# Internet Appendix: "Unemployment Crises" (For Online Publication Only)

## A Data

We detail our procedures to construct the historical samples for unemployment rates in Appendix A.1, vacancy rates in Appendix A.2, and labor productivity in Appendix A.3.

## A.1 Unemployment Rates

Appendix A.1.1 constructs the historical series for civilian unemployment rates, and Appendix A.1.2 for private nonfarm unemployment rates from January 1890 to December 2017.

#### A.1.1 Civilian Unemployment Rates

From January 1948 to December 2017, we use the seasonally adjusted civilian unemployment rate series from Bureau of Labor Statistics at US Department of Labor. From January 1930 to December 1947, we use the Denton (1971) proportional first difference procedure to interpolate Weir's (1992) annual series of civilian unemployment rates.<sup>1</sup> The monthly indicator series in the interpolation is the monthly unemployment rates from NBER macrohistory files (chapter 8: Income and employment).<sup>2</sup> The monthly average of the interpolated series in a given year is set to equal the year's annual value in Weir.

We construct the monthly indicator series from January 1930 to December 1947 as follows:

- From January 1930 to February 1940, we use as the monthly indicator values the seasonally adjusted unemployment rates from NBER macrohistory series m08292a (April 1929–June 1942. Source: National Industrial Conference Board, published by G. H. Moore Business Cycle Indicators, vol. II, p. 35 and p. 123).
- From March 1940 to December 1946, we use as the monthly indicator values the seasonally adjusted unemployment rates from NBER macrohistory series m08292b (March 1940–December 1946. Source: US Bureau of the Census, Current Population Reports, Labor Force series P-50, no. 2, 13, and 19).
- From January 1947 to December 1947, we construct the monthly indicator values as follows. We first obtain the monthly unemployment rates (not seasonally adjusted) from

<sup>&</sup>lt;sup>1</sup>We use the proportional first difference variant of the Quilis (2013) "denton\_uni" routine in Matlab.

<sup>&</sup>lt;sup>2</sup>http://www.nber.org/databases/macrohistory/contents/chapter08.html

January 1947 to December 1966 from NBER macrohistory series m08292c. (Source: Employment and Earnings and Monthly Report on the Labor Force, vol. 13, no. 9, March 1967). We pass the entire 1947–1966 series through the X-12-ARIMA seasonal adjustment program from US Census Bureau. We then take the seasonally adjusted series from January 1947 to December of 1947.

From January 1890 to December 1929, we use the Denton procedure interpolate Weir's (1992) annual civilian unemployment rates with the credit spread series from NBER macrohistory files as the monthly indicator. (Monthly unemployment rates are not available prior to April 1929 in NBER macrohistory files.) We again require the monthly average of the interpolated series in a given year to equal the year's annual value in Weir.

We construct the monthly indicator series from January 1890 to December 1929 as follows:

- We obtain American railroad bond yields, high grade, from NBER macrohistory series m13019 (January 1857–January 1937. Source: Frederick R. Macaulay, 1938, Appendix Table 10, p. A142–A161) and US railroad bond yields index from NBER macrohistory series m13019a (January 1857–December 1934, Source: Frederick R. Macaulay, 1938, Appendix Table 10, column 4, p. A142–A161). Subtracting series m13019 from m13019a yields the railroad credit spread series from January 1857 to December 1934.
- We obtain Moody's seasoned Aaa and Baa corporate bond yields, monthly, from Federal Reserve Bank of St. Louis from January 1919 to June 2018. Subtracting the Aaa yields from the Baa yields gives the credit spread series over the same sample period.
- We quarterly splice the railroad credit spread series to the Moody's credit spread series in the first quarter of 1919. Specifically, we rescale the railroad series so that its average in the first quarter of 1919 equals the average of the Moody's series in the same quarter. Doing so yields an uninterrupted credit spread series from January 1857 to June 2018. We take the values from January 1890 to December 1929 as the monthly indicator.

