Searching for the Equity Premium

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A DSGE model with recursive utility, search frictions, and capital accumulation is a good start to forming a unified theory of asset prices and business cycles

Explaining the Mehra-Prescott (1985) equity premium puzzle in general equilibrium production economies has been challenging

- Rouwenhorst (1995); Jerman (1998); Tallarini (2000)

Finance specifies "exotic" preferences and exogenous cash flow dynamics to match asset prices, while ignoring firms

- Campbell and Cochrane (1999)
- Bansal and Yaron (2004)
- Rietz (1988); Barro (2006)

Macroeconomics analyzes full-fledged DSGE models, while ignoring asset prices with primitive preferences

- Christiano, Eichenbaum, and Evans (2005)

The holy grail of macro-finance: A unified theory of asset prices and business cycles

What are the microfoundations for the exogenous, often complicated cash flow dynamics in finance models?

To what extent do time-varying risk premiums matter quantitatively for macroeconomic dynamics?

How large is the welfare cost of business cycles in a general equilibrium production economy that replicates the equity premium?

Calibrated to the consumption volatility in the Jordà-Schularick-Taylor database, the DSGE model yields a (leverage-adjusted) equity premium of 4.27% per annum, an average interest rate of 1.97%, and a stock market volatility of 12.42%

Strong time series predictability for stock market excess returns and volatilities, some predictability for consumption volatility, and weak to no predictability for consumption growth and real interest rate

Investment absorbs a large amount of shocks, making consumption growth and the interest rate unpredictable

Wage inertia: a wage elasticity to labor productivity of 0.278 in the model versus 0.267 in the historical U.S. 1890–2015 sample

Risk aversion strongly affects quantity dynamics

A timing premium of 16.1%

The welfare cost is huge, 33.6%, and strongly countercyclical

Downward-sloping term structures of equity return and volatility

Core challenge in explaining the equity premium in production economies (Kaltenbrunner and Lochstoer 2010): Dividends tend to be countercyclical in RBC models

- Dividends = profits (output minus wages) minus investment
- With frictionless labor market, wages equal MPL (as procyclical as output)
- Profits no more procyclical than output
- Investment more procyclical than output and profits (consumption smoothing)
- Dividends tend to be countercyclical

The search model overcomes the core challenge in explaining procyclical dividends in general equilibrium production economies

- Dividends = profits (output minus wages) minus investment minus vacancy costs
- With search frictions, wages are inertial, detached from MPL
- Profits more procyclical than output
- Investment (and vacancy costs) more procyclical than output (consumption smoothing)
- Profits more procyclical than investment and vacancy costs, giving rise to procyclical dividends

Outline

A representative household pools income from its employed and unemployed workers before making optimal consumption decisions

A representative firm makes optimal investment and vacancy decisions to maximize its market equity

The labor market as a matching function that yields new hires from the numbers of vacancies and unemployed workers

Wages determined from a generalized Nash bargaining process between the firm and unemployed workers

The household maximizes recursive utility, J_t:

$$J_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left(E_t \left[J_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}$$

in which C_t is consumption, β time preference, ψ the elasticity of intertemporal substitution, and γ risk aversion

The household's stochastic discount factor, M_{t+1}:

$$M_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{J_{t+1}}{E_t \left[J_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}$$

The representative firm uses capital, K_t, and labor, N_t, to product output, Y_t, with the CES production function (Arrow et al. 1961):

$$Y_t = X_t \left[\alpha \left(\frac{K_t}{K_0} \right)^{\omega} + (1-\alpha) N_t^{\omega} \right]^{\frac{1}{\omega}}$$

in which α is the distribution parameter, and $e \equiv 1/(1-\omega)$ the elasticity of substitution between capital and labor

The "normalized" CES function, the scalar $K_0 > 0$ makes the unit of K_t/K_0 comparable to N_t (Klump and La Grandville 2000)

Calibrate K_0 to match $1-\alpha$ to the average labor share in the data

Aggregate productivity, X_t, with $x_t \equiv \log(X_t)$ governed by:

$$x_{t+1} = (1 - \rho_x)\overline{x} + \rho_x x_t + \sigma_x \epsilon_{t+1},$$

in which $\overline{x}$ is unconditional mean, $0 < \rho_x < 1$ persistence, $\sigma_x > 0$ conditional volatility, and ϵ_{t+1} an i.i.d. standard normal shock

Scale $\overline{x}$ to make the average marginal product of labor around one in simulations to ease the interpretation of parameters

The Den Haan-Ramey-Watson (2000) matching function, $\iota > 0$:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{1/\iota}}$$

$\theta_t \equiv V_t/U_t$; the vacancy filling rate: $q(\theta_t) = (1 + \theta_t^{\iota})^{-1/\iota}$

Employment, N_t, evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t$$

in which $q(\theta_t)V_t$ is the number of new hires

Vacancy costs: $\kappa_t V_t$, in which

$$\kappa_t = \kappa_0 + \kappa_1 \, q(\theta_t)$$

Capital accumulates as:

$$K_{t+1} = (1-\delta)K_t + \Phi(I_t, K_t),$$

in which δ is the capital depreciation rate, I_t is investment, and

$$\Phi_t \equiv \Phi(I_t, K_t) = \left[a_1 + \frac{a_2}{1-1/\nu}\left(\frac{I_t}{K_t}\right)^{1-1/\nu}\right]K_t$$

is the installation function with the supply elasticity of capital $\nu > 0$

Set $a_1 = \delta/(1-\nu)$ and $a_2 = \delta^{1/\nu}$ to ensure no adjustment costs in the deterministic steady state (Jermann 1998)

