

# Factors War<sup>1</sup>

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This essay describes the  $q$ -factor model that Prof. Kewei Hou at The Ohio State University, Prof. Chen Xue at University of Cincinnati, and I jointly developed. Our paper, titled “Digesting anomalies: An investment approach,” was published as the leading article in the March 2015 issue of *Review of Financial Studies*. Since publication, the  $q$ -factor model has rapidly become a leading, workhorse model for estimating expected stock returns in empirical finance.

## 1. Background

Modern academic finance consists of two large building blocks, Corporate Finance and Asset Pricing. Corporate Finance studies optimal investment and financing policies of firms, and Asset Pricing studies optimal portfolio choices of investors and the determinants of expected security returns. Modern academic finance was born in the early 1950s. From 1960s to late 1980s, the leading asset pricing model is the Capital Asset Pricing Model, The CAPM, developed by Sharpe (1964) and Lintner (1965). In the early 1990s, in response to empirical failures of the CAPM, Fama and French (1993) propose their three-factor model. Carhart (1997) adds the momentum factor into the three-factor model to form a four-factor model. The Fama-French-Carhart four-factor model has been the dominant model in empirical finance in the past two decades. Subsequent to our work on the  $q$ -factor model in Hou, Xue, and Zhang (2015), Fama-French (2015) incorporate two new factors that resemble our  $q$ -factors into their original three-factor model to form a five-factor model.

The CAPM predicts that a stock’s expected risk premium equals its market risk times the expected market risk premium. Empirically, in the regression of the stock’s realized risk premiums on realized market risk premiums, the intercept measures the stock’s abnormal return (alpha), and the slope measures its market risk (beta). If the CAPM is correct, alpha should not be statistically different from zero.

Originally derived from an individual investor’s optimal portfolio choice problem, the CAPM has a solid theoretical foundation. Alas, the empirical literature in finance and accounting has accumulated a mountain of patterns in the data that the CAPM cannot explain. These patterns are often referred to as anomalies. In Hou, Xue, and Zhang (2016), we examine in total 437 variables, and find that 161-216 variables have statistically significant predictive power for future stock returns, depending on specific empirical procedures.

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<sup>1</sup> This is an English translation of my article published at *Tsinghua Financial Review* 37, 101-104, in Chinese.

The voluminous literature on asset pricing anomalies shows that it is necessary to seek for a better model than the CAPM. Building on the CAPM, Fama and French (1993) add a size factor (SMB) and a value factor (HML). Size is measured as a stock's market equity, and value is measured as the ratio of a stock's book value of equity divided by its market equity. Carhart (1997) subsequently adds a momentum factor (UMD). Although widely used, these models are entirely empirical in nature, devoid of theoretical foundation. As such, it is an open question why these models would work in the data. Data mining, time-varying expected returns, or mispricing?

Our *q*-factor model has four factors: (i) the market factor; (ii) the size factor; (iii) the (real) investment factor, in which investment is measured as the total growth rate of book assets; and (iv) the return-on-equity (ROE) factor, in which ROE is quarterly earnings scaled by one-quarter-lagged book equity, and measures a firm's accounting profitability.

The *q*-factor model has won the "endorsement" from Fama and French (2015), who incorporate similar investment and profitability factors into their three-factor model to form a five-factor model. Although their construction procedure differs somewhat from our original investment and profitability factors, the economic concept underlying their factors is (virtually) identical to ours.

**Table 1 : History, the *q*-factor model versus the Fama-French five-factor model**

<b>The <i>q</i>-factor model:</b>	
Neoclassical factors	July 2007
An equilibrium three-factor model	January 2009
Production-based factors	April 2009
A better three-factor model that explains more anomalies	June 2009
An alternative three-factor model	April 2010, April 2011
Digesting anomalies: An investment approach	October 2012, August 2014
<b>The Fama-French five-factor model:</b>	
A four-factor model for the size, value, and profitability patterns in stock returns	June 2013
A five-factor asset pricing model	November 2013, September 2014