#### A.1.2 Private Nonfarm Unemployment Rates

From January 1890 to December 1947, we use the Denton (1971) proportional first difference procedure to interpolate Weir's (1992, Table D3, last column) annual series of private nonfarm unemployment rates. The monthly indicator from January 1890 to December 1929 is the spliced credit spread series constructed in Appendix A.1.1. From January 1930 onward, the monthly indicator is the monthly unemployment rate series obtained from NBER

macrohistory files (Appendix A.1.1). The monthly average of the interpolated series in a given year is set to that year's annual value in Weir.

From January 1948 to December 2017, we follow Weir (1992) to calculate private nonfarm unemployment rates as (Civilian labor force — Civilian employment)/(Civilian labor force — (Farm employment + Government employment)). In the numerator, both terms should subtract the sum of farm and government employment to yield private nonfarm labor force and private nonfarm employment, respectively. As such, the numerator is simply civilian unemployment, which we measure as series LNS13000000 from the Current Population Survey (CPS, January 1948—December 2017, seasonally adjusted). Civilian labor force is the CPS series LNS11000000 (January 1948—December 2017, seasonally adjusted).

We back out the sum of farm and government employment as the CPS civilian employment (series LNS12000000, January 1948–December 2017, seasonally adjusted) minus private nonfarm employment from the Current Employment Statistics (CES) survey (series CES0500000001, January 1939–December 2017, seasonally adjusted). In all, private nonfarm unemployment rates equal Civilian unemployment/(Civilian labor force — (Civilian employment from CPS — Private nonfarm employment from CES)).

## A.2 Vacancy Rates

We construct a historical series for the vacancy rates from January 1919 to December 2017.

#### A.2.1 Vacancy Series

From December 2000 to December 2017, we obtain the seasonally adjusted job openings (series JTS00000000JOL, total nonfarm, level in thousands) from the Job Openings and Labor Turnover Survey (JOLTS). This series contains government job openings. Because the series for government job openings are not available prior to December 2000 when JOLTS becomes available, we use total nonfarm job openings (instead of total private nonfarm job openings, series JTS10000000JOL) throughout the long historical sample to be consistent.

From January 1995 to November 2000, we use the seasonally adjusted composite print and online help-wanted index from Barnichon (2010). The Barnichon series, ranging from January 1995 to December 2014, is from Regis Barnichon's Web site. We quarterly splice the Barnichon series to the JOLTS series in the first quarter of 2001. Quarterly splicing means that we rescale the Barnichon series so that its monthly average in the first quarter of 2001 equals the monthly average of the JOLTS series in the same quarter.

From January 1951 to December 1994, we use the seasonally adjusted help-wanted advertising index from the Conference Board. The Conference Board series goes from January 1951 to June 2010. We quarterly splice the Conference Board series to the (spliced) Barnichon series in the first quarter of 1995. Quarterly splicing means that we rescale the Conference Board series so that its monthly average in the first quarter of 1995 equals the monthly average of the Barnichon series (already spliced to the JOLTS series per the last paragraph) in the same quarter. We switch to the Barnichon series in January 1995 because advertising for jobs over the internet has become more and more prevalent since the mid-1990s, making the print help-wanted index from the Conference Board increasingly unrepresentative. A comparison between between the Barnichon series and the Conference Board series shows that the two series have diverged significantly since 1996. From January 1919 to December 1950, we use the Metropolitan Life Insurance company (MetLife) help-wanted advertising index. The MetLife series is from the NBER macrohistory files (series m08082a, January 1919–August 1960, not seasonally adjusted). To seasonally adjust the series, we pass the entire series through the X-12-ARIMA program from US Census Bureau. We then quarterly splice the seasonally adjusted MetLife series to the (spliced) Conference Board series in the first quarter of 1951. Quarterly splicing means that we rescale the MetLife series so that its monthly average in the first quarter of 1951 equals the monthly average of the Conference Board series (already spliced to the rescaled Barnichon series per the last paragraph) in the same quarter.

#### A.2.2 Labor Force Series

To convert the vacancy series into a series of vacancy rates, we need a series of the labor force. From January 1948 to December 2017, we obtain the monthly civilian labor force over 16 years of age from the Current Population Survey released by BLS (series LNS11000000, seasonally adjusted, number in thousands). No additional adjustment is necessary.

