

Lecture Notes

Hou, Xue, and Zhang (2015, Review of Financial Studies)
Digested Anomalies: An Investment Approach

Lu Zhang¹

¹Ohio State and NBER

BUSFIN 8250
Autumn 2014, Ohio State

A q -factor model consisting of the market factor, a size factor, an investment factor, and a profitability factor largely summarizes the cross section of average stock returns

$$r_t^i - r_t^f = \alpha_q^i + \beta_{\text{MKT}}^i \text{MKT}_t + \beta_{\text{ME},t}^i + \beta_{\text{I/A},t}^i + \beta_{\text{ROE},t}^i + \epsilon^i$$

- MKT_t , $r_{\text{ME},t}$, $r_{\text{I/A},t}$, and $r_{\text{ROE},t}$ are the market, size, investment, and *ROE* factors, respectively
- β_{MKT}^i , β_{ME}^i , $\beta_{\text{I/A}}^i$, and β_{ROE}^i are factor loadings

Introduction

Properties of the *q*-factors, 1/1972–12/2012

| | \bar{R} | α | β_{MKT} | β_{SMB} | β_{HML} | β_{UMD} |
|------------------|------------------|------------------|----------------------|----------------------|----------------------|----------------------|
| r_{ME} | 0.31 | 0.23 | 0.17 | | | |
| | (2.12) | 0.04 | 0.02 | 0.99 | 0.17 | |
| | | 0.01 | 0.02 | 0.99 | 0.19 | 0.03 |
| $r_{\text{I/A}}$ | 0.45 | 0.52 | -0.15 | | | |
| | (4.95) | 0.33 | -0.06 | -0.02 | 0.39 | |
| | | 0.28 | -0.05 | -0.02 | 0.41 | 0.05 |
| r_{ROE} | 0.58 | 0.63 | -0.11 | | | |
| | (4.81) | 0.77 | -0.09 | -0.33 | -0.20 | |
| | | 0.50 | -0.03 | -0.33 | -0.10 | 0.28 |
| | $r_{\text{I/A}}$ | r_{ROE} | MKT | SMB | HML | UMD |
| r_{ME} | -0.11 | -0.31 | 0.25 | 0.95 | -0.07 | 0.01 |
| $r_{\text{I/A}}$ | | 0.06 | -0.36 | -0.22 | 0.69 | 0.05 |
| r_{ROE} | | | -0.19 | -0.38 | -0.09 | 0.50 |

About one half of nearly 80 anomalies are insignificant with NYSE breakpoints and value-weighted decile returns

In “explaining” 35 significant anomalies, the q -factor model performs well relative to the Fama-French and Carhart models:

- The average magnitude of high-minus-low alphas: .20% in q , .33% in Carhart, and .55% in Fama-French
- The number of significant high-minus-low alphas: 5 in q , 19 in Carhart, and 27 in Fama-French
- The number of rejections by the GRS test: 20 in q , 24 in Carhart, and 28 in Fama-French

- 1 Intuition
- 2 Factors/Testing Portfolios
- 3 Factor Regressions: Alphas
- 4 Factor Regressions: Betas
- 5 Sharpe Ratios

1 Intuition

2 Factors/Testing Portfolios

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4 Factor Regressions: Betas

5 Sharpe Ratios

Two periods, 0 and 1

Heterogenous firms, indexed by $i = 1, \dots, N$

Firm i 's operating profits in dates 0 and 1, $\Pi_{i0}A_{i0}$ and $\Pi_{i1}A_{i1}$:

- A_{i0} and A_{i1} : Productive assets

$$A_{i1} = I_{i0}$$

in which I_{i0} is investment (the depreciation rate is 100%)

- Π_{i0} and Π_{i1} : Profitability (ROE)

M_1 : Stochastic discount factor

Firm i 's value-maximization problem:

$$P_{i0} + D_{i0} \equiv \max_{\{I_{i0}\}} \Pi_{i0} A_{i0} - I_{i0} - \frac{a}{2} \left(\frac{I_{i0}}{A_{i0}} \right)^2 A_{i0} + E_0[M_1 \Pi_{i1} A_{i1}]$$

The first principle for investment:

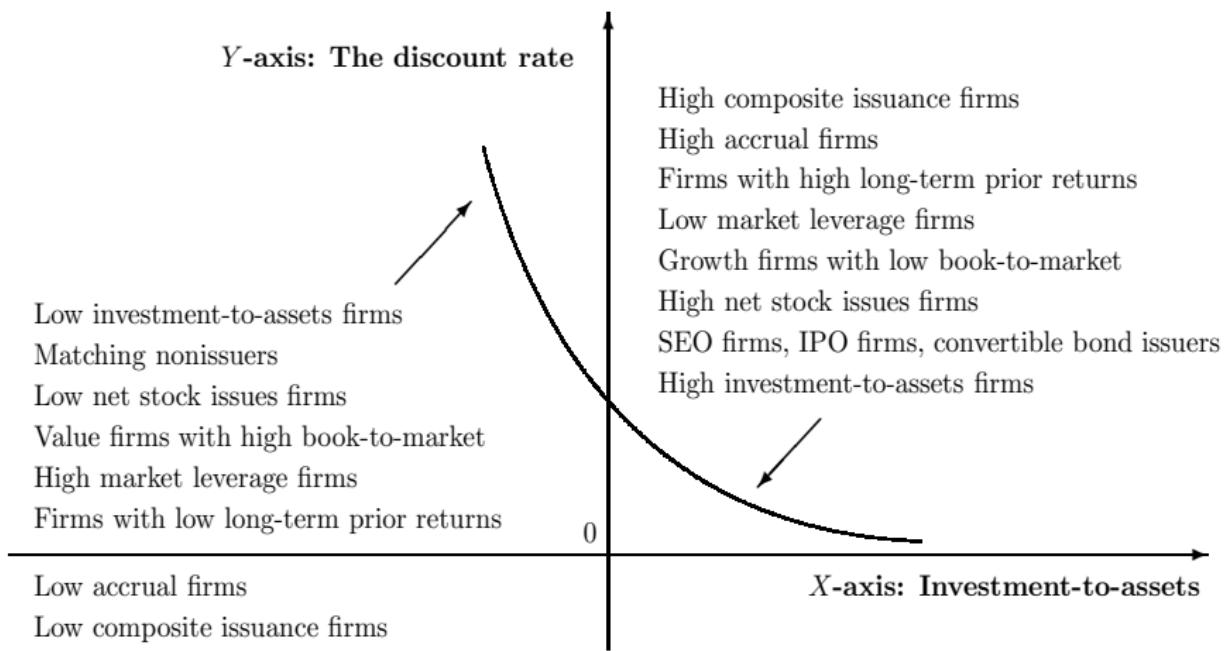
$$1 + a \frac{I_{i0}}{A_{i0}} = E_0[M_1 \Pi_{i1}]$$

