), and share repurchases (item PRSTKC) divided by operating income before depreciation (item OIBDP). We do not calculate the payout ratios for firms with negative earnings but positive distributions. We retrieve data on firms' bond ratings from Standard & Poor's and categorize those firms that never had their public debt rated during our sample period as financially constrained (d(rating)=1). Observations from those firms are only assigned to the constrained subsample in years when the firms report positive debt. The financially unconstrained subsample contains those firms whose bonds have been rated during the sample period (d(rating)=0). We regress daily stock returns on a value-weighted market portfolio over a maximum of 250 days ending on June 30 of year t and calculate idiosyncratic volatility (Ivol) as the standard deviation of the residuals, in monthly percent. Dollar trading volume (Dvol) is the annual volume of trade in a firm's shares from July 1 of year $t-1$ to June 30 of year t, in billions of dollars. At the end of each June, we compute dollar volume for each firm as the sum of last 12 months' daily dollar volume, which is the product of share volume and daily closing price from the Center for Research in Security Prices. Investment-to-assets is the annual change in gross property, plant, and equipment (Compustat annual item PPEGT) plus the annual change in inventories (item INVT) divided by the lagged book value of assets (item AT). Asset growth ($\Delta A/A$) is the change in total assets (item AT) divided by lagged total assets. Investment growth ($\Delta I/I$) is the growth rate of capital expenditure (item CAPX). Net stock issues (*NSI*) are the natural log of the ratio of the split-adjusted shares outstanding at the fiscal year-end in $t-1$ divided by the split-adjusted shares outstanding at the fiscal year-end in $t-2$. The split-adjusted shares outstanding is Compustat shares outstanding (item CSHO) times the Compustat adjustment factor (item AJEX). Abnormal corporate investment (*ACI*) is $3CE_{t-1}/(CE_{t-2}+CE_{t-3}+CE_{t-4})-1$, in which CE_{t-1} is capital expenditures (item CAPX) scaled by its sales (item SALE) for the fiscal year ending in calendar year $t-1$. Net operating assets (*NOA*) are operating assets minus operating liabilities, in which operating assets are calculated as total assets (item AT) minus cash and short-term investment (item CHE). Operating liabilities are total assets minus debt included in current liabilities (item DLC) minus long-term debt (item DLTT) minus minority interests (item MIB) minus preferred stocks (item PSTK) minus common equity (item CEQ). We winsorize all variables at 1% and 99%. In Panel A we calculate the statistics by pooling all the time series and cross-sectional observations. In Panel B we calculate the pairwise cross-sectional Spearman's rank correlations for each year and report time series averaged correlations. The significance of a given correlation (calculated with time series standard errors) is indicated with by * and **, denoting 5% and 1% significance levels, respectively.

Panel A: Descriptive statistics

	Mean	Standard deviation	Minimum	25%	Median	75%	Maximum
Asset size	846.07	2,974.09	1.13	23.35	85.50	383.42	4,4319.00
Payout ratio	0.14	0.27	0.00	0.00	0.04	0.18	3.12
d(rating)	0.53	0.50	0.00	0.00	1.00	1.00	1.00
Ivol	15.51	9.67	3.05	8.70	13.02	19.49	76.31
Dvol	1.20	6.08	0.00	0.00	0.03	0.26	121.35
I/A	0.06	0.22	−0.49	−0.01	0.05	0.13	2.37
$\Delta A/A$	0.12	0.45	−0.63	−0.04	0.07	0.19	7.08
$\Delta I/I$	0.33	1.64	−0.98	−0.46	0.00	0.50	19.00
NSI	0.03	0.13	−0.23	0.00	0.00	0.03	1.13
ACI	−0.20	0.87	−0.99	−0.88	−0.33	0.10	6.88
NOA	0.64	0.39	−0.46	0.46	0.69	0.84	4.03

Panel B: Cross correlations (Spearman)

	Asset size	Payout ratio	d(rating)	Ivol	Dvol	I/A	$\Delta A/A$	$\Delta I/I$	*NSI*	*ACI*	*NCO*
Asset size	1										
Payout ratio	0.45**	1									
d(rating)	−0.37**	−0.21**	1								
Ivol	−0.64**	−0.55**	0.29**	1							
Dvol	0.73**	0.27**	−0.35**	−0.39**	1						
I/A	0.13**	0.00	−0.01	−0.10**	0.21**	1					
$\Delta A/A$	0.17**	0.02**	−0.05**	−0.14**	0.26**	0.73**	1				
$\Delta I/I$	0.12**	0.05**	−0.02**	−0.10**	0.19**	0.54**	0.47**	1			
NSI	0.10**	−0.15**	−0.04**	0.02	0.22**	0.39**	0.47**	0.30**	1		
ACI	0.29**	0.23**	−0.08**	−0.25**	0.26**	0.31**	0.23**	0.54**	0.14**	1	
NOA	0.24**	0.09**	0.00	−0.18**	0.16**	0.56**	0.60**	0.34**	0.36**	0.22**	1

Stock returns at the firm level are volatile. The mean idiosyncratic volatility is 15.5% per month, and the median is 13%. Similar to asset size, dollar trading volume is skewed. The mean volume is 1.2 billion dollars, and the median is only 0.03 billion. The two limits-to-arbitrage proxies have a correlation of -0.39, which is significant at the 1% level. Stocks with high idiosyncratic volatilities have low trading volumes, and stocks with low idiosyncratic volatilities have high trading volumes.

The proxies for limits-to-arbitrage are correlated with those for financing constraints. In June of each year t we calculate the pairwise cross-sectional Spearman's correlations between limits-to-arbitrage proxies measured at the end of June of year t and financing constraints proxies for the fiscal year ending in calendar year $t-1$, and we compute time series average correlations. Small asset firms have high idiosyncratic volatility and low dollar trading volume. The correlations are -0.64 between asset size and idiosyncratic volatility and 0.73 between asset size and trading volume. Low payout firms have high idiosyncratic volatility and low trading volume. The correlations are -0.55 between payout ratio and idiosyncratic volatility and 0.27 between payout ratio and trading volume. Firms without bond ratings have high idiosyncratic volatility and low trading volume. The correlations are 0.29 between the rating dummy and idiosyncratic volatility and -0.35 between the rating dummy and trading volume. All these correlations are significant at the 1% level.

These correlations make sense. Asset size, which is a standard financing constraints measure, can indicate trading frictions. Firms with small asset size are more likely to be thinly traded with lower liquidity and higher transactions costs than firms with big asset size. Further, small asset firms are more likely to be poorly diversified in their operating, investing, and financing activities and have higher idiosyncratic volatilities than big asset firms. Firms with high trading volume are more likely to have easy access to equity markets and low equity financing costs than firms with low trading volume. Finally, to the extent that idiosyncratic volatility affects firms' default probabilities à la Merton (1974), firms with high idiosyncratic volatilities are more likely to have high default probabilities and high costs of debt financing than firms with low idiosyncratic volatilities.

