Searching for the Equity Premium

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November 3, 2020

A DSGE model with recursive utility, search frictions, and capital accumulation is a good start to explaining the equity premium puzzle, while retaining plausible business cycle dynamics

Explaining the Mehra-Prescott (1985) equity premium puzzle in general equilibrium production economies has been challenging

Finance specifies "exotic" preferences and exogenous cash flow dynamics to match asset prices but ignore firms (Campbell and Cochrane 1999; Bansal and Yaron 2004; Barro 2006)

Macroeconomics analyzes full-fledged DSGE models but ignore asset prices with primitive preferences (Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007)

Introduction
The holy grail of macro-finance:
A unified theory of asset prices and business cycles

What are the microfoundations for the exogenous, often complicated cash flow dynamics in finance models (Bansal, Kiku, and Yaron 2012; Nakamura, Steinsson, Barro, and Ursua 2013)?

What are the essential ingredients in the production side that can endogenize the key elements of cash flow dynamics necessary to explain the equity premium?

To what extent do time-varying risk premiums matter quantitatively for macroeconomic dynamics?

How large is the welfare cost of business cycles in a general equilibrium production economy that replicates the equity premium?

Calibrated to the consumption volatility in the Jordà-Schularick-Taylor database, the DSGE model yields a (leverage-adjusted) equity premium of 4.26% per annum, an average interest rate of 1.59%, and a stock market volatility of 11.8%

Strong time series predictability for stock market excess returns and volatilities, some predictability for consumption volatility, and weak to no predictability for consumption growth and real interest rate

Wage inertia: a wage elasticity to labor productivity of 0.256 in the model vs. 0.267 in the historical U.S. 1890–2015 sample

Wage inertia yields strongly procyclical dynamics of profits, sufficient to overcome procyclical investment and vacancy costs to turn dividends procyclical

Time-varying wage inertia yields predictability for market excess returns and volatilities

Investment absorbs a large amount of shocks, making consumption growth and the interest rate unpredictable

Risk aversion strongly affects quantity dynamics

Downward-sloping term structures of equity return and volatility

A reasonable timing premium of 15.3%

The welfare cost is huge, 29.1%, and strongly countercyclical

Petrosky-Nadeau, Zhang, and Kuehn (2018) on endogenous disasters, but limited asset pricing results (no capital)

Bai (2020) on the credit spread puzzle

Gourio (2012): The Rietz-Barro disasters in an RBC model

Kilic and Wachter (2018): Search with the Rietz-Barro disasters

Recent GE studies outside the disasters framework: Croce (2014), Kung and Schmid (2015), Favilukis and Lin (2016), Chen (2017)

Outline

A representative household pools income from its employed and unemployed workers before making optimal consumption decisions

A representative firm makes optimal investment and vacancy decisions to maximize its market equity

The labor market as a matching function that yields new hires from the numbers of vacancies and unemployed workers

Wages determined from a generalized Nash bargaining process between the firm and unemployed workers

The household maximizes recursive utility, J_t:

$$J_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta \left(E_t\left[J_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}$$

in which C_t is consumption, β time preference, ψ the elasticity of intertemporal substitution, and γ risk aversion

The household's stochastic discount factor, M_{t+1}:

$$M_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{J_{t+1}}{E_t\left[J_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}$$

The representative firm uses capital, K_t, and labor, N_t, to product output, Y_t, with the CES production function (Arrow et al. 1961):

$$Y_t = X_t \left[\alpha \left(\frac{K_t}{K_0} \right)^{\omega} + (1-\alpha) N_t^{\omega} \right]^{\frac{1}{\omega}}$$

in which α is the distribution parameter, and $e \equiv 1/(1-\omega)$ the elasticity of substitution between capital and labor

The "normalized" CES function, the scalar $K_0 > 0$ makes the unit of K_t/K_0 comparable to N_t (Klump and La Grandville 2000)

Calibrate K_0 to match $1 - \alpha$ to the average labor share in the data

Aggregate productivity, X_t, with $x_t \equiv \log(X_t)$ governed by:

$$x_{t+1} = (1 - \rho_x)\overline{x} + \rho_x x_t + \sigma_x \epsilon_{t+1},$$

in which $\overline{x}$ is unconditional mean, $0 < \rho_x < 1$ persistence, $\sigma_x > 0$ conditional volatility, and ϵ_{t+1} an i.i.d. standard normal shock

Scale $\overline{x}$ to make the average marginal product of labor around one in simulations to ease the interpretation of parameters

The Den Haan-Ramey-Watson (2000) matching function, $\iota > 0$:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}$$

$\theta_t \equiv V_t/U_t$; the vacancy filling rate: $q(\theta_t) = (1 + \theta_t^\iota)^{-1/\iota}$

Employment, N_t, evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t$$

in which $q(\theta_t)V_t$ is the number of new hires

Vacancy costs: κV_t, in which $\kappa > 0$

Capital accumulates as:

$$K_{t+1} = (1 - \delta)K_t + \Phi(I_t, K_t),$$

in which δ is the capital depreciation rate, I_t is investment, and

$$\Phi_t \equiv \Phi(I_t, K_t) = \left[a_1 + \frac{a_2}{1 - 1/\nu} \left(\frac{I_t}{K_t} \right)^{1 - 1/\nu} \right] K_t$$

is the installation function with the supply elasticity of capital $\nu > 0$

Set $a_1 = \delta/(1 - \nu)$ and $a_2 = \delta^{1/\nu}$ to ensure no adjustment costs in the deterministic steady state (Jermann 1998)

The equilibrium wage rate from Nash bargaining, W_t:

$$W_t = \eta \left(\frac{\partial Y_t}{\partial N_t} + \kappa \theta_t \right) + (1 - \eta)b$$

in which $\eta \in (0, 1)$ is the workers' relative bargaining weight; b the workers' flow value of unemployment

η governs the wage elasticity to labor productivity

The dividends to the firm's shareholders given by:

$$D_t = Y_t - W_t N_t - \kappa V_t - I_t$$

Taking W_t, M_{t+1}, and $q(\theta_t)$ as given, the firm chooses optimal investment and vacancies to maximize:

$$S_t \quad \equiv \quad \max_{\{V_{t+\tau}, N_{t+\tau+1}, I_{t+\tau}, K_{t+\tau+1}\}_{\tau=0}^{\infty}} \quad E_t \left[\sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} \right],$$

subject to employment and capital accumulation and $V_t \geq 0$

The competitive equilibrium consists of investment, I_t, vacancy posting, V_t, multiplier, λ_t, and consumption, C_t, such that:

(i) C_t satisfies the consumption Euler equation;

(ii) I_t satisfies the investment Euler equation, and V_t and λ_t satisfy the intertemporal job creation condition and the Kuhn-Tucker conditions, while taking the stochastic discount factor, M_{t+1}, and the equilibrium wage, W_t, as given;

(iii) the goods market clears:

$$C_t + \kappa V_t + I_t = Y_t$$

From the first-order conditions for I_t and K_{t+1}:

$$\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{1/\nu} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}}\right]\right]$$

Equivalently, $E_t[M_{t+1}r_{Kt+1}] = 1$, in which the investment return:

$$r_{Kt+1} \equiv \frac{\partial Y_{t+1}/\partial K_{t+1} + (1/a_2)(1-\delta+a_1)\left(I_{t+1}/K_{t+1}\right)^{1/\nu} + (1/(\nu-1))(I_{t+1}/K_{t+1})}{(1/a_2)\left(I_t/K_t\right)^{1/\nu}}$$

Let λ_t be the multiplier on $q(\theta_t)V_t \geq 0$, Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0$$

From the first-order conditions with respect to V_t and N_{t+1}:

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda_{t+1}\right)\right]\right]$$

Equivalently, $E_t[M_{t+1}r_{Nt+1}] = 1$, in which the hiring return:

$$r_{Nt+1} \equiv \frac{\partial Y_{t+1}/\partial N_{t+1} - W_{t+1} + (1-s)\left(\kappa/q(\theta_{t+1}) - \lambda_{t+1}\right)}{\kappa/q(\theta_t) - \lambda_t}$$

