Lecture Notes

Hou, Mo, Xue, and Zhang (2021, Review of Finance): "An Augmented q-factor Model with Expected Growth"

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FIN 8250, Autumn 2021
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Augmenting the Hou-Xue-Zhang (2015) q-factor model with an expected growth factor to form the q^5 model:

$$E[R_i - R_f] = \beta^i_{\mathrm{MKT}} E[\mathrm{MKT}] + \beta^i_{\mathrm{Me}} E[R_{\mathrm{Me}}] + \beta^i_{\mathrm{I/A}} E[R_{\mathrm{I/A}}] + \beta^i_{\mathrm{Roe}} E[R_{\mathrm{Roe}}] + \beta^i_{\mathrm{Eg}} E[R_{\mathrm{Eg}}]$$

Stress-testing factor models with a large set of 150 anomalies:

- The q^5 model improves on the q-factor model substantially
- The q-factor model already compares favorably with the Fama-French (2018) 6-factor model

Outline

Outline

Three defining characteristics of neoclassical economics:

- Rational expectations
- Consumers maximize utility; firms maximize market value
- Markets clear

A representative household (investor) maximizes:

$$U(C_t) + \rho E_t[U(C_{t+1})]$$

subject to:

$$\begin{aligned} C_t + \sum_i P_{it} S_{it+1} &= \sum_i (P_{it} + D_{it}) S_{it} \\ C_{t+1} &= \sum_i (P_{it+1} + D_{it+1}) S_{it+1} \end{aligned}$$

The first principle of consumption:

$$E_t[M_{t+1} R_{it+1}] = 1 \quad \Rightarrow \quad \overbrace{E_t[R_{it+1}] - R_{ft} = \beta_{it}^{M} \lambda_{Mt}}^{\textbf{The Consumption CAPM}}$$

An individual firm i maximizes:

$$P_{it} + D_{it} \equiv \max_{\{I_{it}\}} \quad \Pi_{it} A_{it} - I_{it} - \frac{a}{2} \left(\frac{I_{it}}{A_{it}} \right)^2 A_{it} + E_t \left[M_{t+1} \Pi_{it+1} A_{it+1} \right]$$

The first principle of investment:

$$\frac{P_{it+1} + D_{it+1}}{P_{it}} \equiv \underbrace{R_{it+1} = \frac{\Pi_{it+1}}{1 + a(I_{it}/A_{it})}}_{\textbf{The Investment CAPM}}$$

The investment CAPM: Cross-sectionally varying expected returns

The consumption CAPM and the investment CAPM deliver identical expected returns in general equilibrium:

$$R_{ft} + \beta_{it}^{M} \lambda_{Mt} = E_t[R_{it+1}] = \frac{E_t[\Pi_{it+1}]}{1 + a(I_{it}/A_{it})}$$

- Consumption: Risks as sufficient statistics of $E_t[R_{it+1}]$
- Investment: Characteristics as sufficient statistics of $E_t[R_{it+1}]$

$$R_i - R_f = \beta^i_{\text{MKT}} E[\text{MKT}] + \beta^i_{\text{Me}} E[R_{\text{Me}}] + \beta^i_{\text{I/A}} E[R_{\text{I/A}}] + \beta^i_{\text{Roe}} E[R_{\text{Roe}}]$$

- MKT, R_{Me}, $R_{\text{I/A}}$, and R_{Roe}: Market, size, investment, and Roe factors, respectively
- $\beta^i_{\text{MKT}}, \beta^i_{\text{Me}}, \beta^i_{\text{I/A}}$, and β^i_{Roe}: Factor loadings

See global-q.org for our data on *q*-factors and testing portfolios

A linear factor approximation to the nonlinear characteristics model:

$$E_t[R_{it+1}] = \frac{E_t[\Pi_{it+1}]}{1 + a(I_{it}/A_{it})}$$

- All else equal, high investment stocks should earn lower expected returns than low investment stocks
- All else equal, high expected profitability stocks should earn higher expected returns than low expected profitability stocks

In the multiperiod investment framework:

$$R_{it+1} \approx \frac{\Pi_{it+1} + (1-\delta)\left[1 + a\left(I_{it+1}/A_{it+1}\right)\right]}{1 + a\left(I_{it}/A_{it}\right)}$$

The "dividend yield" component, $\Pi_{it+1}/[1 + a\left(I_{it}/A_{it}\right)]$, motivates the q-factor model

The "capital gain" component roughly proportional to investment-to-assets growth, $\left(I_{it+1}/A_{it+1}\right)/\left(I_{it}/A_{it}\right)$

Mechanism: Present value from next period onward as benefits of current investment (analogous to the profitability factor)