Turning to the investment measures, Panel A of Table 1 shows that investment-to-assets, I/A, has a mean of 0.06 per annum and a standard deviation of 0.22. Asset growth, $\Delta A/A$, has a mean of 0.12 per annum and a standard deviation of 0.45. The distribution of investment growth is skewed. The mean is 0.33, but the median is zero. The mean of abnormal corporate investment, *ACI*, is -0.20 and the median is -0.33. Because *ACI* is defined as the growth rate of investment-to-sales relative to its prior three-year moving average, the evidence suggests strong mean reversion in real investment at the firm level. Finally, the distribution of net operating assets is largely symmetric. Its mean is 0.64, which is close to the median of 0.69.

From Panel B, the six anomaly variables are correlated. The pairwise Spearman correlations vary from 0.14 to 0.73 and are all significant at the 1% level. In particular, investment-to-assets is highly correlated with the other measures: 0.73 with asset growth, 0.54 with investment growth, 0.39 with net stock issues, 0.31 with abnormal corporate investment, and 0.56 with net operating assets. *NSI* has a low correlation of 0.14 with abnormal corporate investment but high correlations of 0.47, 0.30, and 0.36 with asset growth, investment growth, and net operating assets, respectively.

4.2. *Testing the investment frictions hypothesis*

For each month from July of year t to June of year $t+1$, we estimate Fama and MacBeth (1973) cross-sectional regressions of monthly percent excess returns on a given investment-related anomaly variable for the fiscal year ending in the calendar year $t-1$. We run the regressions in the full sample as well as in extreme subsamples split by a given financing constraints proxy, and we compare the slopes on the anomaly variable across the extreme subsamples. We split the sample in June of each year t based on a given financing constraints proxy for the fiscal year ending in calendar year $t-1$. Under the q-theory logic, the slopes should be negative and greater in magnitude in the more constrained subsample than in the less constrained subsample.

4.2.1. *Benchmark estimation*

Table 2 reports the detailed results. All six anomaly variables predict returns negatively in the full sample. All variables except for abnormal corporate investment, *ACI*, have slopes that are significant at the 1% level. In particular, investment-to-assets has a slope of -0.69 that is more than 4.9 standard errors from zero. Asset growth has a slope of -0.74 that is more than 8.0 standard errors from zero. Relative to the other variables, *ACI*'s predictive power is substantially weaker. Its slope is -0.05, which is within 1.6 standard errors from zero.

Turning to our key tests, Table 2 shows that the slopes of investment-to-assets are significantly higher in magnitude in the more constrained subsample than in the less constrained subsample. From Column 1, the I/A slope is -0.85 in the small asset tercile and is -0.33 in the big asset tercile. The difference of -0.52 is more than 2.1 standard errors from zero (we use the time series standard error of the slope difference). Using the payout ratio as the financing constraints proxy yields largely similar results. The I/A slope is -0.93 in the low payout ratio tercile and -0.39 in the high payout ratio tercile. The difference of -0.54 is more than 2.4 standard errors from zero. Finally, the I/A slope is -0.86 in the subsample without bond ratings and -0.47 in the subsample with bond ratings. The difference of -0.39 is more than 2.4 standard errors from zero.

The asset growth results are weaker than those for investment-to-assets. Column 2 shows that the $\Delta A/A$ slope is -0.83 in the small asset tercile and is -0.47 in the big asset tercile. The difference of -0.36 is more than 2.3 standard errors from zero. However, using payout ratio yields insignificant difference in the $\Delta A/A$ slope

Table 2
Slopes from Fama and MacBeth (1973) cross-sectional regressions of monthly percent excess returns on anomaly variables in the full sample and subsamples split by financing constraints measures (July 1963–December 2008, 558 months).

For each month from July of year t to June of year $t+1$, we estimate Fama and MacBeth cross-sectional regressions of monthly percent excess returns on a given anomaly variable for the fiscal year ending in calendar year $t-1$ in the full sample as well as in extreme subsamples split by a given financing constraints measure. We split the sample in June of each year t based on a given constraints proxy for the fiscal year ending in calendar year $t-1$. I/A is investment-to-assets, $\Delta A/A$ is asset growth, $\Delta I/I$ is investment growth, *NSI* is net stock issues, *ACI* is abnormal corporate investment, and *NOA* is net operating assets. See the caption of Table 1 for detailed variable definitions. For firms with negative earnings but positive payouts, we do not calculate their payout ratios but categorize them as financially least constrained (along with firms with high payout ratios). We report the slopes and their Fama and MacBeth t-statistics (in parentheses). We also report the t-statistics (in brackets) testing that a given slope is equal across extreme subsamples split by a given financing constraints measure. The time series average numbers of firms in the cross section for the full sample and for different subsamples are in curly brackets. Excess returns are the difference between portfolio returns and one-month Treasury bill rate (from Kenneth French's website).

	I/A (1)	$\Delta A/A$ (2)	$\Delta I/I$ (3)	*NSI* (4)	*ACI* (5)	*NOA* (6)
Full sample	−0.69	−0.74	−0.08	−1.87	−0.05	−0.51
	(−4.92)	(−8.28)	(−5.45)	(−6.98)	(−1.58)	(−5.05)
	{3,148}	{3,148}	{3,148}	{3,148}	{3,117}	{3,148}
Small asset size	−0.85	−0.83	−0.09	−1.27	−0.04	−0.47
	(−5.12)	(−7.82)	(−5.03)	(−3.78)	(−0.97)	(−3.70)
	{1,020}	{1,020}	{1,020}	{1,020}	{1,010}	{1,020}
Big asset size	−0.33	−0.47	−0.05	−1.50	0.02	−0.45
	(−1.63)	(−3.53)	(−1.37)	(−4.70)	(0.44)	(−5.01)
	{1,050}	{1,050}	{1,050}	{1,050}	{1,040}	{1,050}
Small-minus-big	[−2.13]	[−2.39]	[−0.87]	[0.58]	[−1.00]	[−0.11]
Low payout ratio	−0.93	−0.81	−0.10	−1.39	−0.08	−0.50
	(−5.63)	(−7.81)	(−4.81)	(−4.50)	(−2.10)	(−4.45)
	{1,269}	{1,269}	{1,269}	{1,269}	{1,259}	{1,269}
High payout ratio	−0.39	−0.66	−0.06	−2.20	−0.03	−0.56
	(−2.00)	(−5.17)	(−2.49)	(−6.07)	(−0.83)	(−4.24)
	{1,146}	{1,146}	{1,146}	{1,146}	{1,136}	{1,146}
Low-minus-high	[−2.49]	[−1.24]	[−1.37]	[1.91]	[−1.22]	[0.52]
Without bond rating	−0.86	−0.90	−0.10	−1.86	−0.03	−0.50
	(−5.95)	(−9.44)	(−6.11)	(−6.04)	(−0.94)	(−4.86)
	{1,683}	{1,683}	{1,683}	{1,683}	{1,671}	{1,683}
With bond rating	−0.47	−0.50	−0.05	−1.82	−0.09	−0.51
	(−2.61)	(−4.43)	(−2.30)	(−5.85)	(−2.30)	(−4.23)
	{1,466}	{1,466}	{1,466}	{1,466}	{1,446}	{1,466}
Without-minus-with	[−2.49]	[−3.77]	[−2.41]	[−0.11]	[1.61]	[0.21]

across extreme terciles. The slope is −0.81 in the low payout ratio tercile and is −0.66 in the high payout ratio tercile. The difference of −0.15 is within 1.3 standard errors from zero. Using bond rating dummy to measure investment frictions yields more significant results. The $\Delta A/A$ slope is −0.90 in the subsample without bond ratings and −0.50 in the subsample with bond ratings, and the difference is more than 3.7 standard errors from zero.