The equilibrium wage rate from Nash bargaining, W_t:

$$W_t = \eta \left(\frac{\partial Y_t}{\partial N_t} + \kappa_t \theta_t \right) + (1 - \eta) b$$

in which $\eta \in (0, 1)$ is the workers' relative bargaining weight; b the workers' flow value of unemployment

η governs the wage elasticity to labor productivity

The dividends to the firm's shareholders given by:

$$D_t = Y_t - W_t N_t - \kappa_t V_t - I_t$$

Taking W_t, M_{t+1}, and $q(\theta_t)$ as given, the firm chooses optimal investment and vacancies to maximize:

$$S_t \equiv \max_{\{V_{t+\tau}, N_{t+\tau+1}, I_{t+\tau}, K_{t+\tau+1}\}_{\tau=0}^{\infty}} E_t \left[\sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} \right],$$

subject to employment and capital accumulation and $V_t \geq 0$

The competitive equilibrium consists of investment, I_t, vacancy posting, V_t, multiplier, λ_t, and consumption, C_t, such that:

(i) C_t satisfies the consumption Euler equation;

(ii) I_t satisfies the investment Euler equation, and V_t and λ_t satisfy the intertemporal job creation condition and the Kuhn-Tucker conditions, while taking the stochastic discount factor, M_{t+1}, and the equilibrium wage, W_t, as given;

(iii) the goods market clears:

$$C_t + \kappa_t V_t + I_t = Y_t$$

From the first-order conditions for I_t and K_{t+1}:

$$\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{1/\nu} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}}\right]\right]$$

Equivalently, $E_t[M_{t+1} r_{Kt+1}] = 1$, in which the investment return:

$$r_{Kt+1} \equiv \frac{\partial Y_{t+1}/\partial K_{t+1} + (1/a_2)(1-\delta+a_1)\left(I_{t+1}/K_{t+1}\right)^{1/\nu} + (1/(\nu-1))(I_{t+1}/K_{t+1})}{(1/a_2)\left(I_t/K_t\right)^{1/\nu}}$$

Let λ_t be the multiplier on $q(\theta_t)V_t \geq 0$, Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0$$

From the first-order conditions with respect to V_t and N_{t+1}:

$$\frac{\kappa_t}{q(\theta_t)} - \lambda_t = E_t \left[M_{t+1} \left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s) \left(\frac{\kappa_t}{q(\theta_{t+1})} - \lambda_{t+1} \right) \right] \right]$$

Equivalently, $E_t[M_{t+1} r_{Nt+1}] = 1$, in which the hiring return:

$$r_{Nt+1} \equiv \frac{\partial Y_{t+1}/\partial N_{t+1} - W_{t+1} + (1-s)\left(\kappa_t/q(\theta_{t+1}) - \lambda_{t+1}\right)}{\kappa_t/q(\theta_t) - \lambda_t}$$

The stock return of the representative firm, r_{St+1}, as a weighted average of the investment and hiring returns:

$$r_{St+1} = \frac{\mu_{Kt} K_{t+1}}{\mu_{Kt} K_{t+1} + \mu_{Nt} N_{t+1}} r_{Kt+1} + \frac{\mu_{Nt} N_{t+1}}{\mu_{Kt} K_{t+1} + \mu_{Nt} N_{t+1}} r_{Nt+1}$$

The shadow value of capital, $\mu_{Kt} = (1/a_2)(I_t/K_t)^{(1/\nu)}$

The shadow value of labor, $\mu_{Nt} = \kappa_t / q(\theta_t) - \lambda_t$

Parameterize the conditional expectation in job creation condition

Solve for the indirect utility, investment, and conditional expectation functions from recursive utility, investment Euler equation, and job creation condition

Rouwenhorst discrete state on productivity with 17 grid points

Finite element with cubic splines on 50 employment nodes and 50 capital nodes; tensor product on each grid point of productivity

Solve the resulting system of 127,500 equations with the derivative-free fixed point iteration with a small damping parameter

The Jordà-Schularick-Taylor macrohistory database for business cycle and asset pricing moments

Real consumption, output, and investment, as well as asset prices for 17 developed countries

Annual series, 1871–2015

Real consumption growth in the Jordà-Schularick-Taylor macrohistory database

	Sample	$\overline{g}_C$	σ_C	S_C	K_C	$\rho_C^{(1)}$	$\rho_C^{(2)}$	$\rho_C^{(3)}$	$\rho_C^{(4)}$	$\rho_C^{(5)}$
Australia	1871	1.11	5.76	−0.77	6.35	−0.04	0.22	−0.03	0.03	−0.09
Belgium	1914	1.35	8.72	−1.14	13.18	0.26	0.19	0.00	−0.40	−0.22
Canada	1872	1.77	4.62	−1.04	6.27	0.00	0.16	−0.16	−0.04	−0.14
Denmark	1871	1.38	5.27	−0.83	11.44	−0.01	−0.41	0.06	0.18	−0.23
Finland	1871	2.07	5.54	−1.13	9.01	0.16	−0.08	0.02	−0.04	−0.23
France	1871	1.37	6.57	−1.06	13.69	0.39	0.19	−0.06	−0.28	−0.14
Germany	1871	1.67	5.51	−0.57	7.11	0.25	0.24	0.28	−0.07	0.00
Italy	1871	1.47	3.63	0.14	7.62	0.38	0.32	0.10	0.08	0.11
Japan	1875	2.11	6.74	−1.53	20.90	0.21	0.10	0.18	0.20	0.20
Netherlands	1871	1.41	8.18	−0.83	19.86	0.17	0.13	−0.21	−0.21	−0.19
Norway	1871	1.83	3.65	−0.32	12.65	−0.06	−0.34	0.26	0.07	−0.24
Portugal	1911	2.36	4.36	−0.49	3.30	0.22	0.23	−0.02	0.09	−0.16
Spain	1871	1.56	7.92	−2.20	17.20	0.00	−0.02	−0.13	−0.05	0.08
Sweden	1871	1.80	4.20	0.44	7.04	−0.15	−0.17	0.05	0.07	−0.20
Switzerland	1871	1.22	5.85	0.35	7.34	−0.20	−0.10	−0.11	−0.10	0.04
UK	1871	1.33	2.76	−0.34	8.90	0.33	0.02	−0.06	−0.01	−0.11
USA	1871	1.75	3.42	−0.07	3.99	0.08	0.09	−0.11	0.00	−0.10
Mean		1.62	5.45	−0.67	10.34	0.12	0.04	0.00	−0.03	−0.09