Most important, our *q*-factor model predates the Fama-French five-factor model by three to six years. Table 1 reports the historical evolution of the two competing models. It took eight long years for the *q*-factor paper to progress from the conception of initial ideas to formal publication. The first draft, titled "Neoclassical factors," was circulated in July 2007 as NBER working paper #13282. The first draft already used the investment factor to replace the value factor in Fama and French (1993), and the profitability factor to replace the momentum factor in Carhart (1997) as key factors for the cross section of expected stock returns. Afterward, the title of each subsequent draft changed from the last, because I was trying hard to avoid the title in the June-2009 draft. While going through peer reviews at Journal of Finance, I was urged repeatedly to use the title "A better three-factor model that explains more anomalies." Unfortunately, in the first four drafts, including the June-2009 draft, the empirical procedure of the ROE factor was flawed. The same flawed procedure was also used to form monthly sorted testing portfolios. After the procedure

was corrected in the April-2010 draft, the paper limped for one more year at Journal of Finance, and was formally rejected in April 2011. Persisting through the darkest times of my career, I rebuilt my team with Prof. Kewei Hou and Prof. Chen Xue. We put a new draft together in October 2012, with the title “Digesting anomalies: An investment approach” (NBER working paper #18435). After almost two years of rigorous peer reviews and revisions, the paper was accepted in August 2014, and published in March 2015 at Review of Financial Studies.

Fama and French (2013) initially only added a profitability factor into their three-factor model. Subsequent drafts, starting from the November-2013 draft, also added an investment factor to form their five-factor model, presumably to compete with the  $q$ -factor model. Isaac Newton once said: “If I have seen further it is by standing on the shoulders of Giants.” I often tell self-mocking jokes to my MBA students that in the case of the five-factor model, it is Giants who are standing on our dwarf shoulders. And it took a lot out of us not to be crushed.

## 2. Foundation

An important advantage of the  $q$ -factor model over traditional models is its economic, conceptual foundation. In economics, the neoclassical  $q$ -theory of investment is the leading economic theory for explaining firms’ real investment behavior. The theory’s essence is just the Net Present Value (NPV) rule of capital budgeting in Corporate Finance. The NPV of a project is its present value (discounted value of future cash flows) minus its investment costs today. The NPV rule says that a manager should invest in a given project, if and only if the present value of the project is greater than or equal to its investment costs. For a manager, it is possible that there exist many initial projects with nonnegative NPVs. The manager would start with the project with the highest NPV, and work her way down the supply of projects. A good project would have a low discount rate, high profitability, and low investment costs. As the manager invests in more and more projects, their investment costs would become higher and higher, and their profitability lower and lower. For the last infinitesimal project that the manager takes, its NPV equals zero:

$$\text{Investment costs} = \text{Present value} = \text{Profitability}/\text{Discount rate}.$$

The imagination of the  $q$ -factor model is to use the NPV rule, which is a fundamental principle in Corporate Finance, as an Asset Pricing model. Traditional Asset Pricing models, such as the CAPM, are derived from the perspective of individual investors, having no direct bearings with firms’ accounting variables. But after half a century of the CAPM, all we have to show for is the anomalies literature. In contrast, the  $q$ -factor model is motivated from a fundamentally new perspective based on the value maximization of firms. Rewriting the NPV rule yields:

$$\text{Discount rate} = \text{Profitability}/\text{Investment costs}.$$

Relative to profitability, high investment firms incur higher investment costs, meaning that their discount rates and expected returns must be low. Relative to investment, high profitability firms must have high discount rates and high expected returns. In all, investment and profitability are the key determinants of the cross section of expected stock returns.

### 3. Evidence

The  $q$ -factor model performs well in the data. Hou, Xue, and Zhang (2016) show that from January 1967 to December 2014, the investment factor earns on average 0.43% per month, or 5.2% per annum, and the ROE factor earns on average 0.56% per month, or 6.7% per annum. Both  $t$ -statistics are above 5, meaning that the average returns of the  $q$ -factors are very reliable. The Carhart four-factor model cannot explain the  $q$ -factor returns. Regressing our investment factor on the Carhart model yields an alpha of 0.29% per month ( $t=4.57$ ), and regressing our ROE factor on the Carhart model yields an alpha of 0.51% ( $t=5.58$ ). The Fama-French five-factor model cannot explain the  $q$ -factor returns either. The five-factor alpha of our investment factor is 0.12% ( $t=3.35$ ), and the five-factor alpha of our ROE factor is 0.45% ( $t=5.6$ ).