Although going in the right direction, the investment growth results are substantially weaker than those for investment-to-asset and asset growth. From Column 3, the difference in the $\Delta I/I$ slope is only −0.04 across the small and big asset size terciles, and it is within 0.9 standard errors from zero. The slope difference is −0.04 across the low and high payout ratio terciles, and it is within 1.4 standard errors from zero. The slope difference is −0.05 across the subsamples with and without bond ratings, but the difference is more precisely estimated and is more than 2.4 standard errors from zero.

The remaining columns of Table 2 show that the net stock issues, abnormal corporate investment, and net operating assets effects do not conform to the prediction of q-theory with investment frictions. Their slopes from cross-sectional regressions do not differ significantly across extreme financing constraints subsamples. From Column 4, the sign of the *NSI* slope difference even goes in the wrong direction across asset size and payout ratio terciles. The *NSI* slope is −1.27 in the small asset tercile, but −1.50 in the big asset tercile, although the difference is within 0.6 standard errors from zero. The *NSI* slope is −1.39 in the low payout ratio tercile, but −2.20 in the high payout ratio tercile, and the difference is even marginally significant (t=1.91). Although the sign of the *NSI* slope goes in the right direction across bond ratings subsamples, it is only −0.04, and is within 0.2 standard errors from zero.

The *NSI* results show that although the full-sample evidence is supportive of the prediction of q-theory with investment frictions, the prediction is not supported in the tests based on subsamples. As such, it is important to go beyond the sign and the significance of the expected return–investment relation as in most prior studies to examine the deeper prediction about investment frictions in drawing inferences about whether q-theory explains the investment-related anomalies.

From Column 5, the *ACI* slope is −0.04 in the small asset tercile and 0.02 in the big asset tercile. However, the difference of −0.06 is only one standard error from zero. The difference in the *ACI* slope between the low and high payout ratio terciles stands at −0.05, which is within 1.3 standard errors from zero. The evidence with bond ratings goes in the wrong direction from the prediction of q-theory with investment frictions. The *ACI* slope is -0.03 in the more constrained subsample but is -0.09 in the less constrained subsample, although the difference is within 1.7 standard errors from zero. From Column 6, the *NOA* slope differences across the extreme subsamples are all within 0.6 standard errors from zero. The sign of the slope difference goes in the right direction across extreme asset size terciles, but it goes in the wrong direction across subsamples split by payout ratio and by bond ratings.

In summary, the subsample test results are supportive of q-theory with investment frictions only for the investment-to-assets and asset growth anomalies. The evidence is important because it shows that our tests have enough power to reject the null hypothesis of no differences across extreme financing constraints subsamples. However, the tests do not support the q-theory explanation for the investment growth, net stock issues, abnormal corporate investment, or net operating assets anomalies. These results cast doubt on inferences from extant q-theory tests that focus only on the sign and the significance of the relations between average returns and these anomaly variables.

4.2.2. Alternative specifications

We report two alternative specifications of our test design in Table 2. First, we examine the impact of the January effect, which means a general increase in stock prices in January. The January effect is often attributed to buying activities that follow the drop in prices in December when investors sell to create tax losses and offset capital gains. Keim (1983) shows that much of the abnormal returns to small firms occurs in January. However, we are not aware of any prior attempt to examine whether the investment-related anomalies are driven by the January effect.

In Panel A of Table 3 we rerun the tests from Table 2 but without January returns. The first two rows of the panel show that the *ACI* effect of Titman, Wei, and Xie (2004) is driven entirely by the January effect. Without returns in January, the *ACI* slope is close to zero in cross-sectional regressions and is within 0.1 standard errors from zero. The slopes for the other investment-related anomaly variables are somewhat reduced in magnitude, but all of them remain at least 3.7 standard errors from zero. In particular, the *I*/*A* slope reduces from -0.69 to -0.55, which is still more than 3.9 standard errors from zero. The *NSI* slope even increases in magnitude from -1.87 to -2.02, and it is more than 7.6 standard errors from zero.

Dropping January returns weakens the evidence in favor of q-theory as an explanation of the relation between returns and *I*/*A*. The first column of Panel A shows that the *I*/*A* slope is -0.72 in the small asset tercile and -0.30 in the big asset tercile. The difference of -0.42 is within 1.7 standard errors from zero. The *I*/*A* slope is -0.68 in the low payout ratio tercile and -0.32 in the high payout ratio tercile. The difference of -0.36 also is within 1.7 standard errors from zero. And the *I*/*A* slope is -0.74 in the subsample without bond ratings and -0.27 in the subsample with bond ratings. The difference of -0.47 is more than 3.0 standard errors from zero.

Controlling for the January effect does not seem to affect the evidence concerning q-theory with investment frictions for the asset growth anomaly. From the second column of Panel A, the $\Delta A/A$ slope is -0.73 in the small asset tercile and -0.39 in the big asset tercile. The difference of -0.34 is more than 2.1 standard errors from zero. The $\Delta A/A$ slope is -0.64 in the low payout ratio tercile and -0.47 in the high payout ratio tercile. The difference of -0.17 is within 1.5 standard errors from zero. And the $\Delta A/A$ slope is -0.77 in the subsample without bond ratings and -0.35 in the subsample with bond ratings. The difference of -0.42 is more than 3.9 standard errors from zero.

From the third column of Panel A, dropping January returns weakens the differences in the investment growth slope across extreme financing constraints subsamples to insignificant levels. The $\Delta I/I$ slope differs across subsamples with and without bond ratings by -0.04, although it is marginally significant ($t=-1.87$). The slope differences are even smaller in magnitude across extreme asset size and payout ratio terciles, and they are both within 0.5 standard errors from zero. The remaining columns of Panel A show that, as in the benchmark estimation, the *NSI*, *ACI*, and *NOA* slopes do not differ significantly across extreme financing constraints subsamples.