From January 1890 to December 1947, we use Weir's (1992) annual series of civilian labor force (1890–1990, 14 years and older through 1946, 16 and older afterward, number in thousands). We use the Denton proportional first difference procedure to interpolate Weir's annual series to monthly, using a vector of ones as the indicator. Because the labor force is a stock variable, we require the first monthly observation of a given year to equal that year's observation in Weir. We then annually splice the interpolated Weir series to the CPS series in the year of 1948. Annual splicing means that we rescale the interpolated Weir series so that its monthly average in 1948 equals the monthly average of the CPS series in the same year.

#### A.2.3 Vacancy Rates Series

Dividing the vacancy series in Appendix A.2.1 by the labor force series in Appendix A.2.2 yields a long historical series of vacancy rates from January 1919 to December 2017.

# A.3 Labor Productivity

To construct a historical series of labor productivity from January 1890 to December 2017, we calculate the ratio of private nonfarm real output over private nonfarm employment.

#### A.3.1 Nonfarm Business Real Output

We obtain the following raw real output data:

- Private nonfarm real gross domestic product, 1889–1957, annual, Kendrick (1961, Table A-XXIII, p. 338–340, http://www.nber.org/chapters/c2246.pdf).
- Nonfarm business real gross value added in billions of chained (2012) dollars, Table 1.3.6., line 3, annual, 1929–2017, National Income and Product Accounts (NIPA), Bureau of Economic Analysis.
- Nonfarm business real output index, quarterly, from the first quarter of 1947 to the fourth quarter of 2017, from Bureau of Labor Statistics (BLS, series PRS85006043).
- The Miron-Romer (1990, Table 2, p. 336–337) monthly index of industrial production, January 1884–December 1940, not seasonally adjusted. We set the missing value in March 1902 to the average value in February and April of 1902. For seasonal adjustment, we pass the series through the X-12-ARIMA program from US Census Bureau.
- Industrial production index from Federal Reserve Bank of St. Louis (series INDPRO), monthly, January 1919—December 2017. Seasonally adjusted.

We adopt the following procedure to adjust the raw output data:

• We quarterly splice the seasonally adjusted Miron-Romer industrial production series to the Federal Reserve series in the first quarter of 1919. Specifically, we rescale the Miron-Romer series so that its monthly average in the first quarter of 1919 equals the monthly average of the Federal Reserve series in the same quarter. Splicing gives us an interrupted series of industrial production from January 1884 to December 2017.

- We annually splice the Kendrick's (1961) nonfarm business real output series from 1889 to 1929 to the NIPA nonfarm business real output series from 1929 to 1947 in the year 1929. We rescale the Kendrick series so that its value in 1929 equals the value for the NIPA series in the same year. Splicing gives us an uninterrupted annual real output series from 1889 to 1947.
- From January 1889 to December 1947, we use the Denton proportional first difference procedure to interpolate the annual nonfarm business real output, with the monthly industrial production series as the indicators.
- From January 1947 to December 2017, we use the Denton proportional first difference procedure to interpolate the BLS quarterly series of nonfarm business real output, with the monthly industrial production series as the indicators.
- We quarterly splice the above two monthly series of nonfarm business real output in the first quarter of 1947. The pre-1947 series is scaled so that its monthly average in the first quarter of 1947 equals the monthly average of the post-1947 series in the same quarter.

#### A.3.2 Private Nonfarm Employment

We obtain the following raw private nonfarm employment data:

- Private nonfarm employment from Weir (1992, Table D3, annual series, 1890–1947). We calculate private nonfarm employment as total civilian employment minus farm employment minus government employment, all of which are from Weir's Table D3.
- Private nonfarm employment from Current Employment Statistics released by BLS (series CES0500000001, number in thousands, seasonally adjusted, monthly series, January 1939–December 2017).
- Index of factory employment, NBER macrohistory series m08005, monthly, January 1889–December 1923. Not seasonally adjusted. Source: H. Jerome, Migration and Business Cycles, NBER Publication 9, p. 248. For seasonal adjustment, we pass the entire series through the X-12-ARIMA program from US Census Bureau.
- Total production worker employment in manufacturing, NBER macrohistory series m08010b, monthly, number in thousands, January 1919–March 1969. Not seasonally adjusted. Source: BLS Bulletin, Employment and Earnings Statistics for the Unites States, 1909–1960 (for 1919–1958), and 1909–1966 (for 1959–1967); Employment and

Earnings (for September 1967–March 1969). For seasonal adjustment, we pass the entire series through the X-12-ARIMA program.