For the Fama-French five-factor model in the same sample, their profitability factor, denoted RMW, earns on average 0.27% per month ( $t=2.58$ ), or 3.24% per annum. Their investment factor, CMA, earns on average 0.34% per month ( $t=3.63$ ), or 4.08% per annum. The Carhart four-factor model cannot explain the average returns of the new factors. The Carhart alpha of RMW is 0.33% per month ( $t=3.31$ ), and the Carhart alpha of CMA is 0.19% ( $t=2.83$ ). More important, the  $q$ -factor model completely captures the “new” Fama-French factors. Regressing RMW returns on the  $q$ -factor model yields a tiny alpha of 0.04% ( $t=0.42$ ), and regressing CMA returns on the  $q$ -factor model yields a miniscule alpha of 0.01% ( $t=0.32$ ). In all, the evidence from the factor spanning tests shows that the five-factor model cannot explain the  $q$ -factor premiums, but the  $q$ -factor model can entirely explain the five-factor premiums.

I also compare the performance of the  $q$ -factor model with the performance of the Fama-French five-factor model in explaining several classic anomalies, including value, momentum, and quality. A common measure of value is book-to-market equity, Bm, defined as the book equity (total assets minus total debt) divided by the market equity of a public company. The high-minus-low Bm decile earns on average 0.59% per month ( $t=2.84$ ), or 7.1% per annum. The  $q$ -factor model reduces the significant average return to an insignificant alpha of 0.18% per month ( $t=1.15$ ). The investment factor plays a key role in doing so, with a loading of 1.33 ( $t=13$ ). In addition, because of its value factor, the Fama-French five-factor model reduces the average return of the high-minus-low Bm decile to only 0.01% ( $t=0.12$ ).

A popular measure of momentum investing is prior six-month returns,  $R^6$ . The high-minus-low  $R^6$  decile earns on average 0.82% per month ( $t=3.49$ ), or 9.84% per annum. Its  $q$ -factor alpha, however, is only an insignificant 0.24% per month ( $t=0.78$ ). The ROE factor is the key driving force of this result, with a high loading of 0.99 ( $t=5.33$ ). Intuitively, stocks with high prior six-month returns are also more profitable than stocks with low prior six-month returns. Controlling for ROE goes a long way toward explaining momentum profits. In contrast, the Fama-French five-factor model cannot explain momentum, with an alpha of 0.97% ( $t=3.5$ ). As a profitability factor, their RMW is ineffective.

Finally, I measure quality as the four-quarter change in ROE, denoted dRoe1, defined as the latest quarter's ROE minus the ROE from four quarters ago. The high-minus-low dRoe1 decile earns on average 0.76% per month ( $t=5.43$ ), or 9.12% per annum. The  $q$ -factor model reduces the average return to 0.34%, albeit still significant ( $t=2.29$ ). The five-factor model is again ineffective, with an alpha of 0.79% ( $t=5.39$ ). In all, the  $q$ -factor model is more effective than the five-factor model in explaining anomalies.

#### 4. Application

Factor investing has become increasingly popular in the investment management industry. Quantitative investment strategies have long been adopted by hedge funds in search of superior returns. Figure 1 reports the performance of two hypothetical index funds based on  $q$ -factors. Suppose we invest \$1 in the S&P 500 index fund in January 1967. The \$1 investment would grow to \$117 by December 2014. Among the two  $q$ -funds, the big-ME-low-I/A-high-ROE fund value-weights all stocks traded on NYSE, Amex, and NASDAQ that simultaneously have market equity above the median NYSE market equity, investment-to-assets below the bottom 30th percentile at NYSE, as well as ROE above the top 30th percentile at NYSE. With the same \$1 initial investment in January 1967, this fund would grow to \$369 by December 2014. Most important, the small-ME-low-I/A-high-ROE fund value-weights all stocks that simultaneously have market equity below the NYSE median but above its bottom 20th percentile, investment-to-assets below the bottom 30th percentile at NYSE, as well as ROE above the top 30th percentile at NYSE. With the same \$1 initial investment in January 1967, this fund would grow by December 2014 to \$9,402! The annualized return across the 48-year period is 20.5%.