Real consumption growth, the Jordà-Schularick-Taylor macrohistory database, 1950–2015

	$\bar{g}_C$	σ_C	S_C	K_C	$\rho_C^{(1)}$	$\rho_C^{(2)}$	$\rho_C^{(3)}$	$\rho_C^{(4)}$	$\rho_C^{(5)}$
Australia	1.78	2.02	−0.14	3.55	0.17	−0.24	−0.11	0.19	0.30
Belgium	1.89	1.92	0.20	3.42	0.34	0.21	0.41	0.18	0.21
Canada	2.01	1.81	−0.61	4.00	0.31	0.07	0.17	−0.07	−0.26
Denmark	1.24	2.43	−0.03	2.95	0.22	0.01	0.03	−0.17	−0.30
Finland	2.62	3.17	−0.40	3.04	0.40	−0.08	−0.05	−0.05	−0.03
France	2.34	1.79	0.19	2.18	0.65	0.48	0.40	0.42	0.41
Germany	2.81	2.46	0.71	2.98	0.73	0.53	0.50	0.51	0.49
Italy	2.51	2.72	−0.30	2.97	0.67	0.46	0.52	0.48	0.41
Japan	3.90	3.53	0.72	3.00	0.74	0.62	0.69	0.66	0.61
Netherlands	1.92	2.47	−0.16	2.45	0.67	0.32	0.15	0.08	0.13
Norway	2.39	2.19	0.21	3.76	0.23	−0.02	−0.18	−0.14	−0.13
Portugal	3.05	3.56	−0.58	4.03	0.36	0.16	0.08	−0.14	−0.18
Spain	2.79	3.54	0.08	3.20	0.51	0.25	0.20	0.23	0.23
Sweden	1.55	1.92	−0.59	3.12	0.38	0.18	0.08	−0.09	−0.16
Switzerland	1.44	1.42	0.11	2.59	0.61	0.24	0.14	0.10	0.11
UK	1.97	2.09	−0.13	3.11	0.45	0.05	−0.11	−0.11	0.00
USA	2.08	1.73	−0.21	2.49	0.32	0.03	−0.06	0.02	−0.04
Mean	2.25	2.40	−0.05	3.11	0.46	0.19	0.17	0.12	0.11

	Sample	$E[\tilde{r}_S]$	$\tilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\tilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
Australia	1900 (45–47)	7.75	17.08	1.29	4.32	6.46	4.58	12.55
Belgium	1871 (14–19)	6.31	19.88	1.21	8.43	5.10	3.62	14.62
Canada	1900	7.01	17.00	1.60	4.79	5.41	3.84	12.26
Denmark	1875 (15)	7.47	16.43	3.08	5.68	4.39	3.12	11.91
Finland	1896	8.83	30.57	−0.74	10.93	9.57	6.80	22.98
France	1871 (15–21)	3.99	22.22	−0.47	7.78	4.45	3.16	16.75
Germany	1871 (23, 44–49)	8.83	27.59	−0.23	13.22	9.05	6.43	20.22
Italy	1871 (1872–84, 15–21)	6.63	27.21	0.58	10.50	6.05	4.29	20.41
Japan	1886 (46–47)	8.86	27.69	0.00	11.20	8.87	6.29	21.10
Netherlands	1900	6.96	21.44	0.78	4.91	6.19	4.39	15.32
Norway	1881	5.67	19.82	0.90	5.98	4.77	3.39	14.53
Portugal	1880	3.81	25.68	−0.01	9.43	3.82	2.71	19.29
Spain	1900 (36–40)	6.25	21.41	−0.04	6.90	6.29	4.47	15.94
Sweden	1871	8.00	19.54	1.77	5.60	6.23	4.42	14.26
Switzerland	1900 (15)	6.69	19.08	0.89	5.00	5.79	4.11	14.00
UK	1871	6.86	17.77	1.16	4.82	5.70	4.05	12.96
USA	1872	8.40	18.68	2.17	4.65	6.23	4.43	13.66
Mean		6.96	21.71	0.82	7.30	6.14	4.36	16.04

U.S. historical monthly series: Unemployment and labor productivity, 1890–; vacancy, 1919–

Private nonfarm unemployment rates (Lebergott 1964; Weir 1992): Mean, 8.94%; volatility: 24.43% per quarter

Parameters in the monthly benchmark calibration, preferences and productivity

Time discount factor, $\beta = 0.9976$

Risk aversion, $\gamma = 10$

Elasticity of intertemporal substitution, $\psi = 2$

Persistence in log productivity, $\rho_x = 0.95^{1/3}$

Calibrate its conditional volatility, $\sigma_x = 0.015$, to hit average σ_C

Long-run mean of log productivity, $\bar{x} = 0.1945$, to target the marginal product of labor to be one on average

Elasticity of capital-labor substitution, $e = 1/(1-\omega) = 0.4$, per Chirinko and Mallick (2017)

The distribution parameter, $\alpha = 0.25$, to match the average labor share of 0.743 per Gollin (2002)

The capital scaler, $K_0 = 13.98$, to target the labor share of 0.75 at the deterministic steady state (close to its stochastic steady state)