Outline

The Expected Growth Factor

Modeling expected growth via monthly cross-sectional forecasting regressions

Forecast τ-year-ahead investment-to-assets changes with:

- Tobin's q: Erickson and Whited (2000)
- Cash flows: Internal funds (Fazzari, Hubbard, and Petersen 1988); better than earnings in forecasting returns (Ball et al. 2016) and future growth (likely due to intangibles, Penman 2009; Lev, Radhakrishnan, and Zhang 2009)
- dRoe: Capturing short-term dynamics of investment growth (Liu, Whited, and Zhang 2009)

Caveat: Unobservable expected growth depends on its specification, cash flows in particular

The Expected Growth Factor

Monthly cross-sectional regressions of future investment-to-assets changes, 7/1963–12/2018

τ	$\log(q)$	Cop	dRoe	R^2	Pearson	Rank
1	−0.03	0.52	0.77	6.42	0.14	0.21
	(−5.63)	(12.75)	(7.62)		[0.00]	[0.00]
2	−0.07	0.70	0.91	8.61	0.15	0.22
	(−9.76)	(12.34)	(10.07)		[0.00]	[0.00]
3	−0.09	0.75	0.72	8.98	0.15	0.22
	(−12.39)	(12.17)	(8.60)		[0.00]	[0.00]

Relatively reliable out-of-sample correlations with subsequent, realized investment-to-assets changes

The Expected Growth Factor

Properties of the expected growth deciles, 1/1967–12/2018

Low	2	3	4	5	6	7	8	9	High	H−L
Average excess returns, $\overline{R}$										
−0.12	0.20	0.28	0.42	0.45	0.49	0.56	0.64	0.77	0.95	1.07
−0.40	0.84	1.21	2.00	2.36	2.61	3.00	3.54	4.17	4.69	6.48
The expected 1-year-ahead growth, $E_t[\mathrm{d}^1\mathrm{I}/\mathrm{A}]$										
−15.21	−7.67	−5.61	−4.20	−3.03	−1.97	−0.86	0.47	2.52	7.65	22.87
−36.75	−31.37	−25.19	−20.56	−15.96	−11.01	−5.08	3.01	16.53	37.98	45.21
Average future 1-year-ahead realized growth, $\mathrm{d}^1\mathrm{I}/\mathrm{A}$										
−16.69	−12.30	−4.11	−3.56	−1.10	−0.43	−0.32	0.64	1.57	5.96	22.65
−11.71	−8.36	−7.15	−5.22	−2.24	−0.90	−0.71	1.18	3.59	9.07	14.72

$E_t[\mathrm{d}^1\mathrm{I}/\mathrm{A}]$ and $\mathrm{d}^1\mathrm{I}/\mathrm{A}$ aligned at the portfolio level (Corr = 0.64)

The Expected Growth Factor

R_{Eg}, independent 2×3 monthly sorts on size and $E_t[\text{d}^1\text{I/A}]$, 1/1967–12/2018

$\overline{R}_{\text{Eg}}$	α	β_{Mkt}	β_{Me}	$\beta_{\text{I/A}}$	β_{Roe}	R^2
0.84	0.67	−0.11	−0.09	0.21	0.30	0.44
(10.27)	(9.75)	(−6.38)	(−3.56)	(4.86)	(9.13)	

α	β_{Mkt}	β_{Me}	$\beta_{\text{I/A}}$	β_{Roe}	$\beta_{\log(q)}$	R^2
0.67	−0.11	−0.09	0.23	0.30	−0.02	0.44
(9.80)	(−6.40)	(−3.61)	(4.72)	(8.83)	(−0.48)	

α	β_{Mkt}	β_{Me}	$\beta_{\text{I/A}}$	β_{Roe}	β_{Cop}	R^2
0.37	−0.02	−0.02	0.31	0.14	0.60	0.65
(6.35)	(−1.66)	(−0.54)	(9.51)	(4.37)	(10.63)	

α	β_{Mkt}	β_{Me}	$\beta_{\text{I/A}}$	β_{Roe}	β_{dRoe}	R^2
0.63	−0.11	−0.10	0.18	0.23	0.16	0.46
(8.56)	(−6.62)	(−3.93)	(3.57)	(5.00)	(2.41)	

The Expected Growth Factor

Spanning tests: $p_{\text{GRS}} = 0$ for the R_{Me}, $R_{\text{I/A}}$, and R_{Roe} alphas $= 0$, with and without the R_{Eg} alpha, in the Fama-French (2018) 6-factor models, 1/1967–12/2018

