Internet Appendix (for Online Publication Only): "Searching for the Equity Premium"

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Abstract

This Internet Appendix details derivations and supplementary results for our manuscript "Searching for the equity premium" to appear at *Journal of Financial Economics*.

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1 Derivations

1.1 The Stock Return

Equation (4) implies that the marginal products of capital and labor are given by, respectively:

$$\frac{\partial Y_t}{\partial K_t} = \frac{Y_t}{K_t}\frac{\alpha\left(K_t/K_0\right)^{\omega}}{\alpha\left(K_t/K_0\right)^{\omega}+(1-\alpha)N_t^{\omega}}, \tag{S1}$$

$$\frac{\partial Y_t}{\partial N_t} = \frac{Y_t}{N_t}\frac{(1-\alpha)N_t^{\omega}}{\alpha\left(K_t/K_0\right)^{\omega}+(1-\alpha)N_t^{\omega}}, \tag{S2}$$

As such, Y_t is of constant returns to scale, i.e., $K_t\partial Y_t/\partial K_t + N_t\partial Y_t/\partial N_t = Y_t$. From equation (10):

$$\frac{\partial \Phi_t}{\partial I_t} = a_2\left(\frac{I_t}{K_t}\right)^{-\frac{1}{\nu}} \tag{S3}$$

$$\frac{\partial \Phi_t}{\partial K_t} = a_1 + \frac{a_2}{\nu-1}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\nu}} \tag{S4}$$

It follows that $\Phi(I_t, K_t)$ is of constant returns to scale, i.e., $I_t\partial\Phi_t/\partial I_t + K_t\partial\Phi_t/\partial K_t = \Phi_t$.

The Lagrangian for the firm's problem is:

$$\begin{aligned}\mathcal{L} &= \cdots + Y_t - W_tN_t - \kappa_tV_t - I_t - \mu_{Nt}[N_{t+1}-(1-s)N_t - q(\theta_t)V_t] - \mu_{Kt}[K_{t+1}-(1-s)K_t - \Phi(I_t,K_t)] \\ &+\lambda_t q(\theta_t)V_t + E_t\left[M_{t+1}\left(Y_{t+1} - W_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1} - I_{t+1} - \mu_{Nt+1}[N_{t+2}-(1-s)N_{t+1} - q(\theta_{t+1})V_{t+1}]\right.\right. \\ &\left.\left.-\mu_{Kt+1}[K_{t+2}-(1-s)K_{t+1}-\Phi(I_{t+1},K_{t+1})] + \lambda_{t+1}q(\theta_{t+1})V_{t+1}+\cdots\right)\right]\end{aligned} \tag{S5}$$

The first-order conditions with respect to V_t and N_{t+1} are given by, respectively,

$$\mu_{Nt} = \frac{\kappa_t}{q(\theta_t)} - \lambda_t \tag{S6}$$

$$\mu_{Nt} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\mu_{Nt+1}\right]\right] \tag{S7}$$

Combining the two equations yields the intertemporal job creation condition in equation (15). The first-order conditions with respect to I_t and K_{t+1} are given by, respectively,

$$\mu_{Kt} = \frac{1}{\partial\Phi_t/\partial I_t} \tag{S8}$$

$$\mu_{Kt} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \left(1-\delta+\frac{\partial \Phi_{t+1}}{\partial K_{t+1}}\right)\frac{1}{\partial\Phi_{t+1}/\partial I_{t+1}}\right]\right] \tag{S9}$$

Combining equations (S3)–(S9) yields equation (13).

We first show $P_t = \mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}$, in which $P_t = S_t - D_t$ is ex-dividend equity value, with a guess-and-verify approach (Goncalves, Xue, and Zhang 2020). We first assume it holds for $t+1$: $P_{t+1} = \mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2}$. We then show it also holds for t. It then follows that the equation must hold for all periods. We start with recursively formulating equation (12): $P_t = E_t[M_{t+1}(P_{t+1}+D_{t+1})]$. Using $P_{t+1} = \mu_{Kt+1}K_{t+2}+\mu_{Nt+1}N_{t+2}$ to rewrite the right-hand side yields:

$$\begin{aligned}
P_t &= E_t\left[M_{t+1}\left[\mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2} + D_{t+1}\right]\right] \\
&= E_t\left[M_{t+1}\left[\mu_{Kt+1}\left[(1-\delta)K_{t+1} + \Phi_{t+1}\right] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}]\right.\right. \\
&\quad \left.\left. +Y_{t+1} - W_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1} - I_{t+1}\right]\right] \\
&= E_t\left[M_{t+1}\left[\mu_{Kt+1}\left[(1-\delta)K_{t+1} + \frac{\partial\Phi_{t+1}}{\partial I_{t+1}}I_{t+1} + \frac{\partial\Phi_{t+1}}{\partial K_{t+1}}K_{t+1}\right] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}]\right.\right. \\
&\quad \left.\left. +\frac{\partial Y_{t+1}}{\partial K_{t+1}}K_{t+1} + \frac{\partial Y_{t+1}}{\partial N_{t+1}}N_{t+1} - W_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1} - I_{t+1}\right]\right] \\
&= K_{t+1}E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \left(1-\delta+\frac{\partial\Phi_{t+1}}{\partial K_{t+1}}\right)\mu_{Kt+1}\right]\right] + \mu_{Kt+1}\frac{\partial\Phi_{t+1}}{\partial I_{t+1}}I_{t+1} \\
&\quad +N_{t+1}E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\mu_{Nt+1}\right]\right] + \mu_{Nt+1}q(\theta_{t+1})V_{t+1} - \kappa_{t+1}V_{t+1} - I_{t+1} \\
&= \mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1},
\end{aligned} \tag{S10}$$

in which the third equality follows from constant returns to scale for Y_{t+1} and Φ_{t+1}, and the last equality follows from equations (S6), (S7), (S8), (S9), and the Kuhn-Tucker condition (17).

To prove equation (18),

$$\begin{aligned}
r_{St+1} &= \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{\mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2} + D_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} \\
&= \frac{\begin{array}{c}\mu_{Kt+1}\left[(1-\delta)K_{t+1} + \Phi_{t+1}\right] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}] \\ +Y_{t+1} - W_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1} - I_{t+1}\end{array}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} \\
&= \frac{\begin{array}{c}\mu_{Kt+1}\left[(1-\delta)K_{t+1} + \frac{\partial\Phi_{t+1}}{\partial I_{t+1}}I_{t+1} + \frac{\partial\Phi_{t+1}}{\partial K_{t+1}}K_{t+1}\right] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}] \\ +\frac{\partial Y_{t+1}}{\partial K_{t+1}}K_{t+1} + \frac{\partial Y_{t+1}}{\partial N_{t+1}}N_{t+1} - W_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1} - I_{t+1}\end{array}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} \\
&= \frac{\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \left(1-\delta+\frac{\partial\Phi_{t+1}}{\partial K_{t+1}}\right)\mu_{Kt+1}\right]K_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} + \frac{\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\mu_{Nt+1}\right]N_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} \\
&= \frac{\mu_{Kt}K_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Kt+1} + \frac{\mu_{Nt}N_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Nt+1}.
\end{aligned} \tag{S11}$$