We adopt the following procedure to adjust the raw employment data:

- We quarterly splice NBER macrohistory series m08005 to NBER macrohistory series m08010b, both seasonally adjusted, in the first quarter of 1919. In particular, we rescale the seasonally adjusted series m08005 so that its monthly average in the first quarter of 1919 equals the monthly average of the seasonally adjusted series m08010b in the same quarter. Doing so yields an uninterrupted monthly employment series from January 1889 to December 1939.
- We use this monthly employment series as the indicator in the Denton proportional first difference procedure to interpolate Weir's annual series from 1890 to 1939. Because private nonfarm employment is a stock variable, we require the first monthly observation of a given year to equal that year's observation in Weir. Doing so yields a monthly private nonfarm employment series from January 1890 to December 1939.
- We quarterly splice this monthly private nonfarm employment series to the CES monthly series around the first quarter of 1939. We rescale the interpolated Weir series so that its monthly average in the first quarter of 1939 equals the monthly average of the CES series in the same quarter. Doing so yields a monthly series from January 1890 to December 2017.

#### A.3.3 Private Nonfarm Labor Productivity

We first obtain the nonfarm business real output per job from BLS (series PRS85006163, quarterly, 1947Q1-2017Q4, index, base year = 2012).

We then divide the monthly nonfarm business real output series constructed in Appendix A.3.1 by the monthly private nonfarm employment series constructed in Appendix A.3.2 to obtain a monthly labor productivity series from January 1890 to December 2017.

From January 1947 to December 2017, we use the Denton proportional first difference procedure to benchmark our monthly labor productivity series to the BLS quarterly labor productivity series. Specifically, we impose the average of our monthly series within a given quarter to equal that quarter's BLS observation. Alas, the standard Denton procedure induces a transient, artificial movement at the beginning of the series (Dagum and Cholette 2006, Chapter 6). We remove this transient movement with the Cholette (1984) modification.

Finally, from January 1890–December 1947, we quarterly splice (in the first quarter of 1947) our monthly labor productivity series to the benchmarked monthly series from 1947 onward. In particular, we scale the pre-1947 series so that its monthly average in the first quarter of 1947 equals the monthly average of the post-1947 series.

## A.4 Real Wage Rates

We calculate normal wage rates as compensation of employees by industry (NIPA Table 6.2, line 3 [private industries] minus line 5 [farms]) divided by full-time and part-time employees by industries (NIPA Table 6.4, line 3 [private industries] minus line 5 [farms]). Sample is annual from 1929 onward. The index of Consumer Price Index (CPI) is for all urban consumers (series: CUUR0000SA0) from BLS. The series is monthly from January 1913 onward. We take the monthly averages within a year to obtain the year's annual value. Finally, we compute real wage rates as the normal wage rates divided by the corresponding year's CPI.

# **B** Derivations

The wage offer of a worker to the firm,  $W'_t$ , can be expressed as:

$$W_t' = X_t + (1 - \delta)\chi + \beta E_t \left[ (1 - s)S_{Nt+1}^{W'} - (1 - \delta)S_{Nt+1}^{W} \right]. \tag{A.1}$$