A more familiar form from the corporate finance perspective:

$$r_{i1}^S = \frac{P_{i1} + D_{i1}}{P_{i0}} = \frac{\Pi_{i1} A_{i1}}{E_0[M_1 \Pi_{i1} A_{i1}]} = \frac{\Pi_{i1}}{E_0[M_1 \Pi_{i1}]} = \frac{\Pi_{i1}}{1 + a(I_{i0}/A_{i0})}$$

$$E_0[r_{i1}^S] = \frac{E_0[\Pi_{i1}]}{1 + a(I_{i0}/A_{i0})}$$

- All else equal, high investment stocks should earn lower expected returns than low investment stocks
- All else equal, high expected profitability stocks should earn higher expected returns than low expected profitability stocks



High ROE relative to low investment means high discount rates:

- Suppose the discount rates were low
- Combined with high ROE, low discount rates would imply high net present values of new projects (and high investment)
- So discount rates must be high to counteract high ROE to induce low investment

Price and earnings momentum winners and low distress firms tend to have higher ROE and earn higher expected returns

The investment-based model is a characteristics-based model

We implement a linear factor model:

- Returns are better measured than accounting variables, also with higher frequency
- Estimating the economic model directly involves specification errors in the production and investment technologies, aggregation, etc., absent from the factor model

1 Intuition

2 Factors/Testing Portfolios

3 Factor Regressions: Alphas

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Construct $r_{ME,t}$, $r_{I/A,t}$, and $r_{ROE,t}$ with a triple two-by-three-by-three sort on size, investment, and ROE

Variable definitions:

- Size: Stock price times shares outstanding from CRSP
- Investment, I/A: Annual changes in total assets (item AT) divided by lagged total assets
- ROE: Income before extraordinary items (item IBQ) divided by one-quarter-lagged book equity

NYSE breakpoints: 50-50 for size, 30-40-30 for investment, and 30-40-30 for ROE

Timing:

- Annual sort in June on the market equity at the June end
- Annual sort in June of year t on I/A for the fiscal year ending in calendar year $t - 1$
- Monthly sort at the beginning of each month on ROE with the most recently announced quarterly earnings

Timing is consistent with the economic model

Use an extensive array of anomaly portfolios (nearly 80), scope comparable with the largest in the literature

- Green, Hand, and Zhang (2013)
- Harvey, Liu, and Zhu (2013)
- McLean and Pontiff (2013)

NYSE breakpoints and value-weighted decile returns to alleviate the impact of microcaps

- Fama and French (2008)

Factors/Testing Portfolios

Six categories of anomalies, 80 in total

Panel A: Momentum (plus six momentum-reversal variables)

SUE-1, earnings surprise
(1-month holding period),
Foster, Olsen, and Shevlin (1984)
Abr-1, cumulative abnormal stock
returns around earnings announcements
(1-month holding period), Chan,
Jegadeesh, and Lakonishok (1996)
RE-1, revisions in analysts' earnings
forecasts (1-month holding period),
Chan, Jegadeesh, and Lakonishok (1996)
R6-1, price momentum (6-month prior
returns, 1-month holding period),
Jegadeesh and Titman (1993)
R11-1, price momentum, (11-month
prior returns, 1-month holding period),
Fama and French (1996)

SUE-6, earnings surprise
(6-month holding period),
Foster, Olsen, and Shevlin (1984)
Abr-6, cumulative abnormal stock
returns around earnings announcements
(6-month holding period), Chan,
Jegadeesh, and Lakonishok (1996)
RE-6, revisions in analysts' earnings
forecasts (6-month holding period),
Chan, Jegadeesh, and Lakonishok (1996)
R6-6, price momentum (6-month prior
returns, 6-month holding period),
Jegadeesh and Titman (1993)
I-Mom, industry momentum,
Moskowitz and Grinblatt (1999)

Factors/Testing Portfolios

Six categories of anomalies, 80 in total

Panel B: Value versus growth

B/M, book-to-market equity,

Rosenberg, Reid, and Lanstein (1985)

Rev, reversal, De Bondt and Thaler (1985)

EF/P, analysts' earnings forecasts-to-price,

Elgers, Lo, and Pfeiffer (2001)

D/P, dividend yield,

Litzenberger and Ramaswamy (1979)

NO/P, net payout yield, Boudoukh,

Michaely, Richardson, and Roberts (2007)

LTG, long-term growth forecasts

of analysts, La Porta (1996)

A/ME, market leverage,

Bhandari (1988)

E/P, earnings-to-price, Basu (1983)

CF/P, cash flow-to-price,

Lakonishok, Shleifer, and Vishny (1994)

O/P, payout yield, Boudoukh, Michaely, Richardson, and Roberts (2007)

SG, sales growth,

Lakonishok, Shleifer, and Vishny (1994)

Dur, equity duration,

Dechow, Sloan, and Soliman (2004)