1.2 Wages

We extend the derivation in Petrosky-Nadeau, Zhang, and Kuehn (2018) to our setting with capital accumulation. Let $\partial J_t/\partial N_t$ be the marginal value of an employed worker to the representative household, $\partial J_t/\partial U_t$ the marginal value of an unemployed worker to the household, ϕ_t the marginal utility of the household, $\partial S_t/\partial N_t$ the marginal value of an employed worker to the representative firm, and $\partial S_t/\partial V_t$ the marginal value of an unfilled vacancy to the firm. A worker-firm match turns an unemployed worker into an employed worker for the household as well as an unfilled vacancy into an employed worker for the firm. As such, the total surplus from the Nash bargain is:

$$H_t \equiv \left(\frac{\partial J_t}{\partial N_t} - \frac{\partial J_t}{\partial U_t}\right)/\phi_t + \frac{\partial S_t}{\partial N_t} - \frac{\partial S_t}{\partial V_t}. \tag{S12}$$

The equilibrium wage arises from the Nash worker-firm bargain as follows:

$$\max_{\{W_t\}} \quad \left[\left(\frac{\partial J_t}{\partial N_t} - \frac{\partial J_t}{\partial U_t}\right)/\phi_t\right]^{\eta} \left(\frac{\partial S_t}{\partial N_t} - \frac{\partial S_t}{\partial V_t}\right)^{1-\eta}, \tag{S13}$$

in which $0 < \eta < 1$ is the worker's bargaining power. The outcome is the surplus-sharing rule:

$$\left(\frac{\partial J_t}{\partial N_t} - \frac{\partial J_t}{\partial U_t}\right)/\phi_t = \eta H_t = \eta \left[\left(\frac{\partial J_t}{\partial N_t} - \frac{\partial J_t}{\partial U_t}\right)/\phi_t + \frac{\partial S_t}{\partial N_t} - \frac{\partial S_t}{\partial V_t}\right]. \tag{S14}$$

As such, the worker receives a fraction of η of the total surplus from the wage bargain.

1.2.1 Workers

Tradeable assets consist of risky shares and a riskfree asset. Let r_{ft+1} denote the risk-free interest rate, ξ_t the household's financial wealth, χ_t the fraction of the household's wealth invested in the risky shares, $r_{\xi t+1} \equiv \chi_t r_{St+1} + (1-\chi_t) r_{ft+1}$ the return on wealth, and T_t the taxes raised by the government. The household's budget constraint is given by:

$$\frac{\xi_{t+1}}{r_{\xi t+1}} = \xi_t - C_t + W_t N_t + U_t b - T_t. \tag{S15}$$

The household's dividends income, D_t, is included in the current financial wealth, ξ_t.

Let ϕ_t denote the Lagrange multiplier for the household's budget constraint (S15). The household's maximization problem is given by:

$$J_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} - \phi_t \left(\frac{\xi_{t+1}}{r_{\xi t+1}} - \xi_t + C_t - W_t N_t - U_t b + T_t\right), \tag{S16}$$

The first-order condition for consumption yields:

$$\phi_t = (1-\beta)C_t^{-\frac{1}{\psi}} \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left[E_t \left(J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}-1}, \tag{S17}$$

which gives the marginal utility of consumption. Using $N_{t+1} = (1-s)N_t + f(\theta_t)U_t$ and $U_{t+1} = sN_t + (1-f(\theta_t))U_t$, we differentiate J_t in equation (S16) with respect to N_t:

$$\begin{aligned} \frac{\partial J_t}{\partial N_t} &= \phi_t W_t + \frac{1}{1-\frac{1}{\psi}} \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left[E_t \left(J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}-1} \\ &\quad \times \frac{1-\frac{1}{\psi}}{1-\gamma} \beta \left[E_t \left(J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t \left[(1-\gamma) J_{t+1}^{-\gamma} \left[(1-s) \frac{\partial J_{t+1}}{\partial N_{t+1}} + s \frac{\partial J_{t+1}}{\partial U_{t+1}} \right] \right]. \end{aligned} \tag{S18}$$

Dividing both sides by ϕ_t:

$$\frac{\partial J_t}{\partial N_t} / \phi_t = W_t + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[\frac{1}{\left[E_t \left(J_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} E_t \left[J_{t+1}^{-\gamma} \left[(1-s) \frac{\partial J_{t+1}}{\partial N_{t+1}} + s \frac{\partial J_{t+1}}{\partial U_{t+1}} \right] \right]. \tag{S19}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned} \frac{\partial J_t}{\partial N_t} / \phi_t &= W_t + E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left[\frac{J_{t+1}}{\left[E_t \left(J_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} \left[(1-s) \frac{\partial J_{t+1}}{\partial N_{t+1}} + s \frac{\partial J_{t+1}}{\partial U_{t+1}} \right] / \phi_{t+1} \right] \\ &= W_t + E_t \left[M_{t+1} \left[(1-s) \frac{\partial J_{t+1}}{\partial N_{t+1}} + s \frac{\partial J_{t+1}}{\partial U_{t+1}} \right] / \phi_{t+1} \right]. \end{aligned} \tag{S20}$$

Similarly, differentiating J_t in equation (S16) with respect to U_t yields:

$$\begin{aligned} \frac{\partial J_t}{\partial U_t} &= \phi_t b + \frac{1}{1-\frac{1}{\psi}} \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left[E_t \left(J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}-1} \\ &\quad \times \frac{1-\frac{1}{\psi}}{1-\gamma} \beta \left[E_t \left(J_{t+1}^{1-\gamma} \right) \right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t \left[(1-\gamma) J_{t+1}^{-\gamma} \left[f(\theta_t) \frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t)) \frac{\partial J_{t+1}}{\partial U_{t+1}} \right] \right]. \end{aligned} \tag{S21}$$

Dividing both sides by ϕ_t:

$$\frac{\partial J_t}{\partial U_t} / \phi_t = b + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}} \left[\frac{1}{\left[E_t \left(J_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma} E_t \left[J_{t+1}^{-\gamma} \left[f(\theta_t) \frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t)) \frac{\partial J_{t+1}}{\partial U_{t+1}} \right] \right]. \tag{S22}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned}
\frac{\partial J_t}{\partial U_t}/\phi_t &= b + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] \\
&= b + E_t\left[M_{t+1}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right]. \qquad \text{(S23)}
\end{aligned}$$

1.2.2 The Representative Firm

We start by reformulating the firm's problem recursively as:

$$S_t = Y_t - W_t N_t - \kappa_t V_t - I_t + \lambda_t q(\theta_t) V_t + E_t[M_{t+1} S_{t+1}], \qquad \text{(S24)}$$

subject to $N_{t+1} = (1-s)N_t + q(\theta_t)V_t$ and $K_{t+1} = (1-\delta)K_t + \Phi(I_t, K_t)$.

The first-order condition with respect to V_t says:

$$\frac{\partial S_t}{\partial V_t} = -\kappa_t + \lambda_t q(\theta_t) + E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}} q(\theta_t)\right] = 0. \qquad \text{(S25)}$$

Equivalently,

$$\frac{\kappa_t}{q(\theta_t)} - \lambda_t = E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right]. \qquad \text{(S26)}$$

In addition, differentiating S_t with respect to N_t yields:

$$\frac{\partial S_t}{\partial N_t} = \frac{\partial Y_t}{\partial N_t} - W_t + (1-s)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right]. \qquad \text{(S27)}$$

Combining the last two equations yields the job creation condition.