Intuitively,  $W'_t$  increases in labor productivity,  $X_t$ , and the cost of delay to the firm,  $\chi$ . A higher  $\chi$  makes the firm more likely to accept a higher wage offer from the worker to avoid any delay. As  $W'_t$  contains a higher constant proportion because of a higher  $\chi$ ,  $W'_t$  becomes more insulated from labor market conditions. Because  $W'_t$  is the flow value of  $J^{W'}_{Nt}$  from equation (6),  $J^{W'}_{Nt}$  also becomes more insulated. More important, as  $J^{W'}_{Nt}$  enters the second term in equation (7), the equilibrium wage,  $W_t$ , also becomes more insulated to aggregate conditions as a result of a higher  $\chi$ . From the last term in equation (A.1), an increase in the separation rate reduces the wage offer from the worker to the firm,  $W'_t$ . As s rises, the present value of profits produced by the worker drops. To make the firm indifferent, the worker must reduce the wage offer. Also, the worker's offer,  $W'_t$ , increases in the firm's surplus from accepting the offer,  $S^{W'}_{Nt+1}$ . In contrast, the worker's offer would be lower if the firm's surplus,  $S^{W}_{Nt+1}$ , from rejecting the offer to make a counteroffer,  $W_t$ , is higher. However, the quantitative impact of this channel would be negligible if the breakdown probability,  $\delta$ , goes to one. As such, the worker's offer,  $W'_t$ , increases with  $\delta$ .

To prove equation (7), we plug equations (5) and (6) into equation (3) to obtain:

$$W_{t} + \beta E_{t} \left[ (1 - s) J_{Nt+1}^{W} + s J_{Ut+1} \right] = b + \delta \beta E_{t} \left[ f(\theta_{t}) J_{Nt+1}^{W} + (1 - f(\theta_{t})) J_{Ut+1} \right] + (1 - \delta) \beta E_{t} \left[ J_{Nt+1}^{W'} \right]. \tag{A.2}$$

Solving for  $W_t$  yields:

$$W_{t} = b + [\delta f(\theta_{t}) - (1-s)]\beta E_{t} \left[ J_{Nt+1}^{W} \right] + [\delta (1-f(\theta_{t})) - s]\beta E_{t} \left[ J_{Ut+1} \right] + (1-\delta)\beta E_{t} \left[ J_{Nt+1}^{W'} \right].$$
 (A.3)

Rearranging the right-hand side yields equation (7).

To characterize the worker's counteroffer,  $W'_t$ , as in equation (A.1), we first rewrite the firm's value recursively (while marking the dependence of  $S_t$  on  $W_t$  with the notation  $S_t^W$ ):

$$S_t^W = X_t N_t - W_t N_t - \kappa_t V_t + \lambda_t q(\theta_t) V_t + \beta E_t \left[ S_{t+1}^W \right], \tag{A.4}$$

The first-order condition with respect to  $V_t$  yields:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = \beta E_t \left[ S_{Nt+1}^W \right]. \tag{A.5}$$

Also, replacing  $W_t$  with  $W'_t$  in equation (A.4) and differentiating with respect to  $N_t$  yield:

$$S_{Nt}^{W'} = X_t - W_t' + (1 - s)\beta E_t \left[ S_{Nt+1}^{W'} \right]. \tag{A.6}$$

Plugging equation (A.5) into the firm's indifference condition (4) yields:

$$S_{Nt}^{W'} = (1 - \delta) \left[ \frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi \right]. \tag{A.7}$$

Combining with equation (A.6) yields:

$$X_t - W_t' + (1 - s)\beta E_t \left[ S_{Nt+1}^{W'} \right] = (1 - \delta) \left[ \frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi \right].$$
 (A.8)

Isolating  $W'_t$  to one side of the equation:

$$W'_{t} = X_{t} + (1 - \delta)\chi + (1 - s)\beta E_{t} \left[ S_{Nt+1}^{W'} \right] - (1 - \delta) \left[ \frac{\kappa_{0}}{q(\theta_{t})} + \kappa_{1} - \lambda_{t} \right]$$
 (A.9)

$$= X_t + (1 - \delta)\chi + (1 - s)\beta E_t \left[ S_{Nt+1}^{W'} \right] - (1 - \delta)\beta E_t \left[ S_{Nt+1}^{W} \right]$$
 (A.10)

$$= X_t + (1 - \delta)\chi + \beta E_t \left[ (1 - s)S_{Nt+1}^{W'} - (1 - \delta)S_{Nt+1}^{W} \right], \tag{A.11}$$

which is identical to equation (A.1). Leading equation (A.7) by one period, plugging it along with equation (A.5) into equation (A.8), and solving for  $W'_t$  yield:

$$W_t' = X_t - (1 - \delta) \left( \left( \frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t - \chi \right) - (1 - s)\beta E_t \left[ \frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} - \chi \right] \right). \tag{A.12}$$