Supply elasticity of capital, $\nu = 1.2$

Depreciation rate of capital, $\delta = 1.25\%$

Separation rate, $s = 3\%$, between the SIPP and JOLTS estimates

Curvature in the matching function, $\iota = 0.9$, between the Hagedorn-Manovskii and Den Haan-Ramey-Watson values

Bargaining weight of workers, $\eta = 0.015$

Flow value of unemployment, $b = 0.91$, a simple device for small fundamental surplus, see also Ganong, Noel, and Vavra (2020)

Unit vacancy costs, $\kappa_0 = 0.05$ and $\kappa_1 = 0.025$

The low-η-high-b calibration yields a wage elasticity to labor productivity of 0.278

Hagedorn and Manovskii (2008) estimate the wage elasticity to be 0.449 in the postwar 1951–2004 quarterly sample from BLS

From 1929 to 2015, obtain compensation of employees from NIPA Tables 6.2A–D (line 3, private industries, minus line 5, farms)

Obtain the number of full-time equivalent employees from NIPA Tables 6.5A–D (line 3, private industries, minus line 5, farms)

Dividing the compensation of employees by the number of employees yields nominal wage rates (compensation per person)

Deflate nominal wage rates with the personal consumption deflator from NIPA Table 1.1.4 (line 2) to obtain real wage rates

From 1890 to 1929, obtain the average (nominal) hourly compensation of production workers in manufacturing and consumer price index from measuringworth.com (Officer 2009, Table 7.1; Officer and Williamson 2020a, 2020b)

Divide the manhours index by the index of persons engaged in manufacturing from Kendrick (1961, Table D-II) to obtain hours

Multiply the average hourly compensation series with the hours index to obtain the nominal compensation per person; deflate with the Officer-Williamson consumer price index to obtain real wages

Splice this series in 1929 to the NIPA series from 1929 onward to yield an uninterrupted series from 1890 to 2015

Historical 1890–2015 series of labor productivity from Petrosky-Nadeau and Zhang (2021)

Time-aggregate monthly series into annual by taking the monthly average within a given year

Detrend the annual real wages and labor productivity series as log deviations from their HP-trends with a smoothing parameter of 6.25

In our postwar 1950–2015 annual sample, regressing the log real wages on the log labor productivity yields a wage elasticity of 0.406, with a standard error of 0.081

In our 1890–2015 sample, the wage elasticity estimated to be 0.267, with a standard error of 0.066

Real consumption growth, 10,000 artificial samples, each with 1,740 months

	Data	Mean	5th	50th	95th	p
σ_C	5.45	5.43	3.13	5.42	7.77	0.49
S_C	−0.67	0.06	−0.86	0.04	1.03	0.92
K_C	10.34	7.20	4.07	6.56	12.42	0.11
ρ_1^C	0.12	0.23	0.02	0.23	0.42	0.82
ρ_2^C	0.04	−0.04	−0.24	−0.04	0.17	0.26
ρ_3^C	0.00	−0.04	−0.23	−0.04	0.16	0.36
ρ_4^C	−0.03	−0.04	−0.22	−0.04	0.15	0.45
ρ_5^C	−0.09	−0.04	−0.22	−0.04	0.14	0.69

Real output growth, 10,000 artificial samples, each with 1,740 months

	Data	Mean	5th	50th	95th	p
σ_Y	5.10	6.64	4.61	6.61	8.78	0.88
S_Y	−1.06	0.10	−0.56	0.09	0.79	1.00
K_Y	14.09	5.20	3.41	4.86	8.09	0.00
ρ_1^Y	0.18	0.22	0.04	0.22	0.38	0.64
ρ_2^Y	0.00	−0.05	−0.22	−0.05	0.13	0.33
ρ_3^Y	0.00	−0.05	−0.21	−0.05	0.12	0.33
ρ_4^Y	0.01	−0.04	−0.21	−0.05	0.12	0.30
ρ_5^Y	−0.09	−0.04	−0.20	−0.04	0.12	0.67

Unconditional Moments

Real investment growth, 10,000 artificial samples, each with 1,740 months

	Data	Mean	5th	50th	95th	p
σ_I	13.53	8.83	5.55	8.83	12.04	0.01
S_I	−0.05	0.29	−0.51	0.26	1.19	0.76
K_I	10.75	6.57	3.89	6.00	11.07	0.06
ρ_1^I	0.13	0.16	−0.02	0.17	0.33	0.62
ρ_2^I	−0.05	−0.10	−0.28	−0.10	0.08	0.32
ρ_3^I	−0.07	−0.08	−0.26	−0.08	0.09	0.47
ρ_4^I	−0.11	−0.07	−0.24	−0.07	0.11	0.64
ρ_5^I	−0.08	−0.06	−0.23	−0.06	0.11	0.58