In the second alternative specification, we ask how the results change after we control for the standard determinants of cross-sectional returns such as size, book-to-market, and momentum. We measure size as the log market capitalization at the end of June of year t, book-to-market as the log book equity for the fiscal year ending in $t-1$ minus the log market equity at the end of December of year $t-1$, and momentum as the log prior six-month returns (with one-month gap between the holding period and the current month). A caveat with these controls is that they are correlated with real investment. Small firms, growth firms, and momentum winners tend to invest more than big firms, value firms, and momentum losers. These correlations can make the interpretation of any individual slope in multiple regressions less straightforward than the slope in univariate regressions. However, it remains informative to check to what extent our basic inferences are sensitive to the inclusion of these standard controls.

Panel B of Table 3 reports the detailed results. January returns are included in these tests. Including the standard controls reduces the *ACI* effect to insignificance. Its regression slope of -0.02 is within 1.1 standard errors from zero. The five other anomaly variables retain strong predictive power for cross-sectional returns. All the slopes are at least 3.8 standard errors from zero. More important, including the standard controls weakens the evidence in support of q-theory from the benchmark estimation. In particular, the *I*/*A* slope is -0.62 in the low payout ratio tercile and -0.27 in the high payout ratio tercile. The difference of -0.35 is within 1.8 standard errors from zero. In contrast, this slope difference is -0.54 ($t=-2.49$) in the benchmark estimation. The $\Delta A/A$ slope is -0.57 in

Table 3

Slopes from Fama and MacBeth (1973) cross-sectional regressions of monthly percent excess returns on anomaly variables in the full sample and subsamples split by financing constraints measures, robustness checks.

In Panel A for each month from July of year t to June of year $t+1$ (except for January of year $t+1$) we run cross-sectional regressions of monthly percent excess returns on a given anomaly variable for the fiscal year ending in calendar year $t-1$ in the full sample as well as in extreme subsamples split by a given financing constraints measure. We split the sample in June of each year t based on a given constraints proxy for the fiscal year ending in calendar year $t-1$. In Panel B the regressions are run with January returns as well as three controls: the log market capitalization at the end of June of year t, the log book-to-market equity as the log book equity for the fiscal year ending in $t-1$ minus the log market equity at the end of December of year $t-1$, and the log prior six-month returns (with one-month gap between the holding period and the current month). We report the slopes and their Fama and MacBeth t-statistics (in parentheses), as well as the t-statistics (in brackets) testing that a given slope is equal across extreme subsamples split by a given financing constraints measure. For firms with negative earnings but positive payouts, we do not calculate their payout ratios but categorize them as financially least constrained (along with firms with high payout ratios). Excess returns are the difference between portfolio returns and one-month Treasury bill rate (from Kenneth French's website). I/A is investment-to-assets, $\Delta A/A$ is asset growth, $\Delta I/I$ is investment growth, *NSI* is net stock issues, *ACI* is abnormal corporate investment, and *NOA* is net operating assets. See the caption of Table 1 for detailed variable definitions. Book equity is the Compustat book value of stockholders' equity (Compustat annual item SEQ), plus balance sheet deferred taxes (item TXDB) and investment tax credit (item ITCI, if available), minus the book value of preferred stock. Depending on availability, we use redemption (item PSTKRV), liquidation (item PSTKL), or par value (item PSTK), in that order to estimate the book value of preferred stock. Market equity is price per share times the number of shares outstanding (SHROUT) from Center for Research in Security Prices.

	I/A	$\Delta A/A$	$\Delta I/I$	*NSI*	*ACI*	*NOA*
Panel A: No January returns (July 1963–December 2008, 513 months)						
Full sample	−0.55	−0.60	−0.09	−2.02	0.00	−0.39
	(−3.91)	(−6.82)	(−6.24)	(−7.66)	(0.09)	(−3.79)
Small asset size	−0.72	−0.73	−0.08	−1.79	−0.03	−0.44
	(−4.40)	(−6.95)	(−4.80)	(−5.82)	(−0.62)	(−3.35)
Big asset size	−0.30	−0.39	−0.06	−1.62	0.05	−0.44
	(−1.45)	(−2.95)	(−1.61)	(−4.94)	(1.12)	(−4.72)
Small-minus-big	[−1.63]	[−2.16]	[−0.43]	[−0.42]	[−1.22]	[0.06]
Low payout ratio	−0.68	−0.64	−0.09	−1.59	−0.04	−0.43
	(−4.22)	(−6.33)	(−4.33)	(−5.27)	(−1.09)	(−3.72)
High payout ratio	−0.32	−0.47	−0.08	−2.22	−0.01	−0.37
	(−1.65)	(−3.76)	(−3.51)	(−6.21)	(−0.22)	(−2.77)
Low-minus-high	[−1.64]	[−1.41]	[−0.21]	[1.50]	[−0.77]	[−0.50]
Without bond rating	−0.74	−0.77	−0.10	−2.11	0.01	−0.42
	(−5.12)	(−8.12)	(−6.40)	(−6.97)	(0.28)	(−4.05)
With bond rating	−0.27	−0.35	−0.06	−1.88	−0.03	−0.34
	(−1.51)	(−3.16)	(−2.98)	(−6.06)	(−0.73)	(−2.73)
Without-minus-with	[−3.04]	[−3.94]	[−1.87]	[−0.68]	[1.03]	[−0.93]
Panel B: Controlling for size, book-to-market, and prior returns (July 1963–December 2008, 558 months)						
Full sample	−0.49	−0.52	−0.07	−1.28	−0.02	−0.56
	(−3.84)	(−6.43)	(−5.22)	(−5.66)	(−1.03)	(−6.83)
Small asset size	−0.68	−0.57	−0.07	−0.88	−0.07	−0.67
	(−4.28)	(−5.65)	(−4.13)	(−2.84)	(−1.81)	(−5.54)
Big asset size	−0.20	−0.38	−0.04	−1.38	0.02	−0.43
	(−1.06)	(−3.25)	(−1.33)	(−4.94)	(0.59)	(−4.85)
Small-minus-big	[−2.14]	[−1.32]	[−0.58]	[1.39]	[−1.68]	[−1.71]
Low payout ratio	−0.62	−0.51	−0.06	−0.89	−0.05	−0.51
	(−4.43)	(−6.06)	(−3.73)	(−3.19)	(−1.57)	(−5.03)
High payout ratio	−0.27	−0.45	−0.06	−1.73	−0.01	−0.63
	(−1.56)	(−3.83)	(−2.79)	(−5.83)	(−0.38)	(−6.24)
Low-minus-high	[−1.76]	[−0.50]	[−0.17]	[2.35]	[−0.95]	[1.08]
Without bond rating	−0.65	−0.65	−0.08	−1.28	−0.01	−0.59
	(−4.80)	(−7.57)	(−5.23)	(−4.86)	(−0.37)	(−6.36)
With bond rating	−0.23	−0.29	−0.05	−1.28	−0.05	−0.44
	(−1.42)	(−2.74)	(−2.41)	(−4.79)	(−1.49)	(−4.85)
Without-minus-with	[−2.83]	[−3.55]	[−1.25]	[−0.03]	[1.08]	[−1.80]

the small asset tercile and −0.38 in the big asset tercile. The difference of −0.19 is within 1.4 standard errors from zero. In contrast, this slope difference is −0.36 ($t=-2.39$) in the benchmark estimation. Other results involving I/A and $\Delta A/A$ are largely similar to those in the benchmark estimation.