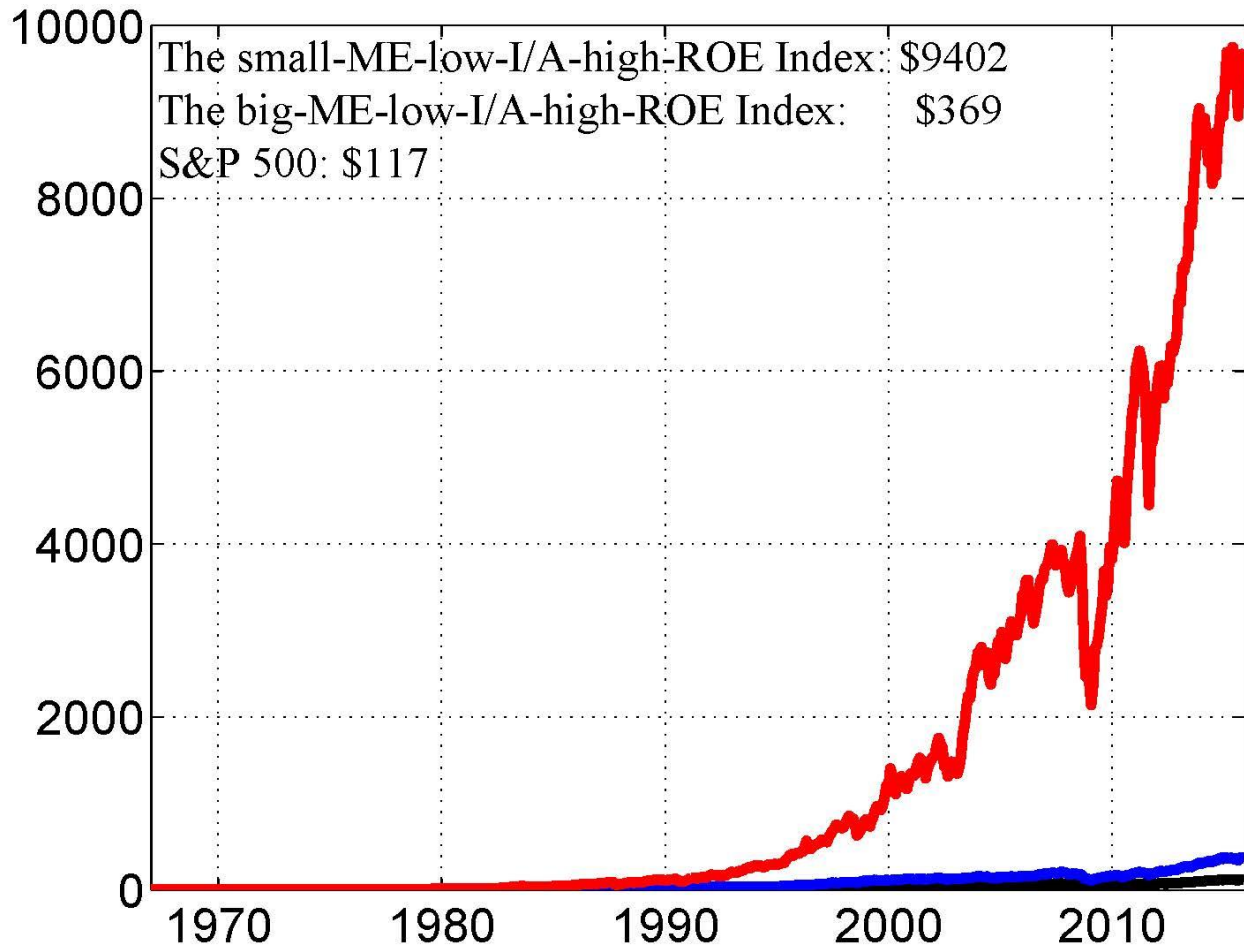
It should be emphasized that this fund comes with risk as well. During the 2007-2009 financial crisis, the cumulative return of this fund drops from around \$4,000 to \$2,000. Afterward, the fund's performance rebounds rapidly, as the crisis subsides. The figure only serves to illustrate the business potential of the  $q$ -factor model in the investment management industry. In practice, a portfolio manager can apply proper risk management techniques to alleviate the downside risk.

#### 5. Conclusion

This essay summarizes the  $q$ -factor model that Prof. Kewei Hou, Prof. Chen Xue, and I jointly developed. In our model, a stock's expected return is described by its exposures to four factors, including the market, size, investment, and ROE factors. Upon publication, the model has rapidly established itself as a leading, workhorse model for estimating expected stock returns in empirical finance. Looking forward, we are applying the  $q$ -factor model to global stock markets, including developed markets other than the U.S., as well as emerging markets such as China.

**Figure 1 : Hypothetical  $q$ -funds, January 1967-December 2014**

This figure reports cumulative returns of three index funds, defined as wealth accumulated from an initial investment of \$1 in January 1967. The black line is for the S&P 500 fund, the blue line for the big-ME-low-I/A-high-ROE fund, and the red line for the small-ME-low-I/A-high-ROE fund. The returns are value-weighted.