1.2.3 The Wage Rate

From equations (S20), (S23), and (S27), the total surplus of the worker-firm relationship is:

$$\begin{aligned}
H_t &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] - b \\
&\quad - E_t\left[M_{t+1}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] + \frac{\partial Y_t}{\partial N_t} - W_t + (1-s)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right] \\
&= \frac{\partial Y_t}{\partial N_t} - b + (1-s)E_t\left[M_{t+1}\left[\left(\frac{\partial J_{t+1}}{\partial N_{t+1}} - \frac{\partial J_{t+1}}{\partial U_{t+1}}\right)/\phi_{t+1} + \frac{\partial S_{t+1}}{\partial N_{t+1}}\right]\right] \\
&\qquad - f(\theta_t)E_t\left[M_{t+1}\left(\frac{\partial J_{t+1}}{\partial N_{t+1}} - \frac{\partial J_{t+1}}{\partial U_{t+1}}\right)/\phi_{t+1}\right] \\
&= \frac{\partial Y_t}{\partial N_t} - b + (1-s-\eta f(\theta_t))E_t\left[M_{t+1}H_{t+1}\right]. \qquad \text{(S28)}
\end{aligned}$$

The sharing rule implies $\partial S_t/\partial N_t = (1-\eta)H_t$, which, combined with equation (S27), yields:

$$(1-\eta)H_t = \frac{\partial Y_t}{\partial N_t} - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right]. \tag{S29}$$

Combining equations (S28) and (S29) yields:

$$\begin{aligned}
\frac{\partial Y_t}{\partial N_t} - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right] &= (1-\eta)\left(\frac{\partial Y_t}{\partial N_t} - b\right) + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right] \\
&\quad -(1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}H_{t+1}\right] \\
W_t &= \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}H_{t+1}\right].
\end{aligned}$$

Using equations (S14) and (S26) to simplify further:

$$W_t = \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + \eta f(\theta_t)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right] \tag{S30}$$

$$W_t = \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + \eta f(\theta_t)\left[\frac{\kappa_t}{q(\theta_t)} - \lambda_t\right]. \tag{S31}$$

If $V_t > 0$, then $\lambda_t = 0$, and equation (S31) reduces to equation (19) because $f(\theta_t) = \theta_t q(\theta_t)$. If $V_t \geq 0$ is binding, $\lambda_t > 0$, but $V_t = 0$ means $\theta_t = 0$ and $f(\theta_t) = 0$. Equation (S31) reduces to $W_t = \eta\partial Y_t/\partial N_t + (1-\eta)b$. Because $\theta_t = 0$, equation (19) continues to hold.

2 Supplementary Results

2.1 The Term Structure of Real Interest Rates

We calculate the prices of real zero-coupon bonds for maturities ranging from 1 month to 10 years. Let $P_{n,t}$ denote the price of an n-period zero-coupon bond. For $n = 1$, $P_{1,t} = E_t[M_{t+1}]$. For $n > 1$, we solve for $P_{n,t}$ recursively from $P_{n,t} = E_t[M_{t+1}P_{n-1,t+1}]$. The log yield-to-maturity is $y_{n,t} \equiv -\log(P_{n,t})/n$. Let $r_{n,t+1} \equiv P_{n-1,t+1}/P_{n,t}$ be the return of buying the n-period zero-coupon bond at time t and selling it at $t{+}1$. Excess returns are in excess of the 1-month interest rate, $r_{n,t+1} - r_{ft+1}$.

To calculate the term structure, we simulate one million months from the model's stationary distribution. The real yield curve is downward sloping in the model. The yield-to-maturity starts at 1.92% per annum for 1-month zero-coupon bond but falls to 1.24% for 1-year, 0.81% for 5-year, and further to 0.59% for 10-year zero-coupon bond. The average yield spread is −1.33% for the 10-year zero-coupon bond relative to the 1-month bond. The real term premium is also negative, −1.55%, for the 10-year zero-coupon bond. Intuitively, long-term bonds earn lower average returns because these bonds are hedges against disaster risks. Disasters stimulate precautionary savings, which in turn drive down real interest rates and push up real bond prices. Because the prices of long-term bonds tend to rise at the onset of disasters, these bonds provide hedges against disaster risks and, consequently, earn lower average returns (Nakamura et al. 2013; Wachter 2013).

Evidence on the slope of the real yield curve seems mixed. A large and liquid market for inflation-indexed bonds (index-linked gilts) has existed in the U.K. since 1982. Evans (1998) and Piazzesi and Schneider (2007) document that real yield curve is downward sloping in the U.K. In the U.S., Treasury inflation-protected securities (TIPS) start trading in 1997. Piazzesi and Schneider show that the TIPS yield curve appears to be upward sloping but caution that interpreting the evidence might be complicated by the relatively short sample and poor liquidity in the TIPS market.

We wish to point out that the downward sloping real yield curve in our model does not necessarily contradict the upward sloping nominal yield curve in the data. Nominal bonds are subject to inflation risks, which are left outside our model. Because long-term bonds are more exposed to inflation risks, a positive inflation risk premium would imply an upward sloping nominal yield curve (Rudebusch and Swanson 2012). We leave this extension to future work.

2.2 Recalibrating the Campbell-Cochrane Model to the Macrohistory Data

2.2.1 The Model

We use the same notations as in Campbell and Cochrane (1999). The consumption growth is:

$$\Delta c_{t+1} = g + v_{t+1}, \tag{S32}$$

in which $c_{t+1} \equiv \log(C_{t+1})$ is log consumption and v_{t+1} is an i.i.d. normal shock with mean zero and variance σ^2. The representative agent maximizes the utility function:

$$E_t \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \tag{S33}$$

in which X_t is the the level of habit, δ the subjective discount factor, and γ the utility curvature parameter. The surplus consumption ratio, S_t, is defined by:

$$S_t \equiv \frac{C_t - X_t}{C_t}. \tag{S34}$$

The log surplus consumption ratio, $s_t \equiv \log(S_t)$, follows an autoregressive process:

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t) v_{t+1}, \tag{S35}$$

in which $\bar{s} \equiv \log(\bar{S})$ is the log of the steady-state surplus consumption ratio $\bar{S}$. ϕ governs the persistence of s_t. $\lambda(s_t)$ is the sensitivity function, which takes the form:

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1 - 2(s_t - \bar{s})} - 1, & s_t \leq s_{\max} \\ 0, & s_t \geq s_{\max} \end{cases} \tag{S36}$$

in which $s_{\max} \equiv \bar{s} + \frac{1}{2}(1 - \bar{S}^2)$ is the s_t value at which the upper expression in (S36) runs into zero. The steady-state surplus consumption ratio, $\bar{S}$, is a function of other parameters of the model:

$$\bar{S} = \sigma\sqrt{\frac{\gamma}{1-\phi}}. \tag{S37}$$

The stochastic discount factor is:

$$M_{t+1} \equiv \delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}. \tag{S38}$$

The dividend growth follows:

$$\Delta d_{t+1} = g + w_{t+1}, \tag{S39}$$

in which $d_{t+1} \equiv \log(D_{t+1})$ is log dividends, and w_{t+1} is an i.i.d. normal shock with mean zero and variance σ_w^2. The correlation between w_t and v_t is ρ.