Panel B shows that controlling for standard cross-sectional determinants of returns does not enhance the ability of q-theory to explain the other anomalies. In particular, the $\Delta I/I$ slope is −0.08 in the subsample without bond ratings and −0.05 in the subsample with bond ratings. The difference is within 1.3 standard errors from zero. In contrast, this difference is −0.05, which is more than 2.4 standard errors from zero in the benchmark estimation. The *NSI* slope even goes in the wrong direction. The slope is −0.89 in the low payout ratio tercile but −1.73 in the high payout ratio tercile. The difference of 0.84 is more than 2.3 standard errors from zero. All the other results are largely similar to those in the benchmark estimation.

In summary, although the estimates generally go in the right direction, the empirical support for the q-theory explanation for the investment-to-assets and asset growth anomalies is not robust to controlling for the January effect and to including the standard controls such as size, book-to-market, and momentum in cross-sectional regressions. As in the benchmark estimation, the investment growth, net stock issues, abnormal corporate investment, and net operating assets anomalies are not supportive of the explanation based on q-theory with investment frictions.

4.3. Does q-theory with investment frictions have explanatory power for anomalies above and beyond limits-to-arbitrage?

Having shown the weak support of the q-theory explanation for the I/A and $\Delta A/A$ anomalies and the absence of support for the other anomalies, we turn our attention to limits-to-arbitrage. We address four related questions. First, do the anomalies vary in magnitude across subsamples split by proxies for limits-to-arbitrage? Second, could the lack of support for q-theory be due to limits-to-arbitrage? Intuitively, mispricing associated with limits-to-arbitrage can drive a wedge between expected returns and average future realized returns. This wedge can cause the empirical tests to fail even if the q-theory prediction holds for expected returns. We address this question by contrasting the significance of the q-theory prediction in the subsample with low limits-to-arbitrage (less contaminated by mispricing) to that in the subsample with high limits-to-arbitrage (more contaminated by mispricing).

Third, is the weak evidence supporting the q-theory explanation for the I/A and $\Delta A/A$ effects robust to controlling for limits-to-arbitrage as the alternative explanation? We address this question by checking whether the evidence in support of q-theory holds for I/A and $\Delta A/A$ regardless of whether proxies for limits-to-arbitrage are high or low. Fourth, is the support of the limits-to-arbitrage hypothesis robust to controlling for q-theory with investment frictions as the alternative explanation? We address this question by checking whether the evidence in support of limits-to-arbitrage holds regardless of whether proxies for investment frictions are high or low.

4.3.1. Do limits-to-arbitrage explain the investment-related anomalies?

At the end of June of each year t we split the sample into terciles based on idiosyncratic volatility and, independently, on the dollar trading volume. Unlike financing constraints proxies that are available only at the last fiscal year-end, we use information up to the end of June of year t to calculate the two limits-to-arbitrage proxies. Within each subsample, we run cross-sectional regressions of monthly percent excess returns from July of year t to June of year $t+1$ on a given investment-related anomaly variable for the fiscal year ending in calendar year $t-1$. We test whether the slope of the anomaly variable varies across the extreme limits-to-arbitrage terciles.

From Table 4, the difference in the I/A slope is -0.91 ($t=-4.20$) across the idiosyncratic volatility subsamples and is -0.73 ($t=-2.75$) across the trading volume terciles. The difference in the $\Delta A/A$ slope is -0.83 ($t=-5.65$) across the idiosyncratic volatility terciles and is -0.44 ($t=-2.19$) across the trading volume terciles. The difference in the NOA slope is -0.32 ($t=-2.39$) across the idiosyncratic volatility terciles and is -0.33 ($t=-2.19$) across the trading volume terciles. In

Table 4

Slopes from Fama and MacBeth (1973) cross-sectional regressions of monthly percent excess returns on anomaly variables in the subsamples split by limits-to-arbitrage measures (July 1963–December 2008, 558 months).

For each month from July of year t to June of year $t+1$, we estimate Fama and MacBeth cross-sectional regressions of monthly percent excess returns on a given anomaly variable for the fiscal year ending in calendar year $t-1$ in the extreme subsamples (terciles) split by a given limits-to-arbitrage measure. We split the sample at the end of June of each year t based on a given proxy measured using data up to June of year t. Ivol denotes idiosyncratic volatility and Dvol is dollar trading volume. I/A is investment-to-assets, $\Delta A/A$ is asset growth, $\Delta I/I$ is investment growth, NSI is net stock issues, ACI is abnormal corporate investment, and NOA is net operating assets. See the caption of Table 1 for detailed variable definitions. We report the slopes and their Fama and MacBeth t-statistics (in parentheses). We also report the t-statistics (in brackets) testing that a given slope is equal across extreme terciles split by a given limits-to-arbitrage measure. The time series average numbers of firms in the cross section for the full sample and for different subsamples are in curly brackets. Excess returns are in excess of one-month Treasury bill rate (from Kenneth French's website).

	I/A (1)	$\Delta A/A$ (2)	$\Delta I/I$ (3)	NSI (4)	ACI (5)	NOA (6)
Low Ivol	−0.10	−0.16	−0.02	−1.49	−0.01	−0.29
	(−0.56)	(−1.24)	(−0.65)	(−4.98)	(−0.32)	(−3.61)
	{1,052}	{1,052}	{1,052}	{1,052}	{1,042}	{1,052}
High Ivol	−1.01	−0.99	−0.10	−1.54	−0.05	−0.61
	(−5.95)	(−9.57)	(−5.10)	(−5.07)	(−1.25)	(−5.06)
	{1,021}	{1,021}	{1,021}	{1,021}	{1,011}	{1,021}
High-minus-low Ivol	[−4.20]	[−5.65]	[−2.72]	[−0.11]	[−0.77]	[−2.39]
Low Dvol	−1.18	−0.94	−0.09	−1.82	−0.12	−0.80
	(−6.08)	(−6.27)	(−4.30)	(−4.92)	(−2.94)	(−5.60)
	{922}	{922}	{922}	{922}	{916}	{922}
High Dvol	−0.45	−0.50	−0.09	−1.54	−0.02	−0.47
	(−2.20)	(−3.40)	(−2.65)	(−4.28)	(−0.40)	(−3.98)
	{954}	{954}	{954}	{954}	{948}	{954}
Low-minus-high Dvol	[−2.75]	[−2.19]	[−0.02]	[−0.61]	[−1.79]	[−2.19]

addition, the difference in the $\Delta I/I$ slope is significant across the idiosyncratic volatility terciles but is insignificant across the trading volume terciles. The difference in the *ACI* slope is marginally significant across the trading volume terciles but is insignificant across the idiosyncratic volatility terciles. Finally, the difference in the *NSI* slope is insignificant across both the idiosyncratic volatility terciles and across the trading volume terciles.