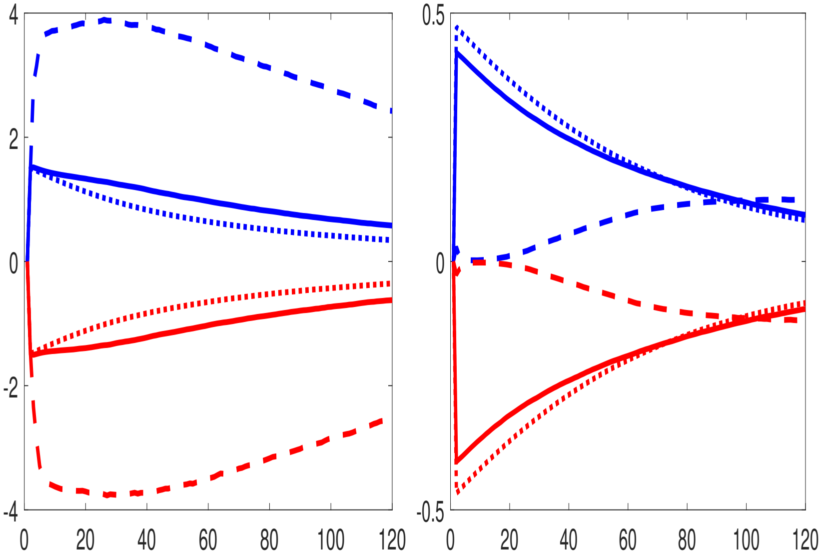
Unconditional Moments

Labor market moments, 10,000 artificial samples, each with 1,740 months

	Data	Mean	5th	50th	95th	p
$E[U]$	8.94	9.40	3.67	7.94	20.20	0.42
S_U	2.13	2.33	0.62	2.03	5.02	0.46
K_U	9.50	10.02	1.92	5.99	30.07	0.30
σ_U	0.24	0.31	0.14	0.31	0.48	0.71
σ_V	0.19	0.33	0.23	0.32	0.49	1.00
σ_θ	0.62	0.35	0.24	0.33	0.53	0.02
ρ_{UV}	−0.57	−0.11	−0.20	−0.10	−0.02	0.00
$e_{w,y/n}$	0.27	0.28	0.25	0.28	0.29	0.84

Asset prices, 10,000 artificial samples, each with 1,740 months

	Data	Mean	5th	50th	95th	p
$E[r_S - r_f]$	4.36	4.27	3.77	4.24	4.86	0.35
$E[r_f]$	0.82	1.97	1.32	2.07	2.24	0.99
σ_S	16.04	12.42	9.82	12.41	15.13	0.02
σ_f	7.30	2.47	1.14	2.47	3.75	0.00



4
2
0
-2
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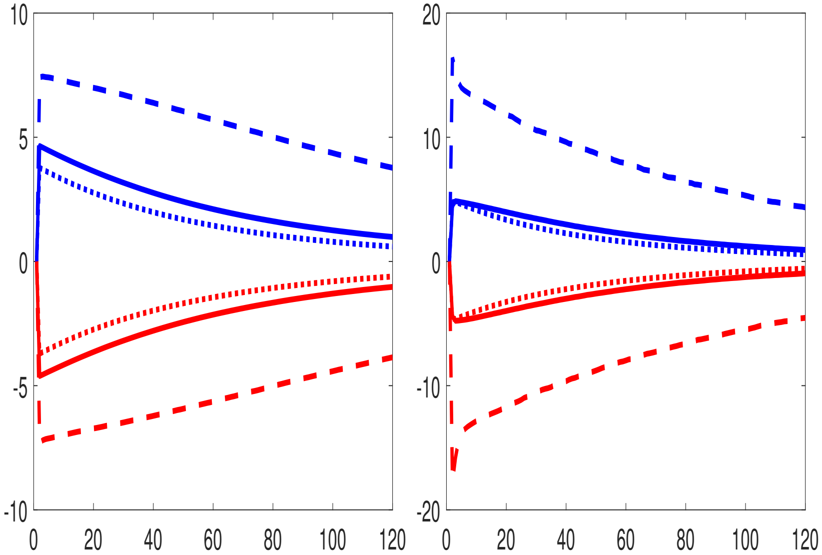
100
120

Impulse responses: Profit and dividend



Unconditional Moments

Disaster dynamics, applying the Barro-Ursúa (2008) peak-to-trough method to the Jordà-Schularick-Taylor data

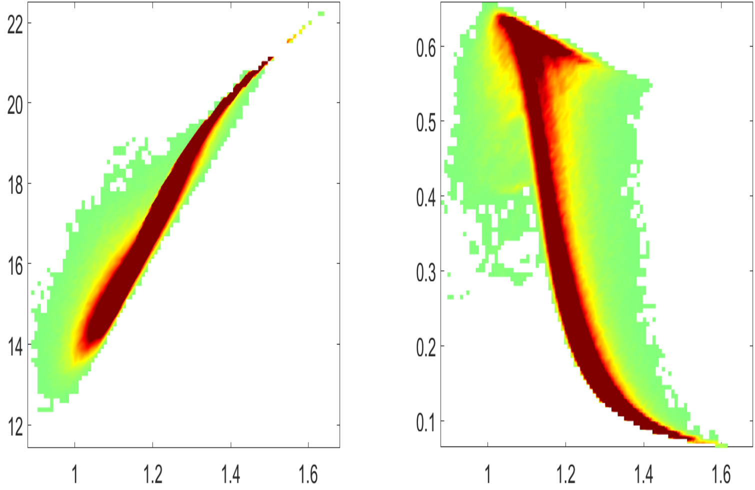
	Data	Mean	5th	50th	95th	p	Data	Mean	5th	50th	95th	p
	Disaster hurdle = 10%						Disaster hurdle = 15%					
	Consumption											
Probability	6.40	6.66	2.29	6.14	12.50	0.47	3.51	4.08	0.72	3.91	8.49	0.52
Size	23.16	23.70	14.89	23.10	34.27	0.49	30.36	30.11	19.23	29.02	44.12	0.42
Duration	4.19	4.10	2.90	4.00	5.67	0.41	4.50	4.49	3.00	4.33	6.50	0.40
	Output											
Probability	5.78	11.45	6.67	11.11	17.24	0.98	2.62	6.52	3.01	6.14	11.34	0.95
Size	22.34	22.85	16.20	22.38	31.01	0.50	32.9	29.04	20.43	28.38	39.75	0.23
Duration	4.14	3.72	2.89	3.67	4.73	0.21	5.04	4.25	3.11	4.17	5.67	0.14

Outline

1. The Model
2. Calibration
3. Unconditional Moments
4. Time-varying Risk Premiums
5. Comparative Statics
6. Additional Implications