Finally, the two parties of the credible bargaining game would agree to accept the equilibrium wage, only if the joint surplus of the match is greater than the joint value of the outside options,  $J_{Ut}$ , as well as the joint present value of continuous delaying:

$$S_{Nt}^W + J_{Nt}^W > \max\left(J_{Ut}, E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} (b - \chi)\right]\right) = J_{Ut}. \tag{A.13}$$

The last equality holds because the flow value of unemployment, b, is higher than  $b - \chi$  (the delaying cost is positive). We verify that this condition holds in simulations. We further characterize the agreement condition (A.13) as follows. Rewriting equation (A.6) with  $W_t$  and combining with equation (A.5) yield  $S_{Nt}^W = X_t - W_t + (1-s) \left[\kappa_0/q(\theta_t) + \kappa_1 - \lambda_t\right]$ . As such, the agreement condition becomes:

$$X_t - W_t + (1 - s) \left( \frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t \right) + J_{Nt}^W > J_{Ut}. \tag{A.14}$$

Although equations (A.1) and (A.13) are easier to interpret, we implement equations (A.12) and (A.14) in our numerical algorithm.

# C Computation

We adopt the globally nonlinear projection algorithm in Petrosky-Nadeau and Zhang (2017). Because of risk neutrality and constant returns to scale, the state space consists of only log productivity,  $x_t$ . Employment,  $N_t$ , is not a state variable, although the unemployment and vacancy rates do depend on  $N_t$ . In particular, we need to solve for the labor market tightness,  $\theta_t = \theta(x_t)$ , the multiplier function,  $\lambda_t = \lambda(x_t)$ , the equilibrium wage,  $W_t = W(x_t)$ , the

worker's wage offer,  $W'_t = W'(x_t)$ , from the following five functional equations:

$$\frac{\kappa_{t}}{q(\theta_{t})} - \lambda(x_{t}) = \beta E_{t} \left[ \exp(x_{t+1}) - W(x_{t+1}) + (1-s) \left( \frac{\kappa_{t+1}}{q(\theta_{t+1})} - \lambda(x_{t+1}) \right) \right]$$

$$(A.15)$$

$$W(x_{t}) = b + (1-\delta)\beta E_{t} \left[ J_{N}^{W'}(x_{t+1}) - J_{U}(x_{t+1}) \right]$$

$$- (1-s-\delta f(\theta_{t})) \beta E_{t} \left[ J_{N}^{W}(x_{t+1}) - J_{U}(x_{t+1}) \right]$$

$$(A.16)$$

$$J_U(x_t) = b + \beta E_t \left[ f(\theta_t) J_N^W(x_{t+1}) + (1 - f(\theta_t)) J_U(x_{t+1}) \right]$$
(A.17)

$$J_N^W(x_t) = W(x_t) + \beta E_t \left[ (1-s)J_N^W(x_{t+1}) + sJ_U(x_{t+1}) \right]$$
(A.18)

$$J_N^{W'}(x_t) = W'(x_t) + \beta E_t \left[ (1-s)J_N^{W'}(x_{t+1}) + sJ_U(x_{t+1}) \right]. \tag{A.19}$$

In addition,  $\theta(x_t)$  and  $\lambda(x_t)$  must also satisfy the Kuhn-Tucker conditions.