In summary, the benchmark estimation finds support for the limits-to-arbitrage hypothesis for the investment-to-assets, asset growth, and net operating assets anomalies, but not for the investment growth, abnormal corporate investment, and net stock issues anomalies.

Table 5 performs two robustness tests for limits-to-arbitrage proxies using the same test design as in Table 3 for financing constraints proxies. Panel A shows that dropping January returns does not materially affect the impact of idiosyncratic volatility on the anomalies. The differences in the $I/A,\Delta A/A,\Delta I/I$, and *NOA* slopes remain significant at the 5% level. However, controlling for the January effect reduces the effect of trading volume on all the anomalies to insignificance. The difference in the *I/A* slope is within 1.8 standard errors from zero, and the difference in the $\Delta A/A$ slope is within 0.9 standard errors across extreme trading volume terciles. In contrast, both differences are more than 2.2 standard errors from zero in the benchmark estimation.

Including standard controls such as size, book-to-market, and momentum into the cross-sectional regressions reduces the impact of idiosyncratic volatility on the $\Delta I/I$ anomaly and the impact of trading volume on the *NOA* anomaly to insignificant levels. From Panel B, the difference in the $\Delta I/I$ slope across the extreme idiosyncratic volatility terciles and the difference in the *NOA* slope across the extreme trading volume terciles are both within 1.5 standard errors from zero. Although somewhat weakened, all the other aspects of the results remain basically unchanged.

In summary, with idiosyncratic volatility as the proxy for limits-to-arbitrage, the empirical support for the mispricing hypothesis is robust for the *I/A*, $\Delta A/A$, and *NOA* anomalies, but not for $\Delta I/I$,*NSI*, and *ACI* anomalies. Although generally going in the right direction, the empirical support for the mispricing hypothesis using trading volume as the limits-to-arbitrage proxy is not

Table 5
Slopes from Fama and MacBeth (1973) cross-sectional regressions of monthly percent excess returns on anomaly variables in the subsamples split by limits-to-arbitrage measures, robustness checks.

In Panel A for each month from July of year t to June of year $t+1$ (except for January of year $t+1$) we run cross-sectional regressions of monthly percent excess returns on a given anomaly variable for the fiscal year ending in calendar year $t-1$ in extreme subsamples (terciles) split by a given limits-to-arbitrage measure. We split the sample at the end of June of each year t based on a given proxy measured using data up to June of year t. In Panel B the regressions are run with January returns as well as three controls: the log market capitalization at the end of June of year t, the log book-to-market equity as the log book equity for the fiscal year ending in $t-1$ minus the log market equity at the end of December of year $t-1$, and the log prior six-month returns (with one-month gap between the holding period and the current month). We report the slopes and their Fama and MacBeth t-statistics (in parentheses), as well as the t-statistics (in brackets) testing that a given slope is equal across extreme terciles split by a given limits-to-arbitrage measure. Excess returns are in excess of one-month Treasury bill rate (from Kenneth French's website). Ivol denotes idiosyncratic volatility and Dvol is dollar trading volume. *I/A* is investment-to-assets, $\Delta A/A$ is asset growth, $\Delta I/I$ is investment growth, *NSI* is net stock issues, *ACI* is abnormal corporate investment, and *NOA* is net operating assets. See the caption of Table 1 for detailed variable definitions. Book equity is the Compustat book value of stockholders' equity (Compustat annual item SEQ), plus balance sheet deferred taxes (item TXDB) and investment tax credit (item ITCI, if available), minus the book value of preferred stock. Depending on availability, we use redemption (item PSTKRV), liquidation (item PSTKL), or par value (item PSTK), in that order to estimate the book value of preferred stock. Market equity is price per share times the number of shares outstanding (SHROUT) from Center for Research in Security Prices.

	I/A	$\Delta A/A$	$\Delta I/I$	*NSI*	*ACI*	*NOA*
Panel A: No January returns (July 1963–December 2008, 513 months)						
Low Ivol	−0.07	−0.06	−0.03	−1.28	−0.01	−0.26
	(−0.40)	(−0.48)	(−1.07)	(−4.14)	(−0.16)	(−3.14)
High Ivol	−0.92	−0.89	−0.10	−1.82	−0.03	−0.60
	(−5.45)	(−8.75)	(−5.04)	(−6.34)	(−0.79)	(−4.90)
High-minus-low Ivol	[−3.82]	[−5.58]	[−2.19]	[−1.37]	[−0.49]	[−2.44]
Low Dvol	−0.90	−0.67	−0.09	−2.14	−0.06	−0.60
	(−4.64)	(−4.57)	(−4.51)	(−6.11)	(−1.56)	(−4.14)
High Dvol	−0.42	−0.50	−0.10	−1.71	0.01	−0.47
	(−1.93)	(−3.30)	(−2.94)	(−4.70)	(0.28)	(−3.76)
Low-minus-high Dvol	[−1.76]	[−0.81]	[0.28]	[−0.95]	[−1.35]	[−0.86]
Panel B: Controlling for size, book-to-market, and prior returns (July 1963–December 2008, 558 months)						
Low Ivol	0.01	−0.11	−0.03	−1.15	0.00	−0.33
	(0.04)	(−0.98)	(−1.49)	(−4.47)	(0.14)	(−4.34)
High Ivol	−0.83	−0.70	−0.08	−0.98	−0.04	−0.71
	(−5.17)	(−7.28)	(−4.08)	(−3.31)	(−1.09)	(−6.07)
High-minus-low Ivol	[−4.09]	[−4.41]	[−1.48]	[0.46]	[−0.91]	[−2.89]
Low Dvol	−0.90	−0.73	−0.07	−1.50	−0.07	−0.71
	(−5.01)	(−5.41)	(−3.57)	(−4.35)	(−2.17)	(−5.58)
High Dvol	−0.25	−0.36	−0.07	−1.38	−0.02	−0.50
	(−1.46)	(−3.40)	(−2.48)	(−4.79)	(−0.60)	(−4.91)
Low-minus-high Dvol	[−2.84]	[−2.26]	[−0.00]	[−0.26]	[−1.08]	[−1.41]

Table 6
Time series average numbers of firms in the cross section for the subsamples split jointly by limits-to-arbitrage measures and financing constraints measures (July 1963–December 2008, 558 months).

We split the full sample by an independent two by two sort on a given limits-to-arbitrage measure and a given financing constraints measure. In June of each year t we sort firms into two groups based on the median of a given limits-to-arbitrage measure and independently sort firms into two groups around the median of a given financing constraints measure for the fiscal year ending in calendar year $t-1$. Taking intersections partitions the full sample into four subsamples. The table reports the time series average number of firms in the cross section for each subsample. I/A is investment-to-assets, $\Delta A/A$ is asset growth, $\Delta I/I$ is investment growth, *NSI* is net stock issues, *ACI* is abnormal corporate investment, and *NOA* is net operating assets. The caption of Table 1 details the variable definitions.