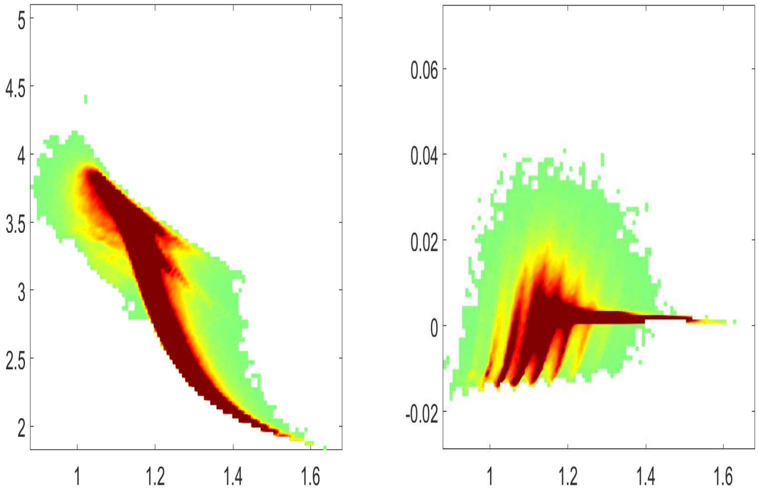
Time-varying Risk Premiums

Price-to-consumption, P_t/C_t, and the equity premium, $E_t[r_{St+1} - r_{ft+1}]$

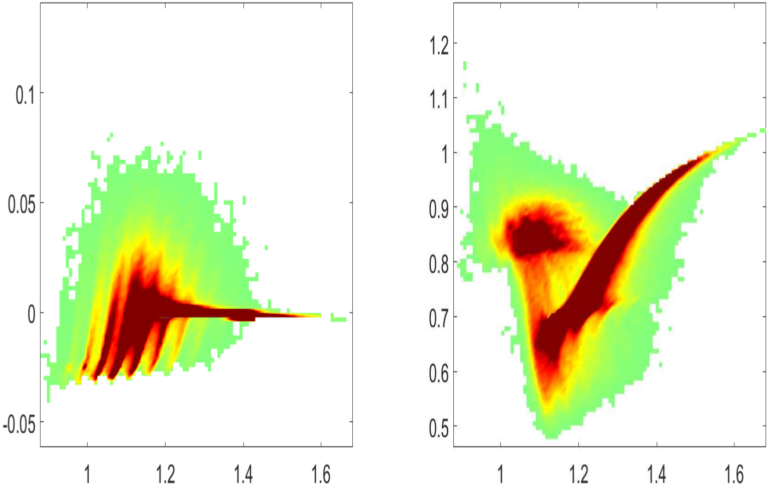


Time-varying Risk Premiums

Stock market volatility, σ_{St}, and the risk free rate, r_{ft+1}



Expected consumption growth, $E_t[g_{Ct+1}]$, and Consumption volatility, σ_{Ct}



	Data			Mean			p		
	1y	3y	5y	1y	3y	5y	1y	3y	5y
	Predicting stock market excess returns								
b	−1.52	−4.71	−6.30	−1.44	−3.86	−5.78	0.55	0.71	0.59
t	−1.22	−1.77	−2.07	−1.78	−2.42	−2.75	0.27	0.29	0.31
R^2	1.87	5.69	9.01	2.62	6.32	9.16	0.55	0.50	0.46
	Predicting consumption growth								
b	−0.34	−1.22	−1.99	−1.37	−2.56	−3.65	0.01	0.10	0.13
t	−0.64	−0.93	−1.09	−2.91	−2.45	−2.72	0.01	0.12	0.16
R^2	2.51	4.12	5.77	7.59	8.44	11.11	0.88	0.69	0.66

	Data 1y	Data 3y	Data 5y	Mean 1y	Mean 3y	Mean 5y	p 1y	p 3y	p 5y
	Predicting stock market volatilities								
b	−17.43	−17.26	−16.16	−12.85	−10.24	−8.72	0.65	0.81	0.86
t	−1.90	−1.80	−1.44	−1.22	−1.57	−1.64	0.73	0.58	0.45
R^2	6.32	15.84	19.02	1.54	3.61	5.35	0.03	0.01	0.03
	Predicting consumption growth volatilities								
b	17.49	18.36	19.73	−35.07	−31.56	−28.72	0.00	0.00	0.00
t	1.61	1.84	2.00	−3.54	−4.31	−4.03	0.00	0.00	0.00
R^2	6.08	13.40	16.34	8.00	17.00	18.95	0.62	0.64	0.59

Outline

Risk aversion, γ, matters for macroeconomic dynamics

A lower b yields lower macro volatilities, risks, and risk premiums

η mostly affects the wage elasticity to labor productivity

higher κ_0 and κ_1 raise $E[U]$ but reduces the equity premium

ν trades σ_C for σ_I, but leaving σ_Y unchanged

A lower δ raises capital, reducing risk premiums

A higher e raises macro volatilities, risks, and risk premiums

A higher α downplays search frictions, reducing risks/risk premiums

		γ 7.5	γ 5	ψ 1.5	ψ 1	γ, ψ 1
σ_C	5.43	4.44	4.03	5.15	4.80	3.93
ρ_{C1}	0.23	0.19	0.16	0.22	0.21	0.17
Prob_C	6.66	5.02	4.41	6.17	5.61	4.11
σ_Y	6.64	5.70	5.15	6.41	6.11	5.21
ρ_{Y1}	0.22	0.19	0.16	0.21	0.20	0.17
Prob_Y	11.45	9.69	8.77	10.97	10.48	8.82
σ_I	8.83	6.44	4.41	8.35	7.72	5.21
ρ_{I1}	0.16	0.14	0.11	0.16	0.16	0.12
$E[U]$	9.40	5.73	4.29	8.47	7.45	4.59
σ_U	0.31	0.36	0.36	0.32	0.33	0.35
σ_V	0.33	0.26	0.23	0.32	0.30	0.23
σ_θ	0.35	0.27	0.24	0.33	0.31	0.24
ρ_{UV}	−0.11	−0.12	−0.14	−0.11	−0.11	−0.13
$e_{w,y/n}$	0.28	0.28	0.28	0.28	0.29	0.28
$E[r_S - r_f]$	4.27	1.57	0.45	3.74	3.23	0.32
$E[r_f]$	1.97	2.63	2.86	1.99	1.90	2.91
σ_S	12.42	10.08	8.22	11.95	11.36	8.96
σ_f	2.47	1.93	1.56	2.98	3.84	2.94

