Lecture Notes

Bai, Hou, Kung, Li, and Zhang (2019, Journal of Financial Economics, "The CAPM Strikes Back? An Equilibrium Model with Disasters")

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Theme

Extending the Zhang (2005) model to general equilibrium

Embedding disasters into a general equilibrium model with heterogeneous firms induces strong nonlinearity in the pricing kernel, helping explain the failure of the (consumption) CAPM

Contribution
Bai et al. (2019)

Our single-factor model, in which a nonlinear consumption CAPM holds exactly by construction, quantitatively reproduces:

- The CAPM fails to explain the value premium in finite samples without disasters, but succeeds in samples with disasters
- The beta "anomaly": A flat relation between the pre-ranking market beta and the average return
- The standard consumption CAPM fails to price the 25 size and book-to-market portfolios

Mechanism: Without disasters, estimated betas only reflect risk in normal times, but the value premium is driven by disaster risk

Literature

Why are the new insights important?

Early investment theories rely on single-factor models, in which the CAPM roughly holds: Gomes, Kogan, and Zhang 2003, Carlson, Fisher, and Giammarino 2004, Zhang 2005, Cooper 2006

Recent models introduce multiple shocks, but inconsistent with the long-sample evidence: Ai and Kiku 2013, Kogan and Papanikolaou 2013, Belo, Lin, and Bazdresch 2014

Disaster models: Rietz 1988, Barro 2006, Gourio 2012, Nakamura, Steinsson, Barro, and Ursua 2013, Wachter 2013

Outline

Outline

Stylized Facts

The book-to-market deciles, July 1963–June 2017 ($p_{\text{GRS}} = 0.03$)

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.44	0.54	0.59	0.54	0.55	0.66	0.62	0.70	0.86	0.91	0.47
t_{R^e}	2.22	3.00	3.26	2.98	3.14	3.88	3.49	3.88	4.41	3.80	2.53
α	−0.11	0.02	0.07	0.03	0.07	0.20	0.15	0.23	0.35	0.32	0.43
t_α	−1.23	0.44	1.17	0.39	0.80	2.21	1.23	2.00	3.03	2.04	1.89
β	1.06	1.00	0.99	0.98	0.91	0.88	0.92	0.91	0.98	1.13	0.07
t_β	41.66	42.06	40.88	32.43	28.19	23.30	19.35	18.26	22.65	17.47	0.86
R^2	0.86	0.91	0.91	0.87	0.83	0.80	0.78	0.76	0.77	0.68	0.00

Stylized Facts

The book-to-market deciles, July 1926–June 2017 ($p_{GRS} = 0.03$)

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.59	0.69	0.69	0.66	0.72	0.79	0.72	0.91	1.06	1.07	0.48
t_{R^e}	3.40	4.28	4.23	3.71	4.19	4.35	3.73	4.49	4.55	3.84	2.50
α	−0.08	0.07	0.05	−0.02	0.07	0.11	0.00	0.16	0.22	0.11	0.19
t_α	−1.21	1.46	1.02	−0.38	0.92	1.32	0.02	1.82	1.94	0.74	0.99
β	1.01	0.95	0.97	1.05	1.00	1.03	1.10	1.14	1.28	1.46	0.45
t_β	52.73	27.62	59.98	22.11	27.29	14.85	17.73	16.11	14.32	14.49	3.87
R^2	0.90	0.91	0.93	0.90	0.89	0.85	0.84	0.83	0.80	0.72	0.14

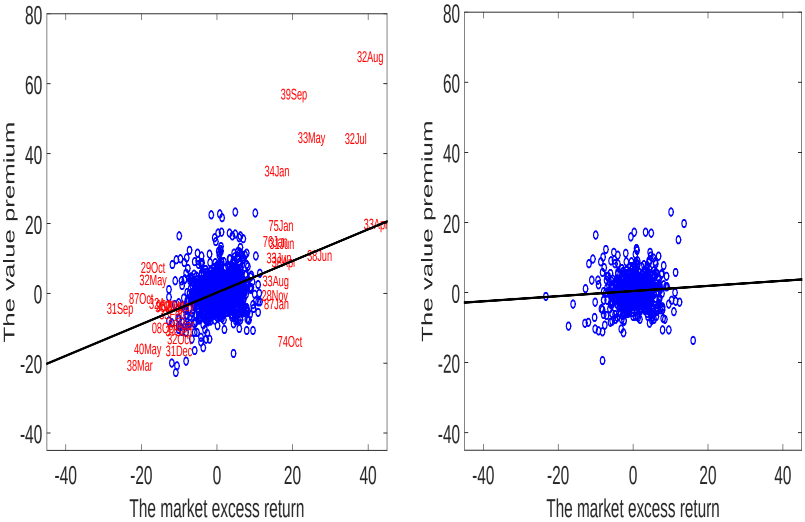
Stylized Facts

The book-to-market deciles, July 1926–June 1963 ($p_{\mathrm{GRS}} = 0.14$)

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.80	0.90	0.84	0.85	0.98	0.99	0.87	1.22	1.35	1.31	0.51
t_{R^e}	2.57	3.06	2.77	2.40	2.89	2.65	2.17	2.88	2.72	2.22	1.30
α	−0.04	0.11	0.02	−0.10	0.07	0.01	−0.18	0.11	0.08	−0.14	−0.10
t_α	−0.44	1.60	0.25	−1.12	0.71	0.07	−1.27	0.89	0.38	−0.50	−0.31
β	0.98	0.91	0.96	1.10	1.06	1.14	1.23	1.30	1.48	1.68	0.71
t_β	46.35	19.18	47.86	16.67	24.69	12.60	17.77	16.90	15.07	14.50	5.31
R^2	0.94	0.92	0.94	0.92	0.93	0.89	0.89	0.89	0.84	0.77	0.31

Stylized Facts

Scatter plots, 1926–2017 versus 1963–2017



Stylized Facts

Large swings in the stock market and the value premium, 3% extreme MKT returns

	MKT	H−L
November 1928	11.81	−0.29
October 1929	−20.12	7.60
June 1930	−16.27	−3.60
May 1931	−13.24	−3.37
June 1931	13.90	14.57
September 1931	−29.13	−4.03
December 1931	−13.53	−16.22
April 1932	−17.96	−2.65
May 1932	−20.51	4.09
July 1932	33.84	44.54
August 1932	37.06	67.95
October 1932	−13.17	−12.80
February 1933	−15.24	−5.70
April 1933	38.85	20.04
May 1933	21.43	44.85
June 1933	13.11	10.40