2.2.2 Numerical Solution

We follow Wachter (2005) and solve for the price-dividend ratio (P_t/D_t) as a function of s_t on a dense grid. Specifically, the grid for s_t consists of an upper segment and a lower segment. Let $S_{g,1}$ denote a vector of 101 equally spaced points between 0 and $S_{\max} \equiv e^{s_{\max}}$ with $S_{\max}$ included and $s_{g,2}$ a vector of 900 equally spaced points between -300 and $\min(\log(S_{g,1}))$. Finally, we form the grid for s_t by concatenating $s_{g,2}$ and $\log(S_{g,1})$.

Let $F_n(s_t)$ denote the time-t price of a claim to the aggregate dividend n periods from now divided by the dividend today. $F_n(s_t)$ follows the recursive relation:

$$F_n(s_t) = E_t\big[M_{t+1}\frac{D_{t+1}}{D_t}F_{n-1}(s_{t+1})\big], \tag{S40}$$

with $F_0(s_t) = 1$. Finally,

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} F_n(s_t). \tag{S41}$$

We solve for $F_n(s_t)$ for n up to 1,000 by iterating on equation (S40) (using $F_0(s_t) = 1$ to start the process). P_t/D_t is then the sum of $F_n(s_t)$ according to equation (S41). Increasing n further has a negligible impact on P_t/D_t.

Equation (S40) can be rewritten as:

$$F_n(s_t) = e^{g+\frac{1}{2}(1-\rho^2)\sigma_w^2} E_t\big[M_{t+1} e^{\rho\frac{\sigma_w}{\sigma} v_{t+1}} F_{n-1}(s_{t+1})\big]. \tag{S42}$$

We use Gauss-Hermite quadrature to numerically evaluate the conditional expectation over the normally distributed v_{t+1} on the right-hand-side of Equation (S42). Specifically, we use 22 integration node points which cover an integration domain between -8 and $+8$ standard deviations of v_{t+1}. To evaluate $F_{n-1}(s_{t+1})$, we use log-linear interpolation following Campbell and Cochrane (1999), by assuming that $\log(F_{n-1}(s_{t+1}))$ is approximately linear in s_{t+1}.

We then compute the stock return as:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{P_{t+1}/D_{t+1} + 1}{P_t/D_t} \times \frac{D_{t+1}}{D_t}. \tag{S43}$$

We simulate a million months of artificial data (after a burn-in period of 1,200 months) to calculate population values for key statistics. We time-aggregate monthly consumption into annual observations by taking the sum of monthly observations within each year. We form annual returns by taking the product of intervening monthly returns within each year.

2.2.3 Quantitative Results

What risk aversion is required to match the equity premium in the Jorda-Schularick-Taylor macrohistory dataset in the Campbell-Cochrane model? To answer this question, we first calibrate σ to match the average consumption volatility of 5.45% of the international panel (Panel A of Table 1). We then calibrate γ to match the average equity premium of 6.14% of the international panel (Panel D of Table 1), while keeping all the other parameters unchanged from those in Table 1 of Campbell and Cochrane (1999). This process yields a value of 6.65 for σ and a value of 7.64 for γ. All the parameter choices are in Table S6.

Our calibration implies a steady-state surplus consumption ratio of $\bar{S} = \sigma\sqrt{\frac{\gamma}{1-\phi}} = 0.494$ and a steady-state risk aversion of $\gamma/\bar{S} = 15.47$.

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Table S1 : Basic Properties of Asset Prices in the Historical Sample, with the Longest Possible Sample for Each Moment

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada's asset prices, which we obtain from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. All (annual) series end in 2015. $E[\widetilde{r}_S]$, $\widetilde{\sigma}_S$, and $E[\widetilde{r}_S - r_f]$ are the average stock market return, stock market volatility, and the equity premium, respectively, without adjusting for financial leverage. $E[r_S - r_f]$ and σ_S are the equity premium and stock market volatility, respectively, after adjusting for financial leverage. $E[r_f]$ is the mean real interest rate, and σ_f the interest rate volatility. All asset pricing moments are in annual percent. We use the longest possible samples of stocks, bills, and bonds described in the second, third, and fourth column, respectively, to calculate each moment. For example, in Australia, the sample for stock market returns starts in 1871, the sample for real interest rates start in 1871, with missing observations from 1945 to 1947, and the sample for long-term government bonds starts in 1900. Other than Italy, which has missing asset prices from 1872 to 1884, all other missing years are in the 20th century.

	Sample, $\widetilde{r}_S$	Sample, r_f	Sample, r_B	$E[\widetilde{r}_S]$	$\widetilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\widetilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
Australia	1871	1871 (45–47)	1900	8.39	15.77	2.02	4.44	6.33	4.49	11.76
Belgium	1871	1871 (15–18)	1871 (14–19)	5.89	21.97	1.68	9.94	5.25	3.73	16.22
Canada	1900	1900	1900	7.01	17.00	1.60	4.79	5.41	3.84	12.26
Denmark	1873	1875	1871 (15)	7.54	16.36	2.98	5.77	4.59	3.26	11.88
Finland	1896	1871	1871	8.83	30.57	0.15	10.50	9.57	6.80	22.98
France	1871	1871 (15–21)	1871	3.21	22.14	−0.47	7.78	4.45	3.16	16.75
Germany	1871	1871 (23, 45–49)	1871 (44–48)	9.44	32.04	−0.23	13.17	9.00	6.39	20.15
Italy	1871	1871 (1872–1884, 15–21)	1871	5.75	26.18	0.58	10.50	6.05	4.29	20.41
Japan	1886 (46–47)	1876	1881	8.86	27.69	−0.41	12.90	8.87	6.29	21.10
Netherlands	1900	1871	1871	6.96	21.44	1.37	5.04	6.19	4.39	15.32
Norway	1881	1871	1871	5.67	19.82	1.10	5.96	4.77	3.39	14.53
Portugal	1871	1880	1871	4.05	25.20	−0.01	9.43	3.82	2.71	19.29
Spain	1900	1871 (36–38)	1900 (37–40)	5.77	21.07	0.70	6.83	6.28	4.46	15.88
Sweden	1871	1871	1871	8.00	19.54	1.77	5.60	6.23	4.42	14.26
Switzerland	1900	1871	1900 (15)	6.50	19.09	1.64	5.88	5.70	4.05	14.04
UK	1871	1871	1871	6.86	17.77	1.16	4.82	5.70	4.05	12.96
USA	1872	1871	1871	8.40	18.68	2.23	4.71	6.23	4.43	13.66
Mean				6.89	21.90	1.05	7.53	6.14	4.36	16.08
Median				6.96	21.07	1.16	5.96	6.05	4.29	15.32

Table S2 : Gollin's (2002) Labor Share Calculations

For the 12 countries that are in both Gollin (2002) and Jordà-Schularick-Taylor macrohistory database, this table reports the labor shares reported in Gollin's Table 2. The three columns correspond to the last three columns labeled "Adjustment 1," "Adjustment 2," and "Adjustment 3," respectively, in Gollin's table.