		b	η	s	ι	κ_0	κ_1	ν	δ	e	α
		0.88	0.025	0.035	0.6	0.075	0.05	1.5	0.01	0.5	0.3
σ_C	5.43	3.24	5.42	5.45	5.54	5.49	5.48	5.23	4.87	5.96	4.62
ρ_{C1}	0.23	0.15	0.23	0.23	0.24	0.23	0.23	0.25	0.18	0.21	0.22
Prob_C	6.66	3.37	7.29	6.51	7.11	6.91	6.78	6.16	5.92	7.00	6.13
σ_Y	6.64	4.53	6.51	6.71	6.78	6.70	6.67	6.64	6.05	7.06	5.86
ρ_{Y1}	0.22	0.15	0.22	0.21	0.23	0.22	0.22	0.23	0.18	0.21	0.21
Prob_Y	11.45	7.96	11.44	11.56	11.95	11.65	11.52	11.33	10.51	11.72	10.68
σ_I	8.83	3.46	8.54	8.92	9.12	8.93	8.85	9.85	7.34	8.84	7.14
ρ_{I1}	0.16	0.10	0.16	0.16	0.17	0.17	0.16	0.16	0.15	0.17	0.16
$E[U]$	9.40	3.45	9.38	9.97	10.58	9.75	9.50	9.22	6.96	9.27	8.18
σ_U	0.31	0.24	0.30	0.29	0.27	0.30	0.31	0.31	0.35	0.36	0.29
σ_V	0.33	0.19	0.33	0.33	0.33	0.33	0.34	0.33	0.30	0.33	0.32
σ_θ	0.35	0.20	0.35	0.35	0.37	0.36	0.36	0.34	0.31	0.35	0.33
ρ_{UV}	−0.11	−0.21	−0.11	−0.11	−0.16	−0.12	−0.11	−0.10	−0.12	−0.11	−0.11
$e_{w,y/n}$	0.28	0.29	0.39	0.26	0.26	0.28	0.28	0.28	0.28	0.27	0.28
$E[r_S - r_f]$	4.27	0.64	3.95	4.16	4.35	4.24	4.25	3.99	2.68	4.29	2.94
$E[r_f]$	1.97	2.81	1.96	2.03	2.06	2.01	1.98	1.97	2.36	1.96	2.18
σ_S	12.42	7.94	11.60	12.35	12.77	12.48	12.40	11.51	10.46	12.51	9.92
σ_f	2.47	1.05	2.35	2.52	2.46	2.49	2.52	2.44	2.07	2.81	1.86

Outline

		γ 7.5	γ 5	ψ 1.5	ψ 1	γ, ψ 1
w_K	92.58	90.11	89.03	92.01	91.34	88.91
$E[r_K]$	5.28	3.74	3.14	4.78	4.13	3.09
σ_K	9.97	6.99	4.63	9.36	8.60	5.54
$E[r_N]$	41.56	15.47	7.37	38.44	37.01	6.20
σ_N	186.07	120.67	85.52	177.21	168.22	82.92
ρ_{KN}	0.71	0.68	0.69	0.71	0.71	0.68
ρ_{KS}	0.97	0.95	0.94	0.96	0.96	0.95
ρ_{NS}	0.72	0.71	0.73	0.72	0.72	0.73

		b	η	s	ι	κ_0	κ_1	ν	δ	e	α
		0.88	0.025	0.035	0.6	0.075	0.05	1.5	0.01	0.5	0.3
w_K	92.58	85.09	93.85	92.99	92.35	92.52	92.55	92.30	92.84	92.51	93.96
$E[r_K]$	5.28	3.12	5.13	5.31	5.39	5.30	5.28	4.96	4.38	5.20	4.49
σ_K	9.97	3.45	9.61	10.08	10.23	10.06	9.99	8.95	8.11	9.89	7.97
$E[r_N]$	41.56	7.40	39.54	39.79	40.77	36.42	36.22	42.48	29.38	47.02	32.59
σ_N	186.07	56.16	164.05	160.81	155.19	138.93	143.74	188.91	151.87	283.14	145.14
ρ_{KN}	0.71	0.79	0.74	0.75	0.79	0.76	0.73	0.70	0.72	0.57	0.74
ρ_{KS}	0.97	0.96	0.97	0.97	0.97	0.97	0.97	0.96	0.97	0.97	0.97
ρ_{NS}	0.72	0.80	0.76	0.76	0.80	0.78	0.76	0.71	0.73	0.58	0.75

Additional Implications

Epstein, Farhi, and Strzalecki (2014): Investor sacrifices 31% of its consumption stream for early resolution of risks in Bansal and Yaron (2004) and 42% in Wachter (2013)

The timing premium, $\pi \equiv 1 - J_0/J_0^\star$, J_0 is the utility with risks resolved gradually, $J_0^\star$ with risks resolved in the next period:

$$J_0^\star = \left[(1-\beta)C_0^{1-\frac{1}{\psi}} + \beta\left(E_t\left[(J_1^\star)^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}$$

in which the continuation utility $J_1^\star$:

$$J_1^\star = \left[(1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}C_t^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-1/\psi}}$$