The stock return of the representative firm, r_{St+1}, as a weighted average of the investment and hiring returns:

$$r_{St+1} = \frac{\mu_{Kt}K_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Kt+1} + \frac{\mu_{Nt}N_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Nt+1},$$

The shadow value of capital, $\mu_{Kt} = (1/a_2)(I_t/K_t)^{(1/\nu)}$

The shadow value of labor, $\mu_{Nt} = \kappa/q(\theta_t) - \lambda_t$

Parameterize the conditional expectation in job creation condition

Solve for the indirect utility, investment, and conditional expectation functions from recursive utility, investment Euler equation, and job creation condition

Rouwenhorst discrete state on productivity with 17 grid points

Finite element with cubic splines on 50 employment nodes and 50 capital nodes; tensor product on each grid point of productivity

Solve the resulting system of 127,500 equations with the derivative-free fixed point iteration with a small damping parameter

The Jordà-Schularick-Taylor macrohistory database for business cycle and asset pricing moments

http://www.macrohistory.net/data

Real consumption, output, and investment, as well as asset prices for 17 developed countries

Annual series, 1871–2015

Calibration

Real consumption growth in the Jordà-Schularick-Taylor macrohistory database

	Sample	$\overline{g}_C$	σ_C	S_C	K_C	$\rho_C^{(1)}$	$\rho_C^{(2)}$	$\rho_C^{(3)}$	$\rho_C^{(4)}$	$\rho_C^{(5)}$
Australia	1871	1.11	5.76	−0.77	6.35	−0.04	0.22	−0.03	0.03	−0.09
Belgium	1914	1.35	8.72	−1.14	13.18	0.26	0.19	0.00	−0.40	−0.22
Canada	1872	1.77	4.62	−1.04	6.27	0.00	0.16	−0.16	−0.04	−0.14
Denmark	1871	1.38	5.27	−0.83	11.44	−0.01	−0.41	0.06	0.18	−0.23
Finland	1871	2.07	5.54	−1.13	9.01	0.16	−0.08	0.02	−0.04	−0.23
France	1871	1.37	6.57	−1.06	13.69	0.39	0.19	−0.06	−0.28	−0.14
Germany	1871	1.67	5.51	−0.57	7.11	0.25	0.24	0.28	−0.07	0.00
Italy	1871	1.47	3.63	0.14	7.62	0.38	0.32	0.10	0.08	0.11
Japan	1875	2.11	6.74	−1.53	20.90	0.21	0.10	0.18	0.20	0.20
Netherlands	1871	1.41	8.18	−0.83	19.86	0.17	0.13	−0.21	−0.21	−0.19
Norway	1871	1.83	3.65	−0.32	12.65	−0.06	−0.34	0.26	0.07	−0.24
Portugal	1911	2.36	4.36	−0.49	3.30	0.22	0.23	−0.02	0.09	−0.16
Spain	1871	1.56	7.92	−2.20	17.20	0.00	−0.02	−0.13	−0.05	0.08
Sweden	1871	1.80	4.20	0.44	7.04	−0.15	−0.17	0.05	0.07	−0.20
Switzerland	1871	1.22	5.85	0.35	7.34	−0.20	−0.10	−0.11	−0.10	0.04
UK	1871	1.33	2.76	−0.34	8.90	0.33	0.02	−0.06	−0.01	−0.11
USA	1871	1.75	3.42	−0.07	3.99	0.08	0.09	−0.11	0.00	−0.10
Mean		1.62	5.45	−0.67	10.34	0.12	0.04	0.00	−0.03	−0.09

Real consumption growth, the Jordà-Schularick-Taylor macrohistory database, 1950–2015

	$\overline{g}_C$	σ_C	S_C	K_C	$\rho_C^{(1)}$	$\rho_C^{(2)}$	$\rho_C^{(3)}$	$\rho_C^{(4)}$	$\rho_C^{(5)}$
Australia	1.78	2.02	−0.14	3.55	0.17	−0.24	−0.11	0.19	0.30
Belgium	1.89	1.92	0.20	3.42	0.34	0.21	0.41	0.18	0.21
Canada	2.01	1.81	−0.61	4.00	0.31	0.07	0.17	−0.07	−0.26
Denmark	1.24	2.43	−0.03	2.95	0.22	0.01	0.03	−0.17	−0.30
Finland	2.62	3.17	−0.40	3.04	0.40	−0.08	−0.05	−0.05	−0.03
France	2.34	1.79	0.19	2.18	0.65	0.48	0.40	0.42	0.41
Germany	2.81	2.46	0.71	2.98	0.73	0.53	0.50	0.51	0.49
Italy	2.51	2.72	−0.30	2.97	0.67	0.46	0.52	0.48	0.41
Japan	3.90	3.53	0.72	3.00	0.74	0.62	0.69	0.66	0.61
Netherlands	1.92	2.47	−0.16	2.45	0.67	0.32	0.15	0.08	0.13
Norway	2.39	2.19	0.21	3.76	0.23	−0.02	−0.18	−0.14	−0.13
Portugal	3.05	3.56	−0.58	4.03	0.36	0.16	0.08	−0.14	−0.18
Spain	2.79	3.54	0.08	3.20	0.51	0.25	0.20	0.23	0.23
Sweden	1.55	1.92	−0.59	3.12	0.38	0.18	0.08	−0.09	−0.16
Switzerland	1.44	1.42	0.11	2.59	0.61	0.24	0.14	0.10	0.11
UK	1.97	2.09	−0.13	3.11	0.45	0.05	−0.11	−0.11	0.00
USA	2.08	1.73	−0.21	2.49	0.32	0.03	−0.06	0.02	−0.04
Mean	2.25	2.40	−0.05	3.11	0.46	0.19	0.17	0.12	0.11

Calibration

	Sample	$\overline{g}_Y$	σ_Y	S_Y	K_Y	$\rho_Y^{(1)}$	$\rho_Y^{(2)}$	$\rho_Y^{(3)}$	$\rho_Y^{(4)}$	$\rho_Y^{(5)}$
Australia	1871	1.45	4.11	−0.90	5.49	0.04	0.27	−0.10	−0.03	−0.05
Belgium	1871	1.63	7.45	1.26	19.01	0.33	0.05	0.00	0.03	−0.29
Canada	1871	1.87	4.97	−0.78	5.11	0.26	0.11	−0.07	−0.15	−0.15
Denmark	1871	1.68	3.66	−1.03	8.13	0.05	−0.17	0.08	0.08	−0.08
Finland	1871	2.06	4.47	−0.78	7.15	0.25	−0.11	0.10	−0.12	−0.17
France	1871	1.64	6.20	−0.60	10.30	0.09	−0.09	0.10	0.19	−0.09
Germany	1871	1.62	10.66	−7.62	78.70	0.30	−0.04	−0.11	−0.16	−0.13
Italy	1871	1.80	4.71	−1.32	13.34	0.27	−0.06	−0.03	0.14	0.01
Japan	1871	2.40	6.18	−2.23	15.50	0.27	0.03	0.16	0.09	0.01
Netherlands	1871	1.54	6.75	0.97	32.58	0.25	−0.12	−0.02	−0.07	−0.16
Norway	1871	2.10	3.53	−0.72	7.21	0.11	−0.08	0.12	0.06	−0.15
Portugal	1871	1.84	4.16	−0.01	4.23	0.01	0.18	0.02	0.18	0.04
Spain	1871	1.86	4.98	−1.58	10.94	0.18	0.05	0.03	0.04	0.14
Sweden	1871	2.02	3.39	−1.32	7.30	−0.08	−0.04	0.02	0.18	−0.17
Switzerland	1871	1.41	3.84	−0.41	4.02	0.13	−0.14	−0.05	0.09	0.05
UK	1871	1.40	2.86	−0.89	5.62	0.35	0.03	−0.18	−0.22	−0.09
USA	1871	1.91	4.77	−0.08	4.83	0.25	0.08	−0.13	−0.19	−0.19
Mean		1.78	5.10	−1.06	14.09	0.18	0.00	0.00	0.01	−0.09