	MKT	H−L
August 1933	12.05	3.76
January 1934	12.60	35.20
September 1937	−13.61	−10.56
March 1938	−23.82	−20.35
April 1938	14.51	9.16
June 1938	23.87	11.15
September 1939	16.88	57.22
May 1940	−21.95	−15.59
October 1974	16.10	−13.57
January 1975	13.66	19.72
January 1976	12.16	15.03
March 1980	−12.90	−8.78
January 1987	12.47	−2.83
October 1987	−23.24	−1.20
August 1998	−16.08	−3.27
October 2008	−17.23	−9.64

Stylized Facts

The beta anomaly, the pre-ranking β deciles, 7/1963–6/2017 ($p_{\text{GRS}} = 0.18$)

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.52	0.52	0.56	0.58	0.69	0.55	0.67	0.55	0.57	0.55	0.03
t_{R^e}	3.85	3.64	3.45	3.38	3.75	2.86	3.14	2.42	2.23	1.72	0.11
α	0.22	0.17	0.13	0.12	0.18	0.01	0.07	−0.08	−0.13	−0.29	−0.52
t_α	2.11	1.76	1.69	1.42	2.17	0.18	0.85	−0.82	−1.10	−1.49	−1.94
β	0.57	0.68	0.82	0.87	0.98	1.03	1.15	1.22	1.34	1.62	1.06
t_β	12.39	17.21	20.57	20.68	28.13	31.21	50.25	41.76	35.41	30.92	11.81
R^2	0.53	0.68	0.77	0.79	0.86	0.86	0.88	0.86	0.84	0.77	0.43

Stylized Facts

The beta anomaly, the pre-ranking β deciles, 7/1928–6/2017 ($p_{GRS} = 0.01$)

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.58	0.63	0.65	0.74	0.83	0.72	0.79	0.73	0.77	0.75	0.16
t_{R^e}	5.03	4.66	4.41	4.46	4.54	3.71	3.74	3.11	2.94	2.44	0.66
α	0.22	0.16	0.13	0.14	0.17	0.01	0.02	−0.13	−0.17	−0.33	−0.55
t_α	2.87	2.22	2.21	2.31	2.49	0.20	0.27	−1.51	−1.68	−2.29	−2.81
β	0.57	0.73	0.83	0.94	1.05	1.11	1.22	1.36	1.48	1.70	1.13
t_β	22.86	30.50	36.61	40.31	41.41	39.61	48.26	36.17	26.65	40.93	18.82
R^2	0.66	0.81	0.85	0.88	0.90	0.90	0.91	0.90	0.88	0.84	0.57

Stylized Facts

The consumption CAPM, 25 size and BM portfolios, annual, 1930–2016

	Low	2	3	4	High	Low	2	3	4	High
	$E[R^e]$					β^C				
Small	6.04	10.65	13.73	16.82	18.56	2.80	0.66	1.63	1.86	1.58
2	9.02	12.32	13.33	14.90	16.03	1.25	1.72	0.88	1.25	1.68
3	9.27	11.83	11.88	13.73	14.72	0.29	1.11	1.77	2.12	2.15
4	8.82	9.68	11.49	12.83	13.16	0.38	0.37	1.32	1.36	0.47
Big	7.46	7.38	8.90	8.36	11.58	1.05	0.59	1.79	2.26	−0.88

Stylized Facts

The consumption CAPM, 25 size and BM portfolios, quarterly, 1947:Q2–2017:Q2

	Low	2	3	4	High	Low	2	3	4	High
	$E[R^e]$					β^C				
Small	1.25	2.58	2.57	3.23	3.65	4.22	4.73	3.43	3.63	3.94
2	1.74	2.58	2.86	3.01	3.38	3.01	2.89	2.91	3.07	3.60
3	1.96	2.61	2.54	2.99	3.26	2.85	2.59	2.57	2.63	2.99
4	2.18	2.18	2.60	2.74	2.93	2.47	2.16	2.54	2.39	3.77
Big	1.90	1.90	2.18	1.98	2.47	2.62	1.94	1.97	2.60	2.80

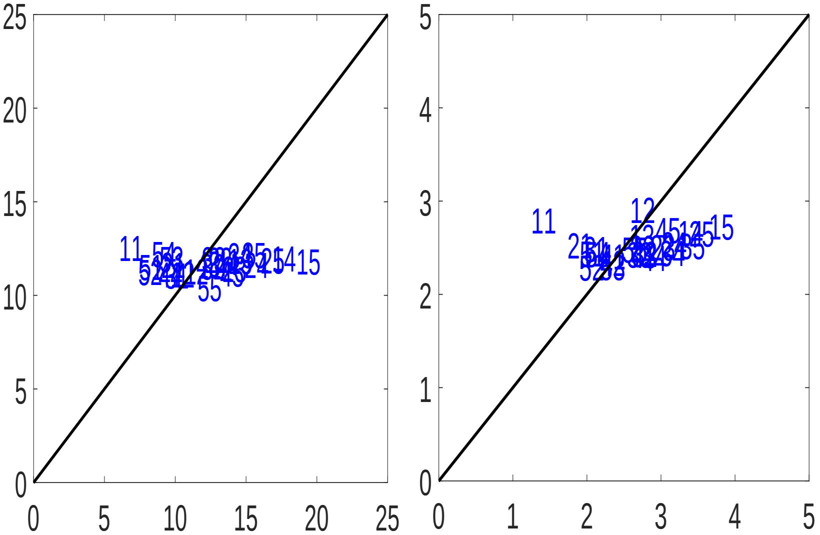
Stylized Facts

The consumption CAPM, 25 size and BM portfolios, second-stage regressions

	Panel A: Annual, 1930–2016		Panel B: Quarterly, 1947:Q2–2017:Q2	
	ϕ_0	ϕ_1	ϕ_0	ϕ_1
Estimates	10.97	0.58	1.88	0.22
t_{FM}	4.14	1.16	3.73	1.12
t_S	3.99	1.13	3.42	1.03
χ^2		152.19		100.00
p_{χ^2}		0.00		0.00
R^2		0.02		0.07