		I/A	$\Delta A/A$	$\Delta I/I$	*NSI*	*ACI*	*NOA*
Panel A: Idiosyncratic volatility (Ivol) as the limits-to-arbitrage proxy							
Small asset	Low Ivol	397	397	397	397	393	397
	High Ivol	1,160	1,160	1,160	1,160	1,150	1,160
Big asset	Low Ivol	1,191	1,191	1,191	1,191	1,180	1,191
	High Ivol	395	395	395	395	391	395
Low payout ratio	Low Ivol	439	439	439	439	433	439
	High Ivol	1,058	1,058	1,058	1,058	1,048	1,058
High payout ratio	Low Ivol	1,146	1,146	1,146	1,146	1,136	1,146
	High Ivol	493	493	493	493	487	493
Without bond rating	Low Ivol	662	662	662	662	657	662
	High Ivol	1,021	1,021	1,021	1,021	1,013	1,021
With bond rating	Low Ivol	928	928	928	928	916	928
	High Ivol	538	538	538	538	530	538
Panel B: Dollar trading volume (Dvol) as the limits-to-arbitrage proxy							
Small asset	Low Dvol	1,034	1,034	1,034	1,034	1,026	1,034
	High Dvol	268	268	268	268	266	268
Big asset	Low Dvol	364	364	364	364	362	364
	High Dvol	1,163	1,163	1,163	1,163	1,156	1,163
Low payout ratio	Low Dvol	766	766	766	766	761	766
	High Dvol	521	521	521	521	517	521
High payout ratio	Low Dvol	628	628	628	628	624	628
	High Dvol	906	906	906	906	901	906
Without bond rating	Low Dvol	953	953	953	953	948	953
	High Dvol	498	498	498	498	495	498
With bond rating	Low Dvol	446	446	446	446	441	446
	High Dvol	934	934	934	934	927	934

robust for any of the six investment-related anomaly variables.

4.3.2. Doubt sorts

To address the remaining three questions, we split the sample jointly by a limits-to-arbitrage measure and an investment frictions measure. In June of each year t, we sort firms into two groups based on the median of a given limits-to-arbitrage proxy observed at the end of June of year t. We also independently sort firms into two groups around the median of a given financing constraints measure for the fiscal year ending in calendar year $t-1$. Taking intersections partitions the full sample into four subsamples. Using the two-by-two sort instead of a three-by-three sort ensures that there are a sufficient number of firms in a given subsample in any given year. Table 6 reports the time series average numbers of firms in the cross section for the subsamples split jointly by limits-to-arbitrage proxies and financing constraints measures. We estimate cross-sectional regressions of monthly percent excess returns from July of year t to June of year $t+1$ on a given anomaly variable for the fiscal year ending in calendar year $t-1$ in each subsample. We calculate the slope differences and their t-statistics across subsamples along a given proxy of limits-to-arbitrage or investment frictions.

To address whether the lack of support for q-theory is due to costly arbitrage that drives a wedge between expected returns and average realized returns, we contrast the significance of the evidence on the impact of investment frictions in the subsample with low limits-to-arbitrage to that in the subsample with high limits-to-arbitrage. If the lack of support for q-theory is caused by limits-to-arbitrage, we should see stronger evidence in support of investment frictions in the low limits-to-arbitrage subsample that is less affected by mispricing than in the high limits-to-arbitrage subsample that is more affected by mispricing.

The evidence reported in the upper halves of Panels A and B in Table 7 says otherwise. The vast majority of the t-statistics reported in brackets suggest that the slope differences across extreme investment frictions subsamples in the low limits-to-arbitrage subsample are insignificant. In particular, only in two (the I/A and *ACI* slope differences across the low and the high payout ratio terciles in the low idiosyncratic volatility subsample) out of 36 specifications are these slope differences significant. This evidence means that the wedge between expected and average returns caused by mispricing is not responsible for the lack of support for q-theory with investment frictions as an explanation for the anomalies.

To address whether the evidence supporting the q-theory explanation for the I/A and $\Delta A/A$ anomalies is

Table 7

Slopes from Fama and MacBeth (1973) cross-sectional regressions of monthly percent excess returns on anomaly variables in the subsamples split jointly by limits-to-arbitrage measures and financing constraints measures (July 1963–December 2008, 558 months).

We run univariate cross-sectional regressions of monthly percent excess returns from July of year t to June of year $t+1$ on a given anomaly variable for the fiscal year ending in calendar year $t-1$ in subsamples split by an independent two by two sort on a given limits-to-arbitrage measure and a given financing constraints measure. In June of each year t we sort firms into two groups based on the median of a given trading frictions measure and independently sort firms into two groups around the median of a given financing constraints measure for the fiscal year ending in calendar year $t-1$. Taking intersections partitions the full sample into four subsamples. I/A is investment-to-assets, $\Delta A/A$ is asset growth, $\Delta I/I$ is investment growth, *NSI* is net stock issues, *ACI* is abnormal corporate investment, and *NOA* is net operating assets. See the caption of Table 1 for detailed variable definitions. We report the slope differences and their Fama and MacBeth t-statistics (in brackets) across the subsamples. "Low Ivol, without-minus-with rating" is the difference between subsamples without and with bond ratings within the half of the sample consisting of firms with lower-than-medium idiosyncratic volatility. "Small asset, high-minus-low Ivol" is the difference between the two idiosyncratic volatility subsamples within the half of the sample consisting of firms with below-median asset size. Other various cuts of the sample are defined analogously. Excess returns are in excess of one-month Treasury bill rate (from Kenneth French's website). For firms with negative earnings but positive payout, we do not calculate their payout ratios but categorize them as least constrained (along with firms with high payout ratios).