	Method 1	Method 2	Method 3
Australia	0.719	0.669	0.676
Belgium	0.791	0.743	0.740
Finland	0.765	0.734	0.680
France	0.764	0.717	0.681
Italy	0.804	0.717	0.707
Japan	0.727	0.692	0.725
Netherlands	0.721	0.680	0.643
Norway	0.678	0.643	0.569
Portugal	0.825	0.748	0.602
Sweden	0.800	0.774	0.723
UK	0.815	0.782	0.719
US	0.773	0.743	0.664
Mean	0.765	0.720	0.677
Median	0.769	0.726	0.681

Table S3 : Dividend Dynamics in the Post-1950 Sample

Real output and dividends are from the Jordà-Schularick-Taylor macrohistory database. "Prop. dev." denotes the HP-filtered proportional deviations from the mean, and "Log dev." log deviations from the HP-trend. ρ_{DY} is the correlation between the cyclical components of dividends and output, and σ_D/σ_Y the volatility of the cyclical component of dividends divided by that of output. We examine three frequencies, annual, 3-year, and 5-year. For the 3-year frequency, we sum up the three annual observations within a given 3-year interval. The 3-year intervals are nonoverlapping. The 5-year series are constructed analogously. The HP smoothing parameters for the 1-, 3-, and 5-year series are $1600/4^4 = 6.25$, $1600/12^4 = 0.077$, and $1600/20^4 = 0.01$, respectively, all of which correspond to 1,600 in the quarterly frequency. All countries start their samples in 1950, except for Switzerland, which starts in 1960. When calculating log deviations, we discard zero-dividend observations.

	1-year frequency				3-year frequency				5-year frequency			
	Prop. dev.		Log dev.		Prop. dev.		Log dev.		Prop. dev.		Log dev.	
	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y
Australia	0.17	10.85	0.21	9.32	0.22	10.37	0.22	8.05	0.67	6.39	0.73	5.00
Belgium	−0.04	15.17	−0.02	11.55	−0.06	13.08	0.00	9.05	0.26	11.68	0.37	9.80
Denmark	0.20	37.70	0.13	15.58	0.34	29.28	0.35	14.64	0.37	14.96	0.41	8.19
Finland	0.05	12.06	0.18	11.12	0.57	8.38	0.87	9.30	0.80	5.40	0.71	7.87
France	−0.11	9.51	−0.06	10.51	0.08	9.71	0.11	9.10	0.20	5.52	0.28	5.89
Germany	−0.23	10.35	0.06	10.68	0.13	9.75	0.26	12.92	−0.33	12.60	0.34	11.17
Italy	0.02	8.13	−0.06	10.66	−0.01	11.85	0.16	11.15	0.27	10.22	0.36	14.63
Japan	0.29	4.83	0.39	5.58	0.20	5.75	0.19	4.65	0.49	4.72	0.56	4.51
Netherlands	−0.00	16.81	0.20	14.90	0.55	13.77	0.39	13.04	0.37	18.40	0.26	13.53
Norway	0.19	33.17	0.08	23.01	0.09	19.02	0.07	16.45	0.57	4.21	0.29	6.40
Portugal	−0.24	6.07	0.07	16.70	0.16	5.77	0.72	26.05	0.51	2.87	0.81	20.83
Spain	−0.05	16.57	0.03	12.37	0.04	6.94	0.06	5.93	0.20	5.25	0.11	4.73
Sweden	−0.03	11.77	0.18	8.80	0.62	9.72	0.82	9.32	0.44	3.88	0.51	5.50
Switzerland	0.03	11.06	0.05	13.17	0.43	8.66	0.34	7.73	0.03	7.99	0.01	9.19
UK	0.63	3.84	0.64	3.70	0.73	4.25	0.74	4.17	0.41	3.59	0.48	3.34
USA	0.65	3.80	0.50	2.96	0.71	4.38	0.65	3.51	0.37	3.30	0.51	2.67
Mean	0.09	13.23	0.16	11.29	0.30	10.67	0.37	10.32	0.35	7.56	0.42	8.33
Median	0.02	10.95	0.10	10.90	0.21	9.72	0.30	9.20	0.37	5.46	0.39	7.13

Table S4 : Predicting Excess Returns and Consumption Growth with Log Price-to-consumption in the post-1950 Sample