Real investment growth in the Jordà-Schularick-Taylor macrohistory database

	Sample	$\overline{g}_I$	σ_I	S_I	K_I	$\rho_I^{(1)}$	$\rho_I^{(2)}$	$\rho_I^{(3)}$	$\rho_I^{(4)}$	$\rho_I^{(5)}$
Australia	1871 (47–49)	1.60	13.56	−0.72	5.06	0.15	0.09	−0.07	−0.16	−0.07
Belgium	1901 (14–20, 40–46)	1.68	10.74	−0.20	3.44	−0.09	−0.06	−0.02	−0.23	0.14
Canada	1872	2.17	18.12	−0.18	10.68	0.27	0.02	−0.18	−0.19	−0.16
Denmark	1871 (15–22)	1.96	10.10	−0.52	6.63	0.21	−0.11	−0.05	0.00	−0.17
Finland	1871	2.40	13.24	−1.49	11.14	0.19	0.01	0.06	−0.27	−0.28
France	1871 (19–20, 45–46)	1.98	19.23	−1.33	16.16	−0.07	−0.31	−0.04	−0.08	0.15
Germany	1871 (14–20, 40–48)	2.69	14.42	−0.56	5.40	0.06	−0.01	−0.10	−0.11	−0.23
Italy	1871	2.50	12.42	1.82	23.10	0.11	−0.14	0.12	0.03	−0.08
Japan	1886 (45–46)	4.21	14.36	−0.77	13.61	0.14	−0.04	−0.07	0.00	0.08
Netherlands	1871 (14–21, 40–48)	1.78	8.23	−0.28	3.70	0.03	0.01	−0.15	−0.04	−0.21
Norway	1871 (40–46)	2.69	13.33	2.08	21.86	−0.13	−0.16	0.02	−0.04	−0.05
Portugal	1954	2.64	9.58	−0.22	3.08	0.22	0.21	0.06	−0.13	0.08
Spain	1871	2.85	13.23	−0.41	4.01	0.23	0.02	−0.23	−0.13	−0.12
Sweden	1871	2.65	12.43	0.10	4.88	0.07	−0.27	−0.08	0.01	−0.11
Switzerland	1871 (14–48)	2.58	11.02	0.69	5.33	0.37	0.17	−0.11	−0.33	−0.22
UK	1871	1.98	11.68	2.82	26.62	0.35	−0.14	−0.12	−0.03	−0.08
USA	1871	2.04	24.37	−1.71	18.02	0.17	−0.11	−0.32	−0.13	−0.02
Mean		2.38	13.53	−0.05	10.75	0.13	−0.05	−0.07	−0.11	−0.08

Asset prices in the Jordà-Schularick-Taylor macrohistory database

	Sample	$E[\tilde{r}_S]$	$\tilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\tilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
Australia	1900 (45–47)	7.75	17.08	1.29	4.32	6.46	4.58	12.55
Belgium	1871 (14–19)	6.31	19.88	1.21	8.43	5.10	3.62	14.62
Canada	1900	7.01	17.00	1.60	4.79	5.41	3.84	12.26
Denmark	1875 (15)	7.47	16.43	3.08	5.68	4.39	3.12	11.91
Finland	1896	8.83	30.57	−0.74	10.93	9.57	6.80	22.98
France	1871 (15–21)	3.99	22.22	−0.47	7.78	4.45	3.16	16.75
Germany	1871 (23, 44–49)	8.83	27.59	−0.23	13.22	9.05	6.43	20.22
Italy	1871 (1872–84, 15–21)	6.63	27.21	0.58	10.50	6.05	4.29	20.41
Japan	1886 (46–47)	8.86	27.69	0.00	11.20	8.87	6.29	21.10
Netherlands	1900	6.96	21.44	0.78	4.91	6.19	4.39	15.32
Norway	1881	5.67	19.82	0.90	5.98	4.77	3.39	14.53
Portugal	1880	3.81	25.68	−0.01	9.43	3.82	2.71	19.29
Spain	1900 (36–40)	6.25	21.41	−0.04	6.90	6.29	4.47	15.94
Sweden	1871	8.00	19.54	1.77	5.60	6.23	4.42	14.26
Switzerland	1900 (15)	6.69	19.08	0.89	5.00	5.79	4.11	14.00
UK	1871	6.86	17.77	1.16	4.82	5.70	4.05	12.96
USA	1872	8.40	18.68	2.17	4.65	6.23	4.43	13.66
Mean		6.96	21.71	0.82	7.30	6.14	4.36	16.04

U.S. historical monthly series: Unemployment and labor productivity, 1890–; vacancy, 1919–

https://ars.els-cdn.com/content/image/1-s2.0-S0304393220300064-mmc2.csv

Private nonfarm unemployment rates (Lebergott 1964; Weir 1992): Mean, 8.94%; volatility: 24.43% per quarter

θ_t volatility: 61.62% (but 38.38% with log deviations)

The U-V correlation: -0.57

Time discount factor, $\beta = 0.9976$

Risk aversion, $\gamma = 10$

Elasticity of intertemporal substitution, $\psi = 2$

Persistence in log productivity, $\rho_x = 0.95^{1/3}$

Calibrate its conditional volatility, $\sigma_x = 0.015$, to hit average σ_C

Long-run mean of log productivity, $\bar{x} = 0.1887$, to target the marginal product of labor to be one on average

Elasticity of capital-labor substitution, $e = 1/(1 - \omega) = 0.4$, per Chirinko and Mallick (2017)

The distribution parameter, $\alpha = 0.25$, to match the average labor share of 0.743 per Gollin (2002)

The capital scaler, $K_0 = 13.75$, to target the labor share of 0.75 at the deterministic steady state (close to its stochastic steady state)

Supply elasticity of capital, $\nu = 1.25$

Depreciation rate of capital, $\delta = 1.25\%$

Separation rate, $s = 3.5\%$, the average in JOLTS

Curvature in the matching function, $\iota = 1.25$ (Den Haan, Ramey, and Watson 2000)

Bargaining weight of workers, $\eta = 0.015$

Flow value of unemployment, $b = 0.91$, a simple device for small fundamental surplus, see also Ganong, Noel, and Vavra (2020)

Unit vacancy cost, $\kappa = 0.01$

The low-η-high-b calibration yields a wage elasticity to labor productivity of 0.256

Hagedorn and Manovskii (2008) estimate the wage elasticity to be 0.449 in the postwar 1951–2004 quarterly sample from BLS

From 1929 to 2015, obtain compensation of employees from NIPA Tables 6.2A–D (line 3, private industries, minus line 5, farms)

Obtain the number of full-time equivalent employees from NIPA Tables 6.5A–D (line 3, private industries, minus line 5, farms)

Dividing the compensation of employees by the number of employees yields nominal wage rates (compensation per person)

Deflate nominal wage rates with the personal consumption deflator from NIPA Table 1.1.4 (line 2) to obtain real wage rates

From 1890 to 1929, obtain the average (nominal) hourly compensation of production workers in manufacturing and consumer price index from measuringworth.com (Officer 2009, Table 7.1; Officer and Williamson 2020a, 2020b)

Divide the manhours index by the index of persons engaged in manufacturing from Kendrick (1961, Table D-II) to obtain hours

Multiply the average hourly compensation series with the hours index to obtain the nominal compensation per person; deflate with the Officer-Williamson consumer price index to obtain real wages

Splice this series in 1929 to the NIPA series from 1929 onward to yield an uninterrupted series from 1890 to 2015

Historical 1890–2015 series of labor productivity from Petrosky-Nadeau and Zhang (2020)

Time-aggregate monthly series into annual by taking the monthly average within a given year

Detrend the annual real wages and labor productivity series as log deviations from their HP-trends with a smoothing parameter of 6.25

In our postwar 1950–2015 annual sample, regressing the log real wages on the log labor productivity yields a wage elasticity of 0.406, with a standard error of 0.081

In our 1890–2015 sample, the wage elasticity estimated to be 0.267, with a standard error of 0.066