	$\overline{R}$	α	MKT	SMB	HML	RMW	CMA	UMD	RMWc
$R_{\text{I/A}}$	0.38	0.10	0.01	−0.04	0.04	0.06	0.81	0.01	
	4.59	2.82	0.84	−2.75	2.16	2.09	33.60	0.83	
		0.10	0.01	−0.04	0.05		0.80	0.01	0.06
		2.57	0.91	−2.68	2.26		31.45	0.82	1.49
R_{Roe}	0.55	0.27	0.00	−0.12	−0.10	0.66	−0.00	0.24	
	5.44	4.32	0.07	−3.71	−2.02	15.43	−0.01	9.58	
		0.23	0.03	−0.10	−0.04		−0.16	0.24	0.71
		2.94	1.37	−2.53	−0.55		−1.88	6.92	8.55
R_{Eg}	0.84	0.71	−0.09	−0.14	−0.01	0.23	0.21	0.12	
	10.27	11.39	−5.44	−6.34	−0.51	5.65	4.50	6.04	
		0.64	−0.06	−0.09	−0.00		0.16	0.11	0.40
		9.87	−3.47	−3.90	−0.04		3.31	5.47	7.02

The Expected Growth Factor

Spanning tests: $p_{\text{GRS}} = 0.68$ (0.00) for the nonmarket 6-factor alphas = 0 in q, $p_{\text{GRS}} = 0.09$ (0.11) in q^5 with RMW (RMWc), 1/1967–12/2018

	$\overline{R}$	α	R_{Mkt}	R_{Me}	$R_{\text{I/A}}$	R_{Roe}	R_{Eg}
UMD	0.64	0.14	−0.08	0.23	−0.03	0.90	
	3.73	0.61	−1.31	1.74	−0.17	5.85	
		−0.16	−0.03	0.27	−0.12	0.77	0.44
		−0.77	−0.53	2.03	−0.69	4.39	2.81
CMA	0.30	0.00	−0.04	0.03	0.96	−0.09	
	3.29	0.08	−3.66	1.72	35.11	−3.41	
		−0.04	−0.04	0.04	0.94	−0.11	0.06
		−0.94	−2.96	1.96	38.15	−3.73	2.16
RMW	0.28	0.03	−0.03	−0.12	0.02	0.54	
	2.76	0.32	−1.23	−1.73	0.20	8.72	
		−0.01	−0.03	−0.11	0.00	0.52	0.06
		−0.17	−1.05	−1.57	0.04	8.04	0.85
RMWc	0.33	0.24	−0.10	−0.18	0.09	0.29	
	4.18	3.75	−5.90	−5.36	2.06	9.97	
		0.11	−0.08	−0.16	0.05	0.23	0.19
		1.80	−4.90	−4.58	1.08	6.85	5.02

Outline

8 competing factor models:

- The q-factor model, the q^5 model
- The Fama-French 5-factor model, the 6-factor model, the alternative 6-factor model with RMWc
- The Stambaugh-Yuan 4-factor model
- The Barillas-Shanken 6-factor model, including MKT, SMB, $R_{\mathrm{I/A}}$, R_{Roe}, the Asness-Frazzini monthly formed HML, UMD
- The Daniel-Hirshleifer-Sun 3-factor model

Use the replicated Stambaugh-Yuan and Daniel-Hirshleifer-Sun models via the common construction (Hou et al. 2019)

Stress Tests

The playing field, monthly Sharpe ratios for factors and factor models, 1/1967–12/2018

Individual factors							
R_{Mkt}	R_{Me}	$R_{\mathrm{I/A}}$	R_{Roe}	R_{Eg}	SMB	HML	CMA
0.11	0.09	0.20	0.22	0.44	0.07	0.11	0.15
RMW	RMWc	UMD	$\mathrm{HML}^{\mathrm{m}}$	MGMT	PERF	FIN	PEAD
0.12	0.19	0.15	0.08	0.20	0.16	0.10	0.32
Factor models							
q	q^5	FF5	FF6	FF6c	BS6	SY4	DHS
0.42	0.63	0.32	0.37	0.43	0.48	0.41	0.42

150 anomalies with NYSE breakpoints and value-weighted returns significant at the 5% level (Hou, Xue, and Zhang 2020)

- Momentum: 39
- Value-versus-growth: 15
- Investment: 26
- Profitability: 40
- Intangibles: 27
- Trading frictions: 3

Stress Tests

Testing deciles, momentum (39)

- Sue1 Earnings surprise (1-month holding period), Foster, Olsen, and Shevlin (1984)
- Abr1 Cumulative abnormal returns around earnings announcements (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
- Abr6 Cumulative abnormal returns around earnings announcements (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
- Abr12 Cumulative abnormal returns around earnings announcements (12-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
- Re1 Revisions in analysts' forecasts (1-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
- Re6 Revisions in analysts' forecasts (6-month holding period), Chan, Jegadeesh, and Lakonishok (1996)
- $R^6 1$ Price momentum (6-month prior returns, 1-month holding period), Jegadeesh and Titman (1993)
- $R^6 6$ Price momentum (6-month prior returns, 6-month holding period), Jegadeesh and Titman (1993)