Factors/Testing Portfolios

Six categories of anomalies, 80 in total

Panel C: Investment

ACI, abnormal corporate investment,
Titman, Wei, and Xie (2004)

NOA, net operating assets, Hirshleifer,
Hou, Teoh, and Zhang (2004)

IG, investment growth,
Xing (2008)

CEI, composite issuance,
Daniel and Titman (2006)

lvG, inventory growth,
Belo and Lin (2011)

OA, operating accruals, Sloan (1996)

POA, percent operating accruals, Hafzalla,
Lundholm, and Van Winkle (2011)

I/A, investment-to-assets,
Cooper, Gulen, and Schill (2008)
ΔPI/A, changes in PPE
plus changes in inventory scaled by assets,
Lyandres, Sun, and Zhang (2008)

NSI, net stock issues,
Pontiff and Woodgate (2008)

NXF, net external financing,
Bradshaw, Richardson, and Sloan (2006)

lvC, inventory changes,
Thomas and Zhang (2002)

TA, total accruals, Richardson, Sloan,
Soliman, and Tuna (2005)

PTA, percent total accruals, Hafzalla,
Lundholm, and Van Winkle (2011)

Factors/Testing Portfolios

Six categories of anomalies, 80 in total

Panel D: Profitability

ROE, return on equity,

Haugen and Baker (1996)

RNA, return on net operating assets,
Soliman (2008)

ATO, asset turnover,
Soliman (2008)

GP/A, gross profits-to-assets,
Novy-Marx (2013)

TES, tax expense surprise,
Thomas and Zhang (2011)

RS, revenue surprise,
Jegadeesh and Livnat (2006)

FP, failure probability,

Campbell, Hilscher, and Szilagyi (2008)

ROA, return on assets,

Balakrishnan, Bartov, and Faurel (2010)

PM, profit margin, Soliman (2008)

CTO, capital turnover,

Haugen and Baker (1996)

F, *F*-score,

Piotroski (2000)

TI/BI, taxable income-to-book income,

Green, Hand, and Zhang (2013)

NEI, number of consecutive quarters

with earnings increases,

Barth, Elliott, and Finn (1999)

O, *O*-score, Dichev (1998)

Panel E: Intangibles and other characteristics

OC/A, organizational capital-to-assets,
Eisfeldt and Papanikolaou (2013)

Ad/M, advertisement expense-to-market,
Chan, Lakonishok, and Sougiannis (2001)

RD/M, R&D-to-market,
Chan, Lakonishok, and Sougiannis (2001)

H/N, hiring rate,
Belo, Lin, and Bazdresch (2014)

G, corporate governance,
Gompers, Ishii, and Metrick (2003)

Ind, industries, Fama and French (1997)

BC/A, brand capital-to-assets,
Belo, Lin, and Vitorino (2014)

RD/S, R&D-to-sales,
Chan, Lakonishok, and Sougiannis (2001)

RC/A, R&D capital-to-assets, Li (2011)

OL, operating leverage,
Novy-Marx (2011)

AccQ, accrual quality, Francis, Lafond,
Olsson, and Schipper (2005)

Factors/Testing Portfolios

Six categories of anomalies, 80 in total

Panel F: Trading frictions

ME, the market equity,
Banz (1981)

Tvol, total volatility,
Ang, Hodrick, Xing, and Zhang (2006)

MDR, maximum daily return,
Bali, Cakici, and Whitelaw (2011)

D- β , Dimson's beta, Dimson (1979)

Disp, dispersion of analysts'
earnings forecasts,
Diether, Malloy, and Scherbina (2002)

1/P, 1/share price,
Miller and Scholes (1982)

IIIq, Absolute return-to-volume,
Amihud (2002)

Ivol, idiosyncratic volatility,
Ang, Hodrick, Xing, and Zhang (2006)

Svol, systematic volatility,
Ang, Hodrick, Xing, and Zhang (2006)

β , market beta,
Frazzini and Pedersen (2014)

S-Rev, short-term reversal, Jegadeesh (1990)

Turn, share turnover,
Datar, Naik, and Radcliffe (1998)

Dvol, dollar trading volume,
Brennan, Chordia, and Subrahmanyam (1998)

Factors/Testing Portfolios

Insignificant anomalies in the broad cross section

| | R6-1 | A/ME | Rev | EF/P | D/P | O/P | SG | LTG | ACI | NXF |
|-------|---------|------------|-------|-------|-------|-------|-------|-------|-------|-------|
| m | 0.48 | 0.43 | -0.39 | 0.45 | 0.27 | 0.35 | -0.27 | 0.01 | -0.27 | -0.30 |
| t_m | 1.43 | 1.82 | -1.57 | 1.73 | 0.94 | 1.53 | -1.34 | 0.02 | -1.70 | -1.55 |
| | TA | RNA | PM | ATO | CTO | F | TES | TI/BI | RS | O |
| m | -0.19 | 0.13 | 0.10 | 0.22 | 0.20 | 0.37 | 0.32 | 0.13 | 0.29 | -0.08 |
| t_m | -1.31 | 0.61 | 0.40 | 1.11 | 1.11 | 1.28 | 1.92 | 0.86 | 1.82 | -0.37 |
| | BC/A | RD/S | RC/A | H/N | G | AccQ | ME | Ivol | Tvol | MDR |
| m | 0.18 | 0.01 | 0.32 | -0.25 | 0.03 | -0.18 | -0.24 | -0.54 | -0.37 | -0.31 |
| t_m | 0.73 | 0.06 | 1.27 | -1.47 | 0.09 | -0.79 | -0.90 | -1.56 | -0.95 | -0.94 |
| | β | D- β | S-Rev | Disp | Turn | 1/P | Dvol | Illiq | | |
| m | -0.13 | 0.07 | -0.31 | -0.33 | -0.12 | -0.00 | -0.26 | 0.27 | | |
| t_m | -0.36 | 0.30 | -1.39 | -1.24 | -0.43 | -0.01 | -1.30 | 1.14 | | |