Calculate $J_0^\star$ via Monte Carlo simulations at the economy's stochastic steady state, $\pi = 16.1\%$

${}_tC \equiv \{C_t, C_{t+1}, \ldots\}$: The consumption stream starting at t

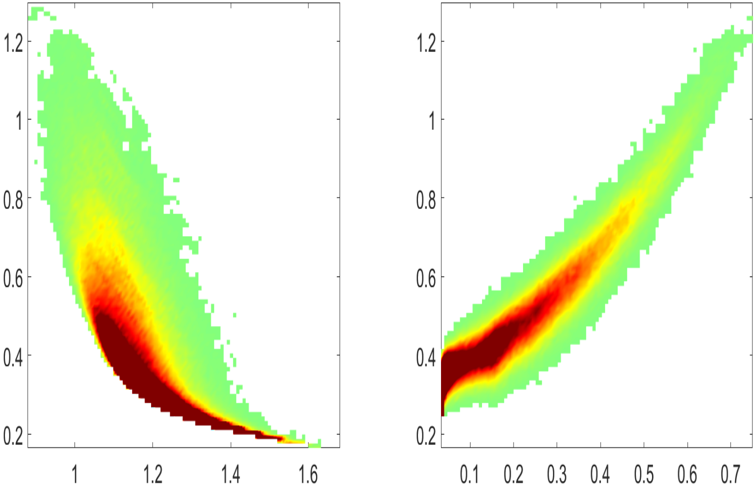
Calculate the welfare cost, $\chi_t \equiv \chi(N_t, K_t, x_t)$, implicitly from:

$$J\left({}_tC(1+\chi_t)\right) = \overline{J} \quad \Rightarrow \quad \chi_t = \frac{\overline{J}}{J_t} - 1$$

in which $\overline{J}$ is the recursive utility derived from the constant consumption at the deterministic steady state, $\overline{C}$

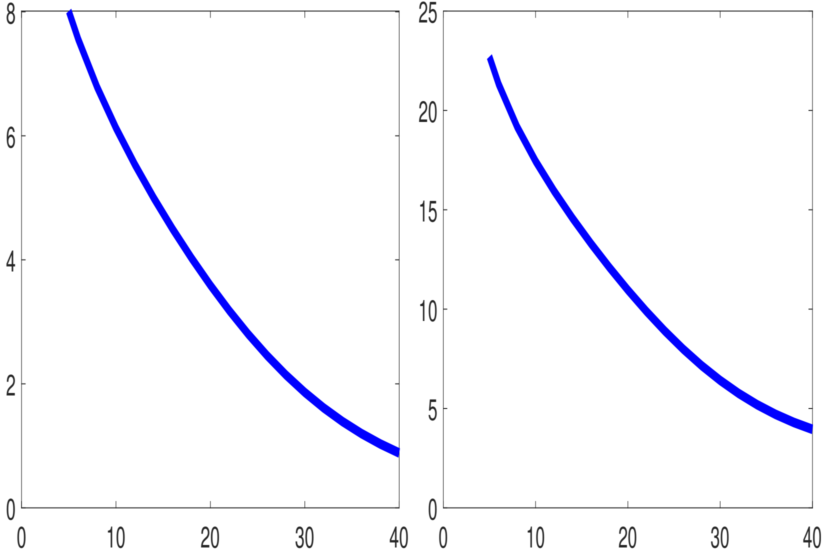
Solve for $\overline{J}$ by iterating on $\overline{J} = \left[(1-\beta)\overline{C}^{1-\frac{1}{\psi}} + \beta\overline{J}^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-1/\psi}}$

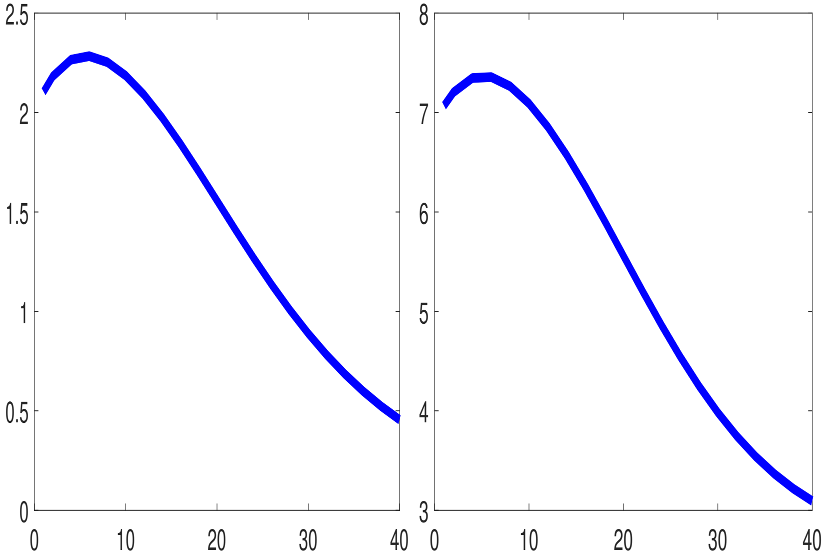
Average χ_t in 1 millon months simulation, 33.6%


1.2
1
0.8
0.6
0.4
0.2
1
1.2
1.4
1.6
1.2
1
0.8
0.6
0.4
0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Additional Implications

Equity term structure, dividend





2.5
2
1.5
1
0.5
0
0
10
20
30
40
8
7
6
5
4
3
0
10
20
30
40

A DSGE model with recursive utility, search frictions, and capital accumulation is a good start to forming a unified theory of asset prices and business cycles