	Data	Mean	5th	50th	95th	p
σ_C	5.45	5.13	2.87	5.13	7.39	0.41
S_C	−0.67	0.03	−1.03	0.03	1.10	0.89
K_C	10.34	8.09	4.38	7.30	14.44	0.18
ρ_{C1}	0.12	0.21	−0.01	0.22	0.40	0.78
ρ_{C2}	0.04	−0.05	−0.26	−0.05	0.17	0.24
ρ_{C3}	0.00	−0.04	−0.24	−0.04	0.16	0.35
ρ_{C4}	−0.03	−0.04	−0.23	−0.04	0.15	0.44
ρ_{C5}	−0.09	−0.04	−0.23	−0.04	0.14	0.67

	Data	Mean	5th	50th	95th	p
σ_Y	5.10	6.43	4.46	6.40	8.48	0.86
S_Y	−1.06	0.09	−0.62	0.08	0.81	0.99
K_Y	14.09	5.45	3.50	5.09	8.64	0.00
ρ_{Y1}	0.18	0.20	0.03	0.21	0.36	0.60
ρ_{Y2}	0.00	−0.06	−0.23	−0.06	0.12	0.31
ρ_{Y3}	0.00	−0.05	−0.22	−0.05	0.12	0.31
ρ_{Y4}	0.01	−0.05	−0.21	−0.05	0.12	0.29
ρ_{Y5}	−0.09	−0.05	−0.21	−0.05	0.12	0.65

	Data	Mean	5th	50th	95th	p
σ_I	13.53	8.59	5.29	8.61	11.83	0.00
S_I	−0.05	0.31	−0.57	0.28	1.26	0.76
K_I	10.75	7.12	4.12	6.47	12.17	0.08
ρ_{I1}	0.13	0.15	−0.04	0.16	0.33	0.58
ρ_{I2}	−0.05	−0.11	−0.29	−0.11	0.08	0.30
ρ_{I3}	−0.07	−0.09	−0.27	−0.09	0.10	0.45
ρ_{I4}	−0.11	−0.07	−0.25	−0.07	0.11	0.62
ρ_{I5}	−0.08	−0.06	−0.24	−0.06	0.12	0.56

	Data	Mean	5th	50th	95th	p
$E[U]$	8.94	8.63	3.81	7.45	17.63	0.37
S_U	2.13	2.64	0.76	2.20	5.85	0.53
K_U	9.50	13.45	2.11	6.77	39.06	0.35
σ_U	0.24	0.32	0.16	0.32	0.48	0.76
σ_V	0.19	0.34	0.23	0.32	0.49	1.00
σ_θ	0.62	0.34	0.23	0.32	0.50	0.01
ρ_{UV}	-0.57	-0.07	-0.16	-0.07	0.01	1.00
$e_{w,y/n}$	0.27	0.26	0.23	0.26	0.27	0.22

	Data	Mean	5th	50th	95th	p
$E[r_S - r_f]$	4.36	4.26	3.52	4.12	5.49	0.34
$E[r_f]$	0.82	1.59	0.07	1.83	2.26	0.87
σ_S	16.04	11.77	9.19	11.74	14.46	0.00
σ_f	7.30	3.13	1.13	3.05	5.37	0.00

Core challenge in explaining the equity premium in production economies (Kaltenbrunner and Lochstoer 2010): Dividends tend to be countercyclical in RBC models

- Dividends = profits (output minus wages) minus investment

- With frictionless labor market, wages equal MPL (as procyclical as output)

- Profits no more procyclical than output

- Investment more procyclical than output and profits (consumption smoothing)

- Dividends tend to be countercyclical

The search model overcomes the core challenge in explaining procyclical dividends in general equilibrium production economies

- Dividends = profits (output minus wages) minus investment minus vacancy costs

- With search frictions, wages are inertial, detached from MPL

- Profits more procyclical than output

- Investment (and vacancy costs) more procyclical than output (consumption smoothing)

- Profits more procyclical than investment and vacancy costs, giving rise to procyclical dividends

Unconditional Moments

Dividend dynamics, data, detrending with proportional deviations from the mean

	Sample	1-year		3-year		5-year	
		ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y
Australia	1870	0.109	6.943	0.231	5.144	0.551	4.596
Belgium	1870	0.183	8.501	0.419	9.797	0.765	5.622
Denmark	1872	0.191	14.885	0.263	12.065	0.027	10.580
Finland	1912	0.083	10.143	0.670	8.464	0.815	4.225
France	1870	0.169	5.590	0.234	5.311	0.479	4.443
Germany	1870	0.018	6.052	0.114	3.821	0.241	1.952
Italy	1870	0.035	5.097	0.262	5.925	0.571	5.603
Japan	1886 (46–47)	0.027	10.673	0.058	6.347	0.110	7.107
Netherlands	1950	−0.001	16.807	0.548	13.768	0.369	18.404
Norway	1880	0.216	10.520	0.348	5.638	0.342	5.413
Portugal	1870	−0.021	3.062	0.043	1.572	0.108	1.570
Spain	1899	0.035	11.598	0.176	6.008	0.266	4.142
Sweden	1871	−0.019	9.309	0.091	5.756	0.565	4.616
Switzerland	1960	0.026	11.061	0.429	8.658	0.034	7.987
UK	1871	0.272	4.314	0.570	3.001	0.114	2.735
USA	1871	0.472	3.176	0.527	3.367	0.307	2.099
Mean		0.112	8.608	0.312	6.540	0.354	5.693

	Sample	1-year		3-year		5-year	
		ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y
Australia	1870	0.121	3.430	0.295	2.565	0.633	2.593
Belgium	1870	0.498	4.706	0.825	5.718	0.923	2.830
Denmark	1872	0.182	6.836	0.226	6.513	0.092	6.031
Finland	1912	0.308	6.883	0.504	5.754	0.437	5.477
France	1870	0.119	2.658	−0.029	3.564	−0.204	3.234
Germany	1870	0.211	20.321	0.552	2.924	0.894	3.652
Italy	1870	0.396	6.373	0.847	10.566	0.764	7.340
Japan	1886 (46–47)	0.612	8.949	0.806	10.513	0.882	7.470
Netherlands	1950	0.203	14.904	0.390	13.042	0.258	13.533
Norway	1880	0.214	8.517	0.440	6.574	0.656	5.580
Portugal	1870	0.007	7.762	0.597	12.343	0.454	14.630
Spain	1899	0.269	8.865	0.562	11.541	0.625	5.039
Sweden	1871	0.152	5.664	0.440	5.112	0.701	5.631
Switzerland	1960	0.051	13.165	0.342	7.725	0.014	9.190
UK	1871	0.094	5.072	0.399	2.788	0.044	2.538
USA	1871	0.415	2.626	0.419	2.682	0.458	2.118
Mean		0.241	7.921	0.476	6.870	0.477	6.055

	1-year	3-year	5-year
ρ_{DY}	0.947	0.954	0.952
σ_D/σ_Y	2.89	2.83	2.74

Dividends = net payouts in the model

Single-shock structure in the model

	Data	Mean	5th	50th	95th	p	Data	Mean	5th	50th	95th	p
	Disaster hurdle = 10%						Disaster hurdle = 15%					
				Panel A: Consumption disasters								
Probability	6.40	5.83	1.55	5.31	11.32	0.37	3.51	3.64	0.71	3.20	7.69	0.45
Size	23.16	23.41	14.52	22.83	33.84	0.48	30.36	29.51	18.81	28.49	43.31	0.38
Duration	4.19	4.10	2.80	4.00	5.80	0.40	4.50	4.49	3.00	4.33	6.81	0.39
				Panel B: Output disasters								
Probability	5.78	10.9	6.14	10.58	16.48	0.97	2.62	6.10	2.33	5.88	10.68	0.94
Size	22.34	22.31	15.91	21.89	30.13	0.46	32.9	28.50	20.07	27.88	38.86	0.20
Duration	4.14	3.73	2.89	3.67	4.78	0.23	5.04	4.25	3.00	4.17	5.75	0.15

Different consumption dynamics accord with observed consumption growth moments but imply vastly different mechanisms