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Factor Regressions

Significant anomalies in the momentum category

| | SUE-1 | SUE-6 | Abr-1 | Abr-6 | RE-1 | RE-6 | R6-6 | R11-1 | I-Mom | ave |
|-----------------|-------|-------|-------|-------|------|------|------|-------|-------|------|
| m | 0.45 | 0.24 | 0.73 | 0.30 | 0.89 | 0.60 | 0.85 | 1.18 | 0.51 | |
| α_{FF} | 0.55 | 0.39 | 0.84 | 0.38 | 1.20 | 0.94 | 1.12 | 1.52 | 0.68 | 0.85 |
| α_C | 0.34 | 0.18 | 0.62 | 0.19 | 0.56 | 0.37 | 0.06 | 0.09 | -0.18 | 0.29 |
| α_q | 0.16 | 0.02 | 0.64 | 0.26 | 0.12 | 0.03 | 0.24 | 0.24 | 0.00 | 0.19 |
| t_m | 3.59 | 2.17 | 5.50 | 3.11 | 3.43 | 2.58 | 3.17 | 3.52 | 2.33 | |
| t_{FF} | 4.50 | 3.62 | 5.93 | 3.89 | 4.81 | 4.52 | 4.47 | 4.99 | 3.25 | |
| t_C | 2.62 | 1.69 | 4.37 | 2.06 | 2.56 | 2.15 | 0.51 | 0.67 | -1.11 | |
| t_q | 1.12 | 0.18 | 4.07 | 2.18 | 0.43 | 0.14 | 0.71 | 0.54 | 0.01 | |
| $ \alpha_{FF} $ | 0.17 | 0.13 | 0.16 | 0.11 | 0.27 | 0.23 | 0.19 | 0.26 | 0.15 | 0.19 |
| $ \alpha_C $ | 0.11 | 0.09 | 0.12 | 0.08 | 0.11 | 0.09 | 0.10 | 0.13 | 0.06 | 0.10 |
| $ \alpha_q $ | 0.05 | 0.07 | 0.13 | 0.07 | 0.10 | 0.11 | 0.08 | 0.13 | 0.13 | 0.10 |
| p_{FF} | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | |
| p_C | 0.00 | 0.00 | 0.00 | 0.01 | 0.16 | 0.12 | 0.00 | 0.00 | 0.45 | |
| p_q | 0.42 | 0.04 | 0.00 | 0.02 | 0.46 | 0.08 | 0.00 | 0.01 | 0.03 | |

Factor Regressions

Significant anomalies in the value minus growth category

| | B/M | E/P | CF/P | NO/P | Dur | ave |
|-----------------|-------|------|-------|------|-------|------|
| m | 0.70 | 0.59 | 0.52 | 0.66 | -0.54 | |
| α_{FF} | 0.01 | 0.05 | 0.01 | 0.52 | -0.06 | 0.13 |
| α_C | -0.01 | 0.01 | -0.06 | 0.49 | -0.08 | 0.13 |
| α_q | 0.21 | 0.17 | 0.22 | 0.36 | -0.27 | 0.25 |
| t_m | 2.88 | 2.63 | 2.44 | 3.23 | -2.59 | |
| t_{FF} | 0.04 | 0.34 | 0.08 | 3.51 | -0.44 | |
| t_C | -0.06 | 0.03 | -0.40 | 3.33 | -0.56 | |
| t_q | 1.15 | 0.76 | 1.04 | 2.38 | -1.32 | |
| $ \alpha_{FF} $ | 0.07 | 0.10 | 0.08 | 0.17 | 0.11 | 0.11 |
| $ \alpha_C $ | 0.06 | 0.09 | 0.07 | 0.15 | 0.08 | 0.09 |
| $ \alpha_q $ | 0.08 | 0.10 | 0.14 | 0.12 | 0.08 | 0.10 |
| p_{FF} | 0.19 | 0.18 | 0.43 | 0.00 | 0.15 | |
| p_C | 0.29 | 0.38 | 0.37 | 0.00 | 0.41 | |
| p_q | 0.35 | 0.13 | 0.02 | 0.00 | 0.72 | |

Factor Regressions

Significant anomalies in the investment category

| | I/A | NOA | $\Delta\text{PI}/\text{A}$ | IG | NSI | CEI | lvG | lvC | OA | POA | PTA | ave |
|-----------------|-------|-------|----------------------------|-------|-------|-------|-------|-------|-------|-------|-------|------|
| m | -0.42 | -0.38 | -0.51 | -0.41 | -0.68 | -0.57 | -0.41 | -0.45 | -0.30 | -0.46 | -0.40 | |
| α_{FF} | -0.15 | -0.52 | -0.41 | -0.26 | -0.64 | -0.50 | -0.29 | -0.38 | -0.37 | -0.32 | -0.29 | 0.38 |
| α_c | -0.09 | -0.41 | -0.36 | -0.20 | -0.54 | -0.40 | -0.19 | -0.30 | -0.33 | -0.25 | -0.27 | 0.30 |
| α_q | 0.14 | -0.38 | -0.26 | 0.05 | -0.26 | -0.22 | -0.03 | -0.28 | -0.56 | -0.12 | -0.10 | 0.22 |
| t_m | -2.45 | -2.55 | -3.43 | -2.93 | -4.13 | -2.96 | -2.77 | -3.05 | -2.32 | -3.02 | -2.57 | |
| t_{FF} | -1.09 | -3.30 | -2.93 | -1.99 | -4.28 | -3.72 | -2.10 | -2.61 | -2.84 | -2.42 | -2.06 | |
| t_c | -0.61 | -2.69 | -2.48 | -1.51 | -3.58 | -2.93 | -1.34 | -1.97 | -2.32 | -1.88 | -1.82 | |
| t_q | 1.08 | -1.90 | -1.85 | 0.39 | -1.75 | -1.50 | -0.20 | -1.84 | -3.90 | -0.87 | -0.67 | |
| $ \alpha_{FF} $ | 0.12 | 0.17 | 0.13 | 0.13 | 0.18 | 0.15 | 0.11 | 0.12 | 0.13 | 0.11 | 0.11 | 0.13 |
| $ \alpha_c $ | 0.10 | 0.14 | 0.12 | 0.11 | 0.15 | 0.15 | 0.10 | 0.10 | 0.12 | 0.11 | 0.10 | 0.12 |
| $ \alpha_q $ | 0.09 | 0.12 | 0.14 | 0.09 | 0.11 | 0.12 | 0.11 | 0.08 | 0.15 | 0.12 | 0.08 | 0.11 |
| p_{FF} | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.01 | 0.00 | 0.00 | 0.01 | |
| p_c | 0.02 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | 0.11 | 0.04 | 0.00 | 0.01 | 0.02 | |
| p_q | 0.01 | 0.00 | 0.00 | 0.01 | 0.02 | 0.01 | 0.08 | 0.56 | 0.00 | 0.00 | 0.11 | |