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start in 1950 and end in 2015. Panel A performs predictive regressions of stock market excess returns on log price-to-consumption, $\sum_{h=1}^{H}[\log(r_{St+h}) - \log(r_{ft+h})] = a + b\log(P_t/C_t) + u_{t+h}$, in which H is the forecast horizon, r_{St+1} real stock market return, r_{ft+1} real interest rate, P_t real market index, and C_t real consumption. r_{St+1} and r_{ft+1} are over the course of period t, and P_t and C_t are at the beginning of t. Excess returns are adjusted for a financial leverage ratio of 0.29. Panel B performs long-horizon predictive regressions of log consumption growth on $\log(P_t/C_t)$, $\sum_{h=1}^{H}\log(C_{t+h}/C_t) = c + d\log(P_t/C_t) + v_{t+h}$. In both regressions, $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values of the slopes are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values of slopes					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market excess returns														
Australia	−4.79	−7.74	−8.35	−9.52	−9.70	−3.03	−4.13	−4.19	−3.06	−2.46	12.17	19.86	21.17	22.51	21.26
Belgium	−2.39	−5.00	−6.91	−9.44	−10.86	−1.45	−1.57	−1.61	−1.89	−2.39	2.46	5.64	8.16	11.80	15.00
Denmark	−0.43	−1.76	−2.32	−3.05	−3.08	−0.17	−0.41	−0.46	−0.67	−0.76	0.08	0.61	0.79	1.13	1.13
Finland	−3.76	−9.48	−14.03	−17.33	−19.08	−1.36	−2.46	−4.08	−5.30	−5.33	3.68	9.66	14.50	18.23	20.25
France	−1.85	−4.05	−5.95	−8.59	−11.47	−0.97	−1.17	−1.09	−1.21	−1.48	1.26	3.10	4.85	7.24	11.90
Germany	−6.24	−11.41	−14.93	−18.14	−19.06	−2.78	−3.21	−3.15	−3.20	−3.40	12.48	20.22	24.99	29.08	29.57
Italy	−0.98	−2.51	−4.20	−5.61	−6.34	−0.52	−0.63	−0.77	−0.80	−0.76	0.32	0.93	1.76	2.40	2.76
Japan	−4.00	−9.60	−13.90	−17.98	−21.83	−2.30	−2.96	−4.35	−5.80	−5.96	8.19	18.14	25.40	31.70	36.39
Netherlands	−3.04	−6.48	−8.91	−11.12	−13.51	−1.68	−1.87	−2.09	−2.46	−3.06	4.13	8.98	12.71	16.31	20.65
Norway	−3.89	−7.14	−8.74	−9.80	−11.69	−1.99	−2.68	−2.70	−2.59	−2.87	4.99	9.69	12.52	14.56	18.57
Portugal	−2.16	−8.22	−14.17	−17.85	−17.39	−0.48	−0.94	−1.30	−1.51	−1.66	0.77	3.93	6.64	7.60	5.75
Spain	−0.32	−2.18	−4.83	−7.32	−9.22	−0.17	−0.54	−0.86	−1.17	−1.32	0.04	0.78	2.22	3.64	4.76
Sweden	−1.57	−3.12	−4.06	−5.13	−6.09	−0.75	−0.84	−0.91	−1.10	−1.24	0.95	1.88	2.46	3.28	4.10
Switzerland	−3.09	−6.51	−8.50	−10.67	−12.95	−1.70	−2.30	−2.85	−3.89	−4.17	4.02	8.50	11.76	15.72	20.05
UK	−6.50	−11.41	−13.92	−14.44	−16.54	−3.01	−4.32	−4.54	−5.95	−6.68	17.37	30.67	38.71	42.28	49.39
USA	−2.89	−5.59	−7.18	−9.65	−12.36	−2.18	−2.27	−2.24	−2.47	−2.79	5.83	10.67	13.61	18.59	23.90
Mean	−2.99	−6.39	−8.81	−10.98	−12.57	−1.53	−2.02	−2.32	−2.69	−2.90	4.92	9.58	12.64	15.38	17.84
Median	−2.96	−6.49	−8.42	−9.73	−12.02	−1.56	−2.07	−2.16	−2.47	−2.63	3.85	8.74	12.14	15.14	19.31
	Panel B: Predicting consumption growth														
Australia	0.40	0.41	0.36	0.81	1.20	1.79	0.93	0.58	1.35	1.85	4.04	1.78	1.08	5.54	10.21
Belgium	0.09	0.09	0.21	0.41	0.54	0.42	0.25	0.43	0.68	0.76	0.24	0.09	0.27	0.62	0.78
Denmark	−0.08	−0.42	−0.69	−1.05	−1.38	−0.27	−0.55	−0.61	−0.79	−0.94	0.10	1.04	1.61	2.57	3.51
Finland	0.31	0.06	−0.40	−0.73	−0.90	0.95	0.08	−0.39	−0.63	−0.72	0.97	0.01	0.37	0.89	1.10
France	0.95	1.81	2.68	3.51	4.37	4.45	3.67	3.84	4.18	4.69	28.51	32.88	37.06	40.26	43.62
Germany	−0.10	−0.47	−1.05	−1.43	−1.84	−0.29	−0.51	−0.69	−0.72	−0.73	0.15	1.07	2.65	3.20	3.74
Italy	1.58	3.04	4.43	5.68	6.84	5.69	4.47	4.07	3.74	3.51	33.87	37.49	40.15	40.67	40.62
Japan	0.51	0.86	1.44	1.86	2.17	1.48	0.89	0.90	0.80	0.71	2.12	1.79	2.65	2.63	2.30
Netherlands	0.67	1.12	1.46	1.87	2.35	2.43	1.49	1.24	1.17	1.14	7.43	6.49	5.94	6.42	7.60
Norway	0.23	0.38	0.56	0.73	1.00	0.78	0.65	0.77	0.95	1.29	1.16	1.26	1.78	2.36	3.78
Portugal	0.19	0.05	0.13	0.62	1.54	0.36	0.04	0.08	0.38	0.98	0.26	0.01	0.03	0.53	2.66
Spain	1.75	3.04	4.02	4.90	5.62	4.78	3.94	3.36	2.99	2.72	24.75	26.39	25.77	24.87	23.66
Sweden	0.00	−0.22	−0.39	−0.56	−0.74	−0.01	−0.44	−0.46	−0.48	−0.53	0.00	0.45	0.74	1.01	1.30
Switzerland	0.22	0.31	0.36	0.35	0.34	1.32	0.84	0.61	0.43	0.33	2.52	1.40	1.00	0.64	0.44
UK	0.45	0.46	0.45	0.23	−0.12	2.14	1.14	0.80	0.30	−0.13	4.78	1.73	0.99	0.19	0.04
USA	0.28	0.16	0.11	0.19	0.22	1.34	0.31	0.15	0.20	0.20	2.69	0.30	0.09	0.19	0.20
Mean	0.47	0.67	0.86	1.09	1.33	1.71	1.07	0.92	0.91	0.95	7.10	7.14	7.64	8.29	9.10
Median	0.30	0.34	0.36	0.51	0.77	1.33	0.74	0.59	0.56	0.73	2.32	1.33	1.35	2.46	3.08

Table S5 : Predicting Volatilities of Stock Market Excess Returns and Consumption Growth with Log Price-to-consumption in the Post-1950 Sample