From 1-million months simulation from the model:

$$
\begin{aligned}
g_{Ct+1} &= E_t[g_{Ct+1}] + \sigma_{Ct}\,\epsilon^g_{t+1} \\
E_{t+1}[g_{Ct+2}] &= 0.288 E_t[g_{Ct+1}] + 0.705\,\sigma_{Ct}\,\epsilon^e_{t+1} \\
\sigma^2_{Ct+1} &= 0.008^2 + 0.964(\sigma^2_{Ct} - 0.008^2) + 0.421 \times 10^{-5}\epsilon^V_{t+1},
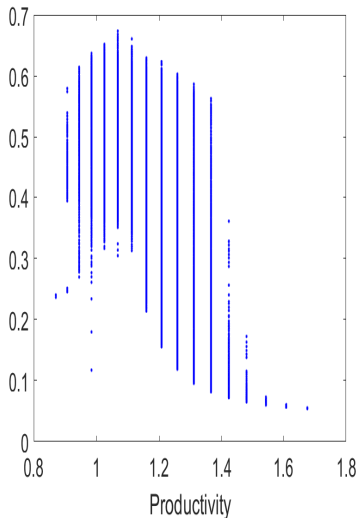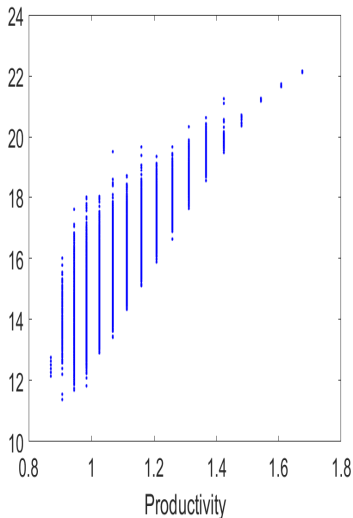\end{aligned}
$$

in which g_{Ct+1} is realized consumption growth, $E_t[g_{Ct+1}]$ expected consumption growth, σ_{Ct} conditional volatility of g_{Ct+1}, and ϵ^g_{t+1}, ϵ^e_{t+1}, and ϵ^V_{t+1} i.i.d. standard normal shocks

Pairwise correlations among ϵ^g_{t+1}, ϵ^e_{t+1}, and ϵ^V_{t+1} close to zero

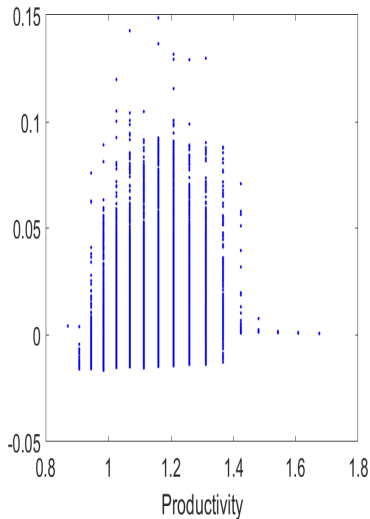# Time-varying Risk Premiums

Price-to-consumption, P_t/C_t, and the equity premium, $E_t[r_{St+1} - r_{ft+1}]$,
1 million months, $\text{Corr}(P_t/C_t, x_t) = 0.97$, $\text{Corr}(E_t[r_{St+1} - r_{ft+1}], x_t) = -0.84$

Time-varying Risk Premiums

Stock market volatility, σ_{St}, and the risk free rate, r_{ft+1},
1 million months, Corr$(\sigma_{St}, x_t) = -0.91$, Corr$(r_{ft+1}, x_t) = 0.23$

Time-varying Risk Premiums

Expected consumption growth, $E_t[g_{Ct+1}]$, and Consumption volatility, σ_{Ct}, 1 million months, $\text{Corr}(E_t[g_{Ct+1}], x_t) = 0.23$, $\text{Corr}(\sigma_{Ct}, x_t) = -0.05$

Time-varying Risk Premiums
Predictive regressions of stock market excess returns on P_t/C_t, the Jordà-Schularick-Taylor historical data

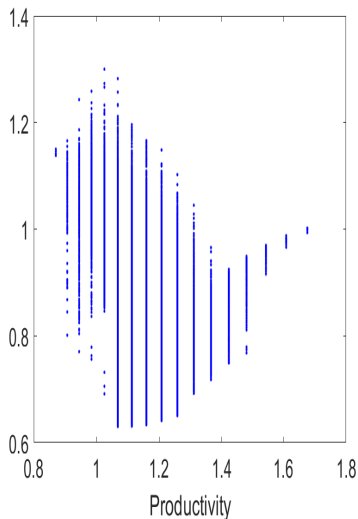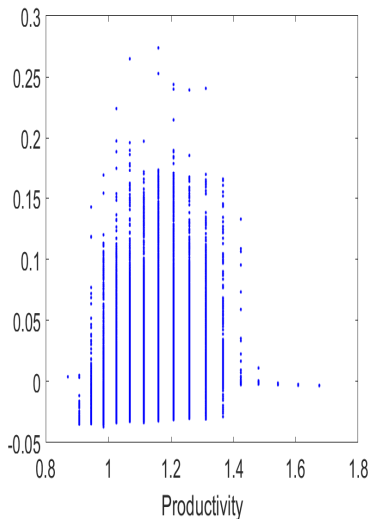|	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Slopes			t-values			R-squares	
Australia	-1.42	-2.92	-3.77	-1.97	-1.80	-1.62	1.80	3.56	4.29
Belgium	-1.30	-4.79	-5.16	-0.82	-1.00	-0.76	0.58	2.47	2.01
Denmark	-0.81	-2.87	-4.24	-0.85	-1.43	-2.14	0.50	2.13	3.76
Finland	-1.38	-5.40	-7.36	-0.77	-1.06	-1.22	0.55	2.55	3.78
France	-0.12	-0.52	-0.43	-0.11	-0.21	-0.11	0.01	0.05	0.02
Germany	-1.04	-2.06	0.13	-0.75	-0.54	0.02	0.19	0.21	0.00
Italy	-0.36	-0.36	0.38	-0.25	-0.10	0.07	0.04	0.01	0.01
Japan	-0.70	-1.60	-1.77	-0.45	-0.45	-0.36	0.19	0.35	0.24
Netherlands	-3.03	-8.88	-13.35	-1.68	-2.11	-2.98	4.15	12.73	20.25
Norway	-1.77	-5.13	-7.92	-1.55	-2.41	-3.24	1.75	5.60	9.84
Portugal	-0.24	-3.87	0.53	-0.08	-0.50	0.06	0.02	0.83	0.01
Spain	-1.02	-4.90	-8.13	-0.74	-1.21	-2.38	0.59	3.13	5.25
Sweden	-1.56	-6.04	-10.50	-1.63	-2.91	-3.20	1.42	6.47	13.08
Switzerland	-3.09	-8.50	-12.95	-1.70	-2.85	-4.17	4.02	11.76	20.05
UK	-2.95	-7.62	-10.51	-2.33	-5.43	-5.92	6.35	18.14	28.03
USA	-3.50	-9.89	-15.75	-3.83	-4.35	-5.16	7.71	21.01	33.59
Mean	-1.52	-4.71	-6.30	-1.22	-1.77	-2.07	1.87	5.69	9.01

Time-varying Risk Premiums

Predictive regressions of stock market excess returns on P_t/C_t,
the Jordà-Schularick-Taylor historical data, 1950–2015