Factor Regressions

Significant anomalies in the profitability category

| | ROE | ROA | GP/A | NEI | FP | ave |
|-----------------|------|------|------|------|-------|------|
| m | 0.80 | 0.62 | 0.34 | 0.39 | -0.67 | |
| α_{FF} | 1.17 | 1.00 | 0.50 | 0.63 | -1.44 | 0.95 |
| α_C | 0.85 | 0.67 | 0.45 | 0.43 | -0.67 | 0.61 |
| α_q | 0.05 | 0.09 | 0.11 | 0.18 | -0.17 | 0.12 |
| t_m | 3.11 | 2.70 | 2.18 | 3.31 | -1.98 | |
| t_{FF} | 5.43 | 5.40 | 3.25 | 6.03 | -6.44 | |
| t_C | 4.03 | 3.59 | 2.85 | 3.73 | -3.79 | |
| t_q | 0.37 | 0.72 | 0.71 | 1.68 | -0.57 | |
| $ \alpha_{FF} $ | 0.24 | 0.23 | 0.14 | 0.23 | 0.23 | 0.21 |
| $ \alpha_C $ | 0.15 | 0.14 | 0.14 | 0.15 | 0.12 | 0.14 |
| $ \alpha_q $ | 0.09 | 0.07 | 0.11 | 0.09 | 0.13 | 0.10 |
| p_{FF} | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | |
| p_C | 0.00 | 0.04 | 0.01 | 0.00 | 0.00 | |
| p_q | 0.05 | 0.75 | 0.38 | 0.05 | 0.00 | |

Factor Regressions

Significant anomalies in the intangibles and trading frictions categories

| | OC/A | Ad/M | RD/M | OL | Svol | ave |
|-----------------|------|------|------|-------|-------|------|
| m | 0.56 | 0.79 | 0.63 | 0.39 | -0.60 | |
| α_{FF} | 0.61 | 0.15 | 0.22 | 0.37 | -0.66 | 0.40 |
| α_C | 0.40 | 0.32 | 0.31 | 0.33 | -0.62 | 0.40 |
| α_q | 0.09 | 0.11 | 0.60 | -0.05 | -0.37 | 0.24 |
| t_m | 4.07 | 2.96 | 2.31 | 2.06 | -2.57 | |
| t_{FF} | 4.52 | 0.79 | 0.93 | 1.91 | -2.88 | |
| t_C | 2.97 | 1.37 | 1.40 | 1.76 | -2.59 | |
| t_q | 0.66 | 0.39 | 2.40 | -0.27 | -1.42 | |
| $ \alpha_{FF} $ | 0.15 | 0.13 | 0.17 | 0.11 | 0.19 | 0.15 |
| $ \alpha_C $ | 0.13 | 0.18 | 0.21 | 0.12 | 0.16 | 0.16 |
| $ \alpha_q $ | 0.11 | 0.11 | 0.27 | 0.12 | 0.11 | 0.14 |
| p_{FF} | 0.00 | 0.18 | 0.02 | 0.07 | 0.01 | |
| p_C | 0.00 | 0.07 | 0.01 | 0.06 | 0.06 | |
| p_q | 0.02 | 0.07 | 0.00 | 0.09 | 0.20 | |

Overall, except for the operating accrual anomaly and the R&D-to-market anomaly, the q -factor model performs as well as, and in most cases outperforms the Fama-French and Carhart models across major categories of anomalies:

- The q -factor model beats the Carhart model and by a bigger margin the Fama-French model in the momentum category
- The q -factor model also outperforms in the investment category and dominates in the profitability category
- The models are comparable in the value versus growth category
- The size factor plays only a limited role in the q -factor model