Stress Tests

Testing deciles, momentum (39)

$R^6 12$	Price momentum (6-month prior returns, 12-month holding period), Jegadeesh and Titman (1993)	$R^{11} 1$	Price momentum (11-month prior returns, 1-month holding period), Fama and French (1996)
$R^{11} 6$	Price momentum, (11-month prior returns, 6-month holding period), Fama and French (1996)	$R^{11} 12$	Price momentum, (11-month prior returns, 12-month holding period), Fama and French (1996)
Im1	Industry momentum, (1-month holding period), Moskowitz and Grinblatt (1999)	Im6	Industry momentum, (6-month holding period), Moskowitz and Grinblatt (1999)
Im12	Industry momentum (12-month holding period), Moskowitz and Grinblatt (1999)	Rs1	Revenue surprise (1-month holding period), Jegadeesh and Livnat (2006)
dEf1	Analysts' forecast change (1-month hold period), Hawkins, Chamberlin, and Daniel (1984)	dEf6	Analysts' forecast change (6-month hold period), Hawkins, Chamberlin, and Daniel (1984)

Stress Tests

Testing deciles, momentum (39)

dEf12	Analysts' forecast change (12-month hold period), Hawkins, Chamberlin, and Daniel (1984)	Nei1	# consecutive quarters with earnings increases (1-month holding period), Barth, Elliott, and Finn (1999)
52w6	52-week high (6-month holding period), George and Hwang (2004))	52w12	52-week high (12-month holding period), George and Hwang (2004)
$\epsilon^6 6$	Six-month residual momentum (6-month holding period), Blitz, Huij, and Martens (2011)	$\epsilon^6 12$	Six-month residual momentum (12-month holding period), Blitz, Huij, and Martens (2011)
$\epsilon^{11} 1$	11-month residual momentum, 1-month, Blitz, Huij, and Martens (2011)	$\epsilon^{11} 6$	11-month residual momentum, 6-month, Blitz, Huij, and Martens (2011)
$\epsilon^{11} 12$	11-month residual momentum, 12-month, Blitz, Huij, and Martens (2011)	Sm1	Segment momentum 1-month, Cohen and Lou (2012)

Stress Tests

Testing deciles, momentum (39)

Sm12	Segment momentum, 12-month, Cohen and Lou (2012)
Ilr6	Industry lead-lag effect in prior returns, 6-month, Hou (2007)
Ile1	Industry lead-lag effect in earnings news, 1-month, Hou (2007)
Cm12	Customer momentum, 12-month Cohen and Frazzini (2008)
Cim1	Customer industries momentum, 1-month, Menzly and Ozbas (2010)
Cim12	Customer industries momentum, 12-month, Menzly and Ozbas (2010)
Ilr1	Industry lead-lag effect in prior returns, 1-month, Hou (2007)
Ilr12	Industry lead-lag effect in prior returns, 12-month, Hou (2007)
Cm1	Customer momentum, 1-month Cohen and Frazzini (2008)
Sim1	Supplier industries momentum, 1-month, Menzly and Ozbas (2010)
Cim6	Customer industries momentum, 6-month, Menzly and Ozbas (2010)

Stress Tests

Testing deciles, value-versus-growth (15)

Bm	Book-to-market equity, Rosenberg, Reid, and Lanstein (1985)	Ep^q1	Quarterly earnings-to-price (1-month holding period)
Ep^q6	Quarterly earnings-to-price (6-month holding period)	Ep^q12	Quarterly earnings-to-price (12-month holding period)
Cp^q1	Quarterly cash flow-to-price (1-month holding period)	Cp^q6	Quarterly cash flow-to-price (6-month holding period)
Nop	Net payout yield, Boudoukh, Michaely Richardson, and Roberts (2007)	Em	Enterprise multiple, Loughran and Wellman (2011)
Em^q1	Quarterly enterprise multiple (1-month holding period)	Sp	Sales-to-price Barbee, Mukherji, and Raines (1996))
Sp^q1	Quarterly sales-to-price (1-month holding period)	Sp^q6	Quarterly sales-to-price (6-month holding period)
Sp^q12	Quarterly sales-to-price (12-month holding period)	Ocp	Operating cash flow-to-price, Desai, Rajgopal, and Venkatachalam (2004)
Ocp^q1	Operating cash flow-to-price (1-month holding period)		

Stress Tests

Testing deciles, investment (26)