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Slopes			t-values			R-squares	
Australia	−4.79	−8.35	−9.70	−3.03	−4.19	−2.46	12.17	21.17	21.26
Belgium	−2.39	−6.91	−10.86	−1.45	−1.61	−2.39	2.46	8.16	15.00
Denmark	−0.43	−2.32	−3.08	−0.17	−0.46	−0.76	0.08	0.79	1.13
Finland	−3.76	−14.03	−19.08	−1.36	−4.08	−5.33	3.68	14.50	20.25
France	−1.85	−5.95	−11.47	−0.97	−1.09	−1.48	1.26	4.85	11.90
Germany	−6.24	−14.93	−19.06	−2.78	−3.15	−3.40	12.48	24.99	29.57
Italy	−0.98	−4.20	−6.34	−0.52	−0.77	−0.76	0.32	1.76	2.76
Japan	−4.00	−13.90	−21.83	−2.30	−4.35	−5.96	8.19	25.40	36.39
Netherlands	−3.04	−8.91	−13.51	−1.68	−2.09	−3.06	4.13	12.71	20.65
Norway	−3.89	−8.74	−11.69	−1.99	−2.70	−2.87	4.99	12.52	18.57
Portugal	−2.16	−14.17	−17.39	−0.48	−1.30	−1.66	0.77	6.64	5.75
Spain	−0.32	−4.83	−9.22	−0.17	−0.86	−1.32	0.04	2.22	4.76
Sweden	−1.57	−4.06	−6.09	−0.75	−0.91	−1.24	0.95	2.46	4.10
Switzerland	−3.09	−8.50	−12.95	−1.70	−2.85	−4.17	4.02	11.76	20.05
UK	−6.50	−13.92	−16.54	−3.01	−4.54	−6.68	17.37	38.71	49.39
USA	−2.89	−7.18	−12.36	−2.18	−2.24	−2.79	5.83	13.61	23.90
Mean	−2.99	−8.81	−12.57	−1.53	−2.32	−2.90	4.92	12.64	17.84

Time-varying Risk Premiums

Predictive regressions of consumption growth on P_t/C_t,
the Jordà-Schularick-Taylor historical data

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Slopes			t-values			R-squares	
Australia	0.75	1.14	1.85	1.40	0.65	0.71	1.69	1.21	1.75
Belgium	−1.03	−0.94	−0.10	−0.91	−0.41	−0.04	1.41	0.30	0.00
Denmark	0.23	0.28	0.20	0.71	0.52	0.29	0.18	0.13	0.05
Finland	−0.91	−2.90	−4.07	−1.14	−1.54	−1.68	2.30	6.56	7.66
France	−0.84	−2.02	−3.18	−2.12	−1.81	−1.95	1.64	1.89	2.67
Germany	−0.95	−2.88	−4.70	−2.15	−1.81	−1.74	2.97	6.17	7.79
Italy	−0.60	−1.74	−2.91	−2.71	−1.96	−1.89	2.74	4.32	5.79
Japan	−1.76	−5.38	−8.78	−4.04	−2.89	−2.40	8.22	14.23	16.95
Netherlands	0.66	1.43	2.32	2.41	1.22	1.14	7.27	5.50	7.48
Norway	−0.35	−1.21	−2.10	−1.36	−2.11	−2.54	0.91	5.58	9.68
Portugal	−1.05	−3.26	−4.95	−2.18	−1.67	−1.53	4.82	10.91	11.55
Spain	−0.10	−0.41	−1.10	−0.14	−0.27	−0.55	0.02	0.08	0.40
Sweden	0.18	0.20	−0.17	0.56	0.28	−0.17	0.18	0.10	0.05
Switzerland	0.22	0.36	0.34	1.32	0.61	0.33	2.52	1.00	0.44
UK	−0.33	−1.53	−3.15	−1.78	−2.77	−4.16	1.44	7.06	17.32
USA	0.48	−0.64	−1.40	1.86	−0.85	−1.23	1.89	0.94	2.70
Mean	−0.34	−1.22	−1.99	−0.64	−0.93	−1.09	2.51	4.12	5.77

Time-varying Risk Premiums

Predictive regressions of consumption growth on P_t/C_t,
the Jordà-Schularick-Taylor historical data, 1950–2015

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Slopes			*t*-values			*R*-squares	
Australia	0.40	0.36	1.20	1.79	0.58	1.85	4.04	1.08	10.21
Belgium	0.09	0.21	0.54	0.42	0.43	0.76	0.24	0.27	0.78
Denmark	−0.08	−0.69	−1.38	−0.27	−0.61	−0.94	0.10	1.61	3.51
Finland	0.31	−0.40	−0.90	0.95	−0.39	−0.72	0.97	0.37	1.10
France	0.95	2.68	4.37	4.45	3.84	4.69	28.51	37.06	43.62
Germany	−0.10	−1.05	−1.84	−0.29	−0.69	−0.73	0.15	2.65	3.74
Italy	1.58	4.43	6.84	5.69	4.07	3.51	33.87	40.15	40.62
Japan	0.51	1.44	2.17	1.48	0.90	0.71	2.12	2.65	2.30
Netherlands	0.67	1.46	2.35	2.43	1.24	1.14	7.43	5.94	7.60
Norway	0.23	0.56	1.00	0.78	0.77	1.29	1.16	1.78	3.78
Portugal	0.19	0.13	1.54	0.36	0.08	0.98	0.26	0.03	2.66
Spain	1.75	4.02	5.62	4.78	3.36	2.72	24.75	25.77	23.66
Sweden	0.00	−0.39	−0.74	−0.01	−0.46	−0.53	0.00	0.74	1.30
Switzerland	0.22	0.36	0.34	1.32	0.61	0.33	2.52	1.00	0.44
UK	0.45	0.45	−0.12	2.14	0.80	−0.13	4.78	0.99	0.04
USA	0.28	0.11	0.22	1.34	0.15	0.20	2.69	0.09	0.20
Mean	0.47	0.86	1.33	1.71	0.92	0.95	7.10	7.64	9.10

Time-varying Risk Premiums

Predictive regressions of stock market volatility on P_t/C_t,
the Jordà-Schularick-Taylor historical data

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Slopes			t-values			R-squares	
Australia	20.04	15.73	15.82	1.89	1.81	1.80	2.15	4.28	6.55
Belgium	11.85	12.20	11.61	1.28	2.05	2.21	1.42	7.26	10.71
Denmark	−35.30	−38.17	−36.44	−3.70	−3.95	−3.43	7.90	21.11	25.79
Finland	6.94	5.56	3.54	0.66	0.97	0.65	0.42	1.26	0.76
France	−58.81	−60.03	−57.57	−6.19	−5.93	−5.17	20.49	42.82	46.37
Germany	−31.59	−35.20	−32.90	−2.89	−3.42	−2.67	5.44	16.61	18.71
Italy	−17.60	−24.80	−24.27	−1.92	−2.86	−2.41	2.45	12.62	17.69
Japan	8.99	9.12	11.75	0.80	0.94	1.26	0.48	1.89	4.74
Netherlands	7.49	11.28	8.97	0.50	1.03	1.15	0.48	4.60	5.30
Norway	−51.27	−54.25	−52.54	−5.44	−7.26	−7.81	20.22	51.02	60.54
Portugal	−50.20	−44.35	−39.43	−4.10	−3.50	−3.39	14.11	27.71	25.37
Spain	−37.40	−34.23	−32.51	−4.00	−4.81	−3.96	10.86	26.06	33.48
Sweden	−23.98	−21.83	−21.98	−2.75	−2.16	−1.79	4.88	9.78	11.85
Switzerland	7.05	9.51	11.03	0.39	0.90	1.30	0.27	3.01	7.64
UK	−35.31	−33.22	−31.62	−4.99	−4.10	−3.23	9.59	21.60	23.91
USA	0.30	6.58	8.06	0.03	1.57	2.51	0.00	1.77	4.86
Mean	−17.43	−17.26	−16.16	−1.90	−1.80	−1.44	6.32	15.84	19.02

Time-varying Risk Premiums

Predictive regressions of stock market volatility on P_t/C_t,
the Jordà-Schularick-Taylor historical data, 1950–2015