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Factor Regressions

q-factor loadings and *q*-characteristics, the momentum category

| | SUE-1 | SUE-6 | Abr-1 | Abr-6 | RE-1 | RE-6 | R6-6 | R11-1 | I-Mom |
|-------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| β_{MKT} | -0.08 | -0.06 | -0.06 | -0.03 | -0.05 | -0.07 | -0.09 | -0.14 | -0.11 |
| β_{ME} | 0.10 | 0.09 | 0.07 | 0.09 | -0.15 | -0.19 | 0.27 | 0.40 | 0.31 |
| $\beta_{I/A}$ | 0.02 | -0.11 | -0.13 | -0.16 | 0.04 | -0.12 | -0.07 | 0.04 | -0.03 |
| β_{ROE} | 0.48 | 0.45 | 0.28 | 0.18 | 1.33 | 1.12 | 1.02 | 1.48 | 0.82 |
| $t_{\beta_{MKT}}$ | -1.82 | -1.53 | -1.31 | -1.20 | -0.76 | -1.24 | -1.17 | -1.43 | -1.72 |
| $t_{\beta_{ME}}$ | 1.94 | 1.27 | 0.67 | 1.82 | -1.42 | -1.98 | 1.43 | 1.74 | 1.86 |
| $t_{\beta_{I/A}}$ | 0.18 | -0.97 | -1.25 | -2.24 | 0.25 | -0.82 | -0.27 | 0.12 | -0.13 |
| $t_{\beta_{ROE}}$ | 5.75 | 5.95 | 3.26 | 2.94 | 10.09 | 9.96 | 5.31 | 5.67 | 4.90 |
| ME | 0.69 | 0.75 | -0.01 | 0.03 | 0.77 | 0.87 | 0.40 | 0.52 | 0.62 |
| I/A | -1.46 | -0.96 | -1.37 | -1.13 | -0.80 | 0.72 | -4.07 | -3.83 | -1.18 |
| ROE | 5.80 | 3.38 | 1.59 | 1.49 | 6.58 | 6.47 | 4.14 | 5.34 | 1.61 |
| t_{ME} | 4.91 | 5.38 | -0.29 | 1.31 | 8.75 | 9.65 | 4.92 | 4.95 | 3.67 |
| $t_{I/A}$ | -3.30 | -2.57 | -2.36 | -2.58 | -1.22 | 1.13 | -5.54 | -4.66 | -1.79 |
| t_{ROE} | 16.46 | 19.07 | 13.38 | 15.47 | 29.77 | 27.86 | 16.00 | 17.06 | 10.24 |

Factor Regressions

q-factor loadings and *q*-characteristics, the value versus growth category

| | B/M | E/P | CF/P | NO/P | Dur |
|--------------------------|---------------|--------------|--------------|---------------|--------------|
| β_{MKT} | -0.03 | -0.12 | -0.15 | -0.18 | 0.11 |
| β_{ME} | 0.46 | 0.25 | 0.19 | -0.32 | -0.23 |
| $\beta_{\text{I/A}}$ | 1.45 | 0.99 | 1.01 | 1.03 | -0.85 |
| β_{ROE} | -0.51 | -0.09 | -0.24 | 0.02 | 0.24 |
| $t_{\beta_{\text{MKT}}}$ | -0.59 | -2.02 | -2.41 | -3.86 | 1.67 |
| $t_{\beta_{\text{ME}}}$ | 5.37 | 1.90 | 1.66 | -4.40 | -1.61 |
| $t_{\beta_{\text{I/A}}}$ | 12.74 | 5.76 | 6.79 | 10.25 | -5.69 |
| $t_{\beta_{\text{ROE}}}$ | -5.98 | -0.66 | -1.78 | 0.19 | 1.87 |
| ME | -2.46 | -0.73 | -0.89 | 1.23 | 0.41 |
| I/A | -9.70 | -1.11 | -5.63 | -14.43 | 3.95 |
| ROE | -5.68 | 0.21 | -0.72 | 1.18 | 0.49 |
| t_{ME} | -10.31 | -4.09 | -4.57 | 7.74 | 5.23 |
| $t_{\text{I/A}}$ | -17.06 | -1.20 | -5.47 | -13.81 | 2.73 |
| t_{ROE} | -29.57 | 1.30 | -4.83 | 7.27 | 2.22 |

Factor Regressions

q-factor loadings and *q*-characteristics, the investment category

| | I/A | NOA | Δ PI/A | IG | NSI | CEI | lvG | lvC | OA | POA | PTA |
|----------------|---------------|--------------|---------------|---------------|--------------|---------------|---------------|--------------|--------------|--------------|--------------|
| β_{MKT} | 0.02 | -0.02 | 0.05 | -0.02 | 0.04 | 0.24 | -0.03 | 0.04 | 0.03 | -0.01 | 0.06 |
| β_{ME} | -0.11 | 0.06 | -0.05 | -0.11 | 0.17 | 0.26 | 0.12 | 0.00 | 0.28 | 0.15 | 0.21 |
| $\beta_{I/A}$ | -1.37 | -0.01 | -0.77 | -0.82 | -0.68 | -1.06 | -0.96 | -0.65 | -0.02 | -0.90 | -0.90 |
| β_{ROE} | 0.15 | -0.01 | 0.16 | -0.07 | -0.32 | -0.12 | 0.05 | 0.18 | 0.29 | 0.05 | 0.04 |
| $t\beta_{MKT}$ | 0.62 | -0.55 | 1.33 | -0.71 | 0.99 | 6.27 | -0.77 | 1.01 | 0.80 | -0.19 | 1.50 |
| $t\beta_{ME}$ | -1.81 | 0.54 | -0.94 | -1.95 | 2.24 | 3.79 | 2.85 | -0.07 | 4.41 | 3.20 | 3.28 |
| $t\beta_{I/A}$ | -15.50 | -0.04 | -6.98 | -10.91 | -6.14 | -13.11 | -11.81 | -5.49 | -0.21 | -9.61 | -8.72 |
| $t\beta_{ROE}$ | 2.29 | -0.12 | 1.93 | -1.06 | -4.07 | -1.42 | 0.56 | 1.95 | 4.59 | 1.04 | 0.55 |
| ME | 0.88 | 0.07 | 0.63 | 0.22 | -1.37 | -1.79 | 0.26 | 0.19 | -0.24 | -0.36 | -0.36 |
| I/A | 83.89 | 55.72 | 61.16 | 34.03 | 27.04 | 14.80 | 37.85 | 44.80 | 10.15 | 11.12 | 16.14 |
| ROE | 1.63 | -1.24 | 0.84 | 0.50 | -1.71 | -1.41 | 0.42 | 1.07 | 0.88 | 1.02 | 0.36 |
| t_{ME} | 7.75 | 1.71 | 7.56 | 6.16 | -6.44 | -7.77 | 4.27 | 4.86 | -5.13 | -9.39 | -6.76 |
| $t_{I/A}$ | 32.74 | 18.28 | 30.77 | 22.38 | 13.85 | 14.37 | 23.42 | 34.89 | 5.19 | 7.90 | 12.93 |
| t_{ROE} | 10.00 | -7.89 | 5.10 | 3.24 | -11.87 | -9.07 | 3.31 | 8.13 | 5.06 | 7.42 | 2.56 |