I/A	Investment-to-assets, Cooper, Gulen, and Schill (2008)	Ia^q6	Quarterly investment-to-assets (6-month holding period)
Ia^q12	Quarterly investment-to-assets (12-month holding period)	dPia	(Changes in PPE and inventory)/assets Lyandres, Sun, and Zhang (2008)
Noa	Net operating assets, Hirshleifer Hou, Teoh, and Zhang (2004)	dNoa	Changes in net operating assets, Hirshleifer, Hou, Teoh, and Zhang (2004)
Ig	Investment growth Xing (2008)	2Ig	Two-year investment growth Anderson and Garcia-Feijoo (2006)
Nsi	Net stock issues Pontiff and Woodgate (2008)	Cei	Composite equity issuance Daniel and Titman (2006)
dIi	% change in investment− % change in industry investment, Abarbanell and Bushee (1998)	dLno	Change in long-term net operating assets, Fairfield, Whisenant, and Yohn (2003)
Ivg	Inventory growth Belo and Lin (2011)	Ivc	Inventory changes Thomas and Zhang (2002)

Stress Tests

Testing deciles, investment (26)

Oa	Operating accruals Sloan (1996)	dWc	Change in net non-cash working capital, Richardson, Sloan, Soliman, and Tuna (2005)
dCoa	Change in current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)	dNco	Change in net non-current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)
dNca	Change in non-current operating assets, Richardson, Sloan, Soliman, and Tuna (2005)	dFin	Change in net financial assets Richardson, Sloan, Soliman, and Tuna (2005)
dFnl	Change in financial liabilities Richardson, Sloan, Soliman, and Tuna (2005)	dBe	Change in common equity, Richardson Sloan, Soliman, and Tuna (2005)
Dac	Discretionary accruals Xie (2001)	Poa	Percent operating accruals, Hafzalla, Lundholm, and Van Winkle (2011)
Pta	Percent total accruals, Hafzalla, Lundholm, and Van Winkle (2011)	Pda	Percent discretionary accruals

Stress Tests

Testing deciles, profitability (40)

Roe1	Return on equity, 1-month, Hou, Xue, and Zhang (2015)	Roe6	Return on equity, 6-month, Hou, Xue, and Zhang (2015)
dRoe1	Change in Roe, 1-month horizon	dRoe6	Change in Roe, 6-month horizon
dRoe12	Change in Roe, 12-month horizon	Roa1	Return on assets, 1-month horizon, Balakrishnan, Bartov, and Faurel (2010)
dRoa1	Change in Roa, 1-month horizon	dRoa6	Change in Roa, 6-month horizon
Ato	Asset turnover Soliman (2008)	Cto	Capital turnover Haugen and Baker (1996)
Rna^q1	Return on net operating assets, 1-month horizon	Rna^q6	Return on net operating assets, 6-month horizon
Ato^q1	Quarterly asset turnover, 1-month horizon	Ato^q6	Quarterly asset turnover, 6-month horizon

Stress Tests

Testing deciles, profitability (40)

Ato^q12	Quarterly asset turnover, 12-month horizon
Cto^q6	Quarterly capital turnover, 6-month horizon
Gpa	Gross profits-to-assets, Novy-Marx (2013)
Gla^q6	Gross profits-to-lagged assets, 6-month horizon
Ole^q1	Operating profits-to-lagged equity, 1-month horizon
Opa	Operating profits-to-assets, Ball, Gerakos, Linnainmaa, and Nikolaev (2015)
Ola^q6	Operating profits-to-lagged assets, 6-month horizon
Cto^q1	Quarterly capital turnover, 1-month horizon
Cto^q12	Quarterly capital turnover, 12-month horizon
Gla^q1	Gross profits-to-lagged assets, 1-month horizon
Gla^q12	Gross profits-to-lagged assets, 12-month horizon
Ole^q6	Operating profits-to-lagged equity , 6-month horizon
Ola^q1	Operating profits-to-lagged assets, 1-month horizon
Ola^q12	Operating profits-to-lagged assets, 12-month horizon

Stress Tests

Testing deciles, profitability (40)

Cop	Cash-based operating profitability, Ball, Gerakos, Linnainmaa, and Nikolaev (2016)	Cla	Cash-based operating profits-to-lagged assets
Cla^q1	Cash-based operating profits-to-lagged assets, 1-month horizon	Cla^q6	Cash-based operating profits-to-lagged assets, 6-month horizon
Cla^q12	Cash-based operating profits-to-lagged assets, 12-month horizon	F^q1	Quarterly F-score, 1-month horizon
F^q6	Quarterly F-score, 6-month horizon	F^q12	Quarterly F-score, 12-month horizon
Fp^q6	Failure probability, 6-month horizon Campbell, Hilscher, and Szilagyi (2008)	O^q1	Quarterly O-score 1-month horizon
Tbi^q12	Quarterly taxable income-to-book income 12-month horizon	Sg^q1	Quarterly sales growth 1-month horizon