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start in 1950 and end in 2015. For a given horizon, H, we measure excess return volatility as $\sigma_{St,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{St+h}|$, in which ϵ_{St+h} is the h-period-ahead residual from the first-order autoregression of excess returns, $\log(r_{St+1}) - \log(r_{ft+1})$ (adjusted for a financial leverage ratio of 0.29). Panel A performs long-horizon predictive regressions of excess return volatilities, $\log \sigma_{St+1,t+H} = a + b \log(P_t/C_t) + u^{\sigma}_{t+H}$. For a given H, consumption growth volatility is $\sigma_{Ct,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{Ct+h}|$, in which ϵ_{Ct+h} is the h-period-ahead residual from the first-order autoregression of log consumption growth, $\log(C_{t+1}/C_t)$. Panel B performs long-horizon predictive regressions of consumption growth volatilities, $\log \sigma_{Ct+1,t+H} = c + d \log(P_t/C_t) + v^{\sigma}_{t+H}$. $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values of slopes					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market volatility														
Australia	1.71	6.76	3.98	4.74	2.83	0.11	0.55	0.31	0.36	0.21	0.01	0.59	0.36	0.62	0.27
Belgium	1.96	2.38	1.97	1.08	−0.63	0.20	0.32	0.28	0.15	−0.10	0.04	0.17	0.19	0.08	0.03
Denmark	13.67	11.12	11.79	11.24	10.74	1.14	0.91	0.82	0.73	0.66	1.95	2.30	3.25	3.28	3.18
Finland	19.65	14.49	14.23	11.41	8.21	1.48	1.48	1.85	2.04	1.72	2.78	5.01	6.96	5.39	3.28
France	−9.75	−10.93	−9.49	−10.00	−10.89	−0.74	−1.43	−1.64	−2.47	−3.97	0.97	3.74	6.17	10.88	16.47
Germany	17.77	15.12	15.12	15.11	13.28	1.07	1.76	1.92	1.93	1.74	1.56	5.04	10.80	16.58	16.29
Italy	−33.16	−28.63	−23.29	−19.12	−18.45	−1.75	−2.07	−2.52	−2.80	−3.46	4.55	10.16	15.51	20.36	27.24
Japan	6.13	12.73	12.65	11.56	10.48	0.41	1.24	1.25	1.22	1.16	0.33	4.57	6.30	6.41	6.37
Netherlands	6.06	8.47	11.33	11.06	8.98	0.42	0.67	1.04	1.20	1.16	0.32	1.44	4.74	5.73	5.26
Norway	−34.27	−29.37	−25.24	−25.28	−26.05	−2.43	−4.10	−3.70	−3.45	−3.36	3.56	12.38	22.44	28.85	35.65
Portugal	−42.17	−42.76	−43.85	−46.49	−46.10	−2.14	−2.42	−2.93	−3.25	−2.89	11.85	22.46	28.69	34.92	31.02
Spain	−18.04	−22.59	−19.14	−18.62	−17.79	−1.41	−2.40	−2.09	−2.05	−1.94	2.42	9.73	11.80	15.50	17.10
Sweden	15.21	17.32	19.29	18.48	19.27	1.48	1.88	2.37	2.46	2.73	3.54	9.61	19.61	22.61	28.04
Switzerland	7.05	11.57	9.51	11.11	11.03	0.39	0.87	0.90	1.18	1.30	0.27	2.01	3.01	5.79	7.64
UK	1.05	6.22	11.23	14.44	16.17	0.07	0.56	1.26	2.11	2.62	0.01	0.88	3.89	7.43	11.03
USA	12.24	10.13	11.34	12.42	13.72	0.83	1.17	1.93	2.60	3.07	1.42	2.42	5.21	9.98	17.62
Mean	−2.18	−1.12	0.09	0.20	−0.32	−0.06	−0.06	0.07	0.12	0.04	2.23	5.78	9.31	12.15	14.16
Median	4.01	7.62	10.37	11.08	8.59	0.30	0.62	0.86	0.95	0.91	1.49	4.15	6.23	8.71	13.66
	Panel B: Predicting consumption growth volatility														
Australia	−4.80	12.99	14.07	13.44	12.20	−0.20	1.60	1.88	2.03	2.09	0.17	4.54	8.03	9.83	9.27
Belgium	−4.34	0.39	5.59	10.58	12.26	−0.33	0.04	0.65	1.26	1.63	0.24	0.00	1.16	5.51	8.86
Denmark	−23.77	−22.00	−16.49	−14.52	−15.41	−1.83	−2.23	−1.65	−1.55	−1.85	3.69	7.15	5.85	6.89	12.03
Finland	−25.16	−14.09	−8.95	−5.96	−5.41	−1.84	−1.03	−0.63	−0.42	−0.40	4.60	3.10	1.74	0.91	0.89
France	16.54	17.63	16.51	16.50	16.37	1.28	1.95	2.11	2.74	3.44	2.07	6.56	9.91	13.57	18.08
Germany	−6.27	−1.99	−0.64	1.33	4.44	−0.47	−0.18	−0.07	0.17	0.57	0.21	0.07	0.01	0.07	1.11
Italy	8.31	5.18	5.95	7.19	9.02	0.73	0.56	0.65	0.93	1.56	0.73	0.79	1.33	2.60	5.04
Japan	1.35	−8.36	−6.69	−7.02	−8.53	0.06	−0.67	−0.55	−0.59	−0.73	0.01	1.09	1.05	1.36	2.26
Netherlands	6.40	9.28	11.58	11.02	10.08	0.54	0.80	0.96	1.02	1.05	0.42	2.10	4.59	5.09	5.23
Norway	−22.89	−23.25	−24.00	−21.67	−19.13	−1.86	−1.76	−2.01	−2.28	−2.66	3.70	8.06	11.96	12.72	12.60
Portugal	−18.51	−12.71	−10.28	−9.25	−9.76	−2.00	−1.38	−1.02	−0.82	−0.77	5.10	3.87	3.57	3.38	3.57
Spain	41.05	34.10	31.91	30.02	29.87	2.88	2.55	2.50	2.32	2.22	11.21	14.65	18.82	23.51	28.32
Sweden	−12.16	−20.44	−17.17	−13.44	−12.42	−0.78	−1.43	−1.41	−1.44	−1.60	0.91	6.08	6.43	5.21	6.27
Switzerland	−13.49	−13.67	−13.37	−9.64	−6.97	−1.01	−1.06	−1.11	−0.84	−0.69	1.40	2.71	3.78	2.63	1.61
UK	−24.73	−16.02	−16.09	−16.80	−16.53	−1.91	−1.85	−2.24	−2.79	−2.77	3.67	4.52	7.87	11.74	14.83
USA	−4.93	−13.20	−10.05	−10.31	−10.96	−0.31	−1.14	−0.98	−1.02	−1.07	0.12	2.32	2.44	3.14	4.42
Mean	−5.46	−4.14	−2.38	−1.16	−0.68	−0.44	−0.33	−0.18	−0.08	0.00	2.39	4.23	5.53	6.76	8.40
Median	−5.60	−10.54	−7.82	−6.49	−6.19	−0.40	−0.85	−0.59	−0.50	−0.55	1.16	3.48	4.19	5.15	5.75

Table S6 : Parameter Choices of the External Habit Model

This table lists the parameter choices of the original Campbell-Cochrane calibration and our calibration. $\star$ indicates annualized values, e.g., $12g$, $\sqrt{12}\sigma$, $12r^f$, ϕ^{12}, and σ^{12}.

Parameter	Variable	Campbell-Cochrane	Our calibration
Mean consumption growth (%)*	g	1.89	1.89
Standard deviation of consumption growth (%)*	σ	1.50	6.65
Log risk-free rate (%)*	r^f	0.94	0.94
Persistence coefficient*	ϕ	0.87	0.87
Utility curvature	γ	2.00	7.64
Standard deviation of dividend growth (%)*	σ_w	11.2	11.2
Correlation between Δd and Δc	ρ	0.2	0.2

Figure S1 : Heatmaps of Key Moments Against Unemployment

From the model's stationary distribution with the benchmark calibration (after a burn-in period of 1,200 monthly periods), we simulate a long sample path with one million months. The equity premium, stock market volatility, and consumption volatility are in monthly percent. In each heatmap, dark red indicates higher density, while light green indicates lower density.