We parameterize the conditional expectation in equation (A.15) as  $\mathcal{E}_t \equiv \mathcal{E}(x_t)$ , and four other functions,  $W(x_t)$ ,  $J_U(x_t)$ ,  $J_N^W(x_t)$ , and  $J_N^{W'}(x_t)$ . Following Christiano and Fisher (2000), we exploit a convenient mapping from  $\mathcal{E}_t$  to policy and multiplier functions to eliminate the need to parameterize the multiplier function separately. After obtaining the parameterized  $\mathcal{E}_t$ , we first calculate  $\tilde{q}(\theta_t) \equiv \kappa_t/\mathcal{E}_t$ . If  $\tilde{q}(\theta_t) < 1$ , the nonnegativity constraint is not binding, we set  $\lambda_t = 0$  and  $q(\theta_t) = \tilde{q}(\theta_t)$ . We then solve  $\theta_t = q^{-1}(\tilde{q}(\theta_t))$ , in which  $q^{-1}(\cdot)$  is the inverse function of  $q(\cdot)$ . If  $\tilde{q}(\theta_t) \geq 1$ , the nonnegativity constraint is binding, we set  $\theta_t = 0$ ,  $q(\theta_t) = 1$ , and  $\lambda_t = \kappa_t - \mathcal{E}_t$ . We approximate the log productivity,  $x_t$ , with the discrete state space method of Rouwenhorst (1995). We use 17 grid points to cover the values of  $x_t$ , which are precisely within four unconditional standard deviations from the mean of zero. We use extensively the approximation toolkit in the Miranda and Fackler (2002) CompEcon Toolbox in Matlab. To obtain an initial guess, we use the loglinear solution from Dynare to a simplified model without the fixed matching cost.

# D Supplementary Results

We furnish additional quantitative results from the model.

# D.1 Characterizing the Stationary Distribution

We simulate the model economy for one million monthly periods from its stationary distribution. To reach the stationary distribution, we start at the initial condition of zero for the log labor productivity and simulate the economy for 6,000 months. Panel A of Figure A.1 presents the scatter plot of the unemployment rate against the log productivity in simula-

tions. The relation is strongly nonlinear. When the log productivity is above its mean of zero, unemployment goes down only slightly. However, when the log productivity is below its mean, unemployment goes up drastically. The correlation between unemployment and log productivity is -0.82. Panel B plots the vacancy rate,  $\theta_t$ , against the log productivity. Although the relation is nonlinear, the nonlinearity is not nearly as strong as that of unemployment in Panel A. The  $V_t$ - $x_t$  correlation is near perfect at 0.96. Panels C and D report the empirical cumulative distribution functions of the unemployment and vacancy rates. Unemployment is highly skewed. Its 2.5 percentile, 4.03%, is close to the median, 5.61%, but the 97.5 percentile is far away, 17.37%. The 99 percentile is 21.36%, and the maximum rate is almost 50%. In contrast, the empirical distribution of the vacancy rates is close to symmetric. Its 2.5, 50, and 97.5 percentiles are 2.62%, 6.06%, and 12.11%, respectively.

# D.2 Comparative Statics on Labor Market Volatilities

Table A.1 reports comparative statics on labor market volatilities in normal periods. The results are largely consistent with those in Hall and Milgrom (2008).

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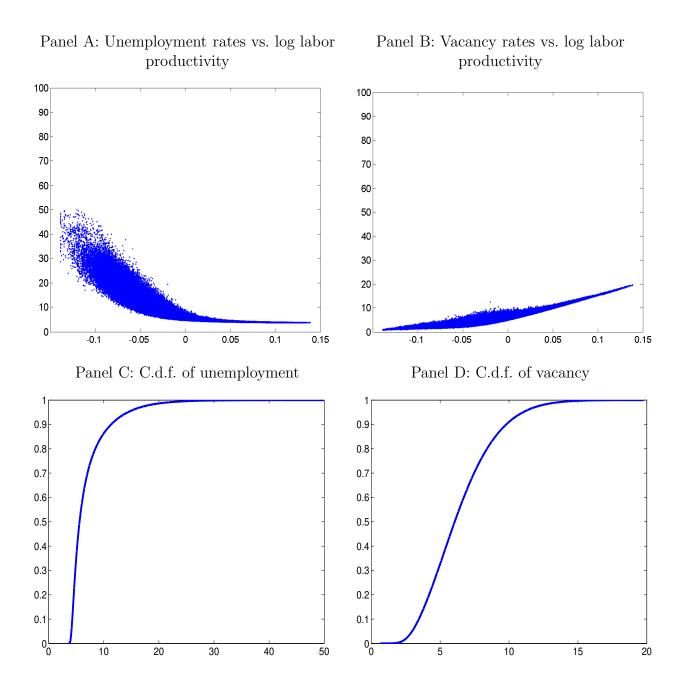
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Table A.1: Comparative Statics, Second Moments in the Labor Market in Normal Periods