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# 资产定价中的因子大战

本文介绍了笔者与俄亥俄州立大学侯恪惟教授、辛辛那提大学薛辰教授共同研究开发的q-因子模型。在该模型中，一只股票的预期风险溢价由四个因子决定：市场因子、市值因子、投资因子和盈利因子。论文正式发表后，q-因子模型已迅速成为金融界实证资产定价中的主导模型之一。展望未来，q-因子模型将拓展到全球金融市场中，比如中国A股市场。

## 资产定价模型概述

现代金融学分两大块，公司金融和资产定价。公司金融研究的主要问题是公司最优实体投资与融资，资产定价研究的问题则是最优证券组合和股票预期收益率的决定因素。现代金融学诞生于20世纪50年代初期。从60年代到80年代后期，主导资产定价模型是Sharpe和Lintner推导的资本资产定价模型（The Capital Asset Pricing Model, CAPM）。Sharpe是1990年诺贝尔经济学奖获得者之一，而CAPM是他获奖的主要贡献。从90年代初期到最近，主导模型是Fama-French的三因子模型。Fama是2013年诺奖获得者之一，而三因子模型则被评选委员会肯定为金融学过去25年最重大的成就之一。Carhart把动量因子加入Fama-French三因子模型而得出Carhart四因子模型。该模型是近20年来运用最多的实证资产定价模型。在我们提出q-因子模型之后，Fama-French把两个与q-因子相似的因子加入他们的三因子模型得出了五因子模型。

在资本资产定价模型中，股票i的预期风险溢价和市场预期风险溢价成正比，正比成多少由该股票的市场风险决定。在实证中把股票i的实际超额收益率回归到市场组合的实际超额收益率。截距是股票i的异常收益率（ $\alpha$ ），而斜率是股票的市场风险（ $\beta$ ）。如果资本资产定价模型是

正确的话， $\alpha$  不应和零有很大区别（统计不应显著）。资本资产定价模型是按照个人投资者最优证券组合原理推导的，具有良好的经济学基础。但是实证金融与会计学的异常文献积累了很多资本资产定价模型不能解释的现象。我们在最近的一篇工作论文中，一共检查了437个异常变量。我们发现在这些变量中一共有161~217个显著变量。也就是说有这么多指标可以用来预期股票的预期未来收益率。利用这些指标可以获得比市场组合更高的收益率。

大量的异常变量表明有必要寻找比资本资产定价模型更好的模型。Fama和French在1993年提出了一个三因子模型，在CAPM基础上，加入了市值因子（SMB）和价值因子（HML）。市值是指股票的市场价值，价值是公司的账面价值除以其市场价值的比率。Carhart在1997年提出的四因子模型进一步加入了动量因子（UMD）。虽然这些模型主导了实证资产定价领域二十多年，但是它们只是实证模型，没有经济学基础。所以人们经常争论为什么这些因子在实践中能工作？是靠运气，还是基本经济学原理？

## q-因子模型概述

我们的q-因子模型有四个因子。第一个是市场因子，包含了宏观经济和总体市场的风险溢价。第二个是市值因子。两个新因子，一个是投

表1 q-因子模型和Fama-French五因子模型的历史演变

q-因子模型:	
新古典因子	2007年7月
一个均衡三因子模型	2009年1月
基于生产的因子	2009年4月
一个更好的能解释更多异常现象的三因子模型	2009年6月
一个替代的三因子模型	2010年4月, 2011年4月
解析异常现象: 基于投资的方法	2012年10月, 2014年8月
Fama-French五因子模型:	
一个解释股票收益率中市值, 价值, 与盈利率效应的四因子模型	2013年6月
一个五因子资产定价模型	2013年11月, 2014年9月

资因子。投资指的是实体投资而非金融投资, 例如建造一个工厂, 买新设备等。还有一个是盈利因子。盈利是会计盈利率, 等于利润除以账面资本。在实证中做一个多元回归, 把股票超额收益率回归到四个因子的收益率。如果我们的模型是正确的话, 异常收益率就应该在统计上为零。但即使为零, 并不证明CAPM异常现象不存在, 也不意味着没有量化投资的余地。异常收益率在q-因子模型中为零说明各种不同的异常变量其实只是投资因子和盈利因子不同的组合。也就是说q-因子模型在实证中非常有用。

q-因子模型得到了Fama-French的认可。他们在2015年把和q因子很相似的投资与盈利因子加入到三因子模型中, 而得到一个五因子模型。虽然他们新因子的具体建造方式和我们的投资与盈利因子略有不同, 这些变量的概念在经济学上基本和q-因子相同。更重要的是, q-因子模型比Fama-French五因子模型要早三到六年。表1描述这两个模型的历史演变。q-因子模型从最初理念的形成为最终论文的发表历时长达八年。第一稿题为《新古典因子》*Neoclassical factors*早在2007年7月就已出炉(美国国家经济研究局工作论文13282)。第一稿就已经用投资