Stylized Facts

The consumption CAPM, 25 size and BM portfolios, annual and quarterly



Outline

The Model

A bird's eye view

Embedding disasters into a general equilibrium production economy:

- Rare disasters in productivity growth (Rietz 1988, Barro 2006)
- Endogenous cross-sectional distribution with asymmetric adjustment costs (Zhang 2005)
- Approximate aggregation (Krusell and Smith 1998)

Value stocks more exposed to disaster risk than growth stocks

The representative household has recursive utility:

$$U_t = \left[(1-\iota) C_t^{1-\frac{1}{\psi}} + \iota \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}},$$

The pricing kernel:

$$M_{t+1} = \iota \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{1-\gamma}}{E_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}$$

Operating profits:

$$\Pi_{it} = (X_t Z_{it})^{1-\xi} K_{it}^{\xi} - f K_{it}$$

Aggregate log productivity growth:

$$g_{xt} = \overline{g} + g_t$$

Firm-specific log productivity:

$$z_{it+1} = (1 - \rho_z)\bar{z} + \rho_z z_{it} + \sigma_z e_{it+1}$$

The Model

Normal times, Rouwenhorst 1995

g_t follows a discretized autoregressive process:

- Five states: $\{g_1, g_2, g_3, g_4, g_5\}$
- Transition matrix: $p_{ij} \equiv \text{Prob}(g_{t+1} = g_i | g_t = g_j)$:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{15} \\ p_{21} & p_{22} & \cdots & p_{25} \\ \vdots & \vdots & \ddots & \vdots \\ p_{51} & p_{52} & \cdots & p_{55} \end{bmatrix}$$

The Model

Disasters, Danthine and Donaldson 1999

Insert the disaster state, $g_0 = \lambda_D < 0$ and the recovery state, $g_6 = \lambda_R > 0$

Modify the transition matrix:

$$P = \begin{bmatrix} \theta & 0 & 0 & \dots & 0 & 1-\theta \\ \eta & p_{11}-\eta & p_{12} & \dots & p_{15} & 0 \\ \eta & p_{21} & p_{22}-\eta & \dots & p_{25} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta & p_{51} & p_{52} & \dots & p_{55}-\eta & 0 \\ 0 & (1-\nu)/5 & (1-\nu)/5 & \dots & (1-\nu)/5 & \nu \end{bmatrix}$$

η: disaster probability, θ: persistence, ν: recovery persistence

Capital accumulation:

$$K_{it+1} = I_{it} + (1-\delta)K_{it}$$

Asymmetric capital adjustment costs:

$$\Phi(I_{it}, K_{it}) = \begin{cases} a^{+}K_{it} + \frac{c^{+}}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & \text{for} \quad I_{it} > 0 \\ 0 & \text{for} \quad I_{it} = 0 \\ a^{-}K_{it} + \frac{c^{-}}{2}\left(\frac{I_{it}}{K_{it}}\right)^2 K_{it} & \text{for} \quad I_{it} < 0 \end{cases}$$

in which $c^{-} > c^{+} > 0$ and $a^{-} > a^{+} > 0$

The cross-sectional distribution of firms, μ_t, including K_{it} and z_{it}:

$$\mu_{t+1} = \Upsilon(\mu_t, X_t, X_{t+1}).$$

Value maximization:

$$V_{it} = \max_{\{\chi_{it}\}} \left(\max_{\{I_{it}\}} \ D_{it} + E_t\left[M_{t+1} V(K_{it+1}, Z_{it+1}; X_{t+1}, \mu_{t+1})\right], \ sK_{it} \right),$$

in which $D_{it} = \Pi_{it} - I_{it} - \Phi(I_{it}, K_{it})$ is net payout

Entry and exit, delisting return, reorganizational costs

$E_t[M_{t+1}R_{it+1}] = 1$ implies:

$$E_t[R_{it+1}] = r_{ft} + \left(-\frac{\text{Cov}_t[R_{it+1}, M_{t+1}]}{\text{Var}_t[M_{t+1}]}\right)\frac{\text{Var}_t[M_{t+1}]}{E_t[M_{t+1}]} \equiv r_{ft} + \beta_{it}^{M}\phi_{Mt}$$

- $r_{ft} \equiv 1/E_t[M_{t+1}]$: The real interest rate
- β_i^M: The true beta
- ϕ_{Mt}: The price of consumption risk

A competitive equilibrium consists of an optimal investment rule, $I(K_{it}, Z_{it}; X_t, \mu_t)$; an optimal exit rule, $\chi(K_{it}, Z_{it}; X_t, \mu_t)$; a value function, $V(K_{it}, Z_{it}; X_t, \mu_t)$; and an equilibrium law of motion for the firm distribution, $\Upsilon(\mu_t, X_t, X_{t+1})$, such that:

- Optimality: $I(K_{it}, Z_{it}; X_t, \mu_t)$, $\chi(K_{it}, Z_{it}; X_t, \mu_t)$, and $V(K_{it}, Z_{it}; X_t, \mu_t)$ solve the firms' problem

- Consistency: The aggregate behavior of the economy is consistent with the optimal behavior of all firms:

$$
\begin{aligned}
Y_t &= \int Y_{it}\, \mu_t(dK_{it}, dZ_{it}) \\
I_t &= \int I_{it}\, \mu_t(dK_{it}, dZ_{it}) \\
K_t &= \int K_{it}\, \mu_t(dK_{it}, dZ_{it}) \\
\Phi_t &= \int \Phi_{it}\, \mu_t(dK_{it}, dZ_{it})
\end{aligned}
$$

- Consistency: The law of motion for the firm distribution, Υ, is consistent with the optimal decisions of firms. Let Θ be any measurable set in the product space of K_{it} and Z_{it}:

$$\mu_{t+1}(\Theta, X_{t+1}) = T(\Theta, (K_{it}, Z_{it}), X_t)\,\mu_t(K_{it}, Z_{it}, X_t),$$

in which $T(\Theta, (K_{it}, Z_{it}), X_t) \equiv$:

$$\iint \mathbf{1}_{\{(I_{it}+(1-\delta)K_{it}, Z_{it+1})\in\Theta\}}\, Q_Z(dZ_{it+1}|Z_{it})Q_X(dX_{t+1}|X_t),$$

and $\mathbf{1}_{\{\cdot\}}$ is an indicator function that takes the value of one if the event described in $\{\cdot\}$ is true, and zero otherwise, and Q_Z and Q_X are the transition functions for Z_{it} and X_t, respectively