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Slopes			t-values			R-squares	
Australia	1.71	3.98	2.83	0.11	0.31	0.21	0.01	0.36	0.27
Belgium	1.96	1.97	−0.63	0.20	0.28	−0.10	0.04	0.19	0.03
Denmark	13.67	11.79	10.74	1.14	0.82	0.66	1.95	3.25	3.18
Finland	19.65	14.23	8.21	1.48	1.85	1.72	2.78	6.96	3.28
France	−9.75	−9.49	−10.89	−0.74	−1.64	−3.97	0.97	6.17	16.47
Germany	17.77	15.12	13.28	1.07	1.92	1.74	1.56	10.80	16.29
Italy	−33.16	−23.29	−18.45	−1.75	−2.52	−3.46	4.55	15.51	27.24
Japan	6.13	12.65	10.48	0.41	1.25	1.16	0.33	6.30	6.37
Netherlands	6.06	11.33	8.98	0.42	1.04	1.16	0.32	4.74	5.26
Norway	−34.27	−25.24	−26.05	−2.43	−3.70	−3.36	3.56	22.44	35.65
Portugal	−42.17	−43.85	−46.10	−2.14	−2.93	−2.89	11.85	28.69	31.02
Spain	−18.04	−19.14	−17.79	−1.41	−2.09	−1.94	2.42	11.80	17.10
Sweden	15.21	19.29	19.27	1.48	2.37	2.73	3.54	19.61	28.04
Switzerland	7.05	9.51	11.03	0.39	0.90	1.30	0.27	3.01	7.64
UK	1.05	11.23	16.17	0.07	1.26	2.62	0.01	3.89	11.03
USA	12.24	11.34	13.72	0.83	1.93	3.07	1.42	5.21	17.62
Mean	−2.18	0.09	−0.32	−0.06	0.07	0.04	2.23	9.31	14.16

Time-varying Risk Premiums

Predictive regressions of consumption growth volatility on P_t/C_t,
the Jordà-Schularick-Taylor historical data

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Slopes			t-values			R-squares	
Australia	3.23	−3.07	−5.52	0.28	−0.27	−0.40	0.06	0.14	0.55
Belgium	48.77	55.42	59.15	2.88	3.41	3.50	11.11	29.61	40.03
Denmark	−2.11	0.21	1.13	−0.17	0.02	0.09	0.02	0.00	0.03
Finland	32.87	38.93	41.42	2.38	3.27	3.79	6.84	25.65	33.31
France	84.04	77.39	76.69	9.11	9.25	8.35	37.34	60.94	68.76
Germany	11.37	13.29	16.28	1.14	0.95	0.95	0.77	2.15	3.75
Italy	6.73	8.51	11.70	0.78	1.06	1.30	0.36	1.60	4.48
Japan	37.88	39.78	39.93	3.66	3.54	3.07	8.50	21.30	24.82
Netherlands	7.04	9.68	8.42	0.60	0.85	0.90	0.58	3.20	3.56
Norway	3.69	4.34	3.63	0.34	0.37	0.32	0.09	0.28	0.29
Portugal	13.68	16.08	19.63	1.43	3.57	5.93	2.03	10.05	18.62
Spain	64.78	59.39	56.29	6.29	5.80	5.46	25.68	49.05	54.04
Sweden	−1.44	3.93	7.43	−0.14	0.39	0.64	0.01	0.26	1.32
Switzerland	−13.49	−13.37	−6.97	−1.01	−1.11	−0.69	1.40	3.78	1.61
UK	0.76	1.50	2.31	0.07	0.22	0.30	0.00	0.06	0.23
USA	−18.05	−18.25	−15.82	−1.89	−1.87	−1.45	2.44	6.30	6.07
Mean	17.49	18.36	19.73	1.61	1.84	2.00	6.08	13.40	16.34

Time-varying Risk Premiums

Predictive regressions of consumption growth volatility on P_t/C_t, the Jordà-Schularick-Taylor historical data, 1950–2015

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Slopes			t-values			R-squares	
Australia	−4.80	14.07	12.20	−0.20	1.88	2.09	0.17	8.03	9.27
Belgium	−4.34	5.59	12.26	−0.33	0.65	1.63	0.24	1.16	8.86
Denmark	−23.77	−16.49	−15.41	−1.83	−1.65	−1.85	3.69	5.85	12.03
Finland	−25.16	−8.95	−5.41	−1.84	−0.63	−0.40	4.60	1.74	0.89
France	16.54	16.51	16.37	1.28	2.11	3.44	2.07	9.91	18.08
Germany	−6.27	−0.64	4.44	−0.47	−0.07	0.57	0.21	0.01	1.11
Italy	8.31	5.95	9.02	0.73	0.65	1.56	0.73	1.33	5.04
Japan	1.35	−6.69	−8.53	0.06	−0.55	−0.73	0.01	1.05	2.26
Netherlands	6.40	11.58	10.08	0.54	0.96	1.05	0.42	4.59	5.23
Norway	−22.89	−24.00	−19.13	−1.86	−2.01	−2.66	3.70	11.96	12.60
Portugal	−18.51	−10.28	−9.76	−2.00	−1.02	−0.77	5.10	3.57	3.57
Spain	41.05	31.91	29.87	2.88	2.50	2.22	11.21	18.82	28.32
Sweden	−12.16	−17.17	−12.42	−0.78	−1.41	−1.60	0.91	6.43	6.27
Switzerland	−13.49	−13.37	−6.97	−1.01	−1.11	−0.69	1.40	3.78	1.61
UK	−24.73	−16.09	−16.53	−1.91	−2.24	−2.77	3.67	7.87	14.83
USA	−4.93	−10.05	−10.96	−0.31	−0.98	−1.07	0.12	2.44	4.42
Mean	−5.46	−2.38	−0.68	−0.44	−0.18	0.00	2.39	5.53	8.40

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Data			Mean			p	
			Panel A: Predicting stock market excess returns						
b	-1.52	-4.71	-6.30	-1.82	-4.91	-7.40	0.33	0.47	0.34
t	-1.22	-1.77	-2.07	-2.36	-3.17	-3.57	0.09	0.08	0.09
R^2	1.87	5.69	9.01	3.86	9.40	13.52	0.77	0.78	0.76
			Panel B: Predicting consumption growth						
b	-0.34	-1.22	-1.99	-1.27	-2.44	-3.52	0.01	0.09	0.13
t	-0.64	-0.93	-1.09	-2.69	-2.49	-2.79	0.01	0.10	0.13
R^2	2.51	4.12	5.77	7.34	8.44	11.27	0.88	0.70	0.68

	1y	3y	5y	1y	3y	5y	1y	3y	5y
		Data			Mean			p	
			Panel C: Predicting stock market volatilities						
b	−17.43	−17.26	−16.16	−15.94	−12.03	−10.15	0.55	0.75	0.81
t	−1.90	−1.80	−1.44	−1.48	−1.84	−1.88	0.64	0.49	0.37
R^2	6.32	15.84	19.02	2.12	4.54	6.34	0.06	0.02	0.04
			Panel D: Predicting consumption growth volatilities						
b	17.49	18.36	19.73	−34.67	−31.47	−28.85	0.00	0.00	0.00
t	1.61	1.84	2.00	−3.36	−4.03	−3.82	0.00	0.00	0.00
R^2	6.08	13.40	16.34	7.69	15.89	17.73	0.58	0.60	0.54

Outline

	Bench-mark	γ 7.5	γ 5	ψ 1.5	ψ 1	γ, ψ 1
σ_C	5.13	4.24	3.94	4.89	4.51	3.83
ρ_{C1}	0.21	0.18	0.15	0.20	0.19	0.16
Prob_C	5.83	4.28	3.82	5.40	4.77	3.54
σ_Y	6.43	5.58	5.17	6.23	5.91	5.21
ρ_{Y1}	0.20	0.18	0.16	0.20	0.19	0.17
Prob_Y	10.90	9.37	8.61	10.47	9.99	8.66
σ_I	8.59	6.27	4.56	8.13	7.36	5.32
ρ_{I1}	0.15	0.13	0.11	0.15	0.14	0.11
$E[U]$	8.63	5.71	4.63	7.90	6.87	4.90
σ_U	0.32	0.35	0.35	0.33	0.34	0.34
σ_V	0.34	0.27	0.24	0.32	0.30	0.24
σ_θ	0.34	0.27	0.25	0.32	0.30	0.24
ρ_{UV}	-0.07	-0.08	-0.09	-0.08	-0.08	-0.09
$e_{w,y/n}$	0.26	0.26	0.26	0.26	0.26	0.26
$E[r_S - r_f]$	4.26	1.55	0.54	3.82	3.17	0.53
$E[r_f]$	1.59	2.45	2.75	1.58	1.54	2.68
σ_S	11.77	9.50	7.99	11.32	10.61	8.68
σ_f	3.13	2.27	1.78	3.74	4.60	3.32