因子来取代Fama-French的价值因子, 用盈利因子来取代Carhart的动量因子, 作为股票收益率横切面的主导因子。之后每一稿的标题都在变化, 原因是我在极力避免2009年6月稿的标题(一位资深审稿人在《金融期刊》*Journal of Finance*再三敦促我用《一个更好的能解释更多异常现象的三因子模型》作为标题)。不幸的是那一稿和之前的几稿中, 盈利因子的建造程序有失误。纠正失误后, 这篇论文继续在《金融期刊》延口残喘了一年, 终于在2011年4月遭拒。之后我重起炉灶, 和二位新合作者侯恪惟与薛辰教授在2012年10月写好新一稿, 题为《解析异常现象: 基于投资的方法》(美国国家经济研究局工作论文18435)。又经过近两年的严格审稿与修改, 终于在2014年8月被《金融研究评论》*Review of Financial Studies*接受, 并在2015年3月发表。

Fama-French起初于2013年6月提出一个四因子模型, 把一个盈利因子加入他们的三因子模型。在当年11月又将一个投资因子加到四因子模型中, 而得出了现在的五因子模型, 与我们的q-因子模型抗衡。在学术界有句俗话, 写好论文需要站在巨人肩膀上。我常在博士生面前自嘲, 说Fama-French五因子模型是巨人站在了我们的肩膀上, 还差点没给我压死。

### q-因子模型的经济基础

和以往的因子模型不同的是, 我们的q-因子模型有很扎实的经济学基础。在经济学理论中, 有一个实体投资经济学理论, 叫q-理论。这其实就是公司金融学中的净现值原则(NPV rule)。净现值原则说的是, 如果项目现值大于投资成本, 则应当投资该项目; 如果项目现值小于投资成本, 则不应当投资。对于公司首席财务官来说, 起初会有很多实体项目可以进行投资, 此时应优先投资最好的项目。好的项目折现率低, 盈利率高, 项目现值也高。随着被投资的好项目越来越多, 投资成本会慢慢变高, 盈利率会越来越低。投资的最后一个项目应该是净现值(项目现值减投资成本)为零,



即投资成本等于项目现值：投资成本 = 项目现值 = 盈利率/折现率。现实生活中经常会有多年的投资项目。以上公式中，我们做了简化，假定项目明年就会完成。盈利率越高，折现率不变，项目现值就越高，投资就越多。当然投资越多，投资成本也会越高。净现值原则体现在最后一个项目上就是投资成本等于项目现值。这是经济学中的边际原则。

q-因子模型最富有想象力和创造力的是把公司金融原则当作资产定价模型。传统资产定价理论从个人投资者最优证券组合的角度出发，和公司变量没有直接关系。但是那一老套做了近半个世纪，结果只是大量异常现象。q-因子模型开辟了一个新的途径。变换一下净现值原则得出：

折现率 = 盈利率/投资成本。相对于盈利率，投资越多的公司，折现率越低，股票预期收益率也越低。相当于投资，盈利率越高的公司，折现率越高，股票预期收益率也越高。不同于传统资产定价理论，我们的理论是基于公司行为，看问题的角度有巨大的转变。总的来说，投资和盈利率是股票预期收益率的关键决定因素。这就是建造投资因子和盈利因子的经济学理论基础。

## q-因子模型的实证

q-因子模型在实证中表现如何呢？从1967年1月到2014年12月，在美国样本中，投资因子每月有0.43%的平均收益率，相当于5.2%的年化收益率。盈利因子每月有0.56%的平均收益率，相当于6.7%的年化收益率。两个平均收益率的t-统计量都超过5，表明非常统计显著。也就是说q-因子的高平均收益率不是靠样本中的运气，而是非常可靠的。Carhart四因子模型不能解释q-因子的平均收益率。把q-模型中的投资因子回归到Carhart四因子上得到的异常收益率是每月0.29%（t=4.57），而把q-模型中的盈利因子回归到Carhart四因子上得到的异常收益率是每月0.51%（t=5.58）。Fama-French五因子模型也不能解释q-因子的平均收益率。把q-模型中的投资因子回归到Fama-French五因子上得到的异常收益率是每月0.12%（t=3.35），而把q-模型中的盈利因子回归到Fama-French五因子上得到的异常收益率是每月0.45%（t=5.6）。