	I/A	$\Delta A/A$	$\Delta I/I$	*NSI*	*ACI*	*NOA*
Panel A: Idiosyncratic volatility (Ivol) as the limits-to-arbitrage proxy						
Low Ivol,	0.06	0.04	−0.06	−0.58	−0.04	0.10
small-minus-big asset	[0.29]	[0.31]	[−1.68]	[−1.26]	[−0.87]	[0.89]
High Ivol,	−0.14	−0.16	0.01	−0.07	−0.01	0.05
small-minus-big asset	[−0.56]	[−1.06]	[0.38]	[−0.15]	[−0.25]	[0.36]
Low Ivol,	−0.40	−0.18	−0.05	−0.31	−0.12	−0.06
low-minus-high payout	[−2.07]	[−1.42]	[−1.58]	[−0.75]	[−2.62]	[−0.55]
High Ivol,	−0.16	−0.15	−0.01	0.47	0.00	−0.02
low-minus-high payout	[−0.70]	[−1.02]	[−0.25]	[0.99]	[0.05]	[−0.14]
Low Ivol,	−0.19	−0.15	−0.04	−0.29	−0.02	0.16
without-minus-with rating	[−1.13]	[−1.14]	[−1.49]	[−0.77]	[−0.41]	[1.69]
High Ivol,	−0.21	−0.33	−0.03	−0.04	0.08	−0.06
without-minus-with rating	[−1.03]	[−2.46]	[−1.10]	[−0.10]	[1.49]	[−0.52]
Small asset,	−0.63	−0.57	−0.01	0.83	0.03	−0.25
high-minus-low Ivol	[−2.85]	[−3.75]	[−0.56]	[1.80]	[0.73]	[−1.94]
Big asset,	−0.43	−0.37	−0.09	0.32	0.01	−0.20
high-minus-low Ivol	[−1.81]	[−2.43]	[−2.18]	[0.74]	[0.09]	[−1.58]
Low payout,	−0.38	−0.43	−0.02	0.54	0.09	−0.18
high-minus-low Ivol	[−1.92]	[−3.11]	[−0.81]	[1.26]	[1.85]	[−1.52]
High payout,	−0.61	−0.46	−0.06	−0.24	−0.03	−0.22
high-minus-low Ivol	[−2.38]	[−2.70]	[−1.84]	[−0.50]	[−0.50]	[−1.58]
Without rating,	−0.59	−0.61	−0.05	0.40	0.03	−0.32
high-minus-low Ivol	[−2.75]	[−4.15]	[−1.62]	[0.96]	[0.67]	[−2.68]
With rating,	−0.57	−0.43	−0.06	0.16	−0.06	−0.09
high-minus-low Ivol	[−2.42]	[−2.74]	[−1.63]	[0.35]	[−1.03]	[−0.68]
Panel B: Dollar trading volume (Dvol) as the limits-to-arbitrage proxy						
Low Dvol,	−0.96	−0.34	−0.06	−0.21	−0.10	−0.18
small-minus-big asset	[−3.08]	[−1.60]	[−1.30]	[−0.35]	[−1.62]	[−0.90]
High Dvol,	0.10	−0.10	−0.01	0.31	−0.10	0.17
small-minus-big asset	[0.28]	[−0.44]	[−0.21]	[0.39]	[−1.27]	[0.86]
Low Dvol,	−0.41	−0.21	−0.04	1.16	−0.03	0.06
low-minus-high payout	[−1.59]	[−1.20]	[−1.42]	[2.01]	[−0.58]	[0.38]
High Dvol,	−0.33	−0.13	−0.02	0.35	−0.05	0.09
low-minus-high payout	[−1.40]	[−0.85]	[−0.58]	[0.65]	[−0.81]	[0.59]
Low Dvol,	−0.57	−0.71	−0.03	−0.62	0.04	−0.18
without-minus-with rating	[−2.02]	[−3.68]	[−0.82]	[−1.10]	[0.82]	[−1.13]
High Dvol,	−0.37	−0.25	−0.06	−0.25	0.08	−0.04
without-minus-with rating	[−1.68]	[−1.64]	[−1.70]	[−0.58]	[1.48]	[−0.28]
Small asset,	−0.80	−0.37	−0.04	−0.51	0.00	−0.28
low-minus-high Dvol	[−2.28]	[−1.57]	[−0.82]	[−0.65]	[0.05]	[−1.37]
Big asset,	0.26	−0.13	0.01	0.01	0.01	0.07
low-minus-high Dvol	[1.00]	[−0.58]	[0.14]	[0.01]	[0.11]	[0.42]
Low payout,	−0.57	−0.38	−0.01	−0.15	−0.03	−0.26
low-minus-high Dvol	[−2.40]	[−2.23]	[−0.36]	[−0.25]	[−0.61]	[−1.68]
High payout,	−0.49	−0.30	0.01	−0.96	−0.05	−0.23
low-minus-high Dvol	[−2.09]	[−1.59]	[0.22]	[−1.94]	[−0.99]	[−1.51]
Without rating,	−0.50	−0.44	0.00	−0.26	−0.10	−0.22
low-minus-high Dvol	[−2.00]	[−2.51]	[0.15]	[−0.53]	[−1.88]	[−1.53]
With rating,	−0.30	0.03	−0.03	0.11	−0.07	−0.08
low-minus-high Dvol	[−1.04]	[0.16]	[−0.74]	[0.21]	[−1.16]	[−0.44]

robust to controlling for limits-to-arbitrage as the alternative explanation, we check whether the evidence in support of q-theory holds regardless of whether proxies for limits-to-arbitrage are high or low. The answer is provided by the first two columns in the upper halves of Panels A and B in Table 7. Although in the vast majority of cases, the sign of the slope differences goes in the right direction as predicted by q-theory, only in five out of 24 specifications are the slope differences in I/A or $\Delta A/A$ significant across extreme investment frictions subsamples. Further, four out of the five significant cases are in the high limits-to-arbitrage subsample (with either high idiosyncratic volatility or low trading volume). As such, the evidence in support of q-theory for the I/A and $\Delta A/A$ effects is not robust to controlling for limits-to-arbitrage proxies.

To address whether the evidence in support of a limits-to-arbitrage explanation for the anomalies is robust to controlling for different levels of investment frictions, we check whether the significance of limits-to-arbitrage holds regardless of whether proxies for investment frictions are high or low. The results provided by the lower halves of Panels A and B in Table 7 suggest that it does. Panel A shows that in 10 out of 12 specifications the slope differences are significant at the 5% level across the high and low idiosyncratic volatility subsamples. Even in the two insignificant cases, the slope differences are more than 1.8 standard errors from zero. Panel B shows that in six out of 12 specifications the slope differences are significant across the low and the high trading volume subsamples. The evidence clearly suggests that limits-to-arbitrage proxies dominate investment frictions proxies in direct comparisons. Again, the support for q-theory with investment frictions as an explanation of anomalies is not robust to controlling for mispricing associated with limits-to-arbitrage.

5. Conclusion

We make two contributions to the literature. First, we use a two-period q-theory model to show theoretically that the expected return–investment relation should be steeper in firms with high investment frictions than in firms with low investment frictions. With frictions, investment entails investment costs, and higher investment entails higher investment costs, causing investment to be less elastic to changes in the discount rate. The higher are the investment costs, the less elastic investment is in responding to changes in the discount rate. A given magnitude change in investment corresponds to a higher magnitude change in the discount rate, meaning that the expected return–investment relation is steeper, the greater the investment costs.

Second, using financing constraints as proxies for investment frictions, we examine the prediction of q-theory that investment costs make the relations of expected returns with investment-to-assets and asset growth steeper. Overall support for the q-theory prediction is weak. Accounting for investment costs does not enable q-theory to explain the investment growth, net stock issues, abnormal corporate investment, or net operating assets anomalies. More important, proxies for limits-to-arbitrage motivated by mispricing dominate proxies for investment frictions motivated by q-theory in direct comparisons, suggesting that mispricing better explains the anomalies in question.

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