	Bench-mark	b 0.85	η 0.025	s 0.0325	ι 1.35	κ 0.025	ν 1.5	δ 0.01	ω -1	α 0.3
σ_C	5.13	2.62	5.19	5.17	5.09	5.24	4.98	4.71	5.78	4.26
ρ_{C1}	0.21	0.14	0.22	0.21	0.21	0.22	0.23	0.17	0.19	0.21
Prob_C	5.83	2.36	6.42	5.89	5.74	5.93	5.46	5.26	6.31	5.10
σ_Y	6.43	4.11	6.37	6.45	6.40	6.52	6.45	5.98	6.97	5.62
ρ_{Y1}	0.20	0.15	0.21	0.21	0.20	0.21	0.21	0.17	0.20	0.20
Prob_Y	10.90	7.44	10.91	10.85	10.80	10.99	10.76	10.17	11.31	9.99
σ_I	8.59	2.55	8.45	8.67	8.54	8.66	9.41	7.30	8.91	6.71
ρ_{I1}	0.15	0.09	0.16	0.16	0.15	0.16	0.15	0.14	0.16	0.15
$E[U]$	8.63	3.45	8.81	8.51	8.50	8.90	8.54	6.86	9.06	7.20
σ_U	0.32	0.07	0.31	0.33	0.32	0.32	0.32	0.35	0.36	0.30
σ_V	0.34	0.16	0.34	0.34	0.34	0.33	0.33	0.30	0.35	0.31
σ_θ	0.34	0.16	0.34	0.34	0.34	0.34	0.33	0.31	0.35	0.31
ρ_{UV}	-0.07	-0.30	-0.07	-0.07	-0.07	-0.08	-0.07	-0.08	-0.08	-0.08
$e_{w,y/n}$	0.26	0.27	0.37	0.26	0.26	0.26	0.26	0.26	0.25	0.26
$E[r_S - r_f]$	4.26	0.45	3.98	4.41	4.30	4.02	4.03	2.57	4.72	2.27
$E[r_f]$	1.59	2.82	1.67	1.49	1.51	1.83	1.62	2.26	1.38	2.29
σ_S	11.77	7.33	11.13	11.91	11.72	11.79	11.05	10.01	12.13	9.15
σ_f	3.13	0.64	2.95	3.09	3.25	2.81	3.11	2.36	3.46	2.23

Risk aversion, γ, matters for macroeconomic dynamics

A lower b yields lower macro volatilities, risks, and risk premiums

η mostly affects the wage elasticity to labor productivity

A higher κ raises $E[U]$ but reduces the equity premium

ν trades σ_C for σ_I, but leaving σ_Y unchanged

A lower δ raises capital, reducing risks/risk premiums

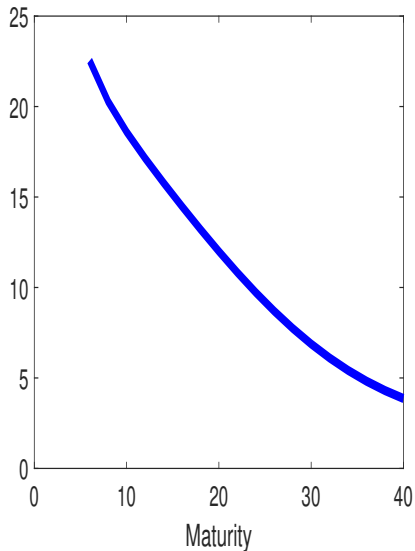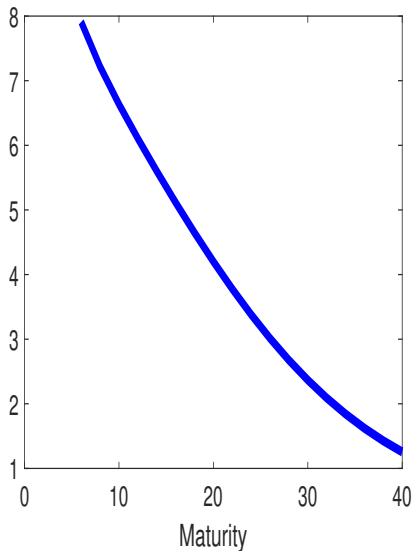A higher e raises macro volatilities, risks, and risk premiums

A higher α downplays search frictions, reducing risks/risk premiums

Results insensitive to s and ι

Outline

Yield-to-maturity starts at 1.53% per annum for 1-month zero-coupon bond but falls to 1.29% for 1-year, 0.95% for 5-year, and further to 0.72% for 10-year bond

The yield spread is -0.81% for the 10-year zero-coupon bond relative to the 1-month bond

The real term premium is also negative, -1.11%, for the 10-year zero-coupon bond

Disasters stimulate precautionary savings, drive down real interest rates, and push up bond prices (hedges against disasters)

The timing premium, $\pi \equiv 1 - J_0/J_0^\star$, J_0 is the utility with risks resolved gradually, $J_0^\star$ with risks resolved in the next period:

$$J_0^\star = \left[(1-\beta)C_0^{1-\frac{1}{\psi}} + \beta \left(E_t \left[(J_1^\star)^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}$$

in which the continuation utility $J_1^\star$:

$$J_1^\star = \left[(1-\beta) \sum_{t=1}^{\infty} \beta^{t-1} C_t^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-1/\psi}}$$

Calculate $J_0^\star$ via Monte Carlo simulations at the economy's stochastic steady state, $\pi = 15.3\%$

$_t C \equiv \{C_t, C_{t+1}, \ldots\}$: The consumption stream starting at t

Calculate the welfare cost, $\chi_t \equiv \chi(N_t, K_t, x_t)$, implicitly from:

$$J\left(_t C(1 + \chi_t)\right) = \overline{J} \quad \Rightarrow \quad \chi_t = \frac{\overline{J}}{J_t} - 1$$
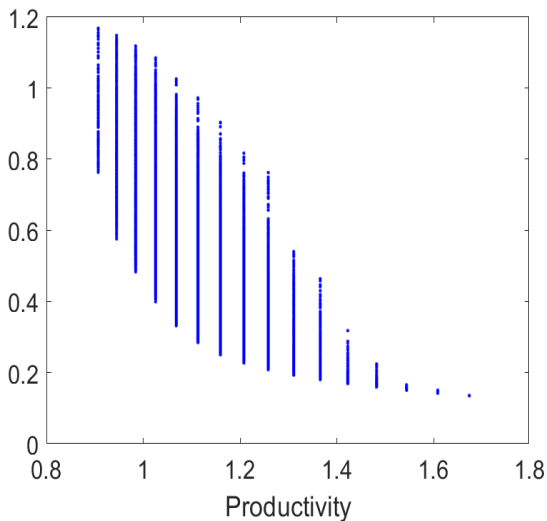
in which $\overline{J}$ is the recursive utility derived from the constant consumption at the deterministic steady state, $\overline{C}$

Solve for $\overline{J}$ by iterating on $\overline{J} = \left[(1 - \beta)\overline{C}^{1 - \frac{1}{\psi}} + \beta \overline{J}^{1 - \frac{1}{\psi}}\right]^{\frac{1}{1 - 1/\psi}}$

Average χ_t in 1 millon months simulation, 29.1%

Additional Implications

The welfare cost is strongly countercyclical, $\mathrm{Corr}(\chi_t, x_t) = -0.76$,
5th, 50th, and 95th percentiles: 18.4%, 24.4%, and 56.3%, respectively

A DSGE model with recursive utility, search frictions, and capital accumulation is a good start to explaining the equity premium puzzle, while retaining plausible business cycle dynamics

Future directions:

- Embed the model structure into a New Keynesian framework

- Extend the model to a multi-country setting

- Incorporate heterogeneous firms into our equilibrium model