Fama-French五因子中的盈利因子，RMW，每月平均收益率为0.27%（t=2.58），年化收益率为3.24%。五因子模型中的投资因子，CMA，每月平均收益率为

0.34%（t=3.63），年化收益率为4.08%。Carhart四因子模型不能解释五因子的平均收益率。把RMW收益率回归到Carhart四因子上得到的异常收益率是每月0.33%（t=3.31），把CMA回归到Carhart四因子上得到的异常收益率是每月0.19%（t=2.83）。最重要的是，q-因子模型完全能够解释RMW和CMA。把RMW收益率回归到q-因子上得到的异常收益率只有每月0.04%（t=0.42），而把CMA回归到q-因子上得到的异常收益率只有每月0.01%（t=0.32）。这些异常收益率基本为零。因此，实证证据表明五因子模型不能解释q-因子收益率，但是q-因子模型却能解释五因子收益率。这些测试表明q-因子模型比五因子模型在实证中更为强大。

下面我再来比较q-因子模型和五因子模型对几个著名异常现象的解释能力。有几个异常现象非常流行，在量化投资领域应用非常广泛。第一个是价值投资，第二个是动量投资，第三个是质量投资。价值投资源于Graham和Dodd在1934年出版的古典投资著作《证券分析》*Security Analysis*。巴菲特就是应用价值理念最成功的投资者。在量化领域，价值投资在20世纪80年代开始被广泛运用，动量投资在90年代开始广泛运用，而质量投资则在近十年开始被应用。

价值投资一个最重要的指标是账面市场价值比，Bm，定义为一家上市公司的帐面股本价值（总资产减去总负债）除以股本市场价值。在每年6月底，我们按照纽约证券交易所Bm分布，把所有在纽约证券交易所，美国证券交易所，和纳斯达克证券交易所交易的股票排序，再分配到10个十分位组合中。我们从本年7月到下年6月持有这些组合，并计算市值加权组合收益率。最后，我们买进10%最高Bm组合，同时卖出10%最低Bm组合，而形成高减低Bm十分位组合。该高减低组合有每月0.59%的平均收益率（t=2.84），相当于7.1%的年化率。q-因子模型把该平均收益率减到0.18%（t=1.15）。投资因子起到关键作用，其载荷为1.33（t=13）。由于Fama-French五因子模型有价值因子，该模型把高减低Bm组合的平均收益率一直降到基本为零。

动量投资最常用的指标是最近6个月的复合收益率，R<sup>6</sup>。在每月初，我们按照纽约证券交易所的R<sup>6</sup>分布，对所有股票进行排序，再分配到10个十分位组合中。我们从下个月开始持有这些组合6个月，并计算市值加权组合收益

率。最后我们买进10%最高 $R^6$ 组合，同时卖出10%最低 $R^6$ 组合，形成高减低 $R^6$ 十分位组合。该高减低组合有每月0.82%的平均收益率（ $t=3.49$ ），相当于9.84%的年化率。 $q$ -因子模型成功地把该显著的平均收益率化解到非显著的每月0.24%（ $t=0.78$ ）。其中盈利因子起到关键作用，其载荷为0.99（ $t=5.33$ ）。Fama-French五因子模型不能解释动量投资。五因子异常收益率高达0.97%（ $t=3.5$ ）。

对质量投资，我们用四季度的股本盈利率变化， $dRoe1$ 。股本盈利率是利润除以上季度账面股本。 $dRoe1$ 则等于最新股本盈利率减去四季度以前的股本盈利率。在每月初，我们按照纽约证券交易所的 $dRoe1$ 分布，对所有股票进行排序，再分配到10个十分位组合中。我们持有这些组合一个月，并计算市值加权组合收益率。最后我们买进10%最高 $dRoe1$ 组合，同时卖出10%最低 $dRoe1$ 组合，形成高减低 $dRoe1$ 十分位组合。该高减低组合有每月0.76%的平均收益率（ $t=5.43$ ）。 $q$ -因子模型把该显著的平均收益率化解到每月0.34%，但还是保持显著