- Market clearing: Aggregate consumption equals aggregate output minus investment:

$$C_t = Y_t - I_t \qquad \Rightarrow \qquad C_t = D_t + fK_t + \Phi_t$$

The fixed costs of production, fK_t, and capital adjustment costs, Φ_t, as compensation to labor and part of consumption, driving a wedge between C_t and D_t (Abel 1999)

Detrending with X_{t-1}, e.g., $\widehat{K}_{it} = K_{it}/X_{t-1}$

Assume the average detrended capital, $\overline{K}_t$, contains all the information in μ_t relevant for forecasting M_{t+1}

The detrended value function, $\widehat{V}(\widehat{K}_{it}, Z_{it}, g_t, \overline{K}_t) =$

$$\max_{\{\chi_{it}\}} \left[\max_{\{\widehat{K}_{it+1}\}} \widehat{D}_{it} + E_t \left[M_{t+1} \widehat{V}(\widehat{K}_{it+1}, Z_{it+1}, g_{t+1}, \overline{K}_{t+1}) \right] \exp(g_{xt}),\ s\widehat{K}_{it} \right]$$

The Model

Approximate aggregation, Krusell and Smith 1998

The equilibrium laws of motion:

$$\begin{aligned}
\log \widehat{C}_t^{(j+1)}(g_t = g_i) &= a_{0i}^{(j+1)} + a_{1i}^{(j+1)} \log \overline{K_t} + a_{2i}^{(j+1)} \left(\log \overline{K_t}\right)^2 \\
\log \overline{K}_{t+1}^{(j+1)}(g_t = g_i) &= b_{0i}^{(j+1)} + b_{1i}^{(j+1)} \log \overline{K_t} + b_{2i}^{(j+1)} \left(\log \overline{K_t}\right)^2
\end{aligned}$$

Check the convergence for the coefficients, for $l = \{0, 1, 2\}$:

$$\max_{i \in [1,7]} |a_{li}^{(j+1)} - a_{li}^{(j)}| < 10^{-2}, \quad \max_{i \in [1,7]} |b_{li}^{(j+1)} - b_{li}^{(j)}| < 10^{-3}$$

Otherwise update the coefficients with the Newton method

$R^2 = 0.9999983$ for $\overline{K}_t$ and 0.99494656 for $\widehat{C}_t$, with $N = 30,000$

Outline

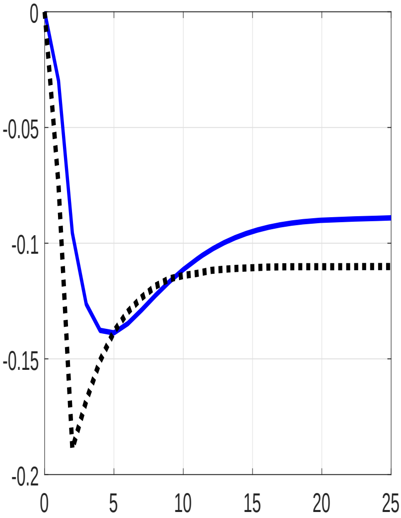
Calibration

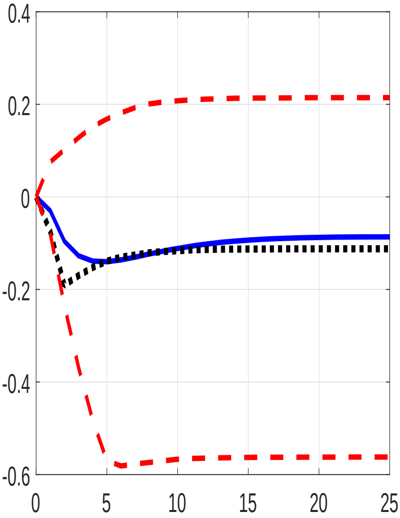
The benchmark monthly calibration

ι	γ	ψ	$\bar{g}$	ρ_g	σ_g
0.9945	5	1.5	0.019/12	0.6	0.003
η	λ_D	θ	λ_R	ν	ξ
0.02/12	-0.0275	$0.914^{1/3}$	0.015	0.964	0.65
δ	f	$\bar{z}$	ρ_z	σ_z	a^+
0.01	0.005	-8.52	0.985	0.5	0.035
a^-	c^+	c^-	s	κ	$\widetilde{R}$
0.05	75	150	0	0.25	-12.33%

Calibration

The impulse response of consumption to a disaster shock mimics the empirical pattern in Nakamura, Steinsson, Barro, and Ursua (2013)





Basic Moments

Aggregate output growth

	Samples with disasters, annual							Samples without disasters, quarterly					
	Data	mean	2.5	50	97.5	p		Data	mean	2.5	50	97.5	p
Vol	4.79	4.41	1.37	4.26	8.50	0.41		0.94	0.50	0.44	0.49	0.65	0.00
Skew	−0.29	−1.89	−4.32	−2.09	2.07	0.15		−0.18	0.02	−0.32	−0.02	1.02	0.88
Kurt	6.14	11.43	2.95	9.54	27.52	0.78		4.51	3.05	2.41	2.90	5.11	0.04
ρ_1	0.54	0.69	0.27	0.73	0.93	0.80	ρ_1	0.37	0.43	0.30	0.42	0.63	0.82
ρ_2	0.19	0.38	−0.15	0.40	0.82	0.74	ρ_4	−0.07	0.11	−0.06	0.09	0.35	0.99
ρ_3	−0.14	0.23	−0.22	0.21	0.72	0.92	ρ_8	−0.02	0.07	−0.09	0.06	0.26	0.82
ρ_4	−0.34	0.14	−0.26	0.12	0.62	0.99	ρ_{12}	−0.12	0.05	−0.10	0.04	0.24	0.99
ρ_5	−0.19	0.09	−0.25	0.07	0.53	0.94	ρ_{20}	0.05	0.02	−0.13	0.02	0.19	0.35