Factor Regressions

q-factor loadings and *q*-characteristics, the profitability category

| | ROE | ROA | GP/A | NEI | FP |
|-------------------|-------|-------|-------|-------|--------|
| β_{MKT} | -0.10 | -0.14 | 0.05 | 0.02 | 0.44 |
| β_{ME} | -0.41 | -0.38 | 0.03 | -0.10 | 0.43 |
| $\beta_{I/A}$ | 0.10 | -0.10 | -0.24 | -0.30 | 0.17 |
| β_{ROE} | 1.50 | 1.31 | 0.52 | 0.63 | -1.61 |
| $t_{\beta_{MKT}}$ | -2.57 | -4.48 | 1.20 | 0.88 | 6.46 |
| $t_{\beta_{ME}}$ | -6.56 | -6.41 | 0.51 | -2.53 | 2.45 |
| $t_{\beta_{I/A}}$ | 1.05 | -1.23 | -2.35 | -3.78 | 0.63 |
| $t_{\beta_{ROE}}$ | 20.71 | 16.86 | 7.08 | 10.83 | -8.79 |
| ME | 2.81 | 2.66 | 0.39 | 2.34 | -3.09 |
| I/A | 3.56 | 5.32 | -1.29 | 5.35 | -3.91 |
| ROE | 16.95 | 14.71 | 3.94 | 4.32 | -8.74 |
| t_{ME} | 10.56 | 10.50 | 10.84 | 11.58 | -10.76 |
| $t_{I/A}$ | 4.16 | 6.88 | -2.05 | 11.45 | -4.18 |
| t_{ROE} | 29.02 | 27.97 | 23.88 | 27.36 | -25.56 |

Factor Regressions

q-factor loadings and *q*-characteristics, intangibles and trading frictions

| | OC/A | Ad/M | RD/M | OL | Svol |
|-------------------|--------|--------|-------|-------|-------|
| β_{MKT} | -0.13 | 0.04 | 0.16 | -0.06 | 0.04 |
| β_{ME} | 0.25 | 0.50 | 0.66 | 0.26 | 0.31 |
| $\beta_{I/A}$ | 0.35 | 1.42 | 0.21 | 0.16 | -0.21 |
| β_{ROE} | 0.51 | -0.27 | -0.58 | 0.54 | -0.43 |
| $t_{\beta_{MKT}}$ | -3.74 | 0.50 | 2.51 | -1.22 | 0.53 |
| $t_{\beta_{ME}}$ | 5.69 | 2.85 | 6.75 | 2.63 | 2.30 |
| $t_{\beta_{I/A}}$ | 3.52 | 6.03 | 1.21 | 1.34 | -1.30 |
| $t_{\beta_{ROE}}$ | 7.12 | -1.37 | -4.10 | 4.85 | -3.54 |
| ME | -1.31 | -1.34 | -4.39 | -1.31 | -0.19 |
| I/A | -13.77 | -10.71 | -3.22 | -5.71 | 0.66 |
| ROE | 1.52 | -3.33 | -2.80 | 1.86 | -0.64 |
| t_{ME} | -9.44 | -9.83 | -9.47 | -8.43 | -3.95 |
| $t_{I/A}$ | -11.38 | -12.17 | -2.54 | -4.70 | 1.37 |
| t_{ROE} | 7.97 | -12.60 | -9.21 | 11.42 | -4.03 |

Factor Regressions

25 size and book-to-market portfolios

| | Low | 2 | 3 | 4 | High | H-L | Low | 2 | 3 | 4 | High | H-L | |
|-----|----------|------|------|------|------|------|--|-------|------|-------|-------|-------|------|
| | <i>m</i> | | | | | | α_{FF} ($ \alpha_{FF} = 0.10$) | | | | | | |
| | Small | 0.08 | 0.72 | 0.84 | 0.95 | 1.11 | 1.02 | -0.54 | 0.02 | 0.13 | 0.18 | 0.16 | 0.70 |
| 2 | 0.32 | 0.69 | 0.86 | 0.87 | 0.99 | 0.67 | -0.21 | 0.00 | 0.09 | 0.07 | 0.04 | 0.25 | |
| 3 | 0.38 | 0.71 | 0.77 | 0.77 | 1.02 | 0.65 | -0.09 | 0.04 | 0.03 | 0.00 | 0.13 | 0.22 | |
| 4 | 0.52 | 0.59 | 0.73 | 0.74 | 0.84 | 0.32 | 0.14 | -0.02 | 0.03 | 0.00 | 0.02 | -0.12 | |
| Big | 0.40 | 0.54 | 0.54 | 0.61 | 0.56 | 0.16 | 0.16 | 0.11 | 0.09 | 0.00 | -0.16 | -0.32 | |
| | t_m | | | | | | t_{FF} ($p_{FF} = 0.00$) | | | | | | |
| | Small | 0.20 | 2.02 | 2.57 | 3.07 | 3.30 | 4.59 | -4.84 | 0.23 | 1.58 | 2.53 | 1.97 | 5.66 |
| 2 | 0.90 | 2.20 | 3.03 | 3.29 | 3.31 | 2.93 | -2.59 | 0.01 | 1.36 | 0.92 | 0.45 | 2.19 | |
| 3 | 1.15 | 2.49 | 3.03 | 3.05 | 3.93 | 2.76 | -1.21 | 0.52 | 0.35 | 0.02 | 1.28 | 1.70 | |
| 4 | 1.73 | 2.35 | 2.91 | 3.08 | 3.24 | 1.43 | 1.74 | -0.23 | 0.27 | 0.03 | 0.17 | -0.87 | |
| Big | 1.70 | 2.53 | 2.71 | 3.04 | 2.43 | 0.79 | 2.66 | 1.29 | 0.92 | -0.02 | -1.31 | -2.32 | |