Stress Tests

Testing deciles, intangibles (27)

Oca	Organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)	Ioca	Industry-adjusted organizational capital-to-assets, Eisfeldt and Papanikolaou (2013)
Adm	Advertising expense-to-market, Chan, Lakonishok, and Sougiannis (2001)	Rdm	R&D-to-market, Chan, Lakonishok, and Sougiannis (2001)
Rdm^q1	Quarterly R&D-to-market, 1-month horizon	Rdm^q6	Quarterly R&D-to-market, 6-month horizon
Rdm^q12	Quarterly R&D-to-market, 12-month horizon	Rds^q6	Quarterly R&D-to-sales 6-month horizon
Rds^q12	Quarterly R&D-to-sales, 12-month horizon	Ol	Operating leverage, Novy-Marx (2011)
Ol^q1	Quarterly operating leverage, 1-month horizon	Ol^q6	Quarterly operating leverage, 6-month horizon
Ol^q12	Quarterly operating leverage, 12-month horizon	Hs	Industry concentration (sales), Hou and Robinson (2006)
Rer	Real estate ratio, Tuzel (2010)	Eprd	Earnings predictability, Francis, Lafond, Olsson, and Schipper (2004)

Stress Tests

Testing deciles, intangibles and trading frictions (3)

Etl	Earnings timeliness, Francis, Lafond, Lafond, Olsson, and Schipper (2004)	Alm^q1	Asset liquidity (market assets) 1-month horizon
Alm^q6	Asset liquidity (market assets) 6-month horizon	Alm^q12	Asset liquidity (market assets) 12-month horizon
R_a^1	12-month-lagged return Heston and Sadka (2008)	R_n^1	Year 1-lagged return, nonannual Heston and Sadka (2008)
$R_a^{[2,5]}$	Years 2–5 lagged returns, annual Heston and Sadka (2008)	$R_a^{[6,10]}$	Years 6–10 lagged returns, annual Heston and Sadka (2008)
$R_n^{[6,10]}$	Years 6–10 lagged returns, nonannual Heston and Sadka (2008)	$R_a^{[11,15]}$	Years 11–15 lagged returns, annual Heston and Sadka (2008)
$R_a^{[16,20]}$	Years 16–20 lagged returns, annual Heston and Sadka (2008)		

Trading frictions (3)

Dtv12	Dollar trading volume, 12-month horizon, Brennan, Chordia, and Subrahmanyam (1998))	Isff1	Idiosyncratic skewness per the FF 3-factor model, 1-month horizon
Isq1	Idiosyncratic skewness, per the q-factor model, 1-month horizon		

Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{\lvert\alpha_{\text{H}-\text{L}}\rvert}$	$\#_{\lvert t\rvert\geq 1.96}$	$\#_{\lvert t\rvert\geq 3}$	$\overline{\lvert\alpha\rvert}$	$\#^{\text{GRS}}_{p<5\%}$
			All (150)		
q	0.28	52	25	0.11	101
q^5	0.19	23	6	0.10	57
FF5	0.43	100	69	0.13	112
FF6	0.30	74	37	0.11	91
FF6c	0.27	59	25	0.11	71
BS6	0.29	63	37	0.13	132
SY4	0.29	64	25	0.11	87
DHS	0.37	70	33	0.14	97

Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{\lvert\alpha_{\text{H}-\text{L}}\rvert}$	$\#_{\lvert t\rvert \geq 1.96}$	$\#_{\lvert t\rvert \geq 3}$	$\overline{\lvert\alpha\rvert}$	$\#^{\text{GRS}}_{p<5\%}$
	Momentum (39)				
q	0.25	11	3	0.10	24
q^5	0.17	4	1	0.09	15
FF5	0.62	37	29	0.15	36
FF6	0.27	19	6	0.10	21
FF6c	0.24	14	5	0.09	18
BS6	0.23	12	4	0.12	33
SY4	0.32	19	6	0.10	23
DHS	0.25	10	3	0.14	26

Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{\lvert\alpha_{\mathrm{H-L}}\rvert}$	$\#_{\lvert t\rvert \geq 1.96}$	$\#_{\lvert t\rvert \geq 3}$	$\overline{\lvert\alpha\rvert}$	$\#^{\mathrm{GRS}}_{p<5\%}$
	Value-versus-growth (15)				
q	0.21	1	0	0.11	8
q^5	0.22	3	0	0.13	7
FF5	0.15	2	0	0.10	7
FF6	0.19	4	0	0.10	9
FF6c	0.17	3	0	0.10	6
BS6	0.23	6	2	0.13	14
SY4	0.24	4	1	0.12	9
DHS	0.78	15	13	0.23	15

Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{\lvert\alpha_{\mathrm{H-L}}\rvert}$	$\#_{\lvert t\rvert\geq 1.96}$	$\#_{\lvert t\rvert\geq 3}$	$\overline{\lvert\alpha\rvert}$	$\#^{\mathrm{GRS}}_{p<5\%}$
	Investment (26)				
q	0.22	9	4	0.10	19
q^5	0.10	1	0	0.08	6
FF5	0.24	10	7	0.09	17
FF6	0.22	10	6	0.09	16
FF6c	0.18	8	2	0.08	7
BS6	0.22	8	6	0.11	24
SY4	0.19	8	3	0.09	17
DHS	0.34	20	4	0.10	22

Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{\lvert\alpha_{\mathrm{H-L}}\rvert}$	$\#_{\lvert t\rvert\geq 1.96}$	$\#_{\lvert t\rvert\geq 3}$	$\overline{\lvert\alpha\rvert}$	$\#^{\mathrm{GRS}}_{p<5\%}$
	Profitability (40)				
q	0.25	16	6	0.10	28
q^5	0.14	5	1	0.09	14
FF5	0.43	32	23	0.12	32
FF6	0.31	26	13	0.10	25
FF6c	0.26	18	7	0.10	21
BS6	0.31	20	12	0.12	37
SY4	0.29	20	9	0.10	24
DHS	0.18	6	1	0.09	13

Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{\lvert\alpha_{\text{H-L}}\rvert}$	$\#_{\lvert t\rvert\geq 1.96}$	$\#_{\lvert t\rvert\geq 3}$	$\overline{\lvert\alpha\rvert}$	$\#^{\text{GRS}}_{p<5\%}$
	Intangibles (27)				
q	0.47	13	11	0.18	19
q^5	0.36	8	4	0.15	13
FF5	0.50	17	9	0.16	18
FF6	0.48	13	11	0.17	18
FF6c	0.50	14	11	0.17	18
BS6	0.49	15	11	0.20	21
SY4	0.38	11	6	0.15	12
DHS	0.60	16	10	0.19	18

Stress Tests

Relative performance of factor models, 1/1967–12/2018

	$\overline{\lvert\alpha_{\text{H}-\text{L}}\rvert}$	$\#_{\lvert t\rvert\geq 1.96}$	$\#_{\lvert t\rvert\geq 3}$	$\overline{\lvert\alpha\rvert}$	$\#^{\text{GRS}}_{p<5\%}$
	Trading frictions (3)				
q	0.24	2	1	0.10	3
q^5	0.19	2	0	0.08	2
FF5	0.22	2	1	0.07	2
FF6	0.20	2	1	0.07	2
FF6c	0.20	2	0	0.07	1
BS6	0.23	2	2	0.09	3
SY4	0.18	2	0	0.09	2
DHS	0.50	3	2	0.18	3

Stress Tests

Explaining the composite score deciles, 1/1967–12/2018

All (150): $\overline{R} = 1.69$ ($t = 9.62$)

	$\alpha_{\text{H}-\text{L}}$	$t_{\text{H}-\text{L}}$	$\overline{\lvert\alpha\rvert}$	p_{GRS}
q	0.86	5.64	0.16	0.00
q^5	0.37	2.62	0.10	0.01
FF5	1.33	7.94	0.25	0.00
FF6	0.94	7.46	0.16	0.00
FF6c	0.82	6.77	0.14	0.00
BS6	0.68	4.85	0.13	0.00
SY4	0.90	7.61	0.16	0.00
DHS	0.74	4.98	0.14	0.00

Stress Tests

Explaining the composite score deciles, 1/1967–12/2018

Momentum (39): $\overline{R} = 1.09$ ($t = 4.21$)

	$\alpha_{\text{H}-\text{L}}$	$t_{\text{H}-\text{L}}$	$\overline{\lvert\alpha\rvert}$	p_{GRS}
q	0.35	1.04	0.10	0.08
q^5	−0.25	−0.85	0.10	0.35
FF5	1.21	3.74	0.27	0.00
FF6	0.33	2.08	0.09	0.06
FF6c	0.29	1.82	0.10	0.04
BS6	0.21	1.26	0.09	0.07
SY4	0.43	1.93	0.10	0.01
DHS	−0.36	−1.49	0.16	0.00

Stress Tests

Explaining the composite score deciles, 1/1967–12/2018

Value-versus-growth (15): $\overline{R} = 0.70$ ($t = 3.47$)