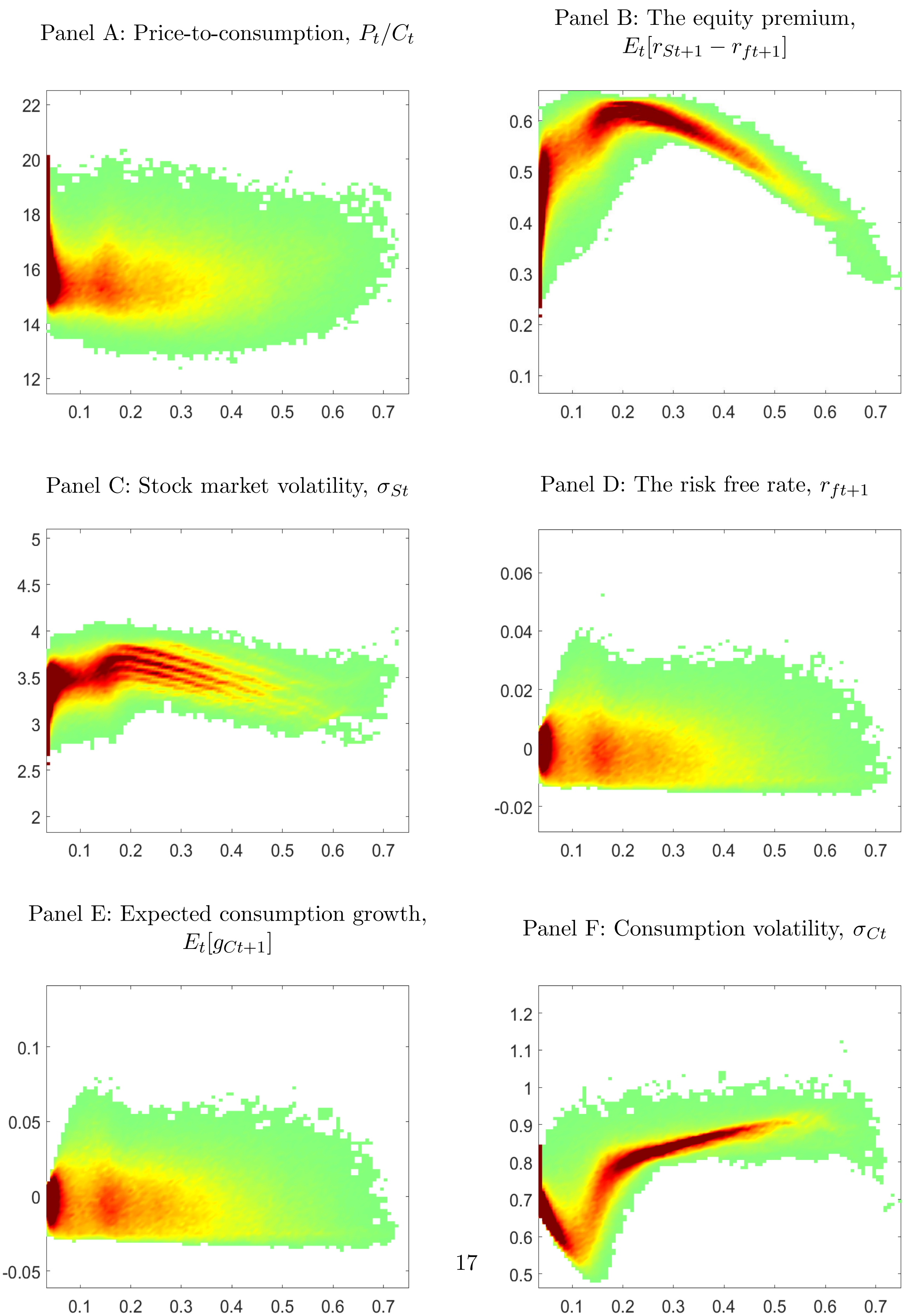
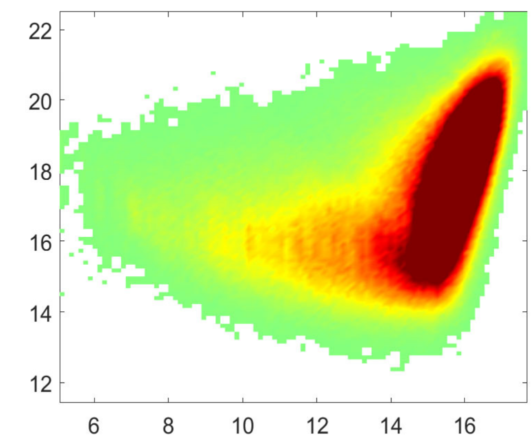


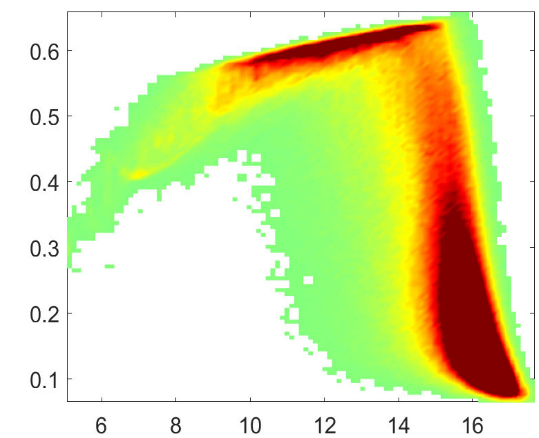
Figure S2 : Heatmaps of Key Moments Against Capital

From the model's stationary distribution with the benchmark calibration (after a burn-in period of 1,200 monthly periods), we simulate a long sample path with one million months. The equity premium, stock market volatility, and consumption volatility are in monthly percent. In each heatmap, dark red indicates higher density, while light green indicates lower density.

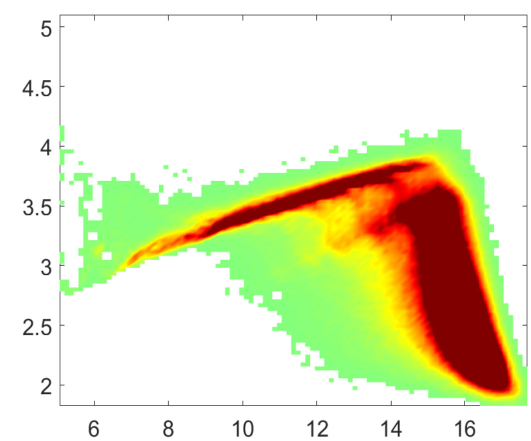
Panel A: Price-to-consumption, P_t/C_t



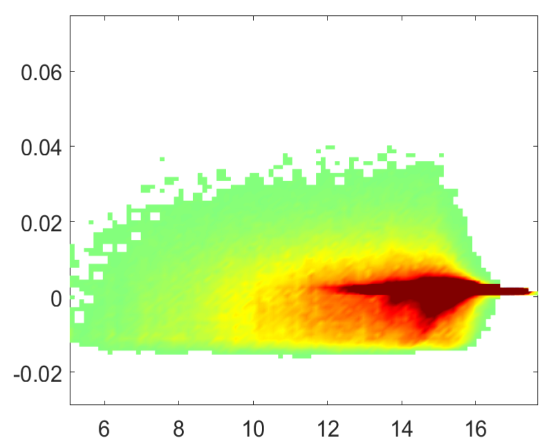
Panel B: The equity premium, $E_t[r_{St+1} - r_{ft+1}]$



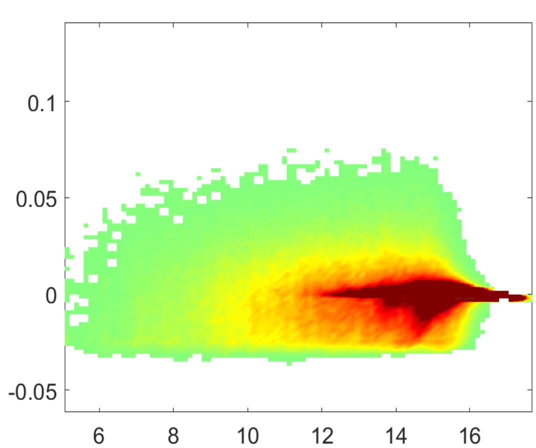
Panel C: Stock market volatility, σ_{St}



Panel D: The risk free rate, r_{ft+1}



Panel E: Expected consumption growth, $E_t[g_{Ct+1}]$



Panel F: Consumption volatility, σ_{Ct}