This table reports six comparative static experiments: (i) the probability of breakdown in bargaining  $\delta = 0.15$ ; (ii) the delaying cost  $\chi = 0.2$ ; (iii) the proportional cost of vacancy  $\kappa_0 = 0.15$ ; (iv) the fixed cost of vacancy  $\kappa_1 = 0.3$ ; (v) the separation rate s = 0.04; and (vi) the curvature parameter of the matching function  $\iota = 0.9$ . In each experiment, all the other parameters remain identical to those in the benchmark calibration. For each experiment, we simulate 25,000 artificial samples (each with 1,536 months) from the model's stationary distribution. We split the samples into two groups: non-crisis samples (in which the maximum unemployment rate is less than 15%) and crisis samples (in which the maximum rate is greater than or equal to 15%). On each non-crisis sample, we implement the same procedure as in Table 1 and report the cross-simulation averages.

	U	V	$\theta$	X	_	U	V	$\theta$	X
	Panel A: $\delta = 0.15$					Panel B: $\chi = 0.2$			
Volatility	0.1	0.132	0.214	0.013		0.068	0.137	0.195	0.013
Persistence	0.819	0.625	0.772	0.774		0.79	0.698	0.769	0.776
Correlation		-0.705	-0.9	-0.859	U		-0.776	-0.896	-0.777
			0.943	0.948	V			0.974	0.96
				0.984	$\theta$				0.947
	Panel C: $\kappa_0 = 0.15$					Panel D: $\kappa_1 = 0.3$			
Volatility	0.119	0.187	0.284	0.013		0.114	0.163	0.257	0.013
Persistence	0.801	0.652	0.764	0.772		0.806	0.63	0.764	0.771
Correlation		-0.712	-0.886	-0.793	U		-0.703	-0.892	-0.814
			0.955	0.952	V			0.948	0.945
				0.959	$\theta$				0.964
	Panel E: $s = 0.04$				=,	Panel F: $\iota = 0.9$			
Volatility	0.117	0.163	0.259	0.013		0.108	0.153	0.24	0.013
Persistence	0.804	0.614	0.759	0.767		0.831	0.604	0.765	0.769
Correlation		-0.706	-0.896	-0.832	U		-0.685	-0.886	-0.852
			0.947	0.945	V			0.945	0.937
				0.97	$\theta$				0.98



**Fig. A.1.** The unemployment-log labor productivity relation, the vacancy-log labor productivity relation, and empirical cumulative distribution functions (C.d.f.) of unemployment and vacancy rates in the model. *Note*: Results are based on the one-million-month simulated data from the model's stationary distribution. The unemployment and vacancy rates are in percent.

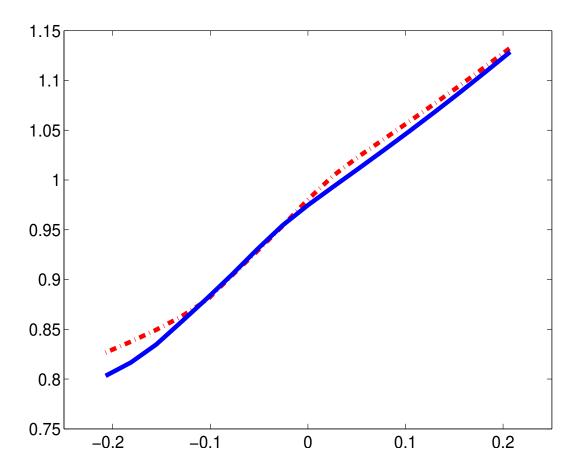


Fig. A.2. The equilibrium wage rate under the benchmark calibration and the low-δ-high- $\chi$  calibration. Note: On the grid of the log labor productivity,  $x_t$ , this figure plots the equilibrium wage rate,  $W_t$ , against  $x_t$ . The blue solid line is for the benchmark calibration, in which the probability of bargaining breakdown,  $\delta = 0.1$ , and the delay cost  $\chi = 0.25$ . The red broken line is for an alternative calibration with  $\delta = 0.075$  and  $\chi = 0.3$ .