（ $t=2.29$ ）。Fama-French五因子模型对解释质量投资基本无效。五因子异常收益率高达0.79%（ $t=5.39$ ）。

### $q$ -因子模型的应用

因子投资在全球金融市场上有长足的应用。量化投资很长时间都是对冲基金最常用的策略之一。近年来几家大投资管理公司，比如黑岩（BlackRock）和道富银行（State Street），都通过在证券交易所交易的基金（Exchange Traded Fund, ETF）大规模从事因子投资。

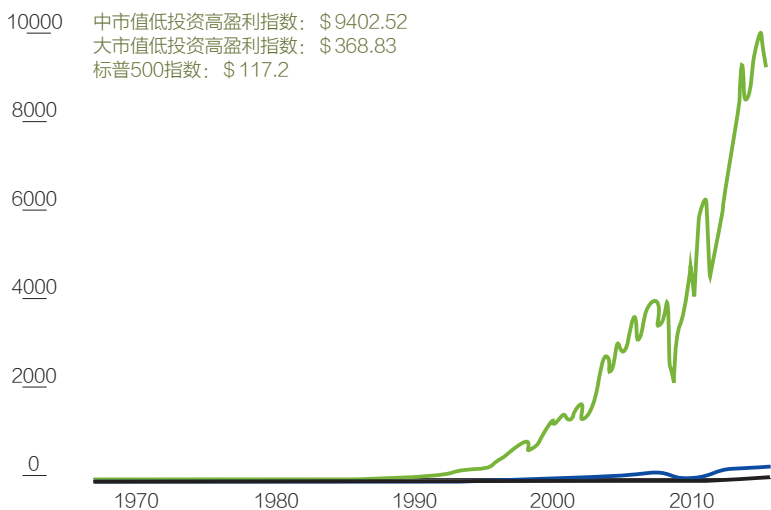
图1报告两只 $q$ -因子模拟指数基金的累积收益率，并与标普500指数基金做比较。假设我们在1967年1月初做1美元的初始投资在标普500基金。到2014年12月底，该初始投资能涨到117.2美元。在两只 $q$ -因子模拟指数基金中，大市值低投资高盈利指数包括所有在纽约证券交易所、美国证券交易所和纳斯达克上交易的股票，同时按照纽约证券交易所股票的分布，是属于大市值，低投资，和高盈利的股票。同样在1967年1月初做1美元的初始投资，该指数到2014年12月底能涨到368.83美元。最重要的是中市值低投资高盈利指数。该指数包括所有同时是中市值、低投资和高盈利的股票。同样是1967年1月初做1美元的初始投资，该指数到2014年12月底能涨到9402.52美元！该48年的累积收益率相当于年化收益率20.5%。

当然中市值低投资高盈利指数不是没有风险的。在2007-2009年的次债危机中，该指数累积收益率从4000美元掉至2000美元左右。但在市场稳定后，该指数反弹的速度非常快。图1只是对 $q$ -因子的潜在商业价值做一个简单说明。在实际操作中，可以对 $q$ -因子指数基金进行恰当的风险控制，以减小像在次债危机中高回撤率的可能性。

展望未来，我们正在把 $q$ -因子模型拓展到全球金融市场中，包括除美国以外的其他发达市场，还有新兴市场，比如中国A-股市场。①

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图1  $q$ -因子模拟指数基金，美国样本，1967年1月到2014年12月



本图描述的是三个指数基金的累积收益率，定义为从1967年1月初始投资1美元累积起来的财富。黑线指的是标普500指数基金；蓝线是大市值低投资高盈利指数；绿线是中市值低投资高盈利指数。大市值是指市值大于纽约证券交易所市值中值的股票；中市值是指小于纽约证券交易所市值中值，但大于该交易所20%最小市值的股票。低投资是指实体投资除以资产规模比率属于纽约证券交易所所30%最低的股票。高盈利是指利润除以资本规模比率属于该交易所30%最高的股票。每月初我们按照纽约证券交易所的市值、投资和盈利分布，把所有纽约证券交易所、美国证券交易所和纳斯达克的股票进行排序。大市值低投资高盈利指数包括所有同时是大市值、低投资和高盈利的股票；中市值低投资高盈利指数包括所有同时是中市值、低投资和高盈利的股票。我们用市值加权来计算指数收益率。