Basic Moments

Aggregate consumption growth

	Samples with disasters, annual							Samples without disasters, quarterly					
	Data	mean	2.5	50	97.5	p		Data	mean	2.5	50	97.5	p
Vol	2.13	4.28	1.30	4.13	8.28	0.87		0.50	0.46	0.40	0.45	0.60	0.09
Skew	−1.48	−1.93	−4.42	−2.14	2.13	0.32		−0.41	0.02	−0.31	−0.03	1.14	0.99
Kurt	8.09	11.66	2.98	9.63	28.82	0.63		4.17	3.10	2.44	2.93	5.83	0.04
ρ_1	0.48	0.69	0.24	0.74	0.93	0.85	ρ_1	0.31	0.44	0.31	0.44	0.66	0.97
ρ_2	0.18	0.39	−0.15	0.42	0.83	0.75	ρ_4	0.10	0.13	−0.05	0.12	0.39	0.61
ρ_3	−0.05	0.24	−0.22	0.23	0.72	0.86	ρ_8	−0.02	0.08	−0.08	0.08	0.30	0.86
ρ_4	−0.19	0.16	−0.24	0.13	0.63	0.95	ρ_{12}	0.08	0.06	−0.10	0.05	0.28	0.35
ρ_5	0.00	0.10	−0.24	0.08	0.55	0.70	ρ_{20}	−0.04	0.03	−0.13	0.03	0.21	0.83

Basic Moments

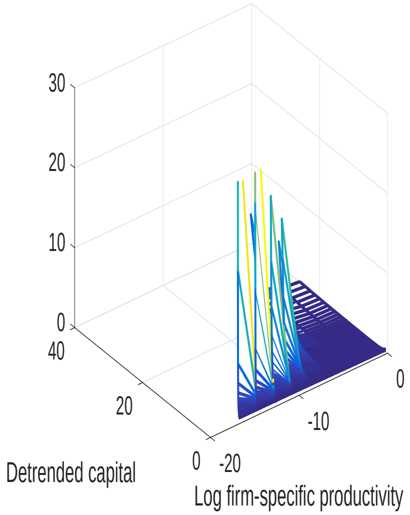
Aggregate investment growth

	Samples with disasters, annual							Samples without disasters, quarterly					
	Data	mean	2.5	50	97.5	p		Data	mean	2.5	50	97.5	p
Vol	13.53	19.56	3.10	12.28	71.84	0.45		2.40	1.09	0.98	1.08	1.33	0.00
Skew	−1.33	−0.17	0.02	−1.56	2.69	0.68		−0.53	−0.20	−0.58	−0.20	0.25	0.96
Kurt	7.07	27.45	6.68	19.50	100.98	0.96		4.73	3.70	2.85	3.41	5.26	0.03
ρ_1	0.41	0.18	0.00	0.23	0.59	0.17	ρ_1	0.46	0.24	0.11	0.24	0.38	0.01
ρ_2	−0.15	−0.06	0.00	0.00	−0.44	0.71	ρ_4	−0.03	−0.00	−0.12	−0.01	0.14	0.63
ρ_3	−0.33	−0.07	0.00	0.00	0.38	0.96	ρ_8	−0.18	−0.01	−0.12	−0.01	0.11	1.00
ρ_4	−0.17	−0.06	−0.00	0.00	−0.07	0.84	ρ_{12}	−0.09	−0.01	−0.13	−0.01	0.11	0.90
ρ_5	−0.05	−0.05	−0.00	−0.05	−0.06	0.57	ρ_{20}	0.03	−0.00	−0.12	0.00	0.11	0.29

Outline

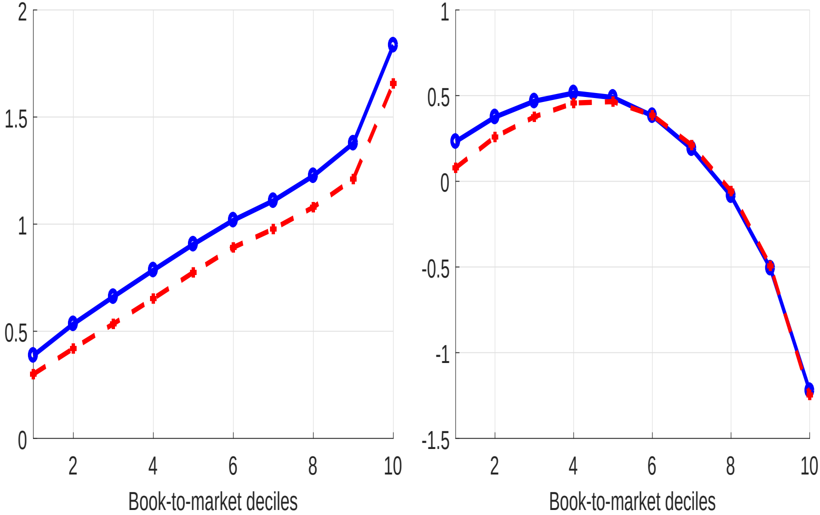
Equilibrium Properties

The true beta: Disaster vs. mean normal state



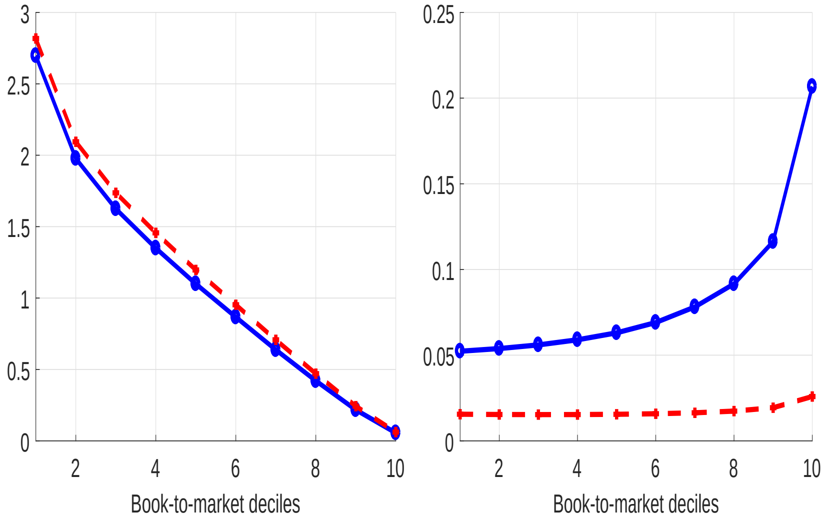
Equilibrium Properties

Value versus growth: $\widehat{K}_{it}$ and z_{it}



Equilibrium Properties

Value versus growth: $\widehat{I}_{it}/\widehat{K}_{it}$ and β^M_{it}



Outline

The CAPM

The book-to-market deciles, no-disaster samples, $p_{\mathrm{GRS}} = 0.00, [0.00, 0.01]$

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.77	0.76	0.75	0.74	0.75	0.76	0.78	0.82	0.91	1.16	0.40
t_{R^e}	23.37	23.02	22.48	22.05	22.08	21.79	22.75	23.93	25.51	28.69	7.72
α	0.10	0.04	−0.02	−0.07	−0.10	−0.13	−0.07	0.02	0.13	0.35	0.25
2.5	−0.04	−0.09	−0.16	−0.20	−0.24	−0.26	−0.20	−0.12	−0.00	0.17	0.02
97.5	0.25	0.18	0.12	0.08	0.05	0.00	0.06	0.16	0.27	0.51	0.49
t_α	1.46	0.57	−0.22	−0.99	−1.37	−1.80	−0.93	0.32	1.83	4.25	2.26
2.5	−0.55	−1.21	−2.21	−2.82	−3.24	−3.62	−2.78	−1.63	−0.01	1.77	0.18
97.5	3.61	2.68	1.68	1.16	0.88	0.02	0.88	2.46	3.87	6.61	4.37
β	0.83	0.90	0.96	1.02	1.06	1.10	1.06	1.00	0.97	1.01	0.18
2.5	0.67	0.74	0.81	0.86	0.89	0.97	0.90	0.84	0.80	0.80	−0.09
97.5	0.98	1.05	1.11	1.18	1.23	1.24	1.22	1.15	1.13	1.20	0.47
t_β	11.04	11.91	12.60	13.23	13.69	14.06	13.58	12.94	11.89	10.64	1.44
2.5	8.56	9.05	10.40	10.63	11.07	11.51	10.92	10.29	9.55	7.66	−0.70
97.5	14.75	16.68	15.84	16.53	16.60	17.57	17.53	17.10	15.10	13.49	3.59
R^2	0.10	0.12	0.13	0.14	0.15	0.16	0.15	0.13	0.12	0.10	0.00

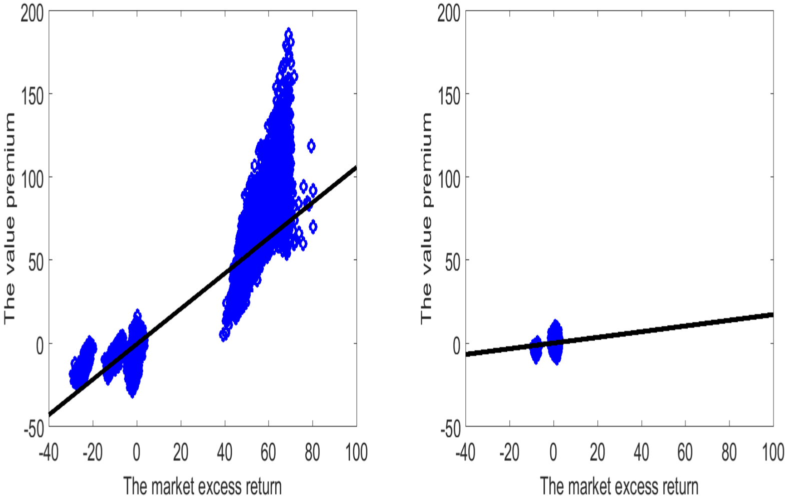
The CAPM

The book-to-market deciles, disaster samples, $p_{GRS} = 0.01, [0.00, 0.20]$

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.75	0.74	0.74	0.74	0.75	0.77	0.81	0.86	0.96	1.20	0.46
t_{R^e}	11.17	10.95	10.73	10.50	10.29	10.02	9.83	9.59	9.29	8.94	4.92
α	0.08	0.06	0.04	0.03	0.01	−0.02	−0.05	−0.09	−0.15	−0.27	−0.35
2.5	−0.03	−0.03	−0.04	−0.05	−0.08	−0.12	−0.19	−0.29	−0.42	−0.70	−0.86
97.5	0.21	0.16	0.13	0.10	0.08	0.06	0.05	0.04	0.02	0.00	0.00
t_α	1.75	1.55	1.22	0.74	0.18	−0.54	−1.10	−1.60	−2.05	−2.32	−2.44
2.5	−0.84	−0.91	−1.03	−1.53	−2.11	−3.01	−3.63	−4.19	−4.16	−4.33	−4.53
97.5	4.43	3.99	3.56	2.92	2.29	1.90	1.39	1.07	0.57	0.05	0.05
β	0.83	0.85	0.87	0.89	0.93	0.99	1.07	1.19	1.40	1.84	1.01
2.5	0.66	0.73	0.79	0.84	0.87	0.90	0.94	1.00	1.13	1.47	0.52
97.5	0.98	0.96	0.94	0.96	1.04	1.18	1.33	1.57	1.85	2.32	1.61
t_β	35.57	42.36	51.84	69.25	74.28	65.01	53.50	38.76	25.28	18.49	7.85
2.5	8.64	12.52	17.67	22.78	18.11	12.68	10.11	8.22	7.10	7.45	3.47
97.5	132.89	133.16	145.36	174.58	184.14	169.89	166.37	139.47	77.09	42.45	17.28
R^2	0.77	0.78	0.79	0.79	0.80	0.81	0.83	0.85	0.86	0.87	0.57

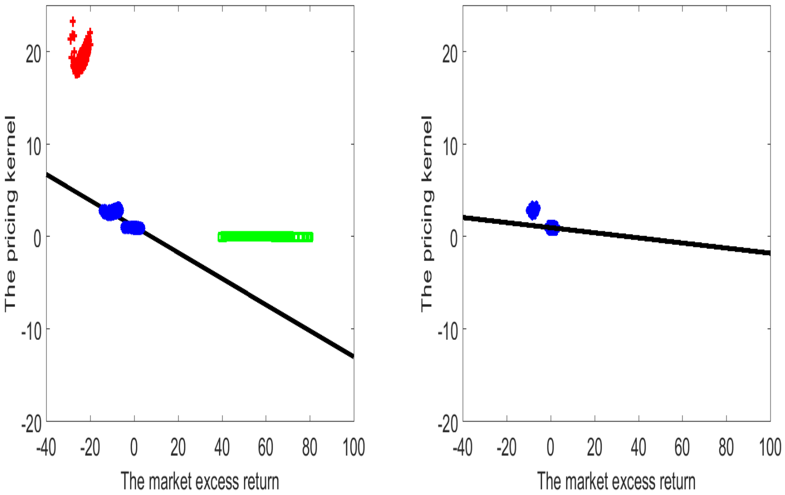
The CAPM

Nonlinearity in the CAPM regressions, disasters versus no disasters



The CAPM

Nonlinearity in the pricing kernel, disasters versus no disasters



The CAPM

The beta anomaly: Deciles formed on rolling market betas, no-disaster samples

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.78	0.81	0.82	0.83	0.81	0.84	0.83	0.81	0.79	0.76	−0.02
t_{R^e}	23.48	23.66	23.62	23.53	21.72	23.38	23.13	22.88	22.50	21.87	−0.48
α	−0.05	0.07	0.11	0.13	0.01	0.13	0.10	0.04	−0.05	−0.25	−0.21
2.5	−0.17	−0.06	−0.04	−0.01	−0.14	−0.01	−0.04	−0.14	−0.20	−0.37	−0.39
97.5	0.07	0.22	0.27	0.29	0.16	0.29	0.25	0.17	0.08	−0.12	−0.02
t_α	−0.69	0.99	1.58	1.82	0.16	1.77	1.27	0.53	−0.67	−3.67	−1.96
2.5	−2.43	−0.84	−0.47	−0.13	−1.77	−0.16	−0.55	−1.80	−2.73	−5.62	−3.91
97.5	1.01	3.24	3.75	4.14	2.08	3.96	3.26	2.39	1.36	−1.91	−0.15
β	1.03	0.92	0.88	0.87	0.99	0.88	0.91	0.96	1.05	1.26	0.23
2.5	0.89	0.75	0.72	0.68	0.83	0.69	0.74	0.81	0.88	1.10	−0.00
97.5	1.17	1.08	1.04	1.02	1.15	1.04	1.08	1.15	1.20	1.41	0.46
t_β	14.06	12.43	11.32	11.06	11.88	11.17	11.18	12.21	13.84	16.87	1.98
2.5	11.01	9.35	8.88	8.28	9.19	8.06	8.43	9.32	10.32	13.45	−0.01
97.5	17.00	16.61	14.24	15.16	15.76	13.54	14.07	14.84	16.66	21.02	4.19
R^2	0.16	0.12	0.11	0.10	0.12	0.10	0.11	0.13	0.15	0.22	0.00

The CAPM

The beta anomaly: Deciles formed on rolling market betas, disaster samples

	L	2	3	4	5	6	7	8	9	H	H−L
$E[R^e]$	0.77	0.79	0.81	0.83	0.82	0.85	0.85	0.85	0.85	0.83	0.06
t_{R^e}	10.48	10.68	10.54	10.26	9.78	9.83	9.57	9.27	8.69	8.31	0.85
α	0.03	0.05	0.04	0.02	−0.02	−0.03	−0.05	−0.09	−0.16	−0.21	−0.24
2.5	−0.12	−0.04	−0.04	−0.06	−0.13	−0.15	−0.22	−0.29	−0.47	−0.55	−0.67
97.5	0.16	0.15	0.12	0.11	0.09	0.09	0.09	0.08	0.07	0.04	0.11
t_α	0.70	1.36	1.17	0.46	−0.49	−0.53	−0.91	−1.23	−1.64	−2.15	−1.74
2.5	−2.92	−1.17	−1.09	−1.75	−3.10	−3.33	−3.80	−4.06	−4.39	−4.78	−4.52
97.5	3.66	3.77	3.26	2.84	2.29	2.33	2.33	2.17	1.84	1.05	1.86
β	0.92	0.92	0.96	1.01	1.05	1.09	1.12	1.16	1.25	1.28	0.37
2.5	0.78	0.84	0.90	0.93	0.94	0.96	0.95	0.95	0.94	0.92	−0.09
97.5	1.12	1.03	1.04	1.08	1.17	1.24	1.31	1.41	1.64	1.72	0.93
t_β	35.79	48.38	62.91	74.19	61.90	48.67	41.79	36.71	28.14	20.98	2.57
2.5	9.71	15.76	21.14	21.10	17.95	13.39	9.96	7.94	5.54	5.57	−2.85
97.5	134.73	167.31	168.44	192.12	192.23	156.44	157.07	154.49	140.85	79.15	7.00
R^2	0.81	0.81	0.82	0.82	0.82	0.84	0.84	0.84	0.85	0.85	0.21

Outline

The Consumption CAPM

First-stage regressions, 25 size and BM portfolios, annual, disaster samples

	Low	2	3	4	High	Low	2	3	4	High
	$E[R^e]$					β^C				
Small	13.69	14.54	15.95	17.90	23.37	−0.64	−0.77	−0.93	−1.15	−1.28
2	12.33	13.29	14.21	15.45	18.90	−0.49	−0.59	−0.72	−0.89	−1.34
3	12.05	12.17	12.42	12.95	14.62	−0.43	−0.47	−0.53	−0.64	−0.74
4	10.57	10.40	10.42	10.85	13.84	−0.32	−0.33	−0.36	−0.46	−0.69
Big	7.96	7.92	8.18	8.86	10.14	−0.07	−0.08	−0.10	−0.22	−0.23

The Consumption CAPM

First-stage regressions, 25 size and BM portfolios, quarterly, no-disaster samples

	Low	2	3	4	High	Low	2	3	4	High
	$E[R^e]$					β^C				
Small	3.16	3.31	3.56	3.92	5.17	0.11	0.12	0.12	0.13	0.27
2	2.89	3.08	3.24	3.45	4.09	0.12	0.12	0.13	0.13	0.18
3	2.84	2.85	2.88	2.96	3.33	0.16	0.13	0.14	0.16	0.25
4	2.53	2.48	2.47	2.53	3.19	0.16	0.18	0.22	0.24	0.24
Big	1.93	1.91	1.96	2.07	2.42	0.74	0.93	1.08	0.94	0.85

The Consumption CAPM

Second-stage regressions, 25 size and BM portfolios

	Annual, disasters		Quarterly, no disasters	
	ϕ_0	ϕ_1	ϕ_0	ϕ_1
Estimates	9.09	−6.48	3.34	−1.19
2.5	5.28	−13.46	3.14	−1.67
97.5	13.70	1.46	3.53	−0.72
t_{FM}	15.57	−6.30	73.94	−13.67
2.5	6.55	−12.84	53.97	−18.11
97.5	52.25	1.48	83.30	−8.26
t_S	8.22	−3.31	44.22	−9.14
2.5	3.81	−5.95	27.35	−10.94
97.5	25.46	1.45	58.20	−6.77
p_{χ^2}		0.01		0.00
2.5		0.00		0.00
97.5		0.04		0.01
R^2		0.61		0.30
2.5		0.01		0.12
97.5		0.95		0.49

The Consumption CAPM

First-stage regressions with M_{t+1}, 25 size and BM portfolios, annual, disasters

	Low	2	3	4	High	Low	2	3	4	High
			$\hat{\beta}^M$					$t_{\hat{\beta}^M}$		
Small	0.04	0.04	0.04	0.05	0.07	8.26	7.87	7.58	7.20	7.08
2	0.03	0.04	0.04	0.04	0.05	8.51	8.25	8.04	7.85	7.71
3	0.03	0.03	0.03	0.04	0.04	8.26	8.53	8.34	8.03	7.49
4	0.03	0.03	0.03	0.03	0.04	8.94	8.79	8.63	8.16	8.47
Big	0.02	0.02	0.02	0.02	0.03	8.79	8.53	8.26	7.76	7.49

The Consumption CAPM

First-stage regressions with M_{t+1}, 25 size and BM portfolios, quarterly, no-disasters

	Low	2	3	4	High	Low	2	3	4	High
			$\hat{\beta}^M$					$t_{\hat{\beta}^M}$		
Small	0.12	0.13	0.14	0.15	0.25	7.78	5.74	5.20	5.32	6.11
2	0.12	0.12	0.13	0.14	0.17	5.09	5.26	5.36	5.49	5.17
3	0.12	0.12	0.12	0.12	0.15	2.99	5.10	5.32	4.95	3.70
4	0.11	0.11	0.11	0.11	0.15	4.29	4.27	4.08	3.53	3.57
Big	0.09	0.10	0.10	0.10	0.12	2.66	2.81	2.91	2.99	2.90

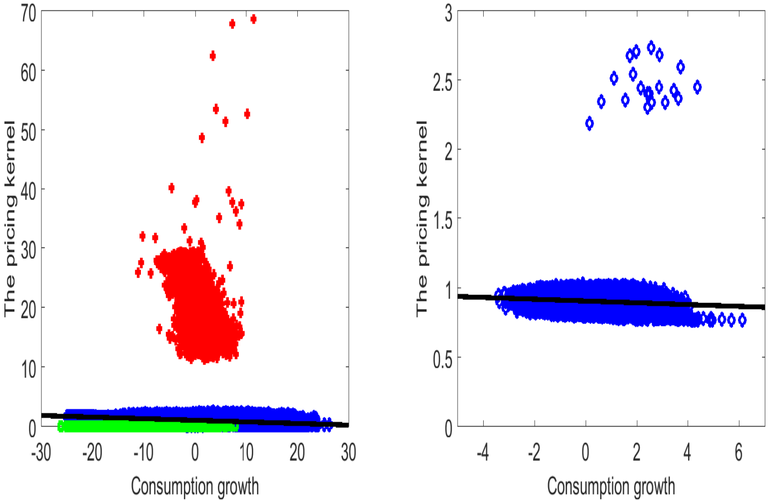
The Consumption CAPM

Second-stage regressions with M_{t+1}, 25 size and BM portfolios

	Annual, disasters		Quarterly, no disasters	
	$\hat{\phi}_0$	$\hat{\phi}_M$	$\hat{\phi}_0$	$\hat{\phi}_M$
Estimates	0.01	5.19	0.02	0.11
2.5	−0.01	0.36	0.01	0.06
97.5	0.06	7.69	0.02	0.26
t_{FM}	2.43	8.35	19.27	15.46
2.5	−1.48	3.17	7.44	8.76
97.5	17.71	18.90	30.21	20.65
t_{S}	0.90	3.56	6.82	5.42
2.5	−0.60	1.54	1.93	3.78
97.5	5.65	7.09	14.23	8.04
p_{χ^2}		0.55		0.51
2.5		0.00		0.00
97.5		1.00		0.98
R^2		0.89		0.43
2.5		0.55		0.13
97.5		0.97		0.79

The Consumption CAPM

Why does the standard consumption CAPM fail? disasters versus no disasters



Cochrane (2005a, p. 67): “Bringing multiple firms in at all is the first challenge for a general equilibrium model that wants to address the cross-section of returns. Since the extra technologies represent nonzero net supply assets, each ‘firm’ adds another state variable to the equilibrium. Many of the above papers circumvent this problem by modeling the discount factor directly as a function of shocks rather than specify preferences and derive the discount factor from the equilibrium consumption process. Then each firm can be valued in isolation. This is a fine short cut in order to learn about useful specifications of technology, but in the end, of course we don't really understand risk premia until they come from the equilibrium consumption process fed through a utility function (my emphasis).”

Bai et al.'s (2015) partial equilibrium results are quantitatively similar, calling for a more pluralistic perspective on GE

Outline

Comparative Statics

Increasing the disaster size and persistence raises the value premium and exacerbates the CAPM failure in no-disaster samples

Raising the disaster probability goes in the same direction, but its quantitative impact is small

The recovery size and persistence have little impact on the value premium or the CAPM performance

Raising the fixed costs parameter increases the value premium but decreases its CAPM alpha in no-disaster samples

Increasing the liquidation value reduces the value premium but the reorganization cost and delisting return have little impact

Increasing the risk aversion or the intertemporal elasticity of substitution strengthens the nonlinear dynamics

Conclusion

Bai et al. (2019)

A general equilibrium heterogeneous firms economy with disasters explains the failure of the (consumption) CAPM with strong nonlinearity in the pricing kernel

The widely documented empirical failures of the consumption CAPM might have more to do with the deficiencies of data and test designs rather than economic theory