Factor Regressions

25 size and book-to-market portfolios

| | Low | 2 | 3 | 4 | High | H-L | Low | 2 | 3 | 4 | High | H-L |
|-------|---|-------|------|-------|-------|-------|---|-------|-------|-------|-------|-------|
| | α_c ($ \overline{\alpha}_c = 0.11$) | | | | | | α_q ($ \overline{\alpha}_q = 0.11$) | | | | | |
| Small | -0.48 | 0.03 | 0.12 | 0.18 | 0.22 | 0.70 | -0.25 | 0.27 | 0.31 | 0.30 | 0.32 | 0.57 |
| 2 | -0.18 | 0.03 | 0.09 | 0.10 | 0.04 | 0.22 | -0.14 | 0.02 | 0.03 | 0.07 | 0.10 | 0.24 |
| 3 | -0.04 | 0.04 | 0.09 | 0.03 | 0.16 | 0.20 | -0.01 | -0.03 | -0.04 | -0.01 | 0.14 | 0.15 |
| 4 | 0.15 | -0.01 | 0.07 | 0.03 | 0.09 | -0.06 | 0.18 | -0.14 | -0.01 | 0.02 | 0.06 | -0.12 |
| Big | 0.17 | 0.07 | 0.07 | -0.03 | -0.13 | -0.31 | 0.10 | -0.04 | 0.06 | -0.01 | -0.04 | -0.13 |
| | t_c ($p_c = 0.00$) | | | | | | t_q ($p_q = 0.00$) | | | | | |
| Small | -4.00 | 0.36 | 1.58 | 2.59 | 2.53 | 5.72 | -1.48 | 2.24 | 3.09 | 3.68 | 2.72 | 2.91 |
| 2 | -2.28 | 0.37 | 1.34 | 1.40 | 0.44 | 1.88 | -1.21 | 0.29 | 0.37 | 0.67 | 0.89 | 1.25 |
| 3 | -0.50 | 0.53 | 0.93 | 0.28 | 1.40 | 1.43 | -0.09 | -0.30 | -0.37 | -0.05 | 1.16 | 0.92 |
| 4 | 1.87 | -0.16 | 0.74 | 0.24 | 0.75 | -0.42 | 1.50 | -1.58 | -0.06 | 0.21 | 0.44 | -0.61 |
| Big | 2.89 | 0.91 | 0.70 | -0.36 | -1.02 | -2.12 | 1.32 | -0.49 | 0.65 | -0.06 | -0.23 | -0.70 |

- 1 Intuition
- 2 Factors/Testing Portfolios
- 3 Factor Regressions: Alphas
- 4 Factor Regressions: Betas
- 5 Sharpe Ratios

Sharpe Ratios

Factors

| Sharpe ratios | | | | | | | Maximum Sharpe ratios | | | |
|---------------|------|------|------|----------|-----------|-----------|-----------------------|------|---------|------|
| MKT | SMB | HML | UMD | r_{ME} | $r_{I/A}$ | r_{ROE} | CAPM | FF | Carhart | q |
| 0.10 | 0.06 | 0.13 | 0.16 | 0.10 | 0.24 | 0.22 | 0.10 | 0.21 | 0.30 | 0.43 |

Sharpe Ratios

Testing portfolios

| | SUE-1 | SUE-6 | Abr-1 | Abr-6 | RE-1 | RE-6 | R6-6 | R11-1 | I-Mom |
|-----------|-------|-------|---------------|-------|------|------|------|-------|-------|
| S_{H-L} | 0.14 | 0.09 | 0.23 | 0.15 | 0.16 | 0.13 | 0.15 | 0.16 | 0.10 |
| S_m | 0.27 | 0.26 | 0.28 | 0.23 | 0.25 | 0.21 | 0.31 | 0.28 | 0.20 |
| | B/M | E/P | CF/P | NO/P | Dur | | | | |
| S_{H-L} | 0.14 | 0.12 | 0.11 | 0.16 | 0.12 | | | | |
| S_m | 0.21 | 0.24 | 0.21 | 0.30 | 0.24 | | | | |
| | I/A | NOA | Δ PI/A | IG | NSI | CEI | IvG | IvC | OA |
| S_{H-L} | 0.11 | 0.12 | 0.17 | 0.14 | 0.21 | 0.14 | 0.13 | 0.14 | 0.10 |
| S_m | 0.24 | 0.29 | 0.25 | 0.26 | 0.30 | 0.32 | 0.23 | 0.25 | 0.23 |
| | ROE | ROA | GP/A | NEI | FP | | | | PTA |
| S_{H-L} | 0.15 | 0.13 | 0.10 | 0.14 | 0.10 | | | | |
| S_m | 0.24 | 0.21 | 0.18 | 0.26 | 0.28 | | | | |
| | OC/A | Ad/M | RD/M | OL | Svol | all | | | |
| S_{H-L} | 0.18 | 0.14 | 0.12 | 0.10 | 0.14 | 0.48 | | | |
| S_m | 0.26 | 0.23 | 0.21 | 0.17 | 0.29 | 1.60 | | | |

The q -factor model largely summarizes the cross section of average stock returns, capturing most (but not all) anomalies that bedevil the Fama-French model

A parsimonious empirical model for estimating expected returns, silent about the rational asset pricing versus mispricing debate:

- Rational asset pricing: q -factors constructed on economic fundamentals; comovement indicated in Sharpe ratios; covariation (betas) between anomalies and q -factors
- Mispricing: simultaneous impact on stocks with similar investment (and similar ROE); high Sharpe ratios