	$\alpha_{\text{H}-\text{L}}$	$t_{\text{H}-\text{L}}$	$\overline{\lvert\alpha\rvert}$	p_{GRS}
q	0.28	1.48	0.13	0.00
q^5	0.38	2.14	0.16	0.00
FF5	0.04	0.30	0.11	0.00
FF6	0.19	1.58	0.10	0.00
FF6c	0.12	1.05	0.10	0.00
BS6	−0.16	−1.17	0.12	0.00
SY4	0.34	2.20	0.14	0.00
DHS	0.98	5.34	0.31	0.00

Stress Tests

Explaining the composite score deciles, 1/1967–12/2018

Investment (26): $\overline{R} = 0.66$ ($t = 4.44$)

	$\alpha_{\text{H}-\text{L}}$	$t_{\text{H}-\text{L}}$	$\overline{\lvert\alpha\rvert}$	p_{GRS}
q	0.25	2.61	0.10	0.00
q^5	0.06	0.54	0.06	0.15
FF5	0.29	3.11	0.08	0.00
FF6	0.27	2.84	0.07	0.01
FF6c	0.27	2.62	0.06	0.06
BS6	0.18	1.73	0.09	0.00
SY4	0.10	1.00	0.07	0.01
DHS	0.55	3.83	0.12	0.00

Stress Tests

Explaining the composite score deciles, 1/1967–12/2018

Profitability (40): $\overline{R} = 0.80$ ($t = 4.64$)

	$\alpha_{\text{H}-\text{L}}$	$t_{\text{H}-\text{L}}$	$\overline{\lvert\alpha\rvert}$	p_{GRS}
q	0.28	2.31	0.07	0.01
q^5	−0.14	−1.21	0.08	0.09
FF5	0.60	5.35	0.12	0.00
FF6	0.43	3.94	0.09	0.00
FF6c	0.30	2.30	0.07	0.09
BS6	0.34	2.61	0.09	0.00
SY4	0.37	2.86	0.09	0.00
DHS	−0.09	−0.56	0.07	0.35

Stress Tests

Explaining the composite score deciles, 1/1967–12/2018

Intangibles (27): $\overline{R} = 0.94$ ($t = 5.27$)

	$\alpha_{\text{H-L}}$	$t_{\text{H-L}}$	$\overline{\lvert\alpha\rvert}$	p_{GRS}
q	0.42	2.62	0.18	0.00
q^5	0.50	3.19	0.19	0.00
FF5	0.43	3.24	0.18	0.00
FF6	0.54	4.25	0.20	0.00
FF6c	0.57	4.17	0.21	0.00
BS6	0.26	1.85	0.15	0.00
SY4	0.46	3.16	0.18	0.00
DHS	0.89	5.24	0.28	0.00

Stress Tests

Explaining the composite score deciles, 1/1967–12/2018

Trading frictions (3): $\overline{R} = 0.23$ ($t = 1.77$)

	$\alpha_{\text{H−L}}$	$t_{\text{H−L}}$	$\overline{\lvert\alpha\rvert}$	p_{GRS}
q	0.16	1.80	0.10	0.00
q^5	0.15	1.60	0.08	0.06
FF5	0.14	1.80	0.08	0.05
FF6	0.12	1.53	0.07	0.07
FF6c	0.12	1.34	0.06	0.28
BS6	0.14	1.60	0.11	0.00
SY4	0.13	1.50	0.09	0.01
DHS	0.57	4.29	0.13	0.00

Outline

Individual Factor Regressions

Examples, 1/1967–12/2018

	Sue1	$R^6 6$	Bm	Oa	dFin	Dac	Cop	Rdm
$\overline{R}$	0.45	0.83	0.43	−0.29	0.27	−0.45	0.68	0.73
$t_{\overline{R}}$	3.50	3.66	2.14	−2.36	2.43	−3.47	3.94	2.96
α_q	0.05	0.30	0.11	−0.57	0.41	−0.74	0.75	0.81
α_{q^5}	−0.07	−0.16	0.05	−0.20	0.14	−0.31	0.11	0.27
t_q	0.39	1.04	0.71	−4.25	2.97	−5.33	5.57	3.64
t_{q^5}	−0.52	−0.64	0.32	−1.30	0.97	−2.16	0.96	1.24
α_{FF6}	0.26	0.19	−0.09	−0.48	0.46	−0.69	0.75	0.68
α_{FF6c}	0.22	0.16	−0.09	−0.32	0.34	−0.59	0.55	0.79
t_{FF6}	2.23	1.92	−0.82	−3.49	3.81	−5.08	6.44	3.24
t_{FF6c}	1.84	1.57	−0.74	−2.13	2.63	−4.12	4.75	3.64

The q^5 model (q + expected growth) outperforms the Fama-French 6-factor model in a large-